

Quantum state exclusion for group-generated ensembles of pure states

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Quantum state exclusion is the task of determining which states from a given set a system was not prepared in. We provide a complete solution to optimal quantum state exclusion for arbitrary sets of pure states generated by finite groups, establishing necessary and sufficient conditions for perfect (zero-error conclusive) exclusion. When perfect exclusion is impossible, we introduce two natural extensions: minimum-error and unambiguous exclusion. For both, we derive the optimal protocols and present analytical expressions for the corresponding failure probabilities and measurements, providing additional insight into how quantum states encode information.

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Introduction. In both scientific and everyday contexts, the ability to confidently exclude what did not occur, based on partial information, is often more efficient than attempting to determine what actually did. This principle is widely applied in classical scenarios, where exclusion serves as a powerful tool for narrowing possibilities and guiding decision-making. For instance, in medical diagnostics, ruling out potential diseases is crucial for refining treatment options, even when the exact disease remains unidentified [1]. Similarly, in engineering system testing, excluding certain failure modes is often more advantageous than trying to pinpoint the exact faulty process [2]. Additionally, in network security, excluding potential vulnerabilities is usually more efficient than attempting to track down the precise nature of a threat [3].

The very same principle extends into the quantum realm through a task named quantum state exclusion (QSE) [4]. Given a quantum system prepared in an unknown state drawn from a predetermined set, the goal is to exclude that the state of the given system is a particular one from that set. This task, also known as quantum antidistinguishability [5], is particularly relevant in the context of quantum state assignments, where the compatibility of different state assignments must be assessed [6,7]. It also plays a crucial role in the debate surrounding the physical reality of quantum systems [8].

In contrast to the traditional approach of quantum state discrimination [9–11], where the goal is to identify the exact state of a system, QSE focuses on ruling out certain states, offering a different perspective on the information that can be extracted from quantum systems. QSE plays a role in quantum resource theories: Any quantum resource can be quantitatively linked to the relative advantage it offers in a

state exclusion task compared to free resources [12,13]. QSE has also been shown to provide an operational interpretation of the Choi rank of a quantum channel [14]. This rank, among other properties, places bounds on the minimum number of Kraus operators required to decompose a channel. The role of contextuality in QSE has been discussed in Ref. [15]. Conversely, noncontextual inequalities have been derived from QSE [16], with implications for communication complexity [17,18]. More recently, Chernoff error exponents for QSE have been calculated in Ref. [19]. Additionally, in quantum state discrimination with multiple copies, it has been shown that the optimal procedure may, in some cases, involve excluding specific states [20]. QSE has also been demonstrated experimentally with single photons in Ref. [21].

Just as in quantum state discrimination, finding optimal protocols for QSE in a fully general setting remains a challenging problem. Beyond the binary case (also known as hypothesis testing), multihypothesis quantum state discrimination has only been solved when a certain degree of symmetry can be invoked [22–31]. This same symmetry principle has been applied in the context of QSE, where it has been used to derive tight bounds for circulant sets of pure quantum states [32].

As far as we know, all existing literature on QSE has focused on perfect exclusion, that is, protocols that enable the error-free conclusive exclusion of one or more states. However, perfect QSE is only possible within a limited region of the parameter space defining the set of states (or hypotheses), with necessary and/or sufficient conditions delineating this region already established. Moreover, many existing results are nonconstructive: While they confirm the existence of an error-free exclusion measurement, they do not provide its explicit form [32].

In this Letter, we present a complete analytical solution to optimal QSE for group-generated sets of pure states, revealing additional facets of the problem. Specifically, we derive the minimum-error probability of exclusion for any finite group and across the entire parameter space, both within and outside the perfect exclusion region, providing a specific

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measurement that achieves this minimum. We also prove that this optimal measurement can always be taken to be sharp. Our approach relies on the Gram matrix [33] formalism, exploits the duality properties of semidefinite programming [34,35], and makes extensive use of group representation theory [36–38].

We also investigate the zero-error (unambiguous) approach [39], which allows for inconclusive answers while forbidding errors, even outside the perfect QSE region. In this context, we provide an analytical expression for the probability of obtaining such inconclusive answers for any group-generated set of pure states.

We show that the minimum probabilities of error and inconclusive answers are both determined by the eigenvalues of the Gram matrix. The same formulas seem to provide a useful upper bound for these probabilities even in general, nonsymmetric cases. This is in agreement with intuition, as symmetry inherently increases indistinguishability and reduces the ability to exclude specific objects, making discrimination and exclusion more challenging.

This Letter is organized as follows. We first present the formulation and structure of the problem, followed by our main results and an outline of the proof of a central lemma, which establishes that the optimal measurements can be constructed from specific rank-one operators. We conclude with a summary of our findings.

Formulation and structure of the problem. This Letter addresses the task of excluding any single preparation from a given ensemble $\mathcal{E} = \{\eta_i, \rho_i\}_i$ of possible states, where $\rho_i = |\psi_i\rangle\langle\psi_i|$ occurs with prior probability η_i . The exclusion is performed through a quantum measurement, represented by a positive operator-valued measure (POVM) $\{\Pi_j\}_j$ on the Hilbert space $\mathcal{H}$ of the states, with each outcome j indicating that the state ρ_j is ruled out. For certain ensembles of states, perfect exclusion is possible; namely, there exists a POVM such that $\text{Tr}(\Pi_i \rho_i) = 0$ for all i . However, as explained in the introduction, we also consider ensembles for which this does not necessarily hold. In analogy with state discrimination, we examine two natural extensions of perfect QSE: minimum-error and unambiguous exclusion.

Minimum-error exclusion aims to minimize the average probability of incorrect exclusion, denoted by P , over the set of all possible POVMs. This optimization can be formulated as a semidefinite program (SDP):

$$P^{\min} = \min_{\{\Pi_i\}} \sum_i \eta_i \text{Tr}(\Pi_i \rho_i),$$

$$\text{subject to } \sum_i \Pi_i = \mathbb{1}, \quad \Pi_i \geq 0 \text{ for all } i. \quad (1)$$

Alternatively, unambiguous exclusion imposes the strict constraint that no errors occur, typically requiring an additional POVM element, Π_Q , associated with an inconclusive outcome. When this outcome is obtained, no definitive decision can be made, so the optimal protocol seeks to minimize its average probability Q . This too can be formulated as an SDP:

$$Q^{\min} = \min_{\{\Pi_i\}} \sum_i \eta_i \text{Tr}(\Pi_Q \rho_i),$$

$$\text{subject to } \Pi_Q + \sum_i \Pi_i = \mathbb{1}, \quad \Pi_Q \geq 0, \quad \Pi_i \geq 0,$$

$$\text{Tr}(\Pi_i \rho_i) = 0 \text{ for all } i. \quad (2)$$

Perfect exclusion corresponds to $P^{\min} = Q^{\min} = 0$. We refer to P and Q as failure probabilities. In this work, we focus on quantum ensembles that are group generated, $\mathcal{E} = \{|\psi_g\rangle = U_g|\psi\rangle \mid g \in \mathcal{G}\}$, where each state in the ensemble is obtained by applying a finite-dimensional unitary (linear or projective) representation, $U: g \mapsto U_g \in \mathcal{U}(\mathcal{H})$, of a finite group $\mathcal{G}$ to a seed state $|\psi\rangle$. Furthermore, we assume that all states are equally probable, i.e., $\eta_g = 1/|\mathcal{G}|$, for all $g \in \mathcal{G}$, where $|\mathcal{G}|$ is the order of $\mathcal{G}$.

Any such representation can be decomposed into a direct sum of irreducible representations (irreps for short), $U_g = \bigoplus_{\mu} U_g^{\mu} \otimes \mathbb{1}_{m_{\mu}}$, where $\mathbb{1}_d$ stands for the d -dimensional identity operator, μ labels the distinct irreps, and m_{μ} denotes their multiplicity. Accordingly, the Hilbert space $\mathcal{H}$ breaks into orthogonal subspaces [36–38] as $\mathcal{H} = \bigoplus_{\mu} \mathcal{H}_{\mu} \otimes \mathbb{C}^{m_{\mu}}$, where $\mathcal{H}_{\mu}$ carries the irrep μ , while $\mathbb{C}^{m_{\mu}}$ denotes the multiplicity space, over which U acts trivially. This decomposition allows us to write the seed state as $|\psi\rangle = 1/\sqrt{|\mathcal{G}|} \sum_{\mu} \sqrt{d_{\mu}} |\psi_{\mu}\rangle$, where d_{μ} is the dimension of the irrep μ and $|\psi_{\mu}\rangle \in \mathcal{H}_{\mu} \otimes \mathbb{C}^{m_{\mu}}$ are non-normalized bipartite states. The factors $1/\sqrt{|\mathcal{G}|}$ and $\sqrt{d_{\mu}}$ have been introduced for later convenience. The states $|\psi_{\mu}\rangle$ can be written in a Schmidt form:

$$|\psi_{\mu}\rangle = \sum_{k=1}^{r_{\mu}} \sqrt{\alpha_k^{\mu}} |v_k^{\mu}\rangle |u_k^{\mu}\rangle, \quad (3)$$

where $r_{\mu} = \min\{d_{\mu}, m_{\mu}\}$, $\alpha_k^{\mu} \geq 0$, for all μ and k , and both $\{|v_k^{\mu}\rangle\}_{k=1}^{r_{\mu}}$ and $\{|u_k^{\mu}\rangle\}_{k=1}^{r_{\mu}}$ are orthonormal bases of $\mathcal{H}_{\mu}$ and $\mathbb{C}^{m_{\mu}}$, respectively. Using the great orthogonality theorem [36–38,40], one can show that the operator $\Omega = \sum_g U_g |\psi\rangle\langle\psi| U_g^{\dagger}$, which represents the (unnormalized) density matrix of the ensemble (ensemble operator for short), is diagonal in this basis:

$$\Omega = \bigoplus_{\mu} \mathbb{1}_{d_{\mu}} \otimes \sum_{k=1}^{r_{\mu}} \alpha_k^{\mu} |u_k^{\mu}\rangle\langle u_k^{\mu}|. \quad (4)$$

As in the case of quantum state discrimination, all the information required for exclusion is encoded in the Gram matrix G of the ensemble [40], whose entries are the overlaps between the states: $G_{g,h} = \langle\psi_g|\psi_h\rangle$, with $g, h \in \mathcal{G}$. The symmetry of the problem—specifically, the representation U that generates the ensemble—determines the structure of the Gram matrix, which belongs to the algebra of the (right-)regular representation of the group [36]. Explicit examples are provided in the Supplemental Material [40].

For any ensemble $\mathcal{E}$, the columns of the square root of the Gram matrix, $S = \sqrt{G}$, define an associated ensemble $\mathcal{E}_S$ from a $|\mathcal{G}|$ -dimensional Hilbert space $\mathcal{H}_S$, which shares the same Gram matrix, G . If $\mathcal{E}$ is generated by a group $\mathcal{G}$, so is $\mathcal{E}_S$, with their states belonging to the (left-)regular representation of $\mathcal{G}$ [40].

The Gram matrix G and the ensemble operator Ω are closely related. Specifically, among the eigenvalues of G —denoted as λ_a , with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{|\mathcal{G}|}$ —those that are strictly positive coincide with the nonzero eigenvalues of Ω [40], which can be read off from Eq. (4).

Results. For ensembles of pure states generated by a finite group $\mathcal{G}$, the minimum-error probability for excluding a state, as defined in Eq. (1), is determined by the eigenvalues $\{\lambda_a\}_{a=1}^{|\mathcal{G}|}$

of the Gram matrix:

$$P^{\min} = \left[\frac{1}{|\mathcal{G}|} \max \left(0, \sqrt{\lambda_1} - \sum_{a>1} \sqrt{\lambda_a} \right) \right]^2. \quad (5)$$

Similarly, for unambiguous QSE, the probability of the inconclusive outcome is given by

$$Q^{\min} = \frac{\text{Tr}(\sqrt{G})}{|\mathcal{G}|} \max \left(0, \sqrt{\lambda_1} - \sum_{a>1} \sqrt{\lambda_a} \right). \quad (6)$$

These results establish that the condition

$$\sqrt{\lambda_1} \leq \sum_{a>1} \sqrt{\lambda_a} \quad (7)$$

is both necessary and sufficient for perfect QSE to be possible in group-generated ensembles. As a special case, Theorem 5.1 in Ref. [32]—which applies to circulant matrices—follows directly from this result, now extended to arbitrary finite groups.

Equations (5) and (6) follow from the lemma below. This lemma also provides the optimal measurements for the two approaches considered in this Letter. Both of them are represented by rank-1 (group-)covariant POVMs.

Note that if the largest eigenvalue λ_1 has multiplicity greater than 1, condition (7) is automatically satisfied, ensuring perfect QSE. This occurs whenever the largest Schmidt coefficient (or eigenvalue of Ω), α_k^μ , corresponds to an irrep μ of dimension greater than 1.

The results above extend to unitary projective representations, where $U_g U_h = e^{i\theta(g,h)} U_{gh}$ for all $g, h \in \mathcal{G}$. Such representations are not merely of academic interest: spin- $\frac{1}{2}$ particles transform under a projective representation of the 3-dimensional rotation group. For instance, the ensemble $\{|\mathbf{n}_\alpha\rangle\}_\alpha$, where α indexes the vertices of a regular tetrahedron in the Bloch sphere, can be viewed as generated by the Pauli matrices $\{\mathbb{1}_2, \sigma_x, \sigma_y, \sigma_z\}$, which form a nontrivial projective representation of $\mathbb{Z}_2 \times \mathbb{Z}_2$ [40]. These states also define a symmetric informationally complete POVM [41] for qubits.

When the phases $\theta(g, h)$ are nontrivial, i.e., they cannot be absorbed into the matrices $\{U_g\}_{g \in \mathcal{G}}$ themselves, the corresponding irreps necessarily have dimension 2 or higher [36–38, 42]. Thus, by the previous argument, perfect exclusion is always possible for sets generated by such nontrivial projective representations [40].

Lemma 1. The optimal POVM for QSE can always be chosen to be covariant with a rank-1 seed. Specifically, it takes the form $\{\Pi_g = U_g |\omega\rangle\langle\omega| U_g^\dagger \mid g \in \mathcal{G}\}$ for minimum-error QSE, with the additional POVM element $\Pi_Q = \mathbb{1} - \sum_{g \in \mathcal{G}} \Pi_g$ for unambiguous QSE. Moreover, if $\sqrt{\lambda_1} > \sum_{a>1} \sqrt{\lambda_a}$, the optimal seed state is

$$|\omega\rangle = \frac{1}{\sqrt{|\mathcal{G}|}} \left(\gamma |v_1^1\rangle |u_1^1\rangle - \sum_{\mu \neq 1} \sqrt{d_\mu} \sum_{k=1}^{r_\mu} |v_k^\mu\rangle |u_k^\mu\rangle \right), \quad (8)$$

where the eigenstate $|v_1^1\rangle |u_1^1\rangle$ corresponds to the largest eigenvalue of Ω in Eq. (4), with $\gamma = 1$ for minimum-error QSE, and

$$\gamma = \sum_{\mu \neq 1} \sum_{k=1}^{r_\mu} d_\mu \frac{\sqrt{\alpha_k^\mu}}{\sqrt{\alpha_1^1}} \quad (9)$$

for unambiguous QSE.

Proof. Consider a POVM of the specified form for some $|\omega\rangle \in \mathcal{H}$. It must hold that $\Phi := \sum_{g \in \mathcal{G}} U_g |\omega\rangle\langle\omega| U_g^\dagger = \mathbb{1}_\psi$, where $\mathbb{1}_\psi$ is the projector onto the span of the orbit of the seed state $|\psi\rangle$. Therefore, applying (the analog of) Eqs. (3) and (4) to Φ , we see that the state $|\omega\rangle$ must be

$$|\omega\rangle = \frac{1}{\sqrt{|\mathcal{G}|}} \sum_{\mu} \sqrt{d_\mu} \sum_{j=1}^{r_\mu} |e_j^\mu\rangle |f_j^\mu\rangle, \quad (10)$$

where $\{|e_j^\mu\rangle\}_{j=1}^{d_\mu}$ and $\{|f_j^\mu\rangle\}_{j=1}^{m_\mu}$ are orthonormal bases of $\mathcal{H}_\mu$ and $\mathbb{C}^{m_\mu}$, respectively, with the extra condition that $\text{span}\{|f_j^\mu\rangle\}_j = \text{span}\{|u_k^\mu\rangle\}_k$ for all μ . The average error probability becomes

$$P = |\langle\omega|\psi\rangle|^2 = \frac{1}{|\mathcal{G}|^2} \left| \sum_{\mu} d_\mu \sum_{j,k=1}^{r_\mu} \sqrt{\alpha_j^\mu} \langle v_j^\mu | e_k^\mu \rangle \langle u_j^\mu | f_k^\mu \rangle e^{i\theta_{j,k}^\mu} \right|^2, \quad (11)$$

where we have written the overlaps in polar form.

To minimize the error probability, we can replace the original ensemble $\mathcal{E}$ by its associated ensemble $\mathcal{E}_S$ and, hence, without loss of generality, take U to be the regular representation. In the decomposition of this representation, all the irreps of $\mathcal{G}$ appear, each with multiplicity equal to its dimension [36–38], i.e., $d_\mu = m_\mu = r_\mu$. Thus, we can always choose mutually unbiased bases $\{|e_j^\mu\rangle\}_j$ and $\{|v_j^\mu\rangle\}_j$, as well as $\{|f_j^\mu\rangle\}_j$ and $\{|u_j^\mu\rangle\}_j$ [43], for the spaces $\mathcal{H}_\mu$ and $\mathbb{C}^{m_\mu}$, respectively, such that

$$|\langle e_j^\mu | v_k^\mu \rangle| = |\langle f_j^\mu | u_k^\mu \rangle| = \frac{1}{\sqrt{d_\mu}}, \quad 1 \leq j, k \leq d_\mu. \quad (12)$$

With this choice, the average error probability can be expressed as

$$P = \frac{1}{|\mathcal{G}|^2} \left| \sum_{\mu} \sum_{j,k=1}^{d_\mu} \sqrt{\alpha_j^\mu} e^{i\theta_{j,k}^\mu} \right|^2. \quad (13)$$

Since each eigenvalue α_k^μ appears in Ω with multiplicity d_μ [see Eq. (4)], we can rewrite the error probability as

$$P = \frac{1}{|\mathcal{G}|^2} \left| \sum_{a=1}^{|\mathcal{G}|} \sqrt{\lambda_a} e^{i\theta_a} \right|^2. \quad (14)$$

If $\sqrt{\lambda_1} \leq \sum_{a>1} \sqrt{\lambda_a}$, the phases in Eq. (14) can be chosen such that the error probability vanishes. This implies that $\text{Tr}(\Pi_g \rho_g) = 0$ for all $g \in \mathcal{G}$, and therefore, the probability of obtaining an inconclusive outcome also vanishes. This completes the proof of Lemma 1 when the above condition holds. Note that the optimal seed is infinitely degenerate, as there are infinitely many phase choices that yield $P^{\min} = Q^{\min} = 0$, corresponding to perfect exclusion.

In contrast, when $\sqrt{\lambda_1} > \sum_{a>1} \sqrt{\lambda_a}$, the error probability P in Eq. (14) does not vanish. Its minimum occurs when $\theta_a = \theta_1 + \pi$ for all $a > 1$. In this case, we adopt the ansatz from Eq. (10), with $|e_j^\mu\rangle \propto |u_j^\mu\rangle$ and $|f_j^\mu\rangle \propto |v_j^\mu\rangle$, replacing the unbiased bases used previously. Under this choice of bases and phases, Eq. (10) gives Eq. (8) with $\gamma = 1$, and Eq. (11) simplifies to the right-hand side of Eq. (5), denoted as P^{primal} .

The optimality of this ansatz follows from the dual formulation of Eq. (1):

$$P^{\text{dual}} = \max_Y \text{Tr}(Y),$$

$$\text{subject to } \frac{\rho_g}{|\mathcal{G}|} - Y \geq 0, \quad \text{for all } g \in \mathcal{G}. \quad (15)$$

To maximize $\text{Tr}(Y)$, we propose a natural choice for the dual operator: $Y = (1/|\mathcal{G}|) \sum_{g \in \mathcal{G}} U_g |\omega\rangle \langle \omega| \psi\rangle \langle \psi| U_g^\dagger$, ensuring $\text{Tr}(Y) = P^{\text{primal}}$. With this choice, the (Holevo-like) constraint in Eq. (15) simplifies to $|\psi\rangle \langle \psi| - |\mathcal{G}|Y \geq 0$, invoking group covariance. This inequality holds since the left-hand side can be written as a convex combination of positive semidefinite operators [40]. This confirms that our choice provides a feasible solution to the dual problem, proving $P^{\text{dual}} = P^{\text{primal}}$ and, in turn, validating Eq. (5).

Next, we address unambiguous QSE, also when the condition $\sqrt{\lambda_1} > \sum_{a>1} \sqrt{\lambda_a}$ holds. With the ansatz (8) and choice (9), the inconclusive POVM element reads $\Pi_Q = \mathbb{1} - \sum_{g \in \mathcal{G}} U_g |\omega\rangle \langle \omega| U_g^\dagger$. This specific choice of the seed state $|\omega\rangle$ ensures that $\text{Tr}(\Pi_Q \rho_g) = |\langle \omega | \psi \rangle|^2 = 0$, and $\Pi_Q = (1 - \gamma^2) |v_1^1\rangle \langle v_1^1| \otimes |u_1^1\rangle \langle u_1^1| \geq 0$, since we can alternatively write $\gamma = (1/\sqrt{\lambda_1}) \sum_{a \neq 1} \sqrt{\lambda_a} < 1$. Thus, our ansatz is a feasible solution of the (primal) SDP in Eq. (2), with the probability of an inconclusive outcome given by $Q^{\text{primal}} = (\alpha_1^1/|\mathcal{G}|)(1 - \gamma^2)$, which simplifies to our main result in Eq. (6).

To show the optimality of this ansatz, we consider the dual formulation of Eq. (2):

$$Q^{\text{dual}} := \max_X 1 - \text{Tr}(X),$$

$$\text{subject to } \sum_{g \in \mathcal{G}} U_g X U_g^\dagger + \nu |\psi\rangle \langle \psi| - \Omega \geq 0, \quad \nu \in \mathbb{R}, \quad (16)$$

which has been particularized for the problem at hand and simplified by exploiting its symmetries. This maximum is not strictly achievable, as the optimal solution lies on the boundary of the feasibility set, which is not contained within the feasibility set itself [34,35]. As a result, unlike in the minimum-error case, there is no natural choice for the dual operator X .

Nevertheless, strong duality holds, and it is possible to construct X such that the dual problem yields an inconclusive probability satisfying $Q^{\text{dual}} = Q^{\text{primal}} - \varepsilon$, where ε is an arbitrary positive real number [40]. This construction ensures that for any such ε , there exists a corresponding ν in Eq. (16) that

makes the solution feasible. Hence, taking the limit $\varepsilon \rightarrow 0^+$, we obtain $Q^{\text{dual}} = Q^{\text{primal}} = Q$, proving the optimality of the ansatz (8) with the specified γ . ■

Conclusions. We have obtained the optimal minimum-error and unambiguous quantum state exclusion protocols for any (finite-)group-generated set of states, offering a constructive proof that provides the explicit form of the optimal POVM in both cases. Several previously known results emerge as particular instances of our findings. A key aspect of our approach is the systematic use of the Gram matrix, which encodes all the information required for discrimination and exclusion, regardless of whether the set is symmetric or not. Our expressions for the failure probabilities, Eqs. (5) and (6), depend solely on the Gram matrix eigenvalues, making them applicable even to nonsymmetric sets. Numerical analysis further suggests that these expressions provide upper bounds on the corresponding probabilities for the optimal QSE protocols, aligning with the heuristic expectation that discrimination and exclusion become more challenging as the states share more symmetries. Our theoretical treatment can also be extended to group-generated ensembles of mixed states, where perfect exclusion is generally not achievable. In such cases, our approach offers a path to estimate the minimum-error probability instead. We are currently exploring these ideas further.

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