

Dispersed-phase forcing reorganizes Lagrangian intermittency in turbulence

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In single-phase turbulence, fluctuations in the Lagrangian velocity increments become more extreme as the timescale τ decreases, leading to the strongest fluctuations as $\tau \rightarrow 0$. Using high-resolution three-dimensional Lagrangian particle tracking experiments, we find that for turbulent flows where bubbles force the flow, Lagrangian intermittency fundamentally changes. The strongest intermittency occurs at intermediate τ , and the flatness of the velocity increments far exceeds that of single-phase turbulence at comparable Reynolds numbers. We demonstrate that this behavior originates from strong deflections of tracer trajectories due to vortex trapping events at the boundaries of bubble wakes. This reveals that unlike in single-phase turbulence where nonlinearity alone generates small-scale internal intermittency, bubble-driven turbulence exhibits a hybrid mechanism in which external intermittency introduced at the bubble scale cooperates with internal intermittency processes to produce extreme fluctuations at intermediate timescales. The mechanism suggests that dispersed-phase forcing can fundamentally reshape Lagrangian intermittency across a broad class of dispersed multiphase and particle-laden turbulent flows.

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Turbulent flows are highly intermittent, with rare, intense fluctuations emerging amid a relatively calm background flow [1–6]. Understanding and predicting this phenomenon remains one of the central challenges in turbulence research [7–12]. Analysis of the spatial velocity increments reveals intermittency from the Eulerian perspective, with strong deviations of their scaling from the predictions of Kolmogorov’s 1941 (K41) theory being observed at small scales [13–15]. Intermittency can also be observed from the complementary Lagrangian perspective using temporal velocity increments, based on the velocity of individual tracer particles as they move through the flow. These again show strong departures from the Lagrangian extension of K41 [16–22], and the deviations are connected to extreme particle accelerations and rapid directional changes of the tracers [23–28]. In single-phase turbulence across a wide range of flows [3,8,17,29], a qualitatively universal property of intermittency in turbulent flows emerges—The probability of extreme fluctuations increases monotonically with decreasing scale (whether decreasing spatial scale or decreasing temporal scale).

The kind of intermittency just described is often referred to as “internal,” since it originates from the intrinsic nonlinear dynamics of the turbulent flow [30]. In contrast, “external intermittency” [31] is associated with the effects of bound-

aries or interfaces, such as a turbulent-nonturbulent interface [32,33]. In most cases, however, external intermittency is controlled by large-scale features of the flow (e.g., turbulent jet geometry), while internal intermittency dominates the small scales. Dispersed multiphase turbulence [34–42] introduces a qualitatively different situation. In such flows, the turbulence is forced through momentum exchange between the inclusions and the carrier fluid, injecting energy at spatial and temporal scales associated with the inclusions. This dispersed-phase forcing can generate external intermittency effects at relatively small scales, where internal intermittency mechanisms are also active. As we demonstrate below, such dispersed-phase forcing gives rise to a hybrid, forcing-induced intermittency that bridges the classical notions of internal and external intermittencies, fundamentally modifying the nature of intermittent fluctuations compared to single-phase turbulence.

In this work, we explore this phenomenon experimentally using high-resolution Lagrangian particle tracking (LPT) of bubble-induced turbulence (BIT). We show that intermittency in BIT departs profoundly from the classical picture for single-phase turbulence: The strongest fluctuations occur not at the smallest scales but at intermediate timescales, and the flatness of the Lagrangian velocity increments far exceeds the values observed for single-phase turbulence at comparable Reynolds numbers. We demonstrate that these extreme fluctuations occur due to a mechanism whereby strong deflections of tracer trajectories are generated during vortex trapping events at the edges of bubble wakes. This mechanism establishes a physical picture of forcing-induced Lagrangian intermittency, in which dispersed-phase forcing reorganizes the scale at which the most intense fluctuations occur, breaking the conventional association of the most intense intermittency occurring at the smallest scales.

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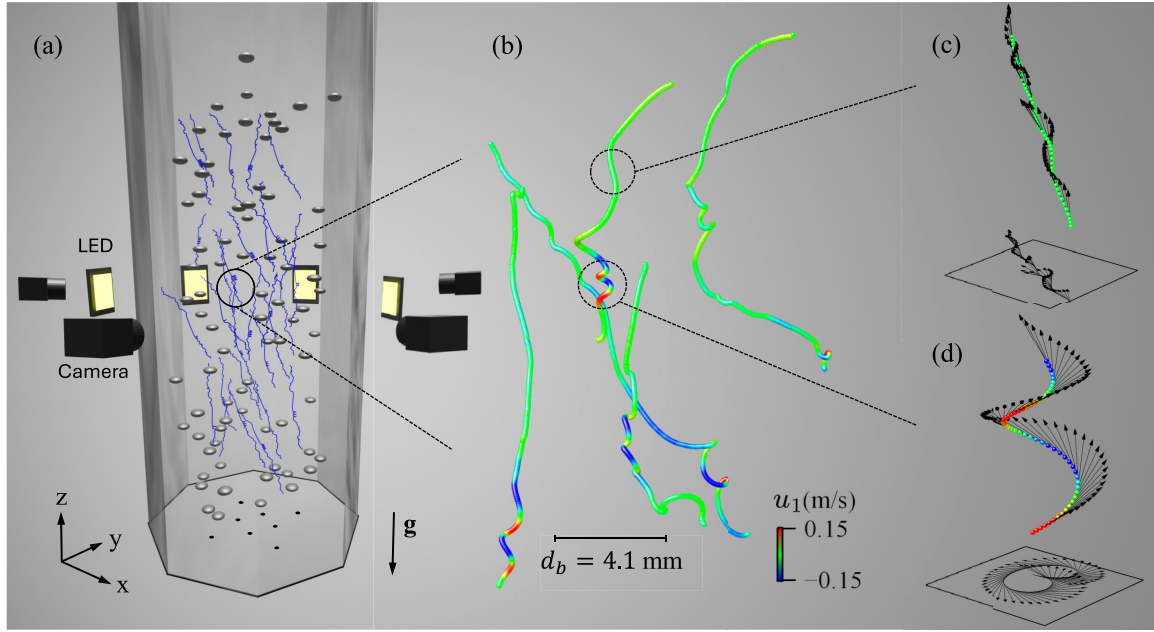


FIG. 1. Experimental setup and typical tracer trajectories. (a) Schematic sketch of the octagonal bubble column, showing the illumination system and the four synchronized high-speed cameras (note that in the actual experiment, the number of bubbles in the column is $O(10^3)$ and approximately 50–100 bubbles are tracked in the field of view). (b) Representative tracer trajectories for the Mid case, with enlarged view showing typical geometries of (c) small-event segments and (d) large-event segments. Arrows indicate the instantaneous motion directions of tracers at each time, illustrating that tracers in large-event segments exhibit pronounced directional changes, whereas those in small-event segments exhibit persistence in their motion. The horizontal planar projections in panels (c) and (d) highlight the corresponding horizontal velocity direction vectors. The scale bar labeled $d_b = 4.1$ mm applies only to panel (b).

I. EXPERIMENTAL SETUP

Experiments were performed in a vertical bubble column operated in the homogeneous bubbly flow regime [Fig. 1(a)]. Using four synchronized high-speed cameras and three-dimensional (3D) LPT, we simultaneously tracked deformable bubbles and tracers, enabling full 3D kinematics of both phases in BIT. Three bubble-swarm cases (Sm, Mid, and La) are considered, corresponding to increasing bubble diameter d_b and void fraction α (see Table I in Appendix A). Measurements were performed sufficiently far above the injection zone to ensure fully developed conditions. Further experimental details are provided in Appendix A and follow Refs. [43–45].

II. LAGRANGIAN INTERMITTENCY AT DIFFERENT SCALES

The Lagrangian tracer velocity increment is defined as $\delta u_i(\tau, t) \equiv u_i(t + \tau) - u_i(t)$, and since our flow in the measurement volume is approximately statistically stationary, the statistics of $\delta u_i(\tau, t)$ depend only on τ . We begin by considering the behavior in the limit $\tau \rightarrow 0$, for which $\delta u_i(\tau, t) = \tau a_i(t) + O(\tau^2)$, where a_i is the i th component of the tracer acceleration. Given this relationship, in Fig. 2 we show results for the probability density functions (PDF) of the horizontal component a_1 (vertical component behaves similarly; see the Supplemental Material [46]). The Subscripts 1 and 2 denote two horizontal directions and 3 is the vertical one (mean bubble rise direction). The results for the PDF from our BIT flow are highly non-Gaussian, exhibiting stretched-exponential tails that closely resemble those in homogeneous

isotropic turbulence (HIT) [47]. The exponential model of Ref. [23] reproduces our BIT data well, with only minor parameter adjustments, which is to be expected given the lower

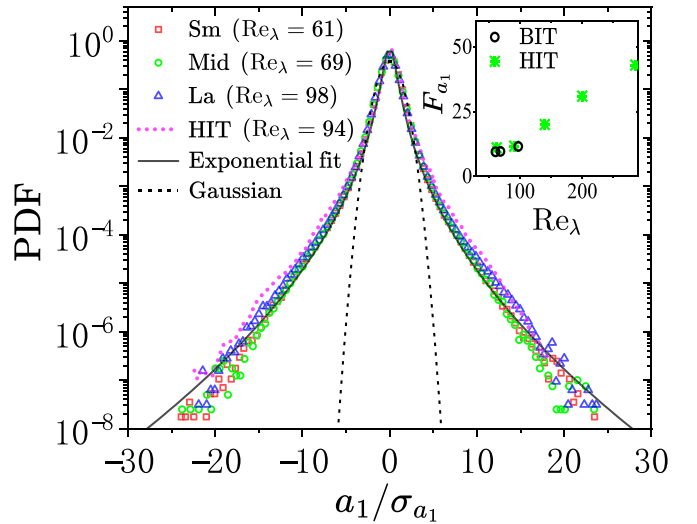


FIG. 2. PDFs of horizontal acceleration in BIT for three cases Sm, Mid, and La. The pink dotted line is HIT data from direct numerical simulations (DNS) [47]. The black solid line denotes the exponential fit function of Ref. [23], $P(a_i) = C \exp[-a_i^2 / ((1 + |a_i| \beta / \sigma |^\gamma) \sigma^2)]$, with modified parameters $\beta = 0.539$, $\gamma = 1.5$, $\sigma = 0.53$, and $C = 0.786$. Black dashed line indicates a Gaussian distribution. Inset: Comparison of acceleration flatness between the three BIT cases and HIT. HIT data at $Re_\lambda = 63$ and 93 are from Ref. [48], and those at $Re_\lambda = 140, 200$, and 285 are from Ref. [23].

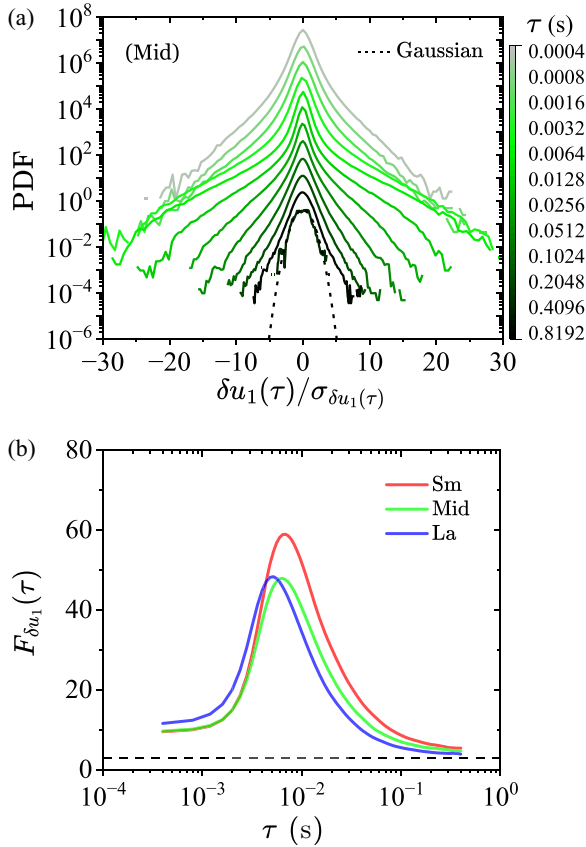


FIG. 3. (a) PDFs of the Lagrangian horizontal velocity increments for case Mid, shown for increasing time lags τ from top to bottom. Increments $\delta u_1(\tau)$ are normalized by their rms, and curves are vertically shifted for clarity. (b) Flatness $F_{\delta u_1}$ for the three BIT cases; the dashed line marks the Gaussian value $F = 3$.

Taylor-Reynolds numbers Re_λ in our BIT flows. The inset of Fig. 2 compares the acceleration flatness $F_{a_1} = \langle a_1^4 \rangle / \langle a_1^2 \rangle^2$ for the three BIT cases with HIT data from simulations [48] and experiments [23]. At comparable Re_λ , the flatness of our BIT flows matches that in HIT and increases with Re_λ in the same manner. It is remarkable to observe such quantitative agreement between our BIT flows and single-phase turbulence, given their dramatic dynamical differences, indicating that dissipative-scale Lagrangian intermittency is almost equal in single-phase turbulence and BIT.

We next examine $\delta u_i(\tau, t)$ at finite τ . Figure 3(a) shows the PDFs of δu_1 for different τ in a representative case Mid; the vertical component behaves similarly [46]. The most striking feature observed is that in BIT the heaviest-tailed PDFs occur at intermediate τ , indicating that the most intermittent events do not occur at the smallest scales. This is in stark contrast to the qualitatively universal property of single-phase turbulence noted earlier that intermittency becomes monotonically stronger with decreasing scale [17,18].

The dependence of intermittency on scale may be seen more clearly by considering the flatness $F_{\delta u_1}(\tau) = \langle [\delta u_1(\tau)]^4 \rangle / \langle [\delta u_1(\tau)]^2 \rangle^2$, where $F = 3$ indicates Gaussian statistics [Fig. 3(b)]. In the limit $\tau \rightarrow 0$, $F_{\delta u_1}$ becomes equal to the flatness of acceleration component a_1 , and we have already observed in the inset of Fig. 2 that these are very similar in

BIT and HIT. At large τ , the HIT results [48] (which are at similar Re_λ to our BIT flows) approach $F_{\delta u_1} \approx 3$. This can be understood based on the observation that when τ is larger than the integral timescale of the flow, $u_1(t + \tau)$ and $u_1(t)$ become uncorrelated, and the PDF of u_1 is close to a Gaussian. A similar argument also explains why our BIT results approach $F_{\delta u_1} \approx 3$ at long times, with the values remaining slightly above 3 in the long time limit due to the PDF of u_1 being less Gaussian in BIT than HIT [49]. However, whereas $F_{\delta u_1}(\tau)$ monotonically decreases with increasing τ for HIT [48], for BIT it initially increases, reaching a maximum at intermediate scales, e.g., $\tau \sim 10^{-2}$ s. The peak value in the BIT cases, $F_{\delta u_1} \approx 60$, far exceeds that in HIT at the same Re_λ , and estimates suggest that achieving comparable flatness in HIT would require $Re_\lambda \gtrsim 500$ [23,50,51]. In BIT, this value occurs already at $Re_\lambda = 61$ for the Sm case, underscoring the profound role that the dispersed phase plays in generating intermittency. These observations raise a central question: What physical mechanism produces such extreme events at intermediate timescales in BIT?

III. CONNECTING INTERMITTENCY TO TRAJECTORY GEOMETRY

To uncover the mechanism generating extreme velocity fluctuations at intermediate τ in BIT, we examine the geometry of tracer trajectories. Figure 1(b) shows representative 3D tracer trajectories, with enlarged views for segments exhibiting small horizontal velocity increments [Fig. 1(c)] and large increments [Fig. 1(d)]. For the trajectories shown, the extreme segments are associated with rapid directional changes of the tracers in spiral vortices [Fig. 1(d)]. To explore this quantitatively across all trajectories, we consider the conditional directional autocorrelation function (DACF; see Appendix B for details)

$$R_{\hat{u}_h}(\tau^*|\cdot) \equiv \langle \hat{u}_h(t) \cdot \hat{u}_h(t + \tau^*) | \cdot \rangle, \quad (1)$$

where $\hat{u}_h$ is the unit vector in the direction of the horizontal projection of the tracer velocity and “ $|\cdot$ ” indicates a condition on the average. We compute $R_{\hat{u}_h}(\tau^* | |\delta u_1(\tau)| \geq \beta)$ and $R_{\hat{u}_h}(\tau^* | |\delta u_1(\tau)| < \beta)$ to analyze trajectory segments associated with large and small velocity increments, respectively, using $\beta = 12 \sigma_{\delta u_1(\tau)}$ for the threshold. Note that τ corresponds to the time variable for the condition on the average (deliberately using the same time-lag variable as is used in the velocity increment $\delta u_i(\tau, t)$), while τ^* is the time variable for the autocorrelation. The purpose of using this DACF is that it can reveal whether a large value of $\delta u_1(\tau, t)$ is connected with an abrupt change in the trajectory direction at a corresponding time $\tau^* = O(\tau)$. Oscillations of $R_{\hat{u}_h}(\tau^*|\cdot)$, and in particular, negative excursions, are a signature of tracer particles moving in horizontal vortices.

The results in Fig. 4(a) show that for segments with small increments, the DACF decays slowly, and the curves for different τ overlap, indicating that the tracers are governed by fluid motions with relatively long timescales. In stark contrast, for segments with large velocity increments, the DACF decays much more rapidly and exhibits pronounced oscillations with negative lobes and a strong dependence of the curves on τ . These findings suggest that large velocity increments

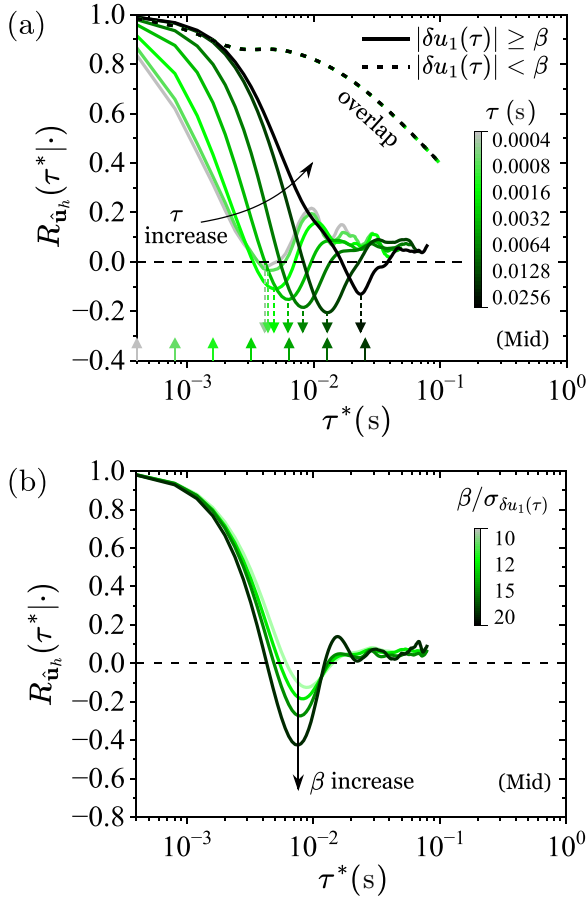


FIG. 4. (a) DACFs of extreme-event and small-event segments in case Mid, with threshold $\beta = 12 \sigma_{\delta u_1(\tau)}$, for different time lags τ . Solid arrows on the abscissa mark the corresponding time lags τ , and dashed arrows mark the corresponding deflection periods T_d (minimum of DACF). (b) DACFs of extreme-event segments in case Mid at $\tau = 0.0064$ s for different thresholds β .

are associated with rapid directional changes of the tracer, consistent with the spiral vortices shown in Figs. 1(b) and 1(d) (see Appendix C for autocorrelations of $\|\mathbf{u}_h\|$).

Turbulent flows are inherently multiscale, and the results in Fig. 4 reveal that a range of temporal flow scales are associated with the large increment events. For the large increment trajectories, the DACF amplitude at a given τ^* reflects the characteristic timescale of the underlying fluid motion carrying the tracer, with a higher DACF value at a given τ^* corresponding to a fluid motion with a longer correlation timescale. Moreover, as shown in Fig. 4(a), the DACF for the large increment trajectories decays more slowly as τ increases, indicating that the associated extreme events are governed by fluid motions with progressively longer timescales.

IV. VORTEX DEFLECTION MECHANISM FOR INTERMITTENCY

The minimum of DACF marks the time (referred to as the deflection period, T_d) at which the tracer has undergone the largest change in its direction, i.e., the maximum “deflection.”

The value of T_d for each curve in Fig. 4(a) is indicated by a downward dashed arrow, while the upward arrows indicate the values of τ . As a simple conceptual model for understanding the results, we consider a flow comprised of horizontal circular vortices, each with their own constant angular velocity Ω . For each vortex, Ω is drawn from a PDF over a finite sample space $[\min(\Omega), \max(\Omega)]$, which also generates a finite range of deflection periods $[\min(T_d), \max(T_d)]$ for tracers in the flow. For a tracer moving around any one of the vortices, the maximum value of $\delta u_1(\tau)$ will occur when $\tau = T_d$. In other words, the largest Lagrangian increment will occur when the tracer experiences the greatest deflection along its trajectory, which we refer to as the vortex deflection mechanism. If we now compute $\delta u_1(\tau)$ for all tracers in this simple flow, then in the regime $\tau \ll \min(T_d)$ the vortex deflection mechanism does not operate and so the values of $\delta u_1(\tau)$ will be relatively small. However, when $\tau \in [\min(T_d), \max(T_d)]$, the vortex deflection mechanism is able to contribute and so can generate large values of $\delta u_1(\tau)$ and hence strong intermittency. The exact time at which the intermittency will be strongest depends on the PDF of T_d , but if it is peaked around a mean value $\langle T_d \rangle$ then the strongest intermittency should occur at $\tau = O(\langle T_d \rangle)$.

The results in Fig. 4(a) show that in the BIT flow the values of τ and T_d start to line up at times close to 10^{-2} s, very close to the time at which the flatness peaks in Fig. 3(b). This shows that the strongest Lagrangian intermittency occurs at times close to T_d , the time at which the tracers experience the maximum deflection in their trajectory. This is in excellent agreement with the simple conceptual model and the idea that the vortex deflection mechanism is responsible for generating the strong Lagrangian intermittency at intermediate timescales in BIT. That this maximum occurs at intermediate times suggests that among the range $[\min(T_d), \max(T_d)]$ in BIT, those of size $\sim 10^{-2}$ s generate the strongest intermittency. From the DACF results across the cases considered, we find approximately $T_d/T_b \in [0.1, 1]$, where $T_b \equiv \sqrt{d_b/g}$ is the bubble timescale. However, the most extreme fluctuations occur at $\tau = 0.0064$ s [corresponding to the time of the peak of the flatness curve in Fig. 3(b)] due to vortices with $T_d \approx 0.0064$ s. This corresponds to $T_d \approx 0.31 T_b$, suggesting that through T_b the bubble diameter plays an important role in determining the timescale at which the Lagrangian intermittency is strongest. Moreover, this timescale also distinguishes the present mechanism from a simple wake-shedding mechanism, since the wake-shedding timescale of millimeter size bubbles is $O(10 T_b)$ [37], much larger than the characteristic timescale identified here.

In Fig. 4(a), it can also be seen that at $\tau \ll 10^{-2}$ s, the results indicate $\tau \ll T_d$ (where T_d is the value for the DACF curve with the corresponding value of τ). According to the conceptual model, the vortex deflection mechanism cannot play a role in this regime. Consistent with this, the values of the flatness in Fig. 3(b) are much smaller at these shorter timescales and the flatness values for BIT approach those for HIT. In this short time regime, the Lagrangian velocity increments are still intermittent, but the vortex deflection mechanism plays a minor role. Instead, it is the nonlinear, internal intermittency mechanism that dominates, which is also the sole intermittency mechanism in the HIT flow.

Figure 4(b) shows the DACF for large increment trajectories for different thresholds β , evaluated for $\tau = 0.0064$ s corresponding to the time of the peak of the flatness curve in Fig. 3(b). As β increases, $R_{\theta_h} \rightarrow -1$, corresponding to a complete reversal of the tracer direction in the horizontal plane. This further demonstrates the strong link between large velocity increments and the deflection of tracer trajectories, with stronger tracer deflections being associated with increasingly extreme Lagrangian velocity increments.

V. SPATIAL RELATION OF EXTREME EVENTS TO BUBBLE WAKES

The preceding analysis demonstrates that Lagrangian intermittency is strongest at intermediate timescales in BIT because of the vortex deflection mechanism. We now want to understand where the associated vortices are located relative to the bubble wakes, and how they are connected to the external intermittency introduced at the small scales by the bubbles. To determine the location of the vortices, we utilize our advanced measurement system that enables simultaneous 3D tracking of both bubbles and tracers (see Appendix A). For each large increment segment at a given τ , we compute the shortest horizontal distance r of the tracer to the nearest bubble trajectory and the vertical distance l to the bubble centroid.

Figures 5(a) and 5(b) show the PDFs of r/d_b and l/d_b for large increment segments ($|\delta u_1(\tau)| \geq 12 \sigma_{\delta u_1(\tau)}$) at several intermediate values of τ . The PDF of r/d_b exhibits a pronounced peak at $r/d_b \simeq 0.5 - 0.6$, identifying the bubble-wake boundary rather than the wake core as the region where most of the large increment segments are located. This boundary contains the strongest shear and velocity gradients owing to the sharp velocity differences between the wake and the surrounding fluid, and these generate the strong vortical motions that are associated with the large velocity increments. The PDFs of l/d_b likewise show that the large increment segments occur close to the bubble, with the PDF decaying for $l/d_b \gtrsim 1$, consistent with the vertical attenuation of the bubble wake at these distances. These results demonstrate that the strong Lagrangian intermittency at intermediate timescales in BIT occurs due to tracer particles becoming trapped in intense vortices that form in the shear-dominated periphery of the bubble wakes. This then is a manifestation of the role of external intermittency at the scale of the bubbles, coupling together with the internal, nonlinear amplification mechanisms to generate strong vortical motion in the flow.

VI. DISCUSSION AND CONCLUSIONS

In classical single-phase turbulence, internal intermittency occurs due to the nonlinear amplification of velocity gradients, while external intermittency occurs due to interfaces or boundaries between turbulent and nonturbulent regions. In contrast, a dispersed multiphase flow such as a bubble-driven flow exhibits a hybrid, forcing-induced form of intermittency: Dispersed bubbles force the local flow as they rise, leading to the formation of wakes and intermittently injecting coherent vortices into the carrier fluid. The formation of these vortices is connected with processes associated with external and in-

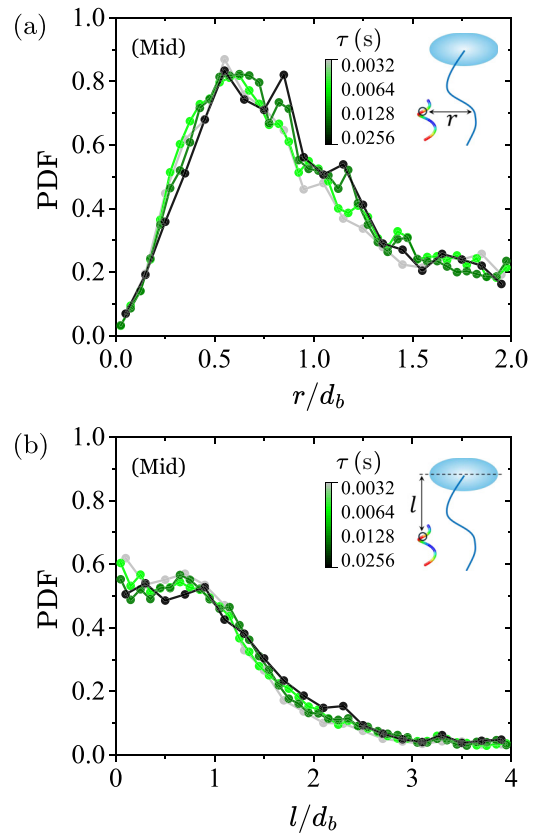


FIG. 5. (a) PDFs of the normalized horizontal distance (depicted in the inset) between tracer positions in large increment segments and the nearest bubble trajectory. (b) PDFs of the normalized vertical distance between large increment segments and the corresponding bubble centroids.

ternal intermittencies—They are generated due to shear at the boundary of the bubble wakes and evolve due to the flows' internal vortex dynamics.

Our experiments reveal how this forcing-induced intermittency manifests itself in the Lagrangian velocity increments of tracer particles in BIT. We find that the strongest Lagrangian intermittency in BIT occurs not at the smallest scales, as in classical single-phase turbulence, but at intermediate scales. By linking velocity increments to the geometry of tracer trajectories using a DACF, we established that the most extreme events arise due to rapid directional changes in the tracer trajectories caused by intense vortices forming at bubble-wake boundaries. This was referred to as the vortex deflection mechanism, and it was shown to lead to the largest velocity increments when the timescale of the increment corresponds to the deflection timescale of the vortex.

These results provide the first experimental evidence for a forcing-induced intermittency in dispersed multiphase turbulence, offering a framework for understanding how a dispersed phase can profoundly alter the behavior of extreme fluctuations in turbulence. Beyond BIT, these findings suggest that any dispersed multiphase or particle-laden flow involving coherent wake interactions at finite scales may similarly alter the intermittency properties, with significant implications for scalar transport, mixing, cavitation inception, and

the prediction of rare events in engineered and environmental multiphase systems [10].

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: EXPERIMENTAL SETUP AND SIMULTANEOUS 3D TRACKING

Experiments were performed in a vertical, octagonal bubble column [Fig. 1(a)] with an inner width of 0.115 m and a water height of 0.9 m. Eight needle gas spargers were mounted uniformly on the bottom plate to generate nearly monodisperse bubble swarms. The bubble diameter was adjusted by the sparger size, yielding the three monodisperse cases listed in Table I (Sm, Mid, La). The resulting bubbles (3–5 mm) rise in the wobbling regime and generate intense turbulence while maintaining a low gas void fraction ($\alpha \approx 0.5\% - 1.2\%$).

The measurement volume ($45 \times 45 \times 90 \text{ mm}^3$) was imaged by four synchronized high-speed cameras at 2500 fps. Polyamide tracer particles with a diameter of $100 \mu\text{m}$ were added to the flow to measure the carrier-phase motion.

Simultaneous three-dimensional tracking of deformable bubbles and tracer particles was achieved using our in-house 3D Lagrangian bubble tracking tool [52] combined with the open-source Shake-The-Box LPT algorithm [53]. Here, bubble tracking is particularly challenging in our flow due to (1) the relatively large bubble sizes and strong deformations, and

TABLE I. Parameters for the three monodispersed cases labeled Sm, Mid, and La, respectively. Here, α is the gas void fraction, d_b the bubble diameter, $\text{Ga} \equiv \sqrt{gd_b^3}/\nu$ the Galileo number, $\text{Eo} \equiv (\rho_l - \rho_g)gd_b^2/\sigma$ the Eötvös number (σ is the surface tension), and $\text{Re}_\lambda \equiv \sqrt{15/(\nu\varepsilon)}u_{\text{rms}}^2$ the Taylor-Reynolds number, with rms of liquid velocity fluctuation u_{rms} . The dissipation rate ε is estimated from compensated second- and third-order structure functions in the K41 scaling range (see Ref. [43] for details). The bubble timescale is defined as $T_b \equiv \sqrt{d_b/g}$. The bubble Reynolds number Re_b is based on d_b and the bubble-to-fluid relative velocity.

Parameter	Sm	Mid	La
α	0.52%	0.78%	1.20%
d_b (mm)	3.5	4.1	4.4
Ga	645	818	910
Eo	1.66	2.28	2.63
Re_b	847	923	925
Re_λ	61	69	98
u_{rms} (m/s)	0.027	0.033	0.045
T_b (ms)	18.9	20.5	21.2

(2) the significant image overlap at the gas void fractions considered. Following the method we recently developed [52], bubbles are first detected via deep-learning segmentation [54]; candidate contours are then matched across four synchronized camera views, enabling robust 3D triangulation even when partial occlusion occurs. A track-to-detect strategy—similar to that commonly used in Shake-The-Box LPT [55]—predicts bubble positions forward in time and eliminates already-assigned detections, allowing nearly all bubbles to be incorporated into continuous trajectories despite dense image conditions. Using this procedure, for example, to compute the PDF at $\tau = 0.0064 \text{ s}$ (Fig. 5) for the Mid case, we simultaneously track 2267 bubbles and 3195 large increment segments over a time period of 19 s. We then use the 3D bubble paths to approximate the temporal evolution of each bubble’s wake core in the dense bubble-swarm cases considered here.

The key experimental parameters for the three cases are summarized in Table I.

APPENDIX B: DIRECTIONAL AUTOCORRELATION FUNCTION

The DACF quantifies the temporal correlation of a tracer’s velocity direction and thereby provides information on the geometry of the tracer trajectory. In Fig. 4, we present results for the conditional DACF, conditional on the magnitude of the Lagrangian increment at some time τ . These results provide information on how extreme Lagrangian increments are connected to the geometry of the tracer trajectory. Here, we provide information on how this quantity is computed from our experimental data.

Step 1: Segment classification. The Lagrangian velocity increment of component u_i at time t is defined as

$$\delta u_i(\tau, t) \equiv u_i(t + \tau) - u_i(t). \quad (\text{B1})$$

For each trajectory, we compute the increment magnitude $|\delta u_i(\tau, t)|$. If $|\delta u_i(\tau, t)| \geq \beta$, the trajectory segment within the interval $[t, t + \tau]$ is marked as being associated with a large velocity increment; otherwise, it is marked as being associated with a small velocity increment.

Step 2: DACF computation. At every instant t , the tracer’s instantaneous direction of motion is given by the unit tracer velocity vector

$$\hat{\mathbf{u}}(t) \equiv \frac{\mathbf{u}(t)}{\|\mathbf{u}(t)\|}. \quad (\text{B2})$$

However, for the purposes of our analysis with the DACF, we use the horizontal velocity vector

$$\hat{\mathbf{u}}_h(t) = \frac{u_1(t)\mathbf{e}_1 + u_2(t)\mathbf{e}_2}{\|u_1(t)\mathbf{e}_1 + u_2(t)\mathbf{e}_2\|}, \quad (\text{B3})$$

where subscripts 1, 2 denote the two horizontal directions (which are statistically equivalent in the flow). The horizontal velocity was used in the analysis because extreme horizontal fluctuations are mainly associated with quasivertical spiral vortices, and the vertical component of the velocity contains a strong contribution associated with the rising motion of the flow that persists in time and therefore masks the vortex-induced deflections of the tracers in the horizontal plane. Projecting onto the horizontal plane thus reveals the tracer

deflections more clearly. For illustration, typical examples of large-event and small-event segments (based on the horizontal component $u_1(t)$) are shown in Figs. 1(c) and 1(d). Qualitatively, tracers in large-event segments exhibit strong directional changes, whereas tracers in small-event segments tend to preserve their direction over relatively long time spans.

Using $\hat{\mathbf{u}}_h(t)$, the conditional DACF is defined as

$$R_{\hat{\mathbf{u}}_h}(\tau^*|\cdot) \equiv \langle \hat{\mathbf{u}}_h(t) \cdot \hat{\mathbf{u}}_h(t + \tau^*) | \cdot \rangle, \quad (\text{B4})$$

where “ $|\cdot$ ” indicates a condition on the average, the angle brackets denote ensemble averaging over all available times t (valid due to statistical stationarity of the flow) and over all tracer trajectories within the selected segment type (valid due to spatial statistical homogeneity of the flow). The condition on the average is set to be $|\delta u_1(\tau)| \geq \beta$ for large increments, while $|\delta u_1(\tau)| < \beta$ is used for small increments as discussed in the main text.

Faster decay of $R_{\hat{\mathbf{u}}_h}(\tau^*|\cdot)$ indicates more rapid directional changes in the tracer. Oscillations of $R_{\hat{\mathbf{u}}_h}(\tau^*|\cdot)$, and in particular, negative excursions, are a signature of tracer particles moving in horizontal vortices.

APPENDIX C: AUTOCORRELATION FUNCTION FOR VELOCITY MAGNITUDE

To more thoroughly test the vortex deflection mechanism, we additionally consider the autocorrelation of the magnitude of the velocity. For the simple vortex discussed earlier to explain the vortex deflection mechanism, this autocorrelation would be constant and equal to one. In a turbulent flow, such a situation will not occur because of the chaotic nature of the flow that naturally leads velocity magnitudes as well as veloc-

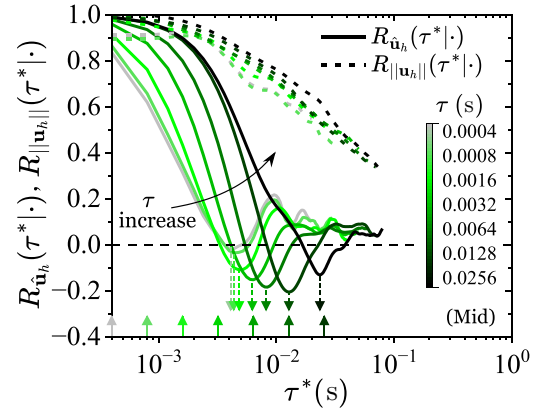


FIG. 6. Comparison between the autocorrelation function of the horizontal velocity direction vector and that of the horizontal velocity magnitude in case Mid. Here, the threshold $\beta = 12 \sigma_{\delta u_1(\tau)}$ is used for the condition on the average.

ity directions to decorrelate along tracer trajectories. However, our deflection mechanism will be approximately confirmed provided that the timescale of the velocity magnitudes is much longer than that associated with the directions (as given by the DACF). In Fig. 6, we show the conditional autocorrelation of the magnitude of the horizontal velocity (dashed lines) together with the DACF results (solid lines). The results clearly show that the velocity directions decorrelate much faster than those of the velocity magnitudes. In particular, the most negative values of the DACF occur at times $O(10^{-2} \text{ s})$, while the autocorrelation of the velocity magnitudes has not reached zero even by times $O(10^{-1} \text{ s})$. These results clearly therefore support our proposed vortex deflection mechanism.

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