

Collective three-body interactions enable a robust quantum speedup

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We show that collective three-body interactions (3BIs), implementable with N atoms loaded inside an optical cavity, offer a significant advantage for preparing complex multipartite entangled states. First, they enable a speedup of order $\sim N$ in preparing generalized Greenberger-Horne-Zeilinger states, outperforming conventional methods based on all-to-all two-body Ising interactions. Second, they saturate the Heisenberg bound in phase estimation tasks using a time-reversal protocol realized through simple rotations and followed by experimentally accessible collective spin measurements. Finally, compared with two-body interactions, in the presence of cavity losses and single particle decoherence, 3BIs feature a high gain in sensitivity for moderate atom numbers and in large ensembles a fast entanglement generation despite constraints in parameter regimes where they are implementable.

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Introduction. A central goal in quantum science is the ability to engineer and control microscopic interactions among particles. Tunable two-body interactions (2BIs) have long served as the foundation of this effort, underpinning entanglement generation, spin-exchange dynamics, and universal quantum gates. Recent advances, however, have pushed beyond this paradigm, enabling the controlled realization of genuine multibody interactions in which three or more particles interact simultaneously. These higher-order couplings have been proposed and realized across diverse platforms, including superconducting qubits [1–3], trapped ions [4–6], ultracold atomic gases [7–11], and Rydberg atom arrays [12–14]. Beyond their fundamental interest, multibody interactions offer powerful routes for accelerating quantum gates [15] and for simulating complex many-body Hamiltonians relevant to chemistry, condensed matter, and high-energy physics [16–21].

Realizing such couplings, however, remains challenging: (1) They typically arise from higher-order processes, leading to slower dynamics and enhanced decoherence; (2) genuine multibody effects are difficult to isolate from dominant lower-order terms; and (3) previous demonstrations have been limited to few-particle systems. A recent experiment [22] addressed these issues by engineering collective three-

body interactions (3BIs) in an ensemble of $N \sim 10^3$ cold atoms coupled to an optical cavity, using momentum states as pseudospins. Here, 3BIs emerge from a cavity-mediated six-photon process—higher-order processes compared to the four-photon process producing two-body couplings. Although the bare three-body coupling is weaker, the cavity’s all-to-all connectivity enhances collective 3BIs to comparable strength with collective 2BIs [23], suggesting faster dynamics in larger arrays. But to date, observations have been restricted to mean-field timescales, leaving the role of 3BIs in genuine quantum dynamics an open question.

In this Letter, we present a detailed analysis of the beyond mean-field (MF) dynamics induced by collective 3BIs and demonstrate they indeed offer a significant advantage for sensing. Due to their intrinsic Z_3 rotational symmetry, generalized Greenberger-Horne-Zeilinger (GHZ) states can be prepared with high fidelity for specific atom numbers, at a speed that is $\sim N$ faster compared to the conventional one-axis twisting (OAT) scheme based on all-to-all two-body Ising interactions. For generic particle numbers, we also observe a rapid growth of multiparticle correlations and quantum Fisher information (QFI), reaching the Heisenberg scaling (HS) desired for phase estimation within experimentally realistic time. Such gain can be extracted using a time-reversal (TR) protocol achievable via simple single-particle rotations [24–27] and measurements of collective single-particle observables.

Even more importantly, we demonstrate that the dynamics remain robust against realistic dissipation inherent to cavity-QED systems. Whereas most previous work on entanglement generation focuses on scalability, our results highlight a complementary advantage: for moderate atom numbers N and

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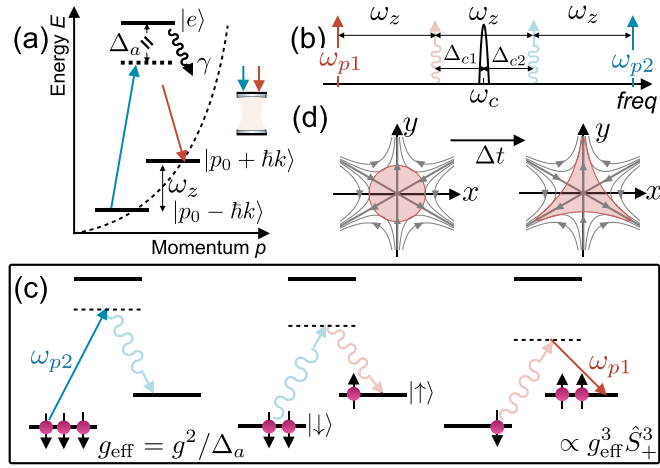


FIG. 1. (a) Energy level diagram of the atomic ensemble used to engineer the 3BIs, where a pair of momentum states, $|p_0 \pm \hbar k\rangle$, serves as the pseudospins $|\uparrow/\downarrow\rangle$ [22]. Inset: schematics of the setup. An optical cavity is driven by two dressing lasers with frequencies $\omega_{p1,2}$ (red and blue arrows). (b) Corresponding Frequency diagram. (c) Two dressing lasers induce a six-photon process that flips three spins collectively between $|\downarrow, \downarrow, \downarrow\rangle$ and $|\uparrow, \uparrow, \uparrow\rangle$ via an intermediate excited state. Virtual cavity photons at two frequencies are shown as red and blue wavy lines (see text). (d) Mean-field dynamics for an initial spin along z . Gray lines show classical trajectories with trifurcation points; bold arrows mark the separatrix.

cooperativity C , collective 3BIs produce a marked enhancement in metrological gain relative to generic 2BI models, especially since these 2BI models exhibit strong deviations from analytic large- NC scaling, for limited cooperativity $NC < 100$ [28]. For larger ensembles, 3BIs further enable faster preparation of target entangled states, an equally critical factor given that coherence times are constrained by technical noise and dissipation.

Model. We consider the experimental setup recently realized in Ref. [22] and illustrated in Figs. 1(a) and 1(b). An ensemble of N cold atoms is confined in a high-finesse optical cavity, where two accessible motional ground states, denoted by $|p_0 \pm \hbar k\rangle$ and separated by an energy $\hbar\omega_z$, serve as the pseudospin $|\uparrow/\downarrow\rangle$. The cavity mode at frequency ω_c is detuned by Δ_a from the optical transition $|\uparrow/\downarrow\rangle$ to $|e\rangle$, with single-photon Rabi frequency $2g$ and spontaneous emission rate γ . The cavity is driven by two simultaneously applied lasers with frequencies ω_{p1} and ω_{p2} (separated by $3\omega_z$). The detuning $|\Delta_a|$ is assumed to be much larger than all other relevant frequency scales. Upon adiabatic elimination of both the excited state and the cavity field, an effective collective 3BI emerges [23]:

$$\hat{H}_3 = \chi_3(\hat{S}_+^3 + \hat{S}_-^3), \quad (1)$$

where $\hat{S}_\pm = \hat{S}_x \pm i\hat{S}_y$ are collective spin raising and lowering operators, and $\hat{S}_{\alpha=x,y,z}$ denote the components of the total spin operator. The basic idea is illustrated in Fig. 1(c). Any group of three atoms in the $|\downarrow\rangle$ state can flip their pseudospin to $|\uparrow\rangle$ and the other way around, in a process that starts when one of the atoms absorbs a photon from the drive with frequency ω_{p2} . Since the $|e\rangle$ level is far from resonance, it is only virtually

populated and the excited atom quickly emits a cavity photon (blue squiggly arrow) that transfers it to $|\uparrow\rangle$. The emitted photon at frequency $\omega_{p2} - \omega_z$ is also blue detuned from the cavity by $\Delta_{c,2}$ and virtually absorbed by the second atom that quickly transitions to $|\uparrow\rangle$, after emitting a second virtual cavity photon (red squiggly arrow) with frequency $\omega_{p1} + \omega_z$, now red detuned from the cavity by $\Delta_{c,1}$. This last virtual photon is finally absorbed by the third atom, which transitions to $|\uparrow\rangle$ by stimulated emitting a photon to the second pump with frequency ω_{p1} . The net process by which three atoms in $|\downarrow\rangle$, all simultaneously flip to $|\uparrow\rangle$, can be made resonant when the energy gain of the atoms is compensated by the pump photons, $\omega_{p2} - \omega_{p1} = 3\omega_z$. The rate at which an atom absorbs/emits a pump photon and subsequently emits a cavity photon is given by the two-photon coupling strength $g_{\text{eff}} = g^2/\Delta_a$.

Under the chosen drive configuration with $\Delta_{c2} = -\Delta_{c1} \equiv \Delta_c$ [22,29], all 2BIs—such as the Two-Axis Twisting (TAT) interaction $\hat{H}_{\text{TAT}} = \chi_2^{\text{TAT}}(\hat{S}_+^2 + \hat{S}_-^2)$ [23] and the exchange interaction $\hat{H}_{\text{ex}} = \chi_2^{\text{ex}}\hat{S}_+\hat{S}_- \approx \chi_2^{\text{ex}}(\vec{S} \cdot \vec{S} - \hat{S}_z^2)$ (or OAT $\hat{H}_{\text{OAT}} = \chi_2^{\text{OAT}}\hat{S}_x^2$), with $\vec{S} \cdot \vec{S} = \sum_{\alpha=x,y,z} \hat{S}_\alpha^2$ [30]—are off resonant or suppressed by destructive interferences, respectively.

MF dynamics. At the MF level, the Bloch vector $\mathbf{S}(t) \equiv \{S_x(t), S_y(t), S_z(t)\}$, with $S_\alpha(t) = \langle \hat{S}_\alpha(t) \rangle$, simply evolves under the nonlinear set of equations, $\dot{S}_+ = -6i\chi_3 S_z S_-^2$ and $\dot{S}_z = -3i\chi_3(S_+^3 - S_-^3)$. The MF flow lines [23,31] can be obtained by initializing the system in a spin coherent state, described by an initial MF Bloch vector $\mathbf{S}(0) \equiv N/2\{x, y, \sqrt{1-x^2-y^2}\}$ and then recording the change $\Delta\mathbf{S} \equiv \mathbf{S}(\Delta t) - \mathbf{S}(0)$ after a short time evolution, Δt . In Fig. 1(d), we show an example of the flow line map near the north-pole direction on the Bloch sphere. Under the MF evolution, the north pole is an unstable fixed point and can be identified as a *trifurcation point*. The dynamics along the three separatrix branches lead to attractive (at angles $\varphi = \pi/6, 5\pi/6, 3\pi/2$) or diverging (at $\varphi = \pi/2, 7\pi/6, 11\pi/6$) trajectories, a feature qualitatively distinct from the nonlinear dynamics arising from 2BIs, where only two attractive/diverging branches define the squeezing/antisqueezing directions. The flow line map of $\hat{H}_3$ was experimentally measured in Ref. [23] showing excellent agreement with theoretical predictions.

Entanglement generation. The utility of $\hat{H}_3$ for the preparation of metrologically useful entangled states can be quantified by the QFI, which tells the best precision that can be achieved in parameter estimation for a given quantum state across all physically realizable measurements and estimators. Quantitatively, for a pure state and a unitary parameter encoding, the QFI can be expressed in terms of the variance of the operator responsible for phase encoding. In our case, the relevant operator is $\hat{S}_z$, and thus $F_Q = 4(\langle \hat{S}_z^2 \rangle - \langle \hat{S}_z \rangle^2)$. We compute the QFI as a function of time for an initial state $|\psi_0\rangle$ prepared along the $\hat{z}$ direction, $|\psi_0\rangle = |S = N/2, M_z = N/2\rangle$, which has no MF dynamics. Here $|S, M_z\rangle$ denote the fully symmetric Dicke states, which are simultaneous eigenstates of the $\vec{S} \cdot \vec{S}$ and $\hat{S}_z$ operators, with eigenvalues $S(S+1)$, $S \in \{0, 1, \dots, N/2\}$ and $M_z \in \{-S, -S+1, \dots, S\}$, respectively. Since the Hamiltonian commutes with $\vec{S} \cdot \vec{S}$, S is conserved. In general, the dynamical evolution is given by $|\psi_t\rangle = \sum_{m=0}^{N/3} c_{N/2-3m}(t) |N/2, N/2-3m\rangle$ and thus only every third state in the fully symmetric Dicke ladder is populated.

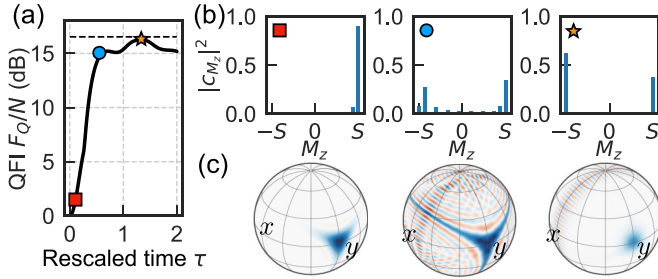


FIG. 2. (a) Quantum Fisher information growth vs time for a system with $N = 45$ atoms, starting from an initial spin coherent state aligned along the z axis. The dashed line indicates the Heisenberg limit (HL). (b) Spin distribution function, $|c_{M_z}|^2$, in the Dicke basis $|S = N/2, M_z\rangle$, and (c) the corresponding spin Wigner function, after evolution under the three-body Hamiltonian $\hat{H}_3$ at $\tau = 0.1$ (red square), 0.66 (blue circle), and 1.3 (yellow star), with rescaled evolution time $\tau = \chi_3 N^{3/2} t$. A final $\pi/2$ rotation about the x axis is applied to facilitate visualization.

In Fig. 2(a), we show the time evolution of the QFI for a system of $N = 45$ atoms and in Fig. 2(c) we show snapshots of the spin Wigner distribution of the state (see Supplemental Material [32]), for three representative time: $\tau = 0.1$ (red square), 0.66 (blue circle), and 1.3 (yellow star) with the rescaled time $\tau \equiv \chi_3 N^{3/2} t$. At $\tau = 0.1$, the state is already non-Gaussian and exhibits a Z_3 rotational symmetry, reflecting the discreteness of the Dicke ladder. This structure, analogous to number-phase duality in the bosonic system, could be useful for quantum error correction [33,34]. At $\tau_{\text{opt},3} = 2/3$, when the QFI reaches its first peak, we observe that the population begins to concentrate around the $|N/2, -N/2\rangle$ states. At longer times, $\tau_{\text{GHZ},3} \approx 1.3$, a generalized GHZ state appears, with only $c_{\pm N/2} \neq 0$, while all other coefficients vanish. For all the states shown, a final $\pi/2$ rotation around the x axis has been applied to make the GHZ structure more visible in Fig. 2(c) [35]. We emphasize these are not ideal GHZ states with $c_{N/2} = c_{-N/2} = 1/\sqrt{2}$, and only exist when $N = 3 + 6m$ with $m \in \mathbb{Z}^+$ and $F_Q \approx 0.9N^2$ [32].

In Fig. 3(a), we plot the QFI as a function of time for atom numbers $N = 301$ to 306 , confirming the initial growth is universal. In Fig. 3(b), we show the QFI at $\tau_{\text{opt},3}$ versus atom number and observe values approaching the HL, $F_Q = N^2$.

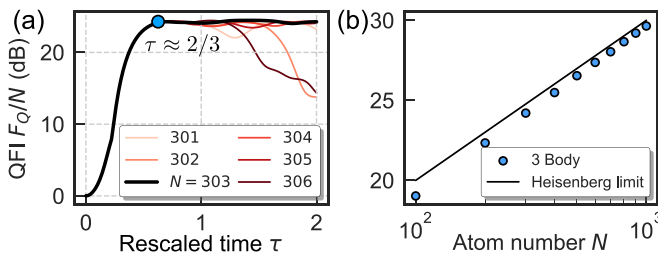


FIG. 3. (a) Time evolution of the QFI for atom numbers ranging from $N = 301$ to 306 . The initial rise of the QFI up to the first peak (blue dots) is well captured by the semiclassical prediction, occurring at $\tau_{\text{opt},3} \approx 2/3$, while the subsequent dynamics reveals sensitivity to the atom number. (b) QFI evaluated at $\tau_{\text{opt},3}$ as a function of atom number. The black line denotes the HL $F_Q = N^2$.

The subsequent dynamics, however, exhibit a strong dependence on the atom number modulo 6.

We compare the entanglement growth timescales with those of conventional 2BIs and find a substantially faster generation of metrologically useful states. Specifically, for 2BIs we use the standard OAT and TAT results: OAT reaches the HS QFI plateau with $F_Q = N^2/2$ at $\chi_2^{\text{OAT}} t_{\text{plat},\text{OAT}} = 1/\sqrt{N}$ and produces a GHZ state at $\chi_2^{\text{OAT}} t_{\text{GHZ},\text{OAT}} = \pi/2$ [36], while TAT attains its optimal QFI at $\chi_2^{\text{TAT}} t_{\text{opt},\text{TAT}} \sim \ln(4N)/2N$ [31].

If the characteristic strength of 2BIs is set by $\chi_2 \sim \chi_2^{\text{OAT},\text{TAT}}$ [22,23,30,32], a simple perturbative argument yields $\chi_3 = \chi_2 g_{\text{eff}}/\Delta_c$, since 3BIs involve the exchange of an additional virtual photon at rate g_{eff} detuned by Δ_c . However, the detunings $\Delta_{a,c}$ and the intracavity photon amplitude $|\alpha|$ are constrained by the requirement that adiabatic elimination remains valid for both the excited state and the cavity mode, imposing $\eta_a |\alpha| \ll 1$ and $\eta_c |\alpha| \ll 1$. Here, $\eta_a = \sqrt{N}g/\Delta_a$ and $\eta_c = \sqrt{N}g_{\text{eff}}/\Delta_c$, related via $\eta_c = \eta_a g/\Delta_c$. The 2BI strength is then $\chi_2 = g|\alpha|^2 \eta_c \eta_a/N$, while the 3BIs satisfy $\chi_3 = g|\alpha|^2 \eta_c^2 \eta_a/N^{3/2}$.

We optimize $\Delta_{a,c}$ and $|\alpha|$ under the above constraints for OAT, TAT, and 3BIs while neglecting dissipation (addressed later). Since the products $\eta_a |\alpha|$ and $\eta_c |\alpha|$ sets the characteristic timescale for 2BIs, we use the same values when evaluating 3BIs to find the ratio

$$\begin{aligned} \frac{t_{\text{GHZ},\text{OAT}}}{t_{\text{GHZ},3}} &\approx 0.3N\eta_c, & \frac{t_{\text{plat},\text{OAT}}}{t_{\text{opt},3}} &= \frac{3}{8}\sqrt{N}\eta_c, \\ \frac{t_{\text{opt},\text{TAT}}}{t_{\text{opt},3}} &= \frac{9 \ln 4N}{16}\eta_c. \end{aligned} \quad (2)$$

These comparisons show that with a reasonable choice of η_c for 3BIs—for example, $\eta_c = 1/2$ —the 3BIs generate entanglement much faster than OAT and even outperform TAT for $N \gtrsim 50$. The resulting timescale is also comparable to recent proposals for creating GHZ states using all-to-all interactions [37–40].

TR protocol. An appealing feature of this scheme is that the rapid generation of QFI from non-Gaussian states for metrological applications can be achieved without requiring complex measurement procedures. Interestingly in our case, the TR protocol [24–26] only requires a single-particle rotation along the y direction of the Bloch sphere that flips $\hat{S}_{x,z} \rightarrow -\hat{S}_{x,z}$ ($\hat{S}_+ \rightarrow -\hat{S}_-$, $\hat{S}_- \rightarrow -\hat{S}_+$), which is illustrated in Fig. 4(a). It begins with the preparation of an initial state, $|\psi_0\rangle$ followed by coherent evolution under the 3BIs $\hat{U} = e^{-it\hat{H}_3}$ for a time t . A phase rotation $\hat{R}(\phi) = e^{-i\phi\hat{S}_z}$ is then applied, after which the dynamics are reversed using $\hat{U}^\dagger$, enabled by the echo pulse. The small unknown phase ϕ is imprinted at the end of the sequence as a proportional change in $\langle \hat{S}_z \rangle$, which can be extracted via standard techniques, such as quantum nondemolition measurements. More explicitly, by coupling the collective spin to a weak probe light field, as the light passes through the ensemble, it acquires a phase shift proportional to the population imbalance. By measuring this phase shift, one can infer the collective population $\langle \hat{S}_z \rangle$, with high precision while preserving the relevant spin projection [30,41]. The TR protocol can also be effectively implemented through a $\pi/3$ rotation about the z axis ($\hat{S}_\pm^3 \rightarrow (\hat{S}_\pm e^{\pm i\pi/3})^3 = -\hat{S}_\pm^3$), $\phi \rightarrow \phi + \pi/3$.

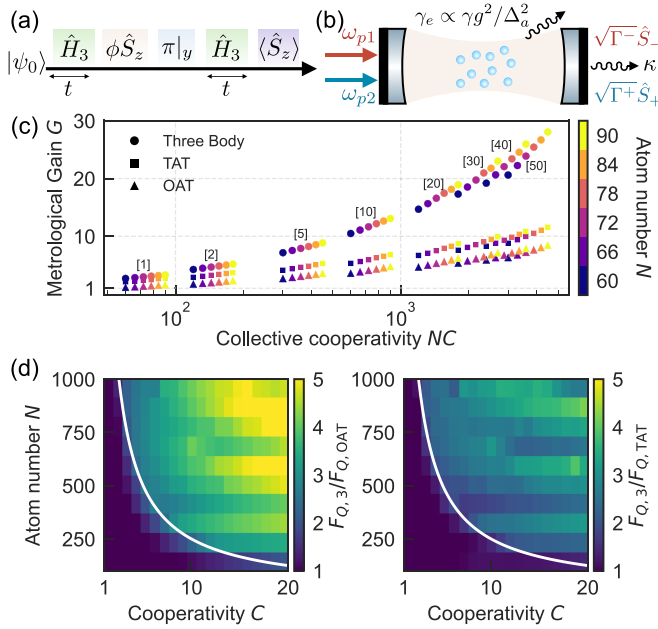


FIG. 4. (a) Time-reversal protocol for quantum-enhanced sensing. (b) Dominant dissipation channels: Spontaneous emission into free space at rate γ_e and photon loss through cavity mirrors at rate κ , leading to two balanced collective decay modes $\hat{S}_\pm$. (c) Metrological gain G vs collective cooperativity NC for 3BIs (circle), TAT (square), and OAT (triangles), optimized over single-particle and collective dissipation; colors denote different atom numbers, while the values in square brackets indicate the cooperativity. (d) Ratio of the QFI for 3BIs to that of OAT (left) and TAT (right) as functions of N and C , including collective dissipation. The QFI is evaluated at the optimal TBI evolution time.

To saturate the QFI, and optimally decode the phase information, it is enough to measure the magnetization $\hat{S}_z$ instead of the fidelity. This conclusion is supported by analyzing the metrological gain relative to the standard quantum limit $\Delta\phi_{\text{SQL}}^2 = 1/N$, $G = \Delta\phi_{\text{SQL}}^2 / \Delta\phi^2$, both numerically and analytically [32].

Dissipative dynamics. In realistic implementations, both the cavity decay rate κ and the single-particle free-space emission rate γ from the excited state, as shown in Fig. 4(b), limit the achievable metrological gain. The key parameter determining the attainable quantum enhancement is the collective cooperativity, defined as NC , with $C = 4g^2/(\kappa\gamma)$. Previous theoretical studies incorporating both collective dephasing and single-particle spin flips in the OAT model, as well as collective decay and spin flips in the TAT and twist-and-turn models, have shown that the maximum achievable gain is constrained by $G \propto \sqrt{NC}$ [42–50].

This scaling originates from the fact that, for the 2BIs considered here [23,30], the collective dissipation rate follows $\Gamma \propto \chi_2\kappa/\Delta_c$, while the effective single-particle dissipation rate scales as $\gamma_e \propto \chi_2\Delta_c/(\kappa C)$. In practice, the cavity detuning Δ_c can be tuned to balance these two decoherence channels, thereby optimizing the achievable metrological gain. These scaling relations are independent of the atomic detuning Δ_a , and for large atom numbers, increasing Δ_a further ensures that the excited state remains adiabatically eliminated.

The situation differs for the 3BIs. We find that the collective dissipation scales as $\Gamma/\chi_3 \propto \Delta_a/(\gamma C)$, while the single-particle emission rate follows $\gamma_e/\chi_3 \propto (\Delta_c/g)^2\Delta_a/(\kappa C)$, exhibiting dependence on both Δ_a and Δ_c . Smaller values of Δ_a and Δ_c are thus favorable for minimizing dissipation; however, they remain constrained by adiabatic elimination conditions. Below, we present two practical scenarios in which the 3BIs offer a clear metrological gain within these constraints. See Appendix for details of the simulated models.

In the first case, we compare the optimal metrological gain of the 3BIs and OAT under the TR protocol for moderate collective cooperativity, $NC \in [60, 4500]$, taking into account both collective and single-particle dissipation. The dynamics are simulated using exact diagonalization [51] for up to $N = 90$ atoms. We only fix $\eta_c = 1/2$ for 3BIs, while separately optimizing the cavity detuning Δ_c and the evolution time to obtain the maximal metrological gain for both models. As expected, the optimal detuning follows the scaling $\Delta_{c,\text{opt}} \propto \sqrt{NC}$ in both cases. As shown in Fig. 4(c), 3BIs provide a pronounced enhancement over OAT within this regime. However, this advantage does not extend to arbitrarily large system sizes: For 3BIs, the atomic adiabatic elimination parameter at the optimal gain, $\eta_a = \eta_c\Delta_{c,\text{opt}}/g \propto \sqrt{N}$, increases with N and eventually breaks down the conditions.

In the second case, our goal is not to optimize the gain or its scaling, but rather to test how rapidly entanglement can be generated in a scalable regime while maintaining the validity of the adiabatic elimination. To ensure a fair comparison, we consider a special case with same $\Delta_{a,c}$ and $|\alpha|$ for OAT, TAT, and 3BIs. To this end, we fix $\Delta_c = g$, which ensures $\eta_c = \eta_a \lesssim 1$, together with $|\alpha| \ll 1$. We initialize the parameters with $N = 100$ and $C = 1$, choosing $\Delta_a = 100g$ such that $\eta_a = 0.1$. As N and C increase, η_a grows as $\sqrt{NC}$, and we impose the upper bound $1/2$ for η_a as indicated by the white line in Fig. 4(d). Below this line, the condition holds, and Δ_a is kept fixed; above it, we scale $\Delta_a \propto \sqrt{NC}$ to preserve the adiabatic condition with $\eta_a = 1/2$. We then simulate the ratio between the QFI of the 3BIs ($F_{Q,3}$) and those of OAT ($F_{Q,\text{OAT}}$) and TAT ($F_{Q,\text{TAT}}$), evaluating all at the optimal evolution time of the 3BIs. Only collective dissipation is included, assuming that single-particle emission remains negligible. The results show that in the region above the white line, the 3BIs enhance the QFI by a factor of 5 relative to OAT and by a factor of 4 relative to TAT, demonstrating a practical metrological gain enabled by rapid entanglement generation.

It could be interesting to explore in the future the metrological performance beyond this regime by considering smaller Δ_c , where the adiabatic elimination of the cavity photons breaks down, leading to even faster dynamics while requiring a full treatment of the spin-boson model.

Conclusion and outlook. Collective three-body interactions allow fast creation of GHZ-type entangled states with Heisenberg-limited sensitivity, far outperforming two-body schemes. This performance persists across a broad parameter range even in the presence of realistic dissipation. While we have focused on cavity implementations, our scheme naturally extends to other boson-mediated platforms, including trapped ions, superconducting circuits, and cold molecules, where precise control of interaction strength and particle number may enable high-fidelity GHZ states. Future directions include

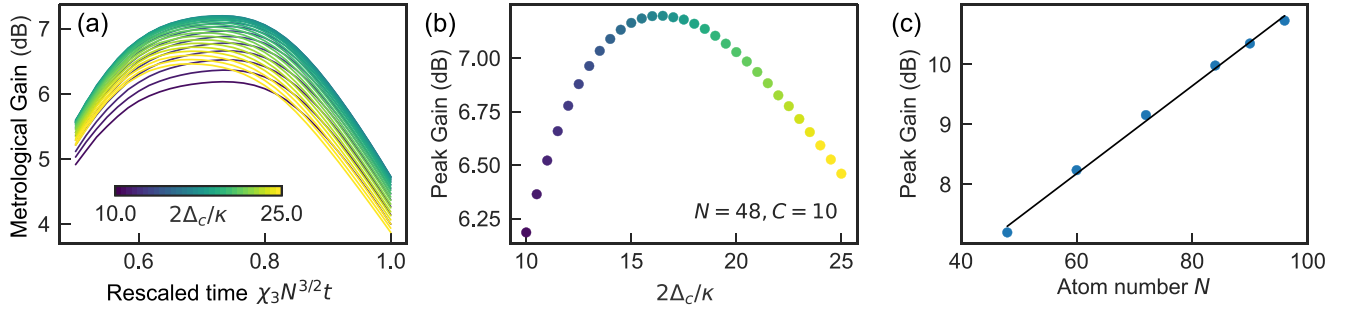


FIG. 5. Optimization procedure for extracting the scaling behavior. (a) Metrological gain as a function of evolution time for fixed $N = 48$ and $C = 10$, with different colors representing various values of $d = 2\Delta_c/\kappa$. For each d , the peak gain is identified. (b) Peak gain as a function of d . (c) Repeating the above procedure for different atom numbers N reveals a linear scaling of the optimal gain with respect to N .

exploring the impact of distinct decoherence channels and exploiting the system's intrinsic Z_3 symmetry, which connects to bosonic codes [33] and may inspire new error-correction strategies. More broadly, collective 3BIs expand the quantum simulation toolbox, paving the way toward the study of complex many-body models such as charge- $4e$ superconductors [19] and lattice gauge theories [52,53].

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

Appendix: Realistic parameters. To numerically compare the metrological performance of OAT, TAT, and 3BIs, we implement the following Hamiltonians and dissipation channels demonstrated in Refs. [22,23,29]:

(1) 3BIs: $\hat{H}_3 = \chi_3(\hat{S}_+^3 + \hat{S}_-^3)$, $\hat{L}_3^+ = \sqrt{\Gamma^+}\hat{S}_+$, $\hat{L}_3^- = \sqrt{\Gamma^-}\hat{S}_-$, with $\chi_3 = |\alpha|^2 g_{\text{eff}}^2 / \Delta_c^2$ and $\Gamma^+ = \Gamma^- = \Gamma$, also illustrated in Fig. 4(b).

(2) OAT: $\hat{H}_{\text{OAT}} = \chi_2^{\text{OAT}} \hat{S}_x^2$, $\hat{L}_2^{\text{OAT}} = \sqrt{\Gamma_2^{\text{OAT}}}\hat{S}_x$, with $\chi_2^{\text{OAT}} = 4\chi_2$ and $\Gamma_2^{\text{OAT}} = 4\Gamma$.

(3) TAT: $\hat{H}_{\text{TAT}} = \chi_2^{\text{TAT}}(\hat{S}_x^2 - \hat{S}_z^2)$, $\hat{L}_2^{\text{TAT}} = \sqrt{\Gamma_2^{\text{TAT}}}(\frac{\sqrt{2}+1}{\sqrt{2}-1}\hat{S}_+ + \hat{S}_-)$, with $\chi_2^{\text{TAT}} = \frac{4}{3}\chi_2$ and $\Gamma_2^{\text{TAT}} = (\frac{\sqrt{2}-1}{\sqrt{3}})^2 \Gamma$.

Here, $\chi_2 = |\alpha|^2 g_{\text{eff}}^2 / \Delta_c$ and $\Gamma = |\alpha|^2 g_{\text{eff}}^2 \kappa / \Delta_c^2$. We assume a total intracavity photon number of $2|\alpha|^2$ when determining the corresponding prefactors. The single-particle emission rate is given by $\gamma_e = 2\gamma|\alpha|^2 g^2 / \Delta_a^2$. We simulate the associ-

ated spin-flip processes within the $|\uparrow / \downarrow\rangle$ manifold, while Ref. [32] presents simulations of single-particle atom loss using the quantum trajectory method [54].

With the cavity cooperativity defined as $C = 4g^2/(\kappa\gamma)$, we consider two parametrization strategies as discussed in the main text. In Fig. 4(c), we first assume that the adiabatic elimination of the excited state remains valid and fix $\eta_c = 1/2$. Under this condition, the relevant ratios between the dissipative rates and the strength of 3BIs are given by

$$\frac{\Gamma}{\chi_3} = \frac{\sqrt{N}}{\eta_c} \frac{\kappa}{\Delta_c}, \quad \frac{\gamma_e}{\chi_3} = \frac{8\sqrt{N}}{\eta_c} \frac{\Delta_c}{\kappa C}. \quad (\text{A1})$$

Accordingly, Δ_c can be optimized to balance collective and single-particle dissipation. Although Γ/χ_3 itself does not explicitly depend on Δ_c , the parameter $\eta_c \propto 1/\Delta_c$ is fixed in our analysis, which indirectly couples Δ_c to the overall dissipation balance.

The optimization procedure for Fig. 4(c) is detailed in Fig. 5. For given values of atom number N and cooperativity C , we scan the evolution time t for various values of $d = 2\Delta_c/\kappa$, as shown in Fig. 5(a). For each d , the optimal metrological gain (versus time) is extracted and plotted in Fig. 5(b). Finally, we repeat this procedure for different values of N and interestingly observe a linear scaling with respect to N even with moderate cooperativity, as shown in Fig. 5(c), along with a $\sqrt{C}$ scaling with cooperativity. Together, these results lead to a novel $N\sqrt{C}$ scaling of the metrological gain. However, as discussed in the main text, this fixed- η_c parametrization is not further scalable with increasing atom number. This limitation motivated instead an alternative analysis introduced in Fig. 6.

We introduce the dimensionless ratios $\beta_\kappa = \Delta_a/\kappa$ and $\beta_\gamma = \Delta_a/\gamma$, which yield $\Delta_a/g = 2\sqrt{\beta_\kappa\beta_\gamma}/C$. The ratios between the relevant rates can then be expressed as

$$\frac{\Gamma}{\chi_3} = \frac{4\beta_\gamma}{C}, \quad \frac{\gamma_e}{\chi_3} = \frac{8\beta_\kappa}{C}, \quad \frac{\chi_2}{\chi_3} = \frac{\Delta_a}{g} = 2\sqrt{\frac{\beta_\kappa\beta_\gamma}{C}} \quad (\text{A2})$$

by assuming $g = \Delta_c$. In the numerical simulations including superradiance, we start with a small atom number $N = 100$ and cooperativity $C = 1$, and define the constants $\beta_\kappa = 500$ and $\beta_\gamma = 5$, which give $\Delta_a/g = 100$. We then increase N and C , testing whether the adiabatic elimination condition $\eta_a \leq 1/2$ is satisfied. In Fig. 6, we plot η_a while scanning N and C , with the contour $\eta_a = 1/2$ marked as the white line. Below

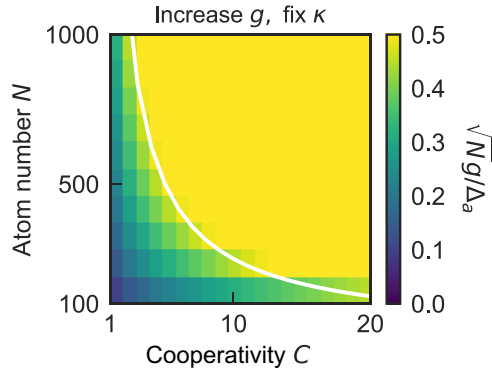


FIG. 6. Adiabatic elimination condition for the excited state $\eta_a = \sqrt{N}g/\Delta_a$ as a function of atom number N and cavity cooperativity C . To increase C , we scale $g \propto \sqrt{C}$ while keeping κ and γ fixed. The white line marks the boundary $\eta_a = 1/2$.

this line, Δ_a is kept fixed, and we take $\Gamma/\chi_3 = 4\beta_\gamma/C$ and $\chi_2/\chi_3 = 2\sqrt{\beta_\kappa\beta_\gamma/C}$ in the simulations. Above the white line, Δ_a is increased by a factor $r = \sqrt{NC/(\beta_\kappa\beta_\gamma)}$, and we choose $\Gamma/\chi_3 = 4r\beta_\gamma/C$ and $\chi_2/\chi_3 = 2\sqrt{N}$. For a given N and C , we first determine the optimal evolution time for the 3BIs. We then evaluate the QFI of OAT and TAT at this same time to provide a direct comparison.

In Fig. 7, we plot the QFI gain as a function of time for $C = 1$ and $C = 10$. For $C = 1$, where Δ_a is kept fixed, the

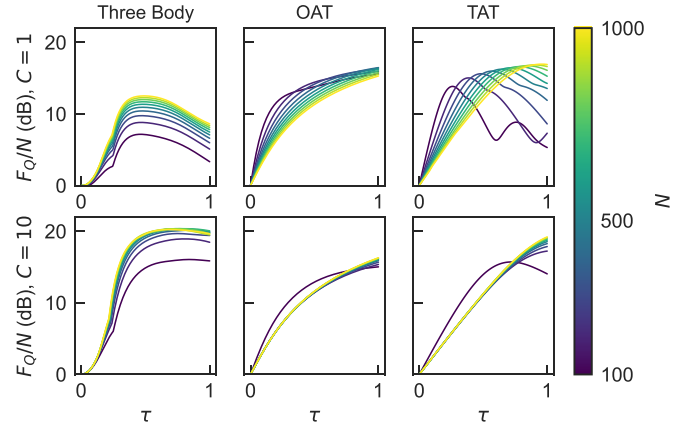


FIG. 7. QFI gain as a function of time for the 3BIs, OAT, and TAT. The top panel corresponds to $C = 1$ and the bottom panel to $C = 10$. Color maps correspond to atom numbers from 100 to 1000.

gain is found to be limited compared with OAT and TAT. This can be attributed to two factors: First, the dynamics is not significantly faster due to the large prefactor $\chi_2/\chi_3 = 100$; and second, the generated state is more susceptible to superradiant decay. In contrast, for $C = 10$, above the white line, we have $\chi_2/\chi_3 = 2\sqrt{N}$ and indeed observe much faster dynamics than in OAT and TAT, accompanied by greater metrological gain.

- [1] A. Fedorov, L. Steffen, M. Baur, M. P. da Silva, and A. Wallraff, Implementation of a Toffoli gate with superconducting circuits, *Nature (London)* **481**, 170 (2012).
- [2] Y. Kim, A. Morvan, L. B. Nguyen, R. K. Naik, C. Jünger, L. Chen, J. M. Kreikebaum, D. I. Santiago, and I. Siddiqi, High-fidelity three-qubit iToffoli gate for fixed-frequency superconducting qubits, *Nat. Phys.* **18**, 783 (2022).
- [3] A. M. Eriksson, T. Sépulcre, M. Kervinen, T. Hillmann, M. Kudra, S. Dupouy, Y. Lu, M. Khanahmadi, J. Yang, C. Castilho-Moreno, *et al.*, Universal control of a bosonic mode via drive-activated native cubic interactions, *Nat. Commun.* **15**, 2512 (2024).
- [4] O. Katz, L. Feng, A. Risinger, C. Monroe, and M. Cetina, Demonstration of three- and four-body interactions between trapped-ion spins, *Nat. Phys.* **19**, 1452 (2023).
- [5] O. Băzăvan, S. Saner, D. J. Webb, E. M. Ainley, P. Drmota, D. P. Nadlinger, G. Araneda, D. M. Lucas, C. J. Ballance, and R. Srinivas, Squeezing, trisqueezing, and quadsqueezing in a spin-oscillator system, *Nat. Phys.* **22**, 757 (2026).
- [6] S. Saner, O. Băzăvan, D. Webb, G. Araneda, D. Lucas, C. Ballance, and R. Srinivas, Generating arbitrary superpositions of nonclassical quantum harmonic oscillator states, *arXiv:2409.03482*.
- [7] H.-N. Dai, B. Yang, A. Reingruber, H. Sun, X.-F. Xu, Y.-A. Chen, Z.-S. Yuan, and J.-W. Pan, Four-body ring-exchange interactions and anyonic statistics within a minimal toric-code Hamiltonian, *Nat. Phys.* **13**, 1195 (2017).
- [8] A. Goban, R. B. Hutson, G. E. Marti, S. L. Campbell, M. A. Perlin, P. S. Julienne, J. P. D'Incao, A. M. Rey, and J. Ye, Emergence of multi-body interactions in a fermionic lattice clock, *Nature (London)* **563**, 369 (2018).
- [9] S. Will, T. Best, U. Schneider, L. Hackermüller, D.-S. Lühmann, and I. Bloch, Time-resolved observation of coherent multi-body interactions in quantum phase revivals, *Nature (London)* **465**, 197 (2010).
- [10] T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl, *et al.*, Evidence for Efimov quantum states in an ultracold gas of caesium atoms, *Nature (London)* **440**, 315 (2006).
- [11] R. J. Fletcher, R. Lopes, J. Man, N. Navon, R. P. Smith, M. W. Zwierlein, and Z. Hadzibabic, Two- and three-body contacts in the unitary Bose gas, *Science* **355**, 377 (2017).
- [12] H. Levine, A. Keesling, G. Semeghini, A. Omran, T. T. Wang, S. Ebadi, H. Bernien, M. Greiner, V. Vuletić, H. Pichler, *et al.*, Parallel implementation of high-fidelity multi-qubit gates with neutral atoms, *Phys. Rev. Lett.* **123**, 170503 (2019).
- [13] M. Kim, Y. Song, J. Kim, and J. Ahn, Quantum Ising Hamiltonian programming in trio, quartet, and sextet qubit systems, *PRX Quantum* **1**, 020323 (2020).
- [14] K. Kim, F. Yang, K. Mølmer, and J. Ahn, Realization of an extremely anisotropic Heisenberg magnet in Rydberg atom arrays, *Phys. Rev. X* **14**, 011025 (2024).

- [15] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2010).
- [16] H.-W. Hammer, A. Nogga, and A. Schwenk, *Colloquium: Three-body forces: From cold atoms to nuclei*, *Rev. Mod. Phys.* **85**, 197 (2013).
- [17] H. Büchler, A. Micheli, and P. Zoller, Three-body interactions with cold polar molecules, *Nat. Phys.* **3**, 726 (2007).
- [18] B. Derrida, Random-energy model: Limit of a family of disordered models, *Phys. Rev. Lett.* **45**, 79 (1980).
- [19] E. Berg, E. Fradkin, and S. A. Kivelson, Charge-4e superconductivity from pair-density-wave order in certain high-temperature superconductors, *Nat. Phys.* **5**, 830 (2009).
- [20] J.-i. Fujita and H. Miyazawa, Pion theory of three-body forces, *Prog. Theor. Phys.* **17**, 360 (1957).
- [21] F. Pinto-Gómez, F. De Soto, and J. Rodríguez-Quintero, Complete analysis of the Landau-gauge three-gluon vertex from lattice QCD, *Phys. Rev. D* **110**, 014005 (2024).
- [22] C. Luo, H. Zhang, C. Maruko, E. A. Bohr, A. Chu, A. M. Rey, and J. K. Thompson, Realization of three- and four-body interactions between momentum states in a cavity, *Science* **390**, 925 (2025).
- [23] C. Luo, H. Zhang, A. Chu, C. Maruko, A. M. Rey, and J. K. Thompson, Hamiltonian engineering of collective XYZ spin models in an optical cavity, *Nat. Phys.* **21**, 916 (2025).
- [24] M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet, *Nat. Phys.* **13**, 781 (2017).
- [25] M. Gärttner, P. Hauke, and A. M. Rey, Relating out-of-time-order correlations to entanglement via multiple-quantum coherences, *Phys. Rev. Lett.* **120**, 040402 (2018).
- [26] R. J. Lewis-Swan, A. Safavi-Naini, J. J. Bollinger, and A. M. Rey, Unifying scrambling, thermalization and entanglement through measurement of fidelity out-of-time-order correlators in the Dicke model, *Nat. Commun.* **10**, 1581 (2019).
- [27] A. Chu, P. He, J. K. Thompson, and A. M. Rey, Quantum enhanced cavity QED interferometer with partially delocalized atoms in lattices, *Phys. Rev. Lett.* **127**, 210401 (2021).
- [28] M. Koppenhöfer and A. A. Clerk, Revisiting the impact of dissipation on time-reversed one-axis-twist quantum-sensing protocols, *Phys. Rev. Res.* **5**, 043279 (2023).
- [29] H. Zhang, A. Chu, C. Luo, J. K. Thompson, and A. M. Rey, Solitons in arbitrary dimensions stabilized by photon-mediated interactions, *Phys. Rev. Lett.* **135**, 173402 (2025).
- [30] C. Luo, H. Zhang, V. P. W. Koh, J. D. Wilson, A. Chu, M. J. Holland, A. M. Rey, and J. K. Thompson, Momentum-exchange interactions in a Bragg atom interferometer suppress Doppler dephasing, *Science* **384**, 551 (2024).
- [31] M. H. M. Arias, I. H. Deutsch, and P. M. Poggi, Phase-space geometry and optimal state preparation in quantum metrology with collective spins, *PRX Quantum* **4**, 020314 (2023).
- [32] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/2pg6-vdlj> for details of the semiclassical analysis, higher-order squeezing, and the time-reversal protocol, which includes Refs. [55–59].
- [33] A. L. Grimsmo, J. Combes, and B. Q. Baragiola, Quantum computing with rotation-symmetric bosonic codes, *Phys. Rev. X* **10**, 011058 (2020).
- [34] C. H. Valahu, M. P. Stafford, Z. Huang, V. G. Matsos, M. J. Millican, T. Chalermputitarak, N. C. Menicucci, J. Combes, B. Q. Baragiola, and T. R. Tan, Quantum-enhanced multi-parameter sensing in a single mode, *Sci. Adv.* **11**, eadw9757 (2025).
- [35] K. Mølmer and A. Sørensen, Multiparticle entanglement of hot trapped ions, *Phys. Rev. Lett.* **82**, 1835 (1999).
- [36] L. Pezzé and A. Smerzi, Entanglement, nonlinear dynamics, and the Heisenberg limit, *Phys. Rev. Lett.* **102**, 100401 (2009).
- [37] X. Zhang, Z. Hu, and Y.-C. Liu, Fast generation of GHZ-like states using collective-spin XYZ model, *Phys. Rev. Lett.* **132**, 113402 (2024).
- [38] C. Yin, Fast and accurate Greenberger-Horne-Zeilinger encoding using all-to-all interactions, *Phys. Rev. Lett.* **134**, 130604 (2025).
- [39] C. Yin, Fast quantum computation with all-to-all Hamiltonians, *arXiv:2509.25345*.
- [40] J. Ma, Y. Shen, J. Huang, and C. Lee, Quantum metrology via Floquet-engineered two-axis twisting and turn dynamics, *Phys. Rev. A* **112**, L040602 (2025).
- [41] G. P. Greve, C. Luo, B. Wu, and J. K. Thompson, Entanglement-enhanced matter-wave interferometry in a high-finesse cavity, *Nature (London)* **610**, 472 (2022).
- [42] Z. Chen, J. G. Bohnet, J. M. Weiner, K. C. Cox, and J. K. Thompson, Cavity-aided nondemolition measurements for atom counting and spin squeezing, *Phys. Rev. A* **89**, 043837 (2014).
- [43] E. Davis, G. Bentsen, and M. Schleier-Smith, Approaching the Heisenberg limit without single-particle detection, *Phys. Rev. Lett.* **116**, 053601 (2016).
- [44] M. A. Norcia, R. J. Lewis-Swan, J. R. Cline, B. Zhu, A. M. Rey, and J. K. Thompson, Cavity-mediated collective spin-exchange interactions in a strontium superradiant laser, *Science* **361**, 259 (2018).
- [45] S. Colombo, E. Pedrozo-Peñañiel, A. F. Adiyatullin, Z. Li, E. Mendez, C. Shu, and V. Vuletić, Time-reversal-based quantum metrology with many-body entangled states, *Nat. Phys.* **18**, 925 (2022).
- [46] I. D. Leroux, M. H. Schleier-Smith, and V. Vuletić, Implementation of cavity squeezing of a collective atomic spin, *Phys. Rev. Lett.* **104**, 073602 (2010).
- [47] Y. A. El-Neaj, C. Alpigiani, S. Amairi-Pyka, H. Araújo, A. Balaž, A. Bassi, L. Bathe-Peters, B. Battelier, A. Belić, E. Bentine, *et al.*, AEDGE: Atomic experiment for dark matter and gravity exploration in space, *EPJ Quantum Technol.* **7**, 6 (2020).
- [48] J. Hu, W. Chen, Z. Vendeiro, A. Urvoy, B. Braverman, and V. Vuletić, Vacuum spin squeezing, *Phys. Rev. A* **96**, 050301 (2017).
- [49] D. Barberena, S. R. Muleady, J. J. Bollinger, R. J. Lewis-Swan, and A. M. Rey, Fast generation of spin squeezing via resonant spin-boson coupling, *Quantum Sci. Technol.* **9**, 025013 (2024).
- [50] Z. Li, S. Colombo, C. Shu, G. Velez, S. Pilatowsky-Cameo, R. Schmied, S. Choi, M. Lukin, E. Pedrozo-Peñañiel, and V. Vuletić, Improving metrology with quantum scrambling, *Science* **380**, 1381 (2023).
- [51] N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, Open quantum systems with local and collective incoherent processes: Efficient numerical simulations using permutational invariance, *Phys. Rev. A* **98**, 063815 (2018).

- [52] M. C. Banuls, R. Blatt, J. Catani, A. Celi, J. I. Cirac, M. Dalmonte, L. Fallani, K. Jansen, M. Lewenstein, S. Montangero, *et al.*, Simulating lattice gauge theories within quantum technologies, [Eur. Phys. J. D **74**, 165 \(2020\)](#).
- [53] M. Aidelsburger, L. Barbiero, A. Bermudez, T. Chanda, A. Dauphin, D. González-Cuadra, P. R. Grzybowski, S. Hands, F. Jendrzejewski, J. Jünemann, *et al.*, Cold atoms meet lattice gauge theory, [Philos. Trans. R. Soc. A **380**, 20210064 \(2022\)](#).
- [54] Y. Zhang, Y.-X. Zhang, and K. Mølmer, Monte-Carlo simulations of superradiant lasing, [New J. Phys. **20**, 112001 \(2018\)](#).
- [55] F. Anders, L. Pezzè, A. Smerzi, and C. Klempt, Phase magnification by two-axis counter twisting for detection-noise robust interferometry, [Phys. Rev. A **97**, 043813 \(2018\)](#).
- [56] L. Pezze, A. Smerzi, M. K. Oberthaler, R. Schmied, and P. Treutlein, Quantum metrology with nonclassical states of atomic ensembles, [Rev. Mod. Phys. **90**, 035005 \(2018\)](#).
- [57] H. Zhang, A. Chu, C. Luo, J. K. Thompson, and A. M. Rey, Control and amplification of Bloch oscillations via photon-mediated interactions, [Phys. Rev. Res. **5**, L032039 \(2023\)](#).
- [58] T. Kielinski, P. O. Schmidt, and K. Hammerer, GHZ protocols enhance frequency metrology despite spontaneous decay, [Sci. Adv. **10**, eadr1439 \(2024\)](#).
- [59] Y. Zhang, J. Z. Castro, and R. J. Lewis-Swan, Harnessing quantum chaos in spin-boson models for all-purpose quantum-enhanced sensing, [Phys. Rev. Res. **7**, 013227 \(2025\)](#).