

Turbulent diffusion and dispersion in a superfluid

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Single-body diffusion and two-body dispersion are fundamental processes in classical turbulence, governing particle mixing and transport. However, their behaviors in superfluid turbulence remain largely unexplored. In this study, we numerically investigate the diffusion and relative dispersion of both quantized vortices and superfluid parcels in the 0 K limit in two distinct turbulence regimes: ultraquantum turbulence, characterized by a randomized vortex tangle, and quasiclassical turbulence, in which locally polarized vortices create large-scale flows resembling classical turbulence. Our results reveal that while vortex segments exhibit similar superdiffusion behavior at short times in both regimes, superfluid parcels behave differently—following the same superdiffusion scaling in ultraquantum turbulence but deviating significantly in quasiclassical turbulence. This contrast provides a key clue to the origin of short-time superdiffusion, a puzzle since its recent discovery. Additionally, we show that two-body dispersion of both vortex segments and superfluid parcels exhibits distinct scaling behaviors in ultraquantum and quasiclassical turbulence, highlighting fundamental differences in these two turbulence regimes. Our findings bridge a critical gap in superfluid turbulence research, offering useful insights into turbulent transport in inviscid quantum fluids.

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I. INTRODUCTION

Turbulent diffusion and dispersion are fundamental to understanding transport phenomena in fluid dynamics [1]. In classical turbulence, the chaotic motion of eddies governs how individual fluid parcels spread over time (single-body diffusion) and how initially close pairs of parcels separate (two-body dispersion), both of which follow well-characterized scaling laws. For instance, for a fluid parcel with position vector $\mathbf{r}(t)$, its mean square displacement (MSD), defined as $\langle \Delta \mathbf{r}^2(t) \rangle = \langle |\mathbf{r}(t) - \mathbf{r}(0)|^2 \rangle$, follows a ballistic scaling, $\langle \Delta \mathbf{r}^2(t) \rangle \approx u_{\text{rms}}^2 t^2$, at short times ($t \ll \tau_L$) and transitions to a diffusive scaling, $\langle \Delta \mathbf{r}^2(t) \rangle = 2\Gamma_T t$, at long times ($t \gg \tau_L$) [2]. Here, u_{rms} is the root-mean-square (rms) velocity, $\tau_L = L/u_{\text{rms}}$ is the integral timescale with L being the integral length scale, and $\Gamma_T = u_{\text{rms}}^2 \tau_L$ is the turbulent eddy diffusivity [3]. Meanwhile, in fully developed turbulence, the mean square separation $\langle \Delta^2(t) \rangle = \langle |\mathbf{r}_1(t) - \mathbf{r}_2(t)|^2 \rangle$ of two initially close fluid parcels follows Richardson-Obukhov t^3 law [4,5], $\langle \Delta^2(t) \rangle = g\epsilon t^3$, for times $\tau_0 < t < \tau_L$ while $\Delta(t)$ remains within the inertial range. Here, ϵ is the turbulent energy dissipation rate, $g \simeq 0.5$ is the Richardson constant, and $\tau_0 = |\Delta(0)|^{2/3}/\epsilon^{1/3}$ is the memory time [6]. Extensive studies have explored these scaling laws in classical fluids [7–13].

In an inviscid superfluid, the situation becomes intriguing. Turbulence in a superfluid can arise from a chaotic tangle

of quantized vortex lines [14], which are topological defects characterized by a quantized circulation $\kappa = h/m$, where h is Planck's constant and m is the mass of the bosons constituting the superfluid [15]. The vortices evolve with time and can reconnect when they intersect [16]. Depending on the structure of the vortex tangle, superfluid turbulence can manifest in two distinct regimes [17,18]. The first, known as quasiclassical turbulence (QCT), emerges when the vortices locally align and form bundles that mimic classical vortices [19]. In this regime, the induced velocity field exhibits classical behavior at length scales larger than the mean intervortex spacing ℓ [20,21]. In contrast, when the vortices are arranged randomly, it gives rise to ultraquantum turbulence (UQT), a regime with no classical counterpart, where the flow field fluctuates at scales comparable to ℓ without large-scale motion [22]. Remarkably, both QCT and UQT can decay even at the 0 K limit in the absence of a viscous thermal component. At scales comparable to ℓ , vortex reconnections create quasiparticles [23] and excite Kelvin waves on the vortices [24]. These waves can cascade energy to smaller length scales through nonlinear interactions, eventually causing quasiparticle emissions [25]. While extensive studies have examined the energy cascade and spectral properties in these turbulence regimes [14,25–28], much less is known on how vortices and superfluid parcels diffuse and disperse [29,30].

In a recent experiment [31], we observed superdiffusion of quantized vortices in UQT produced by thermal counterflow in superfluid helium-4 (He II). At short times, the MSD follows $\langle \Delta \mathbf{r}^2(t) \rangle \propto t^\gamma$ with an exponent $\gamma \simeq 1.6$, before crossing over to normal diffusion with $\gamma \simeq 1$ for $t \gg \tau_\ell$, where $\tau_\ell = \ell/u_{\text{rms}}^{(v)}$ is the characteristic time for a vortex segment to travel before reconnecting with other vortices, with $u_{\text{rms}}^{(v)}$ the rms vortex velocity. Similar scaling behavior was also

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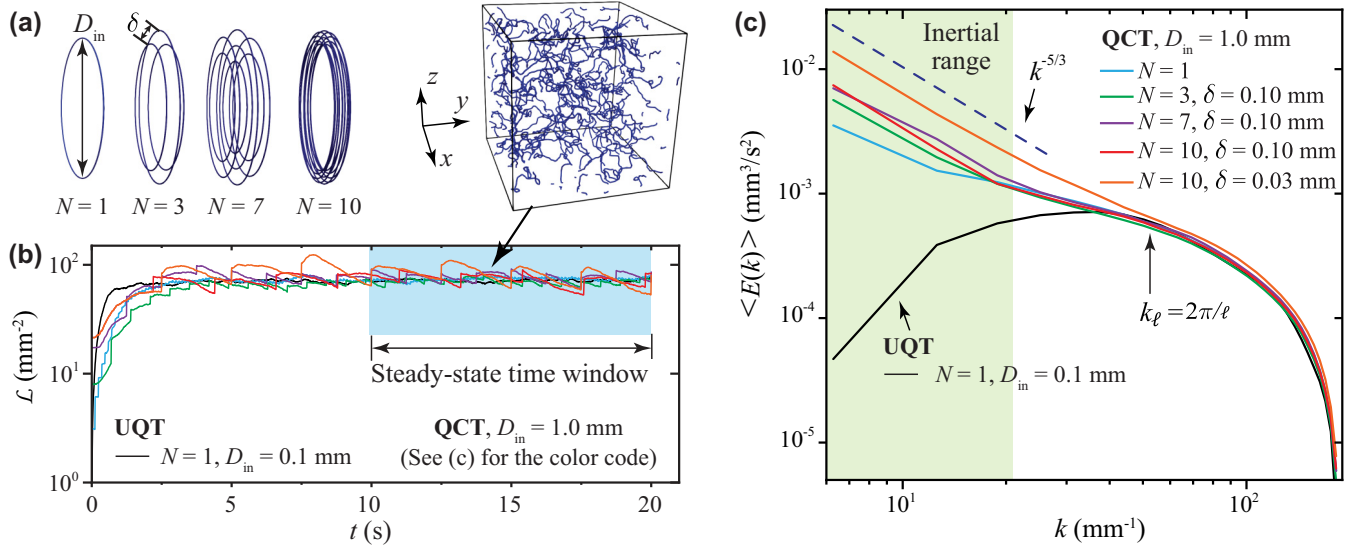


FIG. 1. (a) Schematic showing the configurations of the injected vortex rings and ring bundles. (b) Time evolution of the vortex-line density $\mathcal{L}$. A snapshot of the vortex tangle for the case $D_{in} = 1$ mm, $N = 10$, and $\delta = 0.03$ mm is included. (c) Turbulence energy spectrum $\langle E(k) \rangle$, ensemble-averaged over the steady-state time window for various injection conditions.

reported by Boltnev *et al.* for particles trapped on vortices near a He II free surface [32]. Our subsequent numerical simulations further confirmed that this diffusion law is universal for quantized vortices in UQT, independent of ℓ , and arises from intrinsic temporal correlations of vortex velocities for $t < \tau_\ell$ [33]. However, the mechanism underlying this velocity correlation remains a puzzle. It is also unclear whether similar vortex diffusion persists in QCT, and whether superfluid parcels exhibit the same diffusion characteristics.

With respect to turbulent dispersion, our understanding remains even more limited: Apart from a preliminary numerical study of vortex dispersion in two-dimensional UQT [34], no comprehensive framework exists for dispersion scaling laws in either UQT or QCT, leaving a major gap in superfluid turbulence research.

In this work, we present a systematic numerical study comparing the diffusion and dispersion behaviors of quantized vortices and superfluid parcels in both UQT and QCT at the 0 K limit. Our results reveal that vortex segments exhibit universal short-time superdiffusion in both regimes, whereas superfluid parcels follow the superdiffusion scaling only in UQT but deviate significantly in QCT. Moreover, the relative dispersion of vortices and parcels is found to follow distinct scaling laws in the two turbulence regimes, highlighting their fundamental differences. These findings go far beyond our earlier work [33], establishing the first unified framework for turbulent diffusion and dispersion in superfluids. This framework fills a critical gap in understanding transport in quantum turbulence and has direct implications for applications involving dispersing particles in He II [35–39].

II. GENERATION OF ULTRAQUANTUM AND QUASICLASSICAL TURBULENCE

To study vortex dynamics, we adopt the vortex-filament model [40], where each vortex line is represented as a zero-thickness filament discretized into a series of points. To

generate UQT, we use the method adopted in Ref. [33], where randomly oriented small vortex rings (diameter $D_{in} = 0.1$ mm) are repetitively injected at random locations within a cubical computational box (side length $D = 1$ mm) with periodic boundary conditions in all three directions. The injection occurs at fixed intervals of t_{in} to facilitate the buildup of a vortex tangle. In contrast, generating QCT requires energy injection at large length scales [22]. In this work, we tested injecting individual vortex rings with $D_{in} = 1$ mm as well as vortex-ring bundles consisting of $N = 3, 7$, or 10 coaxial rings arranged in a triangular lattice within the plane containing the central axis. The bundles have an averaged $D_{in} = 1$ mm and an inter-ring distance of $\delta = 0.1$ or 0.03 mm, as illustrated in Fig. 1(a). A filament at $\mathbf{s}$ moves at the local superfluid velocity $\mathbf{u}_s$ given by the Biot-Savart law [40,41]:

$$\frac{d\mathbf{s}}{dt} = \mathbf{u}_s(\mathbf{s}, t) = \frac{\kappa}{4\pi} \int \frac{(\mathbf{s}_1 - \mathbf{s}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3}, \quad (1)$$

where the integration is performed along all the vortices. In our simulations, we adopted $\kappa = 10^{-3}$ cm\$^2\$/s for He II, though the results are expected to be general for superfluids in which vortex dynamics is governed by the same law. Since the integrand diverges at $\mathbf{s}$, we follow Adachi *et al.* and calculate the integral as the sum of the local contribution and the nonlocal contribution [42]:

$$\frac{d\mathbf{s}}{dt} = \beta_l \mathbf{s}' \times \mathbf{s}'' + \frac{\kappa}{4\pi} \int_{\mathbf{s}_1 \neq \mathbf{s}} \frac{(\mathbf{s}_1 - \mathbf{s}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3}, \quad (2)$$

where the prime denotes derivative with respect to the arc length of the vortex filament at $\mathbf{s}$, and β_l is a vortex core-size-dependant coefficient [40] (see Appendix A for details). When two vortices approach to have a separation less than our spatial resolution $\Delta\xi_{min} = 0.008$ mm, we reconnect them at the location of the minimum separation, following the procedures as detailed in Refs. [41,43]. We also remove small vortex loops with lengths less than $5\Delta\xi_{min}$ to account for the cascade loss of the vortices [41]. Figure 1(b) shows the time evolution of

TABLE I. Key flow properties for the UQT and QCT cases. [Note: $u_{\text{rms}}^{(v)}$ and $u_{\text{rms}}^{(p)}$ denote the rms velocities of vortices and superfluid parcels, respectively. The characteristic timescales are defined as $\tau_\ell^{(v)} = \ell/u_{\text{rms}}^{(v)}$ and $\tau_\ell^{(p)} = \ell/u_{\text{rms}}^{(p)}$.]

Case	$u_{\text{rms}}^{(v)}$ (mm/s)	$\tau_\ell^{(v)}$ (s)	$u_{\text{rms}}^{(p)}$ (mm/s)	$\tau_\ell^{(p)}$ (s)
UQT	2.57	0.046	0.21	0.56
QCT	2.50	0.044	0.41	0.27

the vortex-line density $\mathcal{L}$ (defined as the total vortex length per unit volume) for various injection conditions. After an initial transient of 2–3 s, $\mathcal{L}$ settles to a steady level, where the tangle exhibits excellent isotropy (see Appendix B). To ensure a fair comparison, we adjust t_{in} for each case so that the steady-state $\mathcal{L}$ remains about 75 mm^{-2} for all cases, corresponding to a mean intervortex spacing of $\ell = \mathcal{L}^{-1/2} \simeq 0.11 \text{ mm}$. A representative snapshot of the vortex tangle for the case with $D_{\text{in}} = 1 \text{ mm}$, $N = 10$, and $\delta = 0.03 \text{ mm}$ is also shown in Fig. 1(b).

Knowing the vortex-tangle configuration at time t , the energy spectrum $E(k, t)$ as a function of wavenumber k , associated with the superfluid velocity field, can be computed using the expression from Ref. [44] (see Appendix C for details). This spectrum is then ensemble-averaged over the steady-state time window to obtain $\langle E(k) \rangle$. Results for various injection conditions are shown in Fig. 1(c). For UQT, where the energy is injected at a scale comparable to ℓ , the spectrum $\langle E(k) \rangle$ peaks near $k_\ell = 2\pi/\ell$, as expected [22]. For QCT, we observe that in the low- k regime, $\langle E(k) \rangle$ increasingly approaches the classical Kolmogorov scaling $k^{-5/3}$ as bundles of more closely packed rings are used for injection. In contrast, injecting randomly oriented individual rings at large scales fails to deliver sufficient energy density to establish a

clear inertial range, a key consideration for those aiming to generate QCT via ring injection. We also fit the spectrum using $\langle E(k) \rangle = \alpha \epsilon^{2/3} k^{-5/3}$ for the case with $D_{\text{in}} = 1 \text{ mm}$, $N = 10$, and $\delta = 0.03 \text{ mm}$, where the energy dissipation rate $\epsilon = 0.106 \text{ mm}^2/\text{s}^3$ is obtained by equating it to the energy injection rate, determined from the measured energy increase following each bundle injection divided by the injection interval t_{in} (see Fig. 6 in Appendix C). The fitted Kolmogorov constant is $\alpha = 1.43$, which is close to the classical value $\alpha \simeq 1.5$ [45]. In what follows, we focus on this case and the UQT case to comparatively examine their diffusion and dispersion scalings. Table I summarizes the rms velocities of vortex segments $u_{\text{rms}}^{(v)}$ and superfluid parcels $u_{\text{rms}}^{(p)}$, along with the calculated timescales $\tau_\ell^{(v)}$ and $\tau_\ell^{(p)}$.

III. SINGLE-BODY DIFFUSION IN ULTRAQUANTUM AND QUASICLASSICAL TURBULENCE

We begin by examining vortex diffusion using the tagging method described in Ref. [33], where randomly selected vortex-filament points are tracked and their MSD along each axis is analyzed during the steady-state time window. The resulting vortex MSDs in the x direction, $\langle \Delta x^2(t) \rangle = \langle [x(t_0 + t) - x(t_0)]^2 \rangle$ for UQT and QCT are shown in Figs. 2(a) and 2(b), respectively. For UQT, the MSD exhibits superdiffusion scaling $\langle \Delta x^2(t) \rangle \propto t^{\gamma_1}$ with $\gamma_1 \simeq 1.6$ at times on the order of or below the vortex reconnection timescale $\tau_\ell^{(v)}$. This superdiffusion arises from a power-law temporal correlation in vortex velocity [33]. At very short times ($t < 10^{-2} \tau_\ell^{(v)}$), when vortex velocities are fully correlated in time, ballistic scaling $\langle \Delta x^2(t) \rangle \propto t^2$ can emerge. At $t \gg \tau_\ell^{(v)}$, the MSD transitions to normal diffusion, $\langle \Delta x^2(t) \rangle \propto t^{\gamma_2}$ with $\gamma_2 \simeq 1$, as vortex reconnections disrupt the velocity correlations and randomize the vortex motion. Remarkably, the vortex MSD in

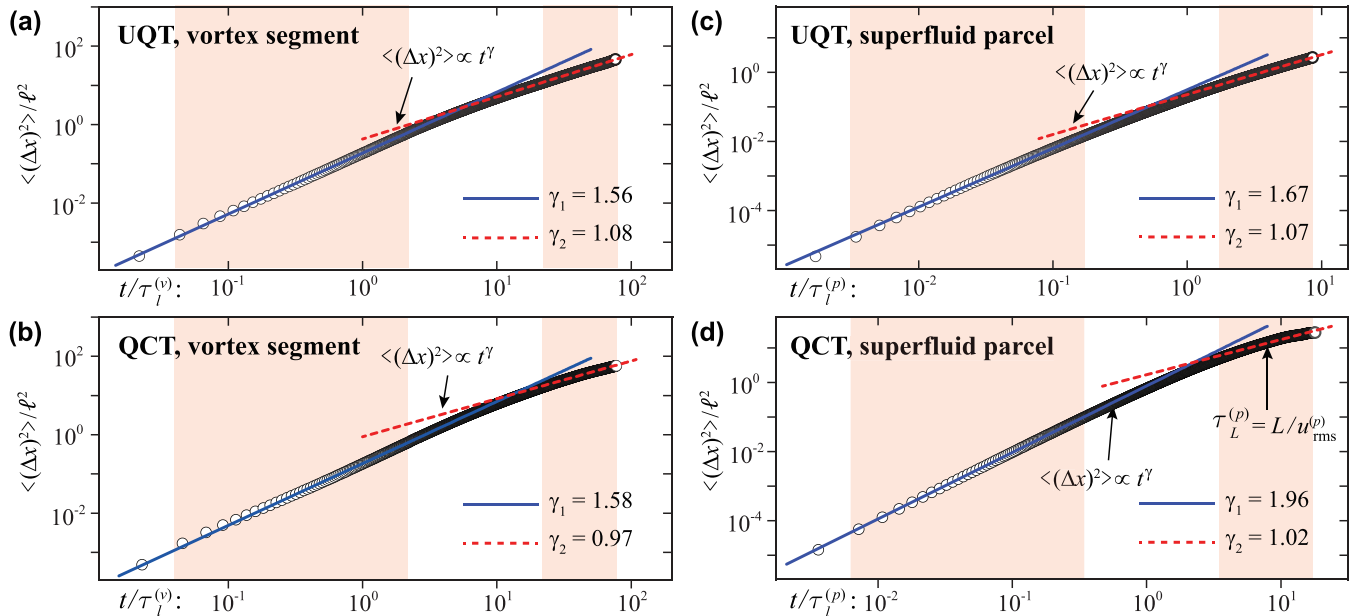


FIG. 2. MSD, $\langle \Delta x^2(t) \rangle$, along the x direction for vortex segments in (a) UQT and (b) QCT, and for superfluid parcels in (c) UQT and (d) QCT. Solid and dashed lines represent power-law fits to the data within the shaded regions. The integral time $\tau_L^{(p)} = L/u_{\text{rms}}^{(p)}$ for parcels in QCT is also indicated in panel (d).

QCT exhibits the same scaling behavior despite fundamental differences between QCT and UQT, suggesting a universal mechanism governing vortex diffusion independent of turbulence form.

Further insight arises from our analysis of the apparent diffusion of superfluid parcels. We begin by randomly selecting parcels at time t_0 . Their initial velocities $\mathbf{u}_s(\mathbf{r})$ are computed using Eq. (1) based on the vortex configuration at t_0 . The parcels are then advected to positions $\mathbf{r}(t_0 + \Delta t) = \mathbf{r}(t_0) + \mathbf{u}_s(\mathbf{r})\Delta t$, and their velocities are updated using the vortex configuration at $t_0 + \Delta t$. Repeating this procedure yields complete parcel trajectories, enabling evaluation of their ensemble-averaged MSDs. Figures 2(c) and 2(d) show the parcel MSDs in the x direction, $\langle \Delta x^2(t) \rangle$, for UQT and QCT, respectively. In UQT, parcels follow the same diffusion scaling as vortex segments, though with smaller $\langle \Delta x^2(t) \rangle$ due to their lower rms velocity. Interestingly, in QCT the parcel MSD exhibits ballistic scaling $\langle \Delta x^2(t) \rangle \propto t^{\gamma_1}$ with $\gamma_1 \simeq 2$ at short times before crossing over to normal diffusion at large times, resembling classical turbulent diffusion and standing in stark contrast to vortex diffusion in the same flow.

To understand these observations, we first note that both vortex segments and superfluid parcels follow the superfluid velocity field $\mathbf{u}_s$. The key difference is that vortex segments experience the local induction term $\beta_1 \mathbf{s}' \times \mathbf{s}''$, whereas most parcels do not, since they are not typically located on the vortices. In UQT, Table I shows that the vortex rms velocity is about an order of magnitude larger than that of parcels, demonstrating the dominance of the local induction contribution over the nonlocal field. As $\mathbf{u}_s$ in UQT fluctuates mainly at scales comparable to ℓ , where the nonlocal flows mimic those produced by the local induction term, both parcels and vortex segments exhibit comparable diffusion scaling.

In QCT, vortex bundling and polarization substantially enhance the nonlocal flows, resulting in a parcel rms velocity nearly twice that in UQT (Table I). These large-scale coherent flows advect parcels and produce fully correlated velocities at short times, thereby giving rise to ballistic parcel diffusion. By contrast, the rms velocity of vortices in QCT remains about the same as in UQT, as the vortex dynamics is still dominated by the local induction effect. The local induction likely yields only power-law temporal velocity correlations, because vortices have mean curvature radii of about ℓ [14], which drive vortex segments into rotations that prevent complete temporal correlation of their velocities.

To support this interpretation, we performed simulations using the local induction approximation (LIA), where vortices evolve solely under the local term, as originally adopted by Schwarz [40]. As shown in Appendix D, the vortex MSDs in both UQT and QCT under LIA reproduce the short-time superdiffusion seen in Figs. 2(a) and 2(b), confirming that this behavior is indeed governed by the local induction effect and pinpointing the origin of this puzzling phenomenon.

IV. TWO-BODY DISPERSION IN ULTRAQUANTUM AND QUASICLASSICAL TURBULENCE

To analyze two-body dispersion, we track randomly selected pairs of vortex segments and superfluid parcels, measuring their separation over time as $|\Delta(t)| = |\mathbf{r}_1(t) - \mathbf{r}_2(t)|$.

The mean square separation $\langle |\Delta(t)|^2 \rangle$ is then ensemble-averaged over many pairs within the steady-state window. Results for parcel pairs with various initial separations $|\Delta(0)|$ are shown in Figs. 3(a) and 3(b) for UQT and QCT, respectively. Similar to classical fluids [8–10], an initial “frozen” regime is observed, where $\langle |\Delta(t)|^2 \rangle$ remains nearly constant due to minimal relative motion. As turbulent fluctuations accumulate, the separations begin to grow. For QCT, we compute the memory time $\tau_0 = |\Delta(0)|^{2/3}/\epsilon^{1/3}$ for each case. When the initial separation is sufficiently small, we indeed observe the Richardson t^3 scaling within the inertial range [highlighted in green in Fig. 3(b)] at $t \gg \tau_0$, followed by a transition to normal diffusion. Fitting the classical expression $\langle |\Delta^2(t)| \rangle = g\epsilon t^3$ to the QCT curve with $|\Delta(0)| = 0.01$ mm yields a Richardson constant of $g \simeq 0.18$, slightly below the classical value of 0.5, likely due to our limited inertial range [46,47]. In UQT, as $|\Delta(0)|$ decreases, the curves also converge toward a power-law scaling $\langle |\Delta(t)|^2 \rangle \propto t^{2.2}$, which transitions to normal diffusion at late times. This anomalous $t^{2.2}$ scaling—distinct from classical turbulence—represents an observation that reflects the fundamental differences in the large-scale flows between UQT and QCT.

We also analyzed the scaling behavior of the reduced mean square separation $\langle |\Delta(t) - \Delta(0)|^2 \rangle$ to eliminate the influence of the initial “frozen” regime. The results for parcel pairs are shown in Figs. 3(c) and 3(d). For pairs with large initial separations, the $\langle |\Delta(t) - \Delta(0)|^2 \rangle/\ell^2$ curves closely follow the single-parcel diffusion curves $6\langle \Delta x^2(t) \rangle/\ell^2$. This is expected, since the two measures are mathematically equivalent when parcel pairs are far apart such that their motions become uncorrelated. As $|\Delta(0)|$ decreases, the dispersion scalings discussed earlier reappear in the same time intervals as in Figs. 3(a) and 3(b). This representation captures both single-particle diffusion and two-particle dispersion in a unified framework, which could serve as a useful tool for data visualization.

The dispersion behavior of vortex segments closely resembles that of superfluid parcels in both UQT and QCT (see Appendix E). Furthermore, since quantized vortices have well-defined cores—unlike those in classical fluids—they can be unambiguously tracked for analyses that are difficult to perform in classical turbulence. For instance, we can distinguish vortex-segment pairs that are initially on the same vortex line from those on different vortex lines, and analyze their separation behavior separately. The results for vortex-segment pairs with an initial separation of $|\Delta(0)| = 0.01$ mm are shown in Fig. 4. For pairs initially on the same vortex line, the dispersion curve exhibits the scaling behavior discussed earlier. However, for pairs initially on different vortex lines, the dispersion closely follows single-body diffusion data $6\langle \Delta x^2(t) \rangle/\ell^2$, indicating uncorrelated motion between the segments. This observation suggests that the t^3 dispersion scaling for vortex segments in QCT is likely associated with stretching of vortices due to large-scale flows.

V. SUMMARY

Our work demonstrates that the short-time superdiffusion scaling of quantized vortices—regardless of turbulence type—is governed by the local induction term. This finding

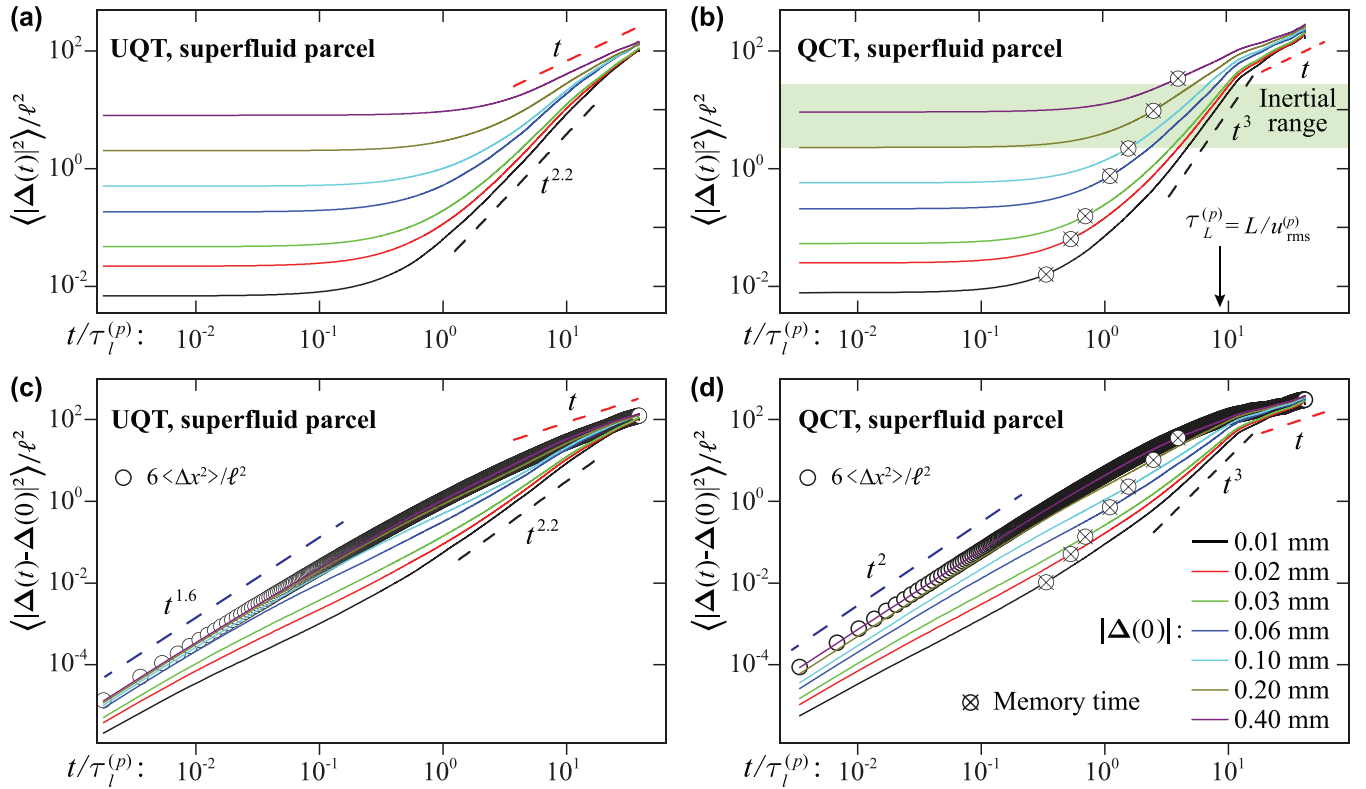


FIG. 3. Mean square separation $\langle |\Delta(t)|^2 \rangle$ and reduced separation $\langle |\Delta(t) - \Delta(0)|^2 \rangle$ for superfluid parcel pairs in (a), (c) UQT and (b), (d) QCT. Solid curves of different colors correspond to different initial pair separations $|\Delta(0)|$, as indicated in panel (d). Open circles denote $6\langle \Delta x^2(t) \rangle / \ell^2$ obtained from single-parcel diffusion data. Circles with crosses in panels (b) and (d) mark the memory time τ_0 . The integral time $\tau_L^{(p)} = L / u_{rms}^{(p)}$ for the parcels in QCT is also indicated in panel (b).

may motivate future analytical derivations of the observed scaling $\langle \Delta x^2(t) \rangle \propto t^{1.6}$ within the LIA framework. In addition, the dispersion scaling observed in UQT, featuring a $t^{2.2}$ behavior, highlights unexplored dynamics that warrant further investigation. These scalings are robust and independent of ℓ , as confirmed by our additional simulations and consistent with Ref. [33]. The method of generating idealized QCT via packed-ring bundle injection is also significant. Thanks to the well-defined structure of quantized vortices, such simulations enable statistical analyses that are often impractical in

classical turbulence—for example, tracking vortex segments to study both inter- and intravortex dynamics. This approach opens opportunities to advance our understanding of classical turbulent transport through quantum analogies.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: VORTEX-FILAMENT METHOD

In this work, we employ the vortex-filament model to investigate vortex dynamics and the resulting superfluid velocity

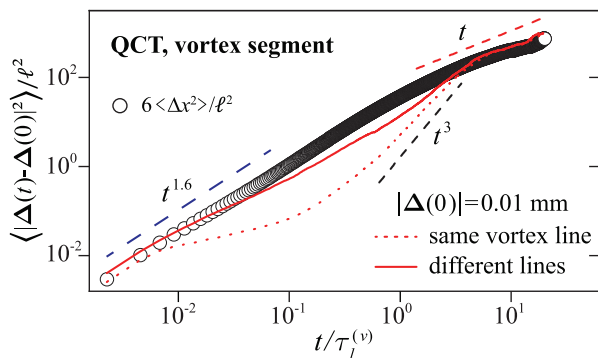


FIG. 4. Dispersion behavior of vortex segments in QCT with an initial separation of $|\Delta(0)| = 0.01$ mm. The red solid and dotted curves represent segment pairs initially located on the same vortex line and on different vortex lines, respectively.

field [40]. In this framework, each quantized vortex line is represented as a zero-thickness filament discretized into a series of points. In our simulations, the point spacing $\Delta\xi$ ranges from $\Delta\xi_{\min} = 0.008$ mm to $\Delta\xi_{\max} = 0.024$ mm. In the zero-temperature limit, where the viscous thermal component is absent, a filament point at position $\mathbf{s}$ moves with the local superfluid velocity $\mathbf{v}_s(\mathbf{s})$, given by

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}_s(\mathbf{s}) = \mathbf{v}_0(\mathbf{s}) + \mathbf{v}_{\text{in}}(\mathbf{s}), \quad (\text{A1})$$

where $\mathbf{v}_0(\mathbf{s})$ is the imposed background superfluid velocity, and $\mathbf{v}_{\text{in}}(\mathbf{s})$ denotes the velocity induced at $\mathbf{s}$ by all vortex filaments, as given by the Biot-Savart law [40]:

$$\mathbf{v}_{\text{in}}(\mathbf{s}) = \frac{\kappa}{4\pi} \int \frac{(\mathbf{s}_1 - \mathbf{s}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3}, \quad (\text{A2})$$

where the integration is carried out along all the vortex filaments. Since this integrand diverges at $\mathbf{s}$ for ideal zero-thickness filaments, we follow the well-established approach [42] and compute the induced velocity as the sum of a local contribution and a nonlocal contribution:

$$\mathbf{v}_{\text{in}}(\mathbf{s}) = \beta_l \mathbf{s}' \times \mathbf{s}'' + \frac{\kappa}{4\pi} \int_{\mathbf{s}_1 \neq \mathbf{s}} \frac{(\mathbf{s}_1 - \mathbf{s}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3}, \quad (\text{A3})$$

where the prime denotes the derivative with respect to the arc length of the vortex filament at $\mathbf{s}$ (i.e., $\mathbf{s}'$ is the unit tangent vector along the filament, and $\mathbf{s}''$ is the unit vector along the binormal direction divided by the local curvature radius [40]). The nonlocal term accounts for contributions from the rest of the filament and all other vortices. The coefficient β_l is given by [40]

$$\beta_l = \frac{\kappa}{4\pi} \ln \left(\frac{2(l_+ l_-)^{1/2}}{e^{1/4} a_0} \right), \quad (\text{A4})$$

where l_+ and l_- are the distances from point $\mathbf{s}$ to its two nearest neighboring points along the same filament, and the cut-off parameter $a_0 \simeq 1 \text{ \AA}$ denotes the vortex core radius in He II.

In our simulations, the background superfluid velocity is set to $\mathbf{v}_0 = 0$. The time evolution of the vortex configuration is obtained by integrating Eq. (A1) using the fourth-order Runge-Kutta method [48] with a time step of $\Delta t = 10^{-4}$ s. We have verified that our spatial and temporal resolutions are sufficient to ensure convergence and independence of the simulation results. During vortex evolution, whenever two vortex filaments approach each other with a minimum separation less than $\Delta\xi_{\min}$, we perform a reconnection of the two filaments at the location of the minimum separation, following procedures detailed in Refs. [41,43]. Furthermore, to maintain a more uniform spatial resolution along the filaments, we adaptively remove or insert vortex-filament points at each time step: A point is removed if the separation between two adjacent points falls below $\Delta\xi_{\min}$, and inserted if it exceeds $\Delta\xi_{\max}$.

At zero temperature, Kelvin waves excited on the vortices can lead to local kinks, promoting reconnections and the formation of small vortex loops. Through nonlinear interactions, these waves cascade to smaller scales, generating increasingly smaller loops in a self-similar breakup process [41]. When loops become sufficiently small, they can propagate ballistically and be absorbed by container walls. To account

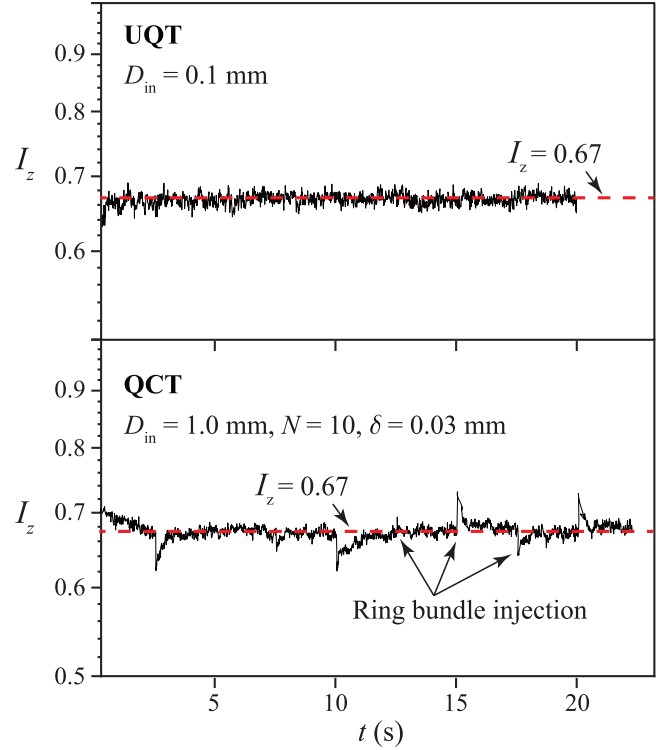


FIG. 5. The vortex-tangle anisotropy I_z as a function of time t for the UQT case with $D_{\text{in}} = 0.1$ mm (top panel) and the QCT case with injection parameters $D_{\text{in}} = 1$ mm, $N = 10$, and $\delta = 0.03$ mm (bottom panel).

for this cascade-induced vortex loss, a common approach in vortex-filament simulations is to remove loops shorter than a given threshold. Following Ref. [41], we remove all vortex loops with lengths less than $5\Delta\xi_{\min}$ in our simulations.

APPENDIX B: ISOTROPICITY OF VORTEX TANGLES

To study the dynamics of both UQT and QCT, it is essential to ensure that the generated turbulence is isotropic. This is crucial for meaningful comparisons with theoretical models developed under the assumption of isotropy. To assess whether the vortex tangles produced in our simulations meet this criterion, we evaluate the dimensionless anisotropy parameter commonly used in the literature [42]:

$$I_i = \frac{1}{\Omega \mathcal{L}} \int_{\mathcal{L}} [1 - (\mathbf{s}' \cdot \hat{\mathbf{e}}_i)^2] d\xi, \quad (\text{B1})$$

where $\mathcal{L}$ is the vortex-line density (i.e., the total vortex length divided by the computational volume Ω), $\mathbf{s}'$ is the unit tangent vector along a vortex segment $d\xi$, and $\hat{\mathbf{e}}_i$ is the unit vector along the i axis ($i = x, y, z$). The integral is performed over all vortex lines within the volume Ω . For an isotropic tangle, one expects $I_x = I_y = I_z = 2/3$, whereas in the extreme case where all vortex lines are oriented perpendicular to the i axis, $I_i = 1$. Figure 5 presents the time evolution of I_z for both the UQT case with $D_{\text{in}} = 0.1$ mm and the QCT case with injection parameters $D_{\text{in}} = 1$ mm, $N = 10$, and $\delta = 0.03$ mm, as described in the main text. In both cases, I_z remains close to $2/3$, indicating near-isotropic behavior. In the QCT case,

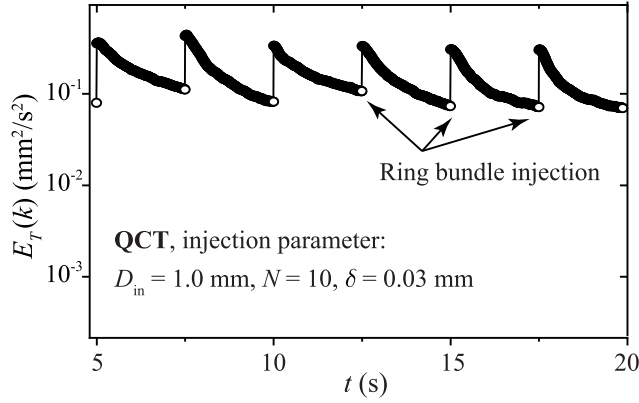


FIG. 6. Total turbulent kinetic energy per unit mass $E_T(t)$ as a function of time t for the QCT case with $D_{\text{in}} = 1$ mm, $N = 10$, and $\delta = 0.03$ mm.

small transient spikes appear in the $I_z(t)$ curve, corresponding to the injection of compact vortex-ring bundles, which momentarily perturb the isotropy. The behaviors of I_x and I_y are found to be very similar to that of I_z in both turbulence regimes, supporting the conclusion that the vortex tangles in our simulations are effectively isotropic.

APPENDIX C: ENERGY SPECTRUM AND DISSIPATION RATE CALCULATION

Given the vortex configuration at time t , the full three-dimensional superfluid velocity field $\mathbf{v}_s$ can be computed using Eq. (A1). Applying a Fourier transform to this velocity field yields the turbulent kinetic energy spectrum per unit fluid mass $E(k, t)$ as a function of wavenumber k . The ensemble-averaged energy spectrum, $\langle E(k) \rangle$, is then obtained by averaging $E(k, t)$ over the steady-state time window shown in Fig. 1:

$$\langle E(k) \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} E(k, t) dt \quad (\text{C1})$$

As discussed in Ref. [44], this energy spectrum can also be calculated from the vortex-filament positions using the following expression:

$$\langle E(k) \rangle = \frac{\kappa^2}{(2\pi)^2} \left\langle \int_0^L \int_0^L \mathbf{s}'_j(\xi_i) \cdot \mathbf{s}'_j(\xi_j) \times \frac{\sin(k|\mathbf{s}(\xi_i) - \mathbf{s}(\xi_j)|)}{k|\mathbf{s}(\xi_i) - \mathbf{s}(\xi_j)|} d\xi_i d\xi_j \right\rangle \quad (\text{C2})$$

where both integrals are performed over all vortex filaments. The resulting energy spectra for both UQT and QCT under various vortex-ring injection conditions are presented in Fig. 1.

In steady state, the energy dissipation rate ϵ equals the energy injection rate. To evaluate ϵ , we calculate the total kinetic energy per unit mass in the computational box as $E_T(t) = \int d^3\mathbf{r} |\mathbf{u}_s(\mathbf{r})|^2 = \int dk E(k, t)$. The result for the QCT case with $D_{\text{in}} = 1$ mm, $N = 10$, and $\delta = 0.03$ mm is shown in Fig. 6. Following each injection of a vortex-ring bundle, the total energy $E_T(t)$ increases by an amount ΔE_T . The energy injection rate and hence the dissipation rate is then

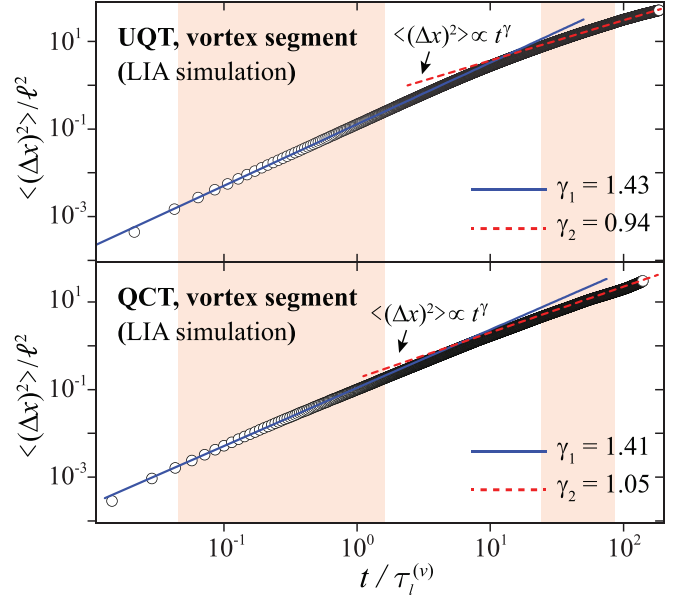


FIG. 7. MSD of vortex segments, $\langle \Delta x^2(t) \rangle$, along the x direction for UQT (top) and QCT (bottom), computed using the LIA. Solid and dashed lines indicate power-law fits to the data within the shaded regions.

given by $\epsilon = \Delta E_T / t_{\text{in}}$, where t_{in} is the injection time interval. Averaging ϵ over all bundle injections during the steady-state time window yields $\epsilon = 0.106 \pm 0.019 \text{ mm}^2/\text{s}^3$. With ϵ determined, we can fit the energy spectrum curve $\langle E(k) \rangle$ in the wavenumber range shown in Fig. 1, where a clear $k^{-5/3}$ scaling is observed, using $\langle E(k) \rangle = \alpha \epsilon^{2/3} k^{-5/3}$. The fitted Kolmogorov constant is $\alpha = 1.43 \pm 0.17$.

APPENDIX D: VORTEX DIFFUSION IN LIA FRAMEWORK

To examine whether the short-time superdiffusion of quantized vortices in both UQT and QCT is primarily governed by the local induction term—i.e., the first term on the right-hand side of Eq. (A3)—we performed numerical simulations using the LIA, in which vortex evolution is driven solely by the local term, as originally adopted by Schwarz [40]. It is worth noting that this analysis serves as a test case, since LIA is known to break down in capturing long-time vortex dynamics. For instance, in thermal counterflow simulations, LIA leads to the formation of artificial layered vortex structures, requiring a nonphysical mixing procedure to maintain a uniform tangle [42]. Moreover, LIA cannot be used to generate true QCT, as the vortex motion is governed only by local curvature effects and does not respond to large-scale flows induced by other vortices. As a result, coherent structures such as locally organized vortex bundles cannot form and maintain under LIA dynamics.

Nonetheless, to probe the apparent diffusion behavior of vortex segments in the absence of nonlocal interactions, we carry out LIA simulations for the UQT case with $D_{\text{in}} = 0.1$ mm, and for the QCT case with injection parameters $D_{\text{in}} = 1$ mm, $N = 1$, and $\delta = 0.03$ mm. The results are presented in Fig. 7. The MSD of vortex segments, $\langle \Delta x^2(t) \rangle$, along the x direction continues to exhibit superdiffusion scaling behavior at short times in both UQT and QCT, with the

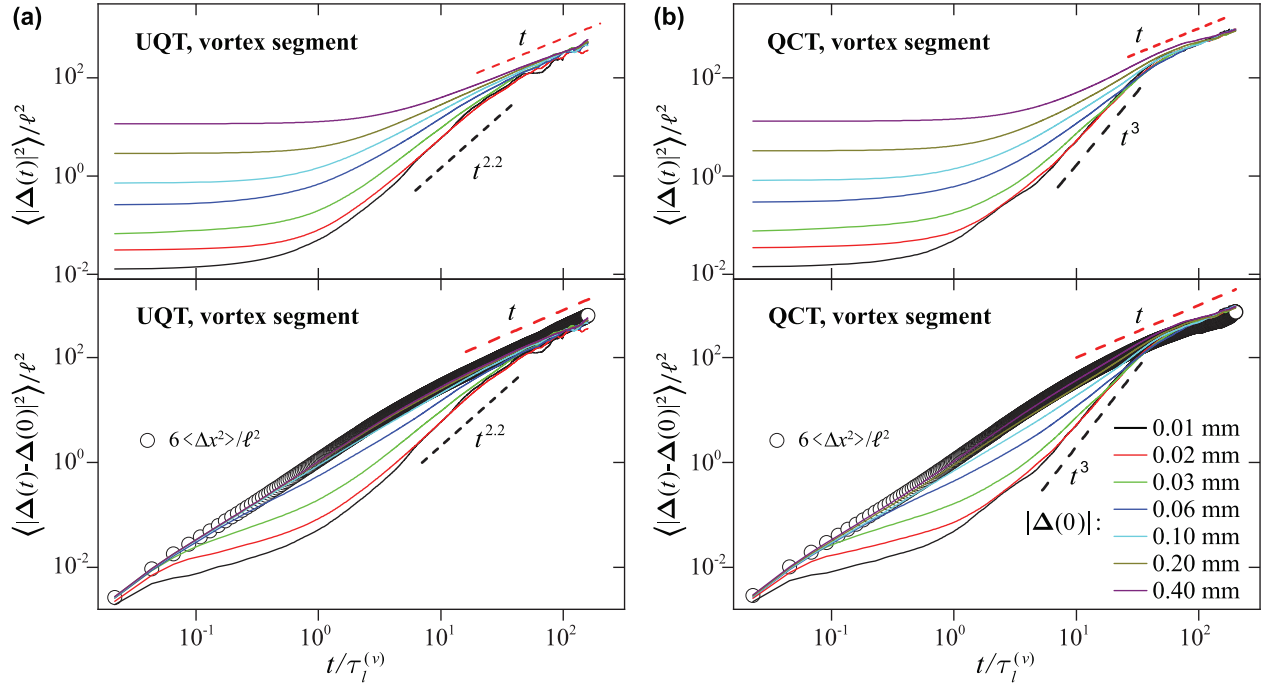


FIG. 8. Simulated vortex-segment pair dispersion in (a) UQT and (b) QCT. The solid curves for different colors correspond to different initial pair separation $|\Delta(0)|$ ranging from 0.01 to 0.4 mm. Open circles represent $6\langle \Delta x^2(t) \rangle / \ell^2$, obtained from diffusion data of individual vortex segments.

diffusion power-law exponent γ_1 slightly reduced compared to simulations using the full Biot-Savart law. This small difference reflects the contribution of the nonlocal term in the full Biot-Savart formulation. At later times, a transition to normal diffusion with a power-law exponent $\gamma_2 \simeq 1$ is observed.

APPENDIX E: VORTEX-PAIR DISPERSION IN ULTRAQUANTUM AND QUASICLASSICAL TURBULENCE

We analyzed the two-body dispersion behavior of quantized vortices in both UQT and QCT by tracking randomly selected pairs of vortex-filament points. The mean square separation, $\langle |\Delta(t)|^2 \rangle$, ensemble-averaged over many pairs within the steady-state time window, was evaluated for the UQT case with $D_{\text{in}} = 0.1$ mm and the QCT case with injection parameters $D_{\text{in}} = 1$ mm, $N = 1$, and $\delta = 0.03$ mm. The upper panels of Fig. 8 present the results for ensembles of vortex pairs with various initial separations $|\Delta(0)|$ in both turbulence regimes.

As with the dispersion of superfluid parcels discussed in the main text, the vortex-pair separation initially enters a “frozen” regime, where $\langle |\Delta(t)|^2 \rangle$ remains nearly constant due to minimal relative motion. As turbulent fluctuations accumulate, the pair separation begins to grow. In UQT, as $|\Delta(0)|$

decreases, the dispersion curves collapse onto a common power-law behavior, $\langle |\Delta(t)|^2 \rangle \propto t^{2.2}$, similar to that observed for superfluid parcels. This scaling persists until a transition to normal diffusion occurs at later times. In QCT, for sufficiently small initial separations $|\Delta(0)|$, we observe the classical Richardson scaling, $\langle |\Delta(t)|^2 \rangle \propto t^3$. It should be noted that we are examining the dispersion of vortex-segment pairs, not passive tracer particles in a hydrodynamic flow. Therefore, quantities such as the inertial range and the turbulent kinetic energy dissipation rate ϵ , which characterize the superfluid velocity field, are not applicable in this context. As a result, the classical Richardson expression, $\langle |\Delta(t)|^2 \rangle = g\epsilon t^3$, cannot be directly used to fit the vortex-pair dispersion curves to evaluate the Richardson constant g .

To exclude the influence of the initial “frozen” regime, we also analyzed $\langle |\Delta(t) - \Delta(0)|^2 \rangle$ for vortex-segment pairs. The results for both UQT and QCT are shown in the lower panels of Fig. 8. For pairs with large initial separations, the normalized dispersion $\langle |\Delta(t) - \Delta(0)|^2 \rangle / \ell^2$ closely matches the normalized diffusion data $6\langle \Delta x^2(t) \rangle / \ell^2$ for individual vortex segments. This is expected when the motions of the two segments are effectively uncorrelated. As the initial separation decreases, the curves converge and recover the dispersion scaling discussed earlier.

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