

Surface code stabilizer measurements for Rydberg atoms

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We consider stabilizer measurements for surface codes with neutral atoms and identify gate protocols that minimize logical error rates in the presence of a fundamental error source—spontaneous emission from Rydberg states. We demonstrate that logical error rates are minimized by protocols that prevent the propagation of Rydberg leakage errors and not by protocols that minimize the physical two-qubit error rate. We provide laser-pulse-level gate protocols to counter these errors. These protocols significantly reduce the logical error rate for implementations of surface codes involving one or two species of atoms. Our work demonstrates the importance of optimizing quantum gates for logical errors in addition to gate fidelities and opens the way to the efficient realization of surface codes with neutral atoms.

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Quantum error correction (QEC) exploits redundancy to encode a logical qubit with an arbitrarily low error rate into several noisy physical qubits, which is possible as long as the physical error rate is below a given threshold [1–5]. In surface codes [6–8], near-faultless logical qubits are prepared and maintained by repeated measurement of so-called stabilizer operators on the physical qubits. In this way, up to $\lfloor d/2 \rfloor$ physical single-qubit errors in d rounds of stabilizer measurements can always be corrected, where d is the so-called *distance* of the code. Recently, several quantum computing platforms have surpassed the threshold for QEC by minimizing noise at the level of single- and two-qubit gates [9–16]. The errors on any quantum computing platform can be categorized as either errors in the computational subspace or *leakage* errors, where the qubit leaves the computational subspace. While errors in the computational subspace can be corrected with hardware-agnostic error correction codes, mitigating the effect of leakage errors requires hardware specific protocols [17–19]. In neutral atom quantum computers [13–15, 20–28], two-qubit operations implemented by laser-exciting electrons to strongly interacting Rydberg states using time-optimal (TO) control methods [29, 30] have recently surpassed gate fidelities $\mathcal{F} \simeq 99.5\%$ [13, 15, 16]. These fidelities are above the threshold against errors in the computational subspace for QEC with surface or color codes, and logical qubits and small logical circuits have been experimentally realized [14]. However, a particularly detrimental error in this platform is Rydberg leakage, where an atom is erroneously left in a Rydberg state at the end of a gate. A Rydberg leakage error on a single qubit can prevent the execution of two-qubit gates

on all of its neighbors, via the Rydberg blockade mechanism [18, 19]. It is thus a key open question how to perform stabilizer measurements that minimize logical error rates under realistic noise models including leakage errors.

In this work, we study how Rydberg decay, the dominant error sources in many experimental implementations [13, 27, 31] and a large source of Rydberg leakage errors, impacts the error rate of a logical qubit encoded in a surface code. We discuss two different architectures of neutral atom quantum computers, involving one or two species of stationary atoms as data and ancilla qubits, respectively. For both architectures, we find that for small enough Rydberg decay rates γ the logical error rate is dominated by Rydberg leakage errors. The latter can cause errors in neighboring qubits during a stabilizer measurement, so that already $\lceil d/4 \rceil$ decay events are sufficient to cause a logical error. We propose and characterize pulse-level gate protocols that negate these errors by inhibiting the propagation of Rydberg excitations, so that again $\lfloor d/2 \rfloor$ errors can be corrected and the logical error rate is significantly reduced. These protocols differ significantly from those that minimize the average time that the atoms spend in the Rydberg state and thus the infidelity $1 - \mathcal{F}$ of individual two-qubit gates and stabilizer measurement. Our work demonstrates the need to optimize fidelities at the logical—not only the physical—level, and paves the way for efficient implementation of surface-code quantum error correction on neutral-atom platforms.

We consider a rotated surface code [32] of distance d made of $2d^2 - 1$ stationary atoms arranged in a square lattice [Fig. 1(a)]. Stabilizers are given by products of Pauli X or Z operators on four data atoms on the corner of a plaquette and read out using one ancilla qubit in the center of the plaquette. We assume that all stabilizers are measured independently, and stabilizer measurements are only performed simultaneously if they involve disjoint sets of atoms. Let us consider a single Z -stabilizer measurement (X -stabilizer measurements are performed in the same way, with Hadamard gates inserted on all data atoms before/after the measurement): Each of the five atoms on the plaquette is modeled as a three-level system,

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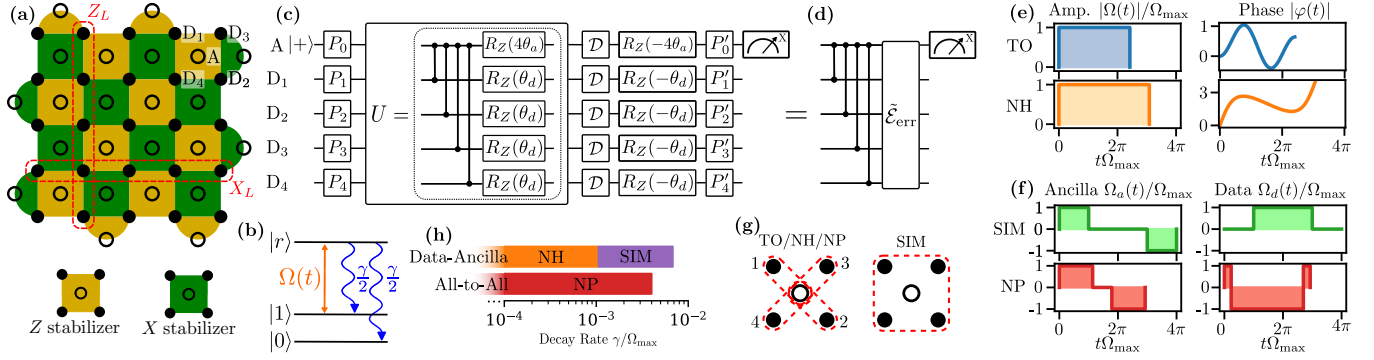


FIG. 1. (a) A distance $d = 5$ surface code with d^2 data qubit (solid circles) and $d^2 - 1$ ancilla qubits (open circles). Z stabilizers are marked by yellow and X stabilizers by green plaquettes. The logical operators X_L (Z_L) consist of X (Z) operators along a row (column) of data qubits. (b) Each atom is modeled as a three-level system with states $|0\rangle$, $|1\rangle$, $|r\rangle$, a time-dependent drive $\Omega(t)$ with phase $\varphi(t)$ and amplitude $|\Omega(t)| \leq \Omega_{\max}$ coupling $|1\rangle$ and $|r\rangle$, and a decay from $|r\rangle$ to $|0\rangle$ and $|1\rangle$ with decay rate $\gamma/2$ each. (c) Circuit to measure a Z stabilizer (see main text). (d) The realistic stabilizer measurement is equivalent to four CZ gates followed by an error channel $\tilde{\mathcal{E}}_{\text{err}}$. (e) Amplitude and phase of Ω for the symmetric time-optimal, no-hopping (NH) protocols. (f) Laser pulse $\Omega_a(t)$ and $\Omega_d(t)$ applied on the ancilla and data atoms, respectively, for the simultaneous (SIM) and no-phase (NP) protocols. (g) For the TO/NH/NP protocols, four CZ gates are applied subsequently in the indicated order. For the SIM protocol, only one global gate is applied on all five atoms simultaneously. (h) Summary of which protocol has the lowest logical error rate as a function of γ in the $d \rightarrow \infty$ limit.

with computational basis states $|0\rangle$, $|1\rangle$ and Rydberg state $|r\rangle$ [Fig. 1(b)]. The dynamics is governed by the Hamiltonian ($\hbar = 1$)

$$H = \sum_{ij=0}^4 B_{ij} |r_i r_j\rangle \langle r_i r_j| + \sum_{i=0}^4 \frac{\Omega_i(t)}{2} |r_i\rangle \langle 1_i| + \text{H.c.}, \quad (1)$$

where B_{ij} is the interaction strength between atoms i and j prepared in the Rydberg state $|r\rangle$ and $\Omega_i(t)$ is the (complex) time-dependent Rabi frequency of a laser incident on atom i coupling $|1\rangle$ and $|r\rangle$. We consider either $B_{ij} = 0$ or $B_{ij} = \infty$, corresponding, respectively, to the absence of interactions or a perfect Rydberg blockade between atoms i and j is impossible. Nevertheless, we show in Supplemental Material [33] that our results remain valid for gates implemented in the experimentally relevant strong but finite blockade regime.

In this work, we adopt the simplest possible model of Rydberg decay and assume that $|r\rangle$ decays only to the states $|0\rangle$ and $|1\rangle$ with a branching ratio of 1:1, described by the Lindblad operators $L_i^{(q)} = \sqrt{\gamma/2} |q_i\rangle \langle r_i|$ for $i \in \{0, \dots, 4\}$ and $q \in \{0, 1\}$, where γ is the decay rate of the Rydberg state. We show in Supplemental Material [33] that results do not change qualitatively for other branching ratios. In most experiments, there are additional decay channels to long-lived states outside of the computational subspace, which require repumping to the qubit states after the stabilizer measurement [18]. Since for the purpose of a Rydberg blockade gate any low-lying noncomputational state inhibits subsequent controlled-Z (CZ) operations and thus acts operationally like $|0\rangle$, these decay channels can be incorporated in our model by increasing the decay probability to $|0\rangle$. Moreover, through stimulated absorption or emission of blackbody radiation (BBR), it is also possible to cause the excitation of a different Rydberg state. While not directly captured in our model, we show in Supplemental Material [33] that in the presence of BBR decay

the protocols developed in this work perform similarly to, and sometimes better than, standard protocols.

To measure a stabilizer, we follow the circuit shown in Fig. 1(c): First, the ancilla atom is prepared in $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Then, laser pulses Ω_0 and $\Omega_1(t), \dots, \Omega_4(t)$ are applied on the ancilla and the four data atoms, respectively, where we assume that for all pulses $|\Omega_j(t)| \leq \Omega_{\max}$, with $\Omega_{\max}$ the maximal achievable Rabi frequency. The pulses are chosen such that in the decay-free case ($\gamma = 0$) a unitary U is implemented, which, up to single-qubit gates $R_Z(\theta_a) = \exp(i\theta_a |1\rangle \langle 1|)$ and $R_Z(\theta_d)$ on ancilla and data atoms, respectively, corresponds to four CZ gates, one between each data atom and the ancilla atom (however, see discussion for the SIM protocol below). For simplicity, we assume that all population remaining in the Rydberg state at the end of the pulses is then removed by applying the channel $\mathcal{D}(\rho) = \Pi \rho \Pi + \langle r | \rho | r \rangle \Pi/2$ on each atom, with $\Pi = |0\rangle \langle 0| + |1\rangle \langle 1|$. This could be realized either by simply waiting long enough for the Rydberg state to decay with high probability or by coupling the Rydberg state to a short-lived intermediate state that decays to $|0\rangle$ and $|1\rangle$ [18]. Thus, our model captures the effect of Rydberg leakage errors during one stabilizer measurement, but not between different stabilizer measurements. The effect of leakage errors propagating across multiple stabilizer rounds can be readily included in our discussion and corrected using, e.g., leakage reduction units [34–36] as we discuss in Supplemental Material [33]. Finally, single-qubit gates are applied on each atom to compensate for the single-qubit rotation induced by U , and the ancilla qubit is measured in the X basis. The noisy stabilizer measurement can now be seen as four ideal CZ gates between the ancilla and the data qubits, followed by a five-qubit error channel $\tilde{\mathcal{E}}_{\text{err}}$ and the measurement of the ancilla qubit [Fig. 1(d)]. We note that this equivalence is exact for the twirled noise channel. We indeed simulate $\tilde{\mathcal{E}}_{\text{err}}$ using a Clifford simulator by applying randomized compiling [37] and assuming that random Pauli gates P and the Clifford conjugate gates $P' = U P U^\dagger$ are inserted before and after the

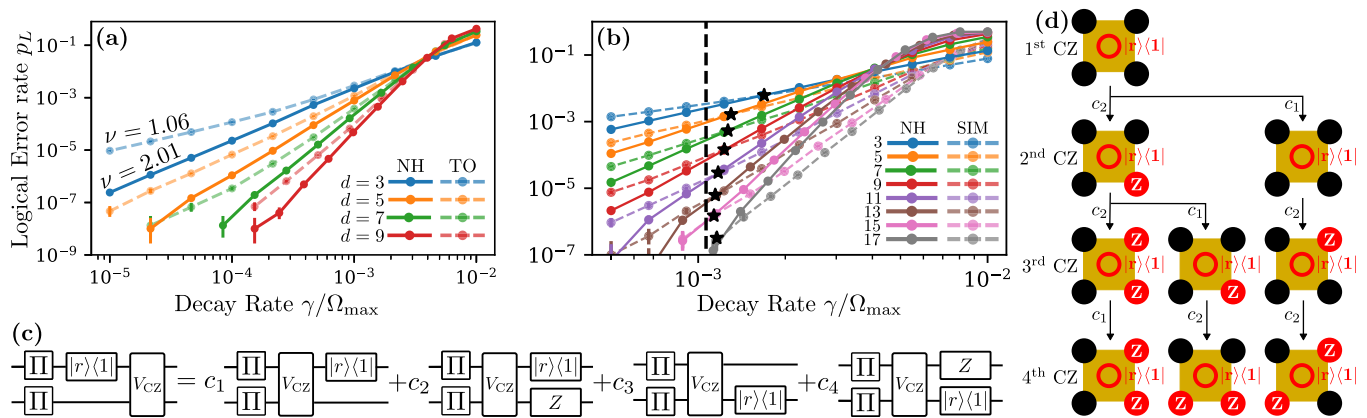


FIG. 2. Data-ancilla blockade. (a) Logical error rate p_L as a function of the decay rate γ (in units of $\Omega_{\max}$) for the NH (solid lines) and TO (dashed lines) protocols. (b) Logical error rate for the NH (solid lines) and SIM (dashed lines) protocol. Black stars show the crossover points between these two protocols. The black dashed line shows the asymptotic crossing point $\gamma_{\times} = 1.1 \times 10^{-3} \Omega_{\max}$ in the limit $d \rightarrow \infty$ (see Supplemental Material [33]). (c) Four different ways how a $|r\rangle\langle 1|$ error can propagate through a V_{CZ} gate. Coefficients $c_1, \dots, c_4$ are calculated in the main text. The projectors Π restrict the state before the $|r\rangle\langle 1|$ error to the computational subspace. (d) Three possible ways how a $|r\rangle\langle 1|$ error on the ancilla qubit after the first gate (first row) can spread. The n th row shows error after n V_{CZ} gates.

unitary U , respectively. This ensures that $\tilde{\mathcal{E}}_{\text{err}}$ is a Pauli channel, i.e., of the form

$$\tilde{\mathcal{E}}_{\text{err}}(\rho) = \sum_Q \lambda_Q Q \rho Q, \quad (2)$$

where Q is summed over all five-qubit Pauli strings and λ_Q is the probability of Pauli error Q . We note that omitting P and P' from the measurement circuit while still assuming that $\tilde{\mathcal{E}}_{\text{err}}$ is of the form of Eq. (2) corresponds to the Pauli twirling approximation [38]. In the following, we compute error probabilities λ_Q by solving the Lindblad master equation given by H and $L_i^{(q)}$. We then use the stabilizer circuit simulator STIM [39] together with a minimum weight perfect matching decoder [40] to calculate the logical error rate p_L of d rounds of stabilizer measurements using the computed λ_Q .

In order to exemplify the role of Rydberg leakage errors in the stabilizer measurements, we first discuss a so-called *data-ancilla* blockade model, where we assume that for atoms on the same plaquette there is only a Rydberg blockade between the ancilla and all data atoms, but not between two data atoms. This model can be approximately realized in dual species arrays [25,41,42]. We then discuss the *all-to-all* blockade model, where we assume a Rydberg blockade between all atoms on a plaquette. This model can be approximately realized in single-species arrays [24,31].

In both blockade models, the stabilizer measurement can be realized by applying four subsequent CZ gates, each with the same pulse $\Omega_a(t)$ on the ancilla atom and $\Omega_d(t)$ on the data atom. The gate infidelity under Rydberg decay is minimized if each CZ gate is implemented using a TO gate [29,30], in which the same laser pulse $\Omega(t) = \Omega_a(t) = \Omega_d(t)$ with constant amplitude $\Omega_{\max}$ and time-dependent phase $\varphi(t)$ is applied *symmetrically* on both atoms [Fig. 1(e)]. This implements a $V_{CZ} := \text{CZ} \cdot [R_Z(\theta_a) \otimes R_Z(\theta_d)]$ gate corresponding to an exact CZ gate followed by single-qubit rotations [where $R_Z(\theta_a)$ acts on the ancilla and $R_Z(\theta_d)$ acts on the data atom, respectively], with $\theta_a = \theta_d = 2.17$ [29].

Figure 2(a) shows the logical error rate p_L for the TO protocol (dashed lines) for distances between $d = 3$ and $d = 9$ in the data-ancilla blockade model. In particular, for $d = 3$ we find that for small enough decay rates γ the logical error rate scales as $p_L \sim \gamma^\nu$ with $\nu \approx 1$, meaning that already a single decay event can cause a logical error. This contradicts the expectation that a $d = 3$ surface code should be able to correct a single decay event. The same trend persists at larger distances. The ν values extracted from the fits of displayed higher-distance decoding curves are $\nu_{\text{TO}}^{d=5} = 2.12$ and $\nu_{\text{NH}}^{d=5} = 2.87$, $\nu_{\text{TO}}^{d=7} = 2.90$ and $\nu_{\text{NH}}^{d=7} = 3.83$, and $\nu_{\text{TO}}^{d=9} = 4.10$ and $\nu_{\text{NH}}^{d=9} = 4.96$.

To understand why the TO protocol performs worse than expected, it is sufficient to consider a single decay event during the first of the four V_{CZ} gates: Due to the fraction of the laser pulse that is applied after the decay event, it is possible that the ancilla qubit is found in $|r\rangle$ at the end of the gate. Instead of a Pauli X , Y , or Z error, this corresponds to an $|r\rangle\langle 1|$ error that occurs right after the first V_{CZ} gate on the ancilla qubit. This latter type of Rydberg error can be detrimental, as V_{CZ} only acts as a CZ gate on the computational subspace, but its action on states outside of the computational subspace depends on the gate protocol. In particular, we find that an $|r\rangle\langle 1| \otimes I$ error followed by a V_{CZ} gate is equivalent to a V_{CZ} gate followed by a linear combination of $|r\rangle\langle 1| \otimes I$, $|r\rangle\langle 1| \otimes Z$, $I \otimes |r\rangle\langle 1|$, and $Z \otimes |r\rangle\langle 1|$ errors, with coefficients c_1, c_2, c_3 , and c_4 , respectively [Fig. 2(c)]. In Supplemental Material [33], the coefficients c_1 to c_4 for a symmetric protocol with $\theta_a = \theta_d =: \theta$ in which the same pulse $\Omega(t)$ is applied to both atoms are calculated as

$$c_{1/2} = (e^{-2i\theta} \mp ie^{-3i\theta} \sin \theta)/2, \quad (3)$$

$$c_{3/4} = \pm (e^{-3i\theta} \cos \theta)/2, \quad (4)$$

where the $-$ sign is used for c_1 and c_4 and the $+$ sign is used for c_2 and c_3 . In particular, for the TO protocol we find that all

c_i are nonzero and thus all four propagation channels occur with a nonzero amplitude. Hence, a single $|r\rangle\langle r|$ error after the first V_{CZ} gate can propagate in 3^4 different ways. Three of those error propagation paths are shown in Fig. 2(d), where each of the four rows shows possible errors after each of the four V_{CZ} gates. We see that a single $|1\rangle\langle r|$ error on the ancilla qubit can lead to pairs of Z errors on the data qubits that are aligned horizontally, vertically, or diagonally. The pair of vertically aligned Z errors is particularly detrimental, since the logical Z operator is oriented vertically [Fig. 1(a)] and thus only $\lceil d/4 \rceil$ of those errors suffice to make a logical Z error. This is in contrast to independent single-qubit errors, where $\lceil d/2 \rceil$ errors are necessary for a logical error. Note that, since also horizontally or diagonally aligned errors can occur, this behavior cannot be fixed by changing the order in which the V_{CZ} gates are applied to the data qubits.

Equations (3) and (4) show, however, a way in which the spreading of the $|1\rangle\langle r|$ error can be prevented: For $\theta = \pi/2$, we obtain $c_1 = -1$, $c_2 = c_3 = c_4 = 0$, so that the $|1\rangle\langle r|$ error commutes with the V_{CZ} gate, up to a global phase. Using quantum optimal control methods [43] and the methodology developed in Ref. [29], we determine the shortest possible pulse $\Omega_1(t) = \Omega_2(t)$ that implements a V_{CZ} gate with $\theta = \pi/2$ [see Fig. 1(e)]. Similar to the TO pulse [29], this pulse requires a continuous variation of the laser phase with time only, while the amplitude is kept fixed at $\Omega_{\max}$, which is expected to be experimentally advantageous [13]. When applied to the whole plaquette, this continuous pulse forms the basis of the *no-hopping* protocol for stabilizer measurements. Note that in the NH protocol, the time spent in the Rydberg state is increased by less than 10% compared to the TO protocol. The solid lines in Fig. 2(a) show the logical error rate for the NH protocol: For large decay rates $\gamma \gtrsim 10^{-3}\Omega_{\max}$, the NH protocol yields essentially the same logical error rates as the TO protocol since both protocols spend a similar time in the Rydberg state. However, for small decay rates $\gamma \lesssim 10^{-3}\Omega_{\max}$, the NH protocol significantly reduces the logical error rate for all distances d , demonstrating that in this regime the propagation of Rydberg leakage errors becomes the dominant error source. For $d = 3$, ν takes the expected value $\nu = 2.01 \approx 2$, showing that two decay events are necessary to produce one logical error in the NH protocol. We expect that in the currently experimentally accessible region $10^{-4}\Omega_{\max} \lesssim \gamma \lesssim 10^{-3}\Omega_{\max}$ [13,25–28] the logical error rate can then be improved by more than one order of magnitude. Since the exponent ν scales linearly with the distance d (see Supplemental Material [33]) and the logical error rate scales as $p_L \sim \gamma^\nu$, at fixed $0 < \gamma < 1$ even larger improvements are expected for larger distances. Finally, we note that, although the NH derivation assumes an ideal infinite blockade (so that $\theta = \pi/2$ ensures $c_1 = -1$, $c_2 = c_3 = c_4 = 0$), we show in Supplemental Material [33] that the protocol's favorable logical-error scaling persists for experimentally relevant, strong but finite blockade strengths such as $B = 20\Omega_{\max}$.

The data-ancilla blockade also allows for a stabilizer measurement protocol (called the SIM protocol from now on) in which the four CZ gates are not applied subsequently, but *simultaneously*, by applying a π pulse on the ancilla qubit, followed by a 2π pulse applied on all data qubits, and a final π pulse on the ancilla qubit [Fig. 1(f)]. This is similar to the

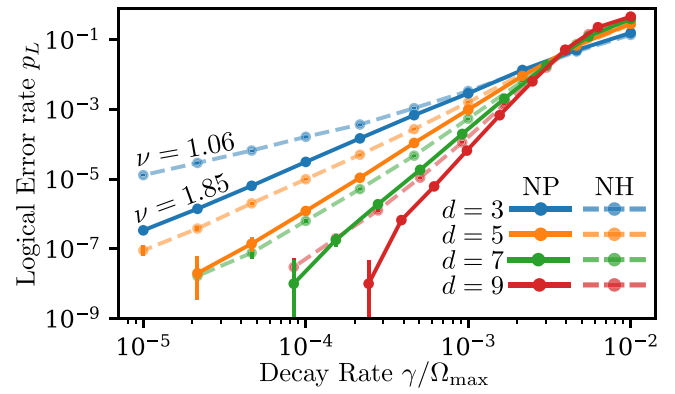


FIG. 3. All-to-all blockade. Logical error rate p_L as a function of the decay rate γ (in units of $\Omega_{\max}$) for the NP (solid lines) and NH (dashed lines) protocols.

original proposal by Jaksch *et al.* [44] for Rydberg blockade gates, though here the 2π pulse is applied to all qubits, which reduces the time spent in the Rydberg state by $\sim 40\%$ compared to the NH protocol. Figure 2(b) shows that for a decay rate $\gamma > \gamma_\times = 1.1 \times 10^{-3}\Omega_{\max}$ the SIM protocol also reduces the logical error rate compared to the NH protocol. However, in the SIM protocol a single decay event of the ancilla qubit during the 2π pulse on the data qubits can lead to *any* five-qubit error Q , so that again only $\lceil d/4 \rceil$ decay events are necessary for a logical error (see Supplemental Material [33]). We thus find that for $\gamma < \gamma_\times$ the NH protocol achieves the lower logical error rate [see Fig. 1(h)].

Finally, we briefly turn to the all-to-all blockade model, in which there is also a Rydberg blockade between data qubits on the same plaquette. In this case, we find that the NH protocol is not sufficient to restore the $\lceil d/2 \rceil$ scaling (see Fig. 3). Similar to the data-ancilla case, we show that this behavior is due to the propagation of Rydberg leakage errors on data qubits and can only be prevented through a gate protocol with $\theta_d \in \{0, \pi\}$ (see Supplemental Material [33]). This is only possible with an *asymmetric* protocol ($\Omega_a \neq \Omega_d$), since we saw above that symmetric protocols have to satisfy $\theta_a = \theta_d = \pi/2$. Using quantum optimal control, we find the shortest protocol with (1) $\theta_d \in \{0, \pi\}$ and (2) $c_1 = 1$, $c_2 = c_3 = c_4 = 0$, called no-phase protocol [see Fig. 1(f)], which indeed restores the expected $\lceil d/2 \rceil$ scaling (see Fig. 3). Consistently, the slopes ν extracted from the fits of displayed decoding curves are $\nu_{\text{NP}}^{d=3} = 1.85$ and $\nu_{\text{TO}}^{d=3} = 1.06$, $\nu_{\text{NP}}^{d=5} = 2.95$ and $\nu_{\text{TO}}^{d=5} = 2.14$, $\nu_{\text{NP}}^{d=7} = 3.92$ and $\nu_{\text{TO}}^{d=7} = 2.83$, and $\nu_{\text{NP}}^{d=9} = 5.05$ and $\nu_{\text{TO}}^{d=9} = 3.48$. We note that also the original π - 2π - π protocol [44] satisfies conditions (1) and (2), but has a 30% longer Rydberg time and thus a larger logical error rate than the NP protocol (see Supplemental Material [33]).

In conclusion, we have established that, due to the propagation of Rydberg leakage errors, gate protocols that minimize the two-qubit gate infidelity do not necessarily achieve the lowest possible logical error rates. For the data-ancilla and all-to-all blockade models, we identified the relevant error mechanisms and mitigated them by introducing two protocols for performing stabilizer measurements with Rydberg atoms, which significantly reduce the logical error rate. Interestingly, both protocols require breaking the symmetry between ancilla and data atoms, either by an asymmetric blockade model or by

an asymmetric laser pulse. Similar results are also expected for other quantum error correction codes, such as quantum low-density parity-check (LDPC) codes, when implemented on stationary arrays of neutral atoms [45]. We expect that our protocols also mitigate Rydberg leakage errors arising due to other errors sources, such as Rydberg dephasing. The precise effect of this and other error sources will be subject of future work. Thus, in a broader context and in line with the goals of fault-tolerant quantum computing, our work demonstrates the necessity to optimize stabilizer measurements beyond two-qubit gate fidelities to achieve the lowest possible logical error rate. This requirement becomes increasingly urgent as platforms approach 99.9% two-qubit gate fidelities and efforts intensify to reduce QEC cycle times from milliseconds to microseconds, a regime in which the error mechanisms analyzed in this work are expected to become dominant.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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