

Dynamical law behind eye movements: Distinguishing between Lévy and intermittent strategies

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Foraging is a complex spatiotemporal process that is often described with stochastic models. Two particular ones, Lévy walks and intermittent search, became popular in this context. Researchers from the two communities, each advocating for either Lévy or intermittent approach, independently analyzed foraging patterns and reported agreement between empirical data and the model they used. We resolve this Lévy-intermittent dichotomy for eye-gaze trajectories collected in a series of experiments designed to stimulate free foraging for visual information. By combining analytical results, statistical quantifiers, and basic machine learning techniques, we devise a method to score the performance of the models when they are used to approximate an individual gaze trajectory. Our analysis indicates that the intermittent search model consistently yields higher scores and thus approximates the majority of the eye-gaze trajectories better.

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I. INTRODUCTION AND SCOPE

The trajectories produced by foraging animals often appear to be randomlike and, in theoretical ecology, stochastic walks are considered one of the most relevant frameworks to model and analyze foraging [1–4]. Among the multitude of models, two have become especially popular over the last two decades: Lévy walks [5,6] and intermittent search [7,8].

In its basic formulation, a Lévy walk (LW) is realized by performing a sequence of relocations, each carried out in a random direction with a constant velocity [5,6]. The duration of each relocation is drawn from a probability density function (PDF) characterized by a power-law tail. LWs have proven to be useful as simple classifiers to distinguish subjects with specific diseases, e.g., attention deficit hyperactivity disorder, and typically developed subjects [9]. Moreover, LW models have been addressed to derive frameworks that able to characterize Lévy components in noisy signals [10] and in complex systems in general [11]. In the specific context of searching strategies, it has been stated that this type of stochastic walks is able to reproduce foraging patterns of living organisms, ranging from chemotactic bacteria [12] and marine predators [13,14] to human gatherers [15–17], visual foraging [18], and

even extinct animals [19]. In the case of renewable [20] or finite-lived [21] targets, LWs were advocated as the most optimal foraging strategy, which is considered to be a reason for the universality of the ‘Lévy foraging’ [22]. This hypothesis, however, prompted significant criticism, both with respect to the data analysis methodology and theoretical reasoning used to support it [23–28].

An intermittent search (IS) is composed of two alternating phases, a diffusion (D), which is associated with the searching phase, and a ballistic (B), associated with relocation [7,8,29–31]. Durations of the phases are governed by the corresponding switching rates λ_{DB} (from D to B) and λ_{BD} . Experimental data collected on various species have been analyzed and a good agreement with the IS model was stated [29].

Recently, it was demonstrated [32] that some synthetic organisms develop IS strategies rather than Lévy-like ones. This result supports the idea that, while LWs are often adapted as search strategies (e.g., for robots [33–36]), IS might offer greater efficiency when specific constraints are taken into account.

While the two models are essentially different in their statistical properties, distinguishing between these two strategies is not trivial, since the two models produce trajectories that might look very similar [8]. The mean squared displacement (MSD) is often used to classify the process [12,16,37,38]. However, in the realm of limited data, both in terms of quantity and duration, the MSD alone may not suffice to identify the process. For instance, similar finite-time behavior of the MSD can be closely reproduced with both LW and IS. Therefore, a stricter classifier is required to effectively address the Lévy-intermittent dichotomy in an ensemble of finite-length trajectories.

We address this problem in two steps. First, we design an experiment to induce free foraging that allows one to collect

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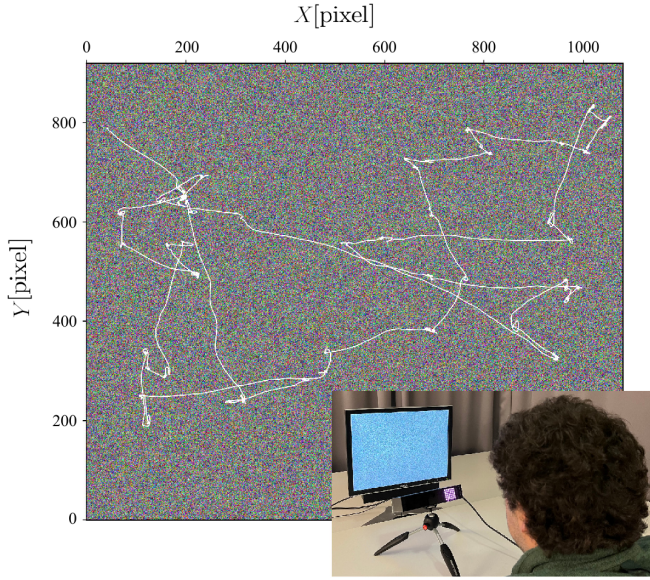


FIG. 1. Experiment: A subject is presented a random set of 1080×920 colored pixels for 180 s and asked to find a (nonexisting) hidden image. The eye-gaze location is recorded with a rate of 10^3 frames/s. Here, a 15-s record fragment is shown.

a statistically significant number of trajectories. Second, analyzing the collected data, we answer the following question: Which model, LW or IS, best describes individual trajectories?

To answer the question, we introduce a moment-based metric to quantify the goodness of fit [39] of a model and develop a machine-learning-assisted algorithm to retrieve model parameters. We find out that, according to the metrics, most of the trajectories are better described by IS rather than LWs.

II. EXPERIMENT

The idea to stimulate visual search and analyze recorded gaze trajectories with specific models has already been addressed [40–42]. Our experiment differs in two key aspects. Namely, (1) here the search is maximally unconstrained and homogeneous, and (2) we classify not the ensemble but each trajectory individually.

In the experiment, a set of randomly colored pixels, with no hidden picture or geometrical shape, is shown to a participant (Fig. 1). The participant is asked to find an unspecified hidden visual element during 180 s, a reasonable duration to balance the level of participant engagement and the need to sample long enough trajectories. The text prompt used was as follows: “You will see images with hidden patterns. Find the patterns concealed in the images. You don’t have to press any key on the keyboard or click the mouse. Press “spacebar” when you are ready.” Our aim was to provide conditions for the least constrained visual foraging, confined to a finite area. The hardware included an EyeLink Portable Duo eye-tracker [43], with a spatial resolution of 0.1 pixels and a sampling frequency up to 1 kHz.

We collected and analyzed data from 120 subjects, each one with, at least, 99% of their points “available,” with a recording time step of $\Delta t = 16$ ms, resulting in a trajectory

with at least $N = 11\,250$ points. We label as “not available” (NA), measurements either of subject’s blinking or the eye-tracker lost track of the cornea. All legal and ethical requirements were fulfilled.

III. SELECTING BETWEEN LÉVY AND INTERMITTENT STRATEGIES

A planar uniform LW process [44] is described as follows: During time τ_i , a pointlike walker moves with a constant velocity v_L in a random direction specified by angle ϕ , which is sampled from a uniform distribution on the interval $[0, 2\pi]$. After completing this constant-velocity event, an angle is sampled, and a duration τ_{i+1} is drawn from the PDF

$$\psi(\tau) = \frac{\gamma}{\tau_0} \left(1 + \frac{\tau}{\tau_0}\right)^{-(1+\gamma)}, \quad (1)$$

with $\gamma \in]0, 2]$ (for $\gamma > 2$, the resulting spread converges to normal diffusion in the asymptotic limit). Thus, the LW process is parameterized with three positive numbers $\{v_L, \gamma, \tau_0\}$. Potentially, $\psi(\tau)$ can be made more flexible, e.g., by modifying its time parameter τ_0 . Our analysis reveals that such modifications do not play a significant role in the classification.

Planar IS [29] consists of the two alternating phases mentioned above, the diffusion, a two-dimensional Wiener process, parameterized by a diffusivity D , and the ballistic phase by a constant velocity v_B . Together with the two switching rates, the IS is therefore parameterized by the four quantities $\{v_B, D, \lambda_{BD}, \lambda_{DB}\}$.

We base our analysis on the second and fourth moments, $m_2(t_s)$ and $m_4(t_s)$, respectively, computed within time windows of different size t_s . To estimate the moments from the recorded time series (X_t, Y_t) , we first calculate gaze velocities $v_X(t, t_s) = [X_{t+t_s} - X_t]/t_s$ and $v_Y(t, t_s) = [Y_{t+t_s} - Y_t]/t_s$. The

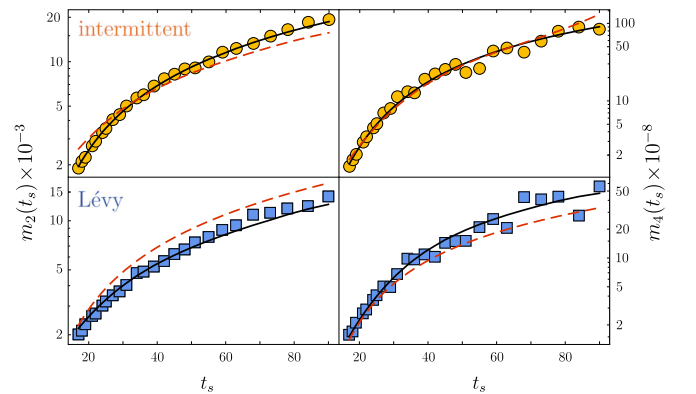


FIG. 2. Moments $m_2(t_s)$ (left), and $m_4(t_s)$ (right), calculated for two eye-gaze trajectories. The top plots show an example where the IS model (solid line) fits better the empirical values (circle, symbols) than the LW model (dashed line). The bottom plots show an example where the LW model (solid line) better fits the empirical values (squares) than the IS strategy (dashed lines). Time is in units of ms and the second and fourth moments are in units of $(\text{pixels})^2/(\text{ms})^2$ and $(\text{pixels})^4/(\text{ms})^4$, respectively.

respective moments are

$$m_k(t_s) = \langle \|\vec{v}(t, t_s)\|^k \rangle_t \quad (2)$$

for $k = 2, 4$ and $\|\vec{v}(t, t_s)\| = (v_x^2(t, t_s) + v_y^2(t, t_s))^{1/2}$. The average is computed by sliding over the data series with a window of the length t_s (see Fig. 2). The above setup corresponds to the so-called “equilibrated initial conditions” [45], indicating that the process has already achieved stationarity.

At large values of time, the second- and fourth-order moments of the LW scale, respectively, as t^2 and t^4 for $\gamma < 1$, as $t^{3-\gamma}$ and $t^{5-\gamma}$ for $1 < \gamma < 2$, and as t and t^2 for $\gamma > 2$ (the latter case is similar to ordinary Brownian motion). As for IS at large times, both the second- and fourth-order moments also scale as t and t^2 , respectively. Thus, the logarithm of

However, the analytic expression of $m_2(t_s)$ alone does not allow to extract all four parameters of the IS when analyzing synthetic data. This is the reason why we also include $m_4(t_s)$, which reads

$$\langle \mathbf{r}^4(t) \rangle = C_1^{(4)} t^2 + C_2^{(4)} t - C_3^{(4)} - C_4^{(4)} e^{-\beta t} + (C_5^{(4)} t^2 + C_6^{(4)} t - C_7^{(4)}) e^{-\beta \alpha t}, \quad (4)$$

with

$$\begin{aligned} C_1^{(4)} &= 8\alpha^2 \left(2D + \frac{1-\alpha}{\alpha^2} D_I \right)^2, & C_2^{(4)} &= 8 \frac{1-\alpha}{\alpha^3 \beta^2} \left[\left(D - \frac{\alpha+1}{2\alpha^2} D_I \right)^2 - \frac{5}{8\alpha^4} D_I^2 \right], \\ C_3^{(4)} &= 4 \frac{1-\alpha}{\alpha^4 \beta^4} \left[\left(2D - \frac{\alpha^2 + \alpha + 1}{\alpha^3} D_I \right)^2 - \frac{6\alpha^3 - 6\alpha^2 - \alpha - 2}{2\alpha^6} \right], \\ C_4^{(4)} &= 32(1-\alpha)\alpha\beta \left(2D + \frac{1}{1-\alpha} D_I \right)^2, & C_5^{(4)} &= 4 \frac{1-\alpha}{\alpha^2} D_I^2, \\ C_6^{(4)} &= \frac{16}{\alpha^2 \beta} D_I^2, & C_7^{(4)} &= \frac{8}{\alpha^2 \beta^2} D_I \left(8D + \frac{11\alpha^2 - 12\alpha + 3}{\alpha^2(1-\alpha)} D_I \right)^2. \end{aligned}$$

To optimize model parameters, we introduce a metric to quantify the distance between the values of $m_k(t_s)$, estimated from the experimental data, and with each model “mod” $\in \{\text{LW, IS}\}$:

$$d_k^{\text{mod}}(t_s) = \left| \log \left(\frac{m_k^{\text{exp}}(t_s)}{m_k^{\text{mod}}(t_s)} \right) \right|. \quad (5)$$

It satisfies all the requirements to be a metric: It is zero when the moment values are identical, positive otherwise, symmetric, and it obeys the triangle inequality. However, since it is not differentiable at zero, we use the square of this metric for optimization. Clearly, the minima of the squared metric coincide with the minima of the original metric (see the Appendixes).

Using the logarithm of the moments ensures that the weight is evenly distributed across the values of the moments. The distance is calculated for 25 windows of size $t_s = \lfloor 2^{n/10} \rfloor$ ms. The largest value $t_s = 90$ ms is chosen as a compromise between capturing the long-term dynamics and minimizing the influence of boundary effects. We consider $\bar{d}_k^{\text{mod}}$ as the average of $d_k^{\text{mod}}(t_s)$ over t_s .

To optimize the parameters of the intermittent search model, we use analytic expressions for $m_k^{\text{IS}}(t_s)$ as follows. We

all these functions will be proportional to $\ln t$ at large t . In our studies, the fundamental fact is that, beyond asymptotic behavior, the behavior of the system at small time values is significant. Therefore, an explicit expression of the moments is needed for proper analysis.

Indeed, differently from LWs, the planar IS allows for full analytical evaluation, and we derived the explicit expressions for the moments. Namely, the second moment reads

$$\langle \mathbf{r}^2(t) \rangle = C_1^{(2)} t - C_2^{(2)} (1 - e^{-\beta \alpha t}), \quad (3)$$

with the values of the constants given by

$$C_1^{(2)} = 2 \frac{1-\alpha}{\alpha} D_I + 4\alpha D, \quad C_2^{(2)} = 2 \frac{1-\alpha}{\alpha^2 \beta} D_I,$$

where $\beta = \lambda_{BD} + \lambda_{DB}$, $\alpha = \lambda_{BD}/\beta$ and $D_I = v_B^2/\beta$.

first minimize $\bar{d}_4^{\text{IS}}$, using the dual annealing algorithm [46] to find an initial guess of all parameters. Since estimations based on $\bar{d}_4^{\text{IS}}$ minimization alone are highly sensitive to the noise inherent when dealing with finite samples, we then minimize iteratively $\bar{d}_4^{\text{IS}}$ and $\bar{d}_2^{\text{IS}}$. Distance $\bar{d}_2^{\text{IS}}$ is used to update the estimates of D and λ_{DB} while keeping v_B and λ_{BD} unchanged, whereas we use $\bar{d}_4^{\text{IS}}$ to update the estimate of v_B and λ_{BD} . This Sinkhorn-like [47] procedure is repeated iteratively until convergence.

For the LW model, since the velocity v_L enters the definitions of the moments in a trivial way, it is here used as a scaling parameter. The two remaining parameters, τ_0 and γ , are estimated with a grid search algorithm (see the Appendixes).

To quantify how well each model approximates eye-gaze trajectories, we use the adjusted coefficient of determination [48],

$$\bar{R}_{\text{mod},k}^2 = 1 - \frac{\bar{d}_k^{\text{mod}}}{\sigma_k^2} \frac{n-1}{n-p-1}, \quad (6)$$

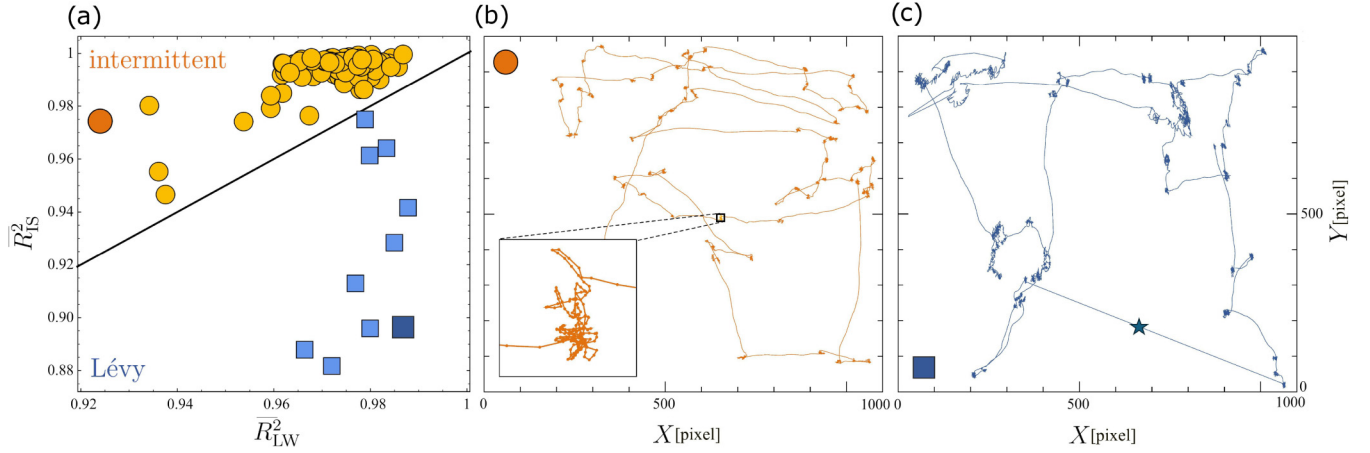


FIG. 3. Classification of gaze trajectories. (a) Out of the 120 collected trajectories, 110 ($\circ$) were classified as produced by intermittent searches and 10 ($\square$) as produced by Lévy walks. Classification is based on the adjusted coefficients of determination and a score function [cf. Eqs. (6) and (7)]. Panels (b) and (c) show 15-s fragments of the trajectories with the highest and lowest scores [large circle and square in panel (a), respectively]. Inset in panel (b) shows a 90 ms fragment. The star in panel (c) marks a blink event.

where $n = 25$, σ_k^2 represents the variance of $\log[m_k^{\text{exp}}(t_s)]$ over the t_s values, and $p = 3, 4$ is the number of free parameters of LW and IS models, respectively. Finally, we define the score function,

$$\Gamma = \bar{R}_{IS}^2 - \bar{R}_{LW}^2, \quad (7)$$

with $\bar{R}_{\text{mod}}^2 = (\bar{R}_{\text{mod},2}^2 + \bar{R}_{\text{mod},4}^2)/2$. We choose model IS if $\Gamma > 0$ and LW otherwise.

IV. DISCUSSION: ASSESSING THE BEST MODEL FOR EYE-GAZE TRAJECTORIES

Figure 3 presents the results of the classification of 120 trajectories collected in the experiment: 110 trajectories are classified as IS and only 10 as LWs. This finding corroborates the conventional (qualitative) description of eye-gaze dynamics as processes consisting of two alternating phases, ballisticlike saccades and diffuselike fixations [49–51]. However, we notice that both IS and LW present good fitness, showing values of $\bar{R}_{IS}^2$ and $\bar{R}_{LW}^2$ close to 1. The values of their differences Γ are comparable with the values obtained with the synthetic data.

It is noteworthy that, from the LW perspective, nearly all trajectories [except for the four leftmost in Fig. 3(a)] have $\bar{R}_{LW}^2$ values within the same range. It is the $\bar{R}_{LW}^2$ coefficient that reveals that most of the trajectories are actually much better approximated by the intermittent search model.

Figure 4 presents the values of the parameters of each model for every individual trajectory, indicating again, which model fits best with different symbols. From the LW perspective, the trajectories are characterized by values of the exponent $\gamma \sim 2$ [Fig. 4(a), i.e., all classified LWs lie between diffusive and superdiffusive regimes [6]], which should be a result of the specific setup of our experiment. Under different conditions, one could expect gaze trajectories with other exponent values when being modeled as LWs [26].

From the IS perspective, trajectories are characterized by distributions of the four parameters $\{D, v_B, \lambda_{BD}, \lambda_{DB}\}$ [Fig. 4(b)]. The duration of saccades is known to lie in the range between 8 and 60 ms [49,52–54], which is in good agreement with estimated values of $1/\lambda_{BD}$, shown in Fig. 4. In the same way, the distribution of values of $1/\lambda_{DB}$ reproduces fixation durations, known to lie in the range between 90 and 800 ms. Figure 4(b) clearly shows that the very few cases where LW better fits the moments of a specific eye-gaze trajectory correspond to IS parameter values, which typically span the observed range of values. All in all, while in both cases the coefficient of determination is always significantly high, typically larger than 0.88, the IS model not only “wins” in the large majority of cases, but it also achieves values of the coefficient of determination very close to one (see the Appendixes).

To better understand the connection of the four IS parameters optimally describing eye-gaze trajectories, we plot in Fig. 5(a) a ternary heatmap, with v_B , λ_{BD} , and λ_{DB} in the three axes of the ternary plot and D following a color scale. Notice that, due to geometric constraints of a ternary plot, variables must be normalized. We normalized each parameter to the respective range of observed values, from the minimum to the maximum. For these set of eye-gaze trajectories, these ranges are $v_B \in [7.73, 48.0]$, $\lambda_{BD} \in [0.048, 0.300]$, $\lambda_{DB} \in [0.00087, 0.0069]$, and $D \in [0.16, 1.18]$. While no clear structure is observed, one identifies a tendency for low values of the diffusivity to match with low values of v_B as well as a positive correlation between both switching rates.

We have also benchmarked the algorithm with 1000 synthetic trajectories of each model, with the parameter range observed in Fig. 4. The trajectories were sampled with the same frequency and with the same number of record points as in the experiment. We observe that over 99% of LW are well classified, with only six of them being labeled as IS (see the Appendixes). On the other hand, of the 1000 simulated IS, 20 were mislabeled as LW. Thus, in a similar range of parameters, as we found in our eye-tracking experiment, we observe that

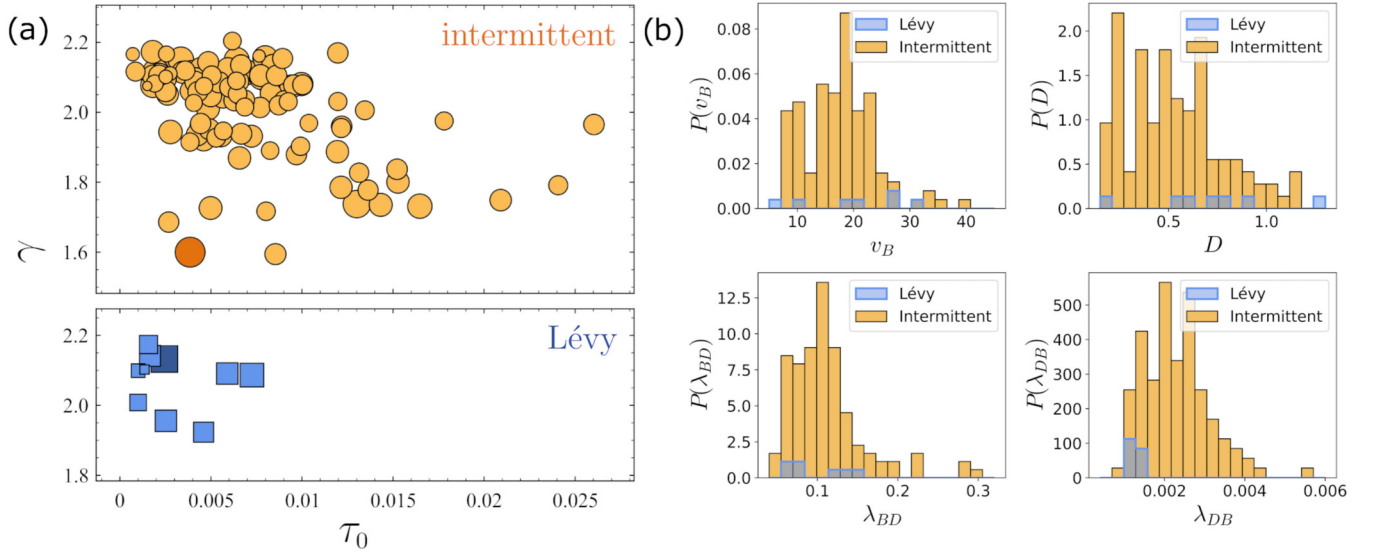


FIG. 4. Optimized parameters of the Lévy walks (a) and intermittent search (b) models for the two gaze trajectories with the largest absolute values of Γ . The area of a circle [a square] is proportional to $1/(1 - \bar{R}_{IS}^2)$ [$1/(1 - \bar{R}_{LW}^2)$]. In the case of Lévy walks, the velocity plays the role of a time scaling parameter.

the Γ classifier presents a high accuracy in determining if a given trajectory is better approximated by a LW or IS. See the left and middle plots of Fig. 6.

Finally, after determining the parameters of the LW and IS, in both empirical and synthetic trajectories, we can check which of the models is the best approximation of gaze trajectories. The right plot in Fig. 6 shows again the coefficient of determination of both models applied to the eye-gaze trajec-

tries (cf. Fig. 3). The clear distinction between synthetic LWs and IS processes (middle plot) and the only slight deviation of a few eye-gaze trajectories as LWs, using the same classification framework, provides evidence that eye-gaze trajectories seem to typically follow IS processes.

V. CONCLUSIONS

We have analyzed eye-gaze trajectories collected in an experiment stimulating free search for visual information. Using an algorithm designed to quantify the ability of IS and LW models to approximate a given trajectory, based on the second and fourth moments of eye-gaze displacements, we found that the IS model scores significantly higher than the LW model in most cases. Our results depend on the classification setup, which is based on a comparison of the performance of each model to fit both moments.

We use the models in their most basic formulations. Both can be further modified, e.g., by adding a stochastic component to the ballistic phases [55], introducing a vertical/horizontal nonuniformity, substituting the uniform planar LWs by intermittent Lévy walks [56], or, in an extreme case, considering composite random walks [57,58]. Moreover, the fact that the search is restricted to a bounded domain (the computer screen) is not taken into account, even though this might influence statistical characteristics of the process. While these amendments might bring the models close to the realm of the experiment, we do not consider them here because our objective is to formulate a well-posed dichotomous problem. Moreover, it should be noticed that we assume ergodicity [cf. Eq. (2)] and we based our analysis on the estimation of moments, instead of, e.g., cumulants. The reason why we choose to estimate moments is twofold. First, they can be estimated directly from the data, with minimal processing. Second, cumulants are functions of the respective moments of the same or lower order. In principle, the analytical expressions should

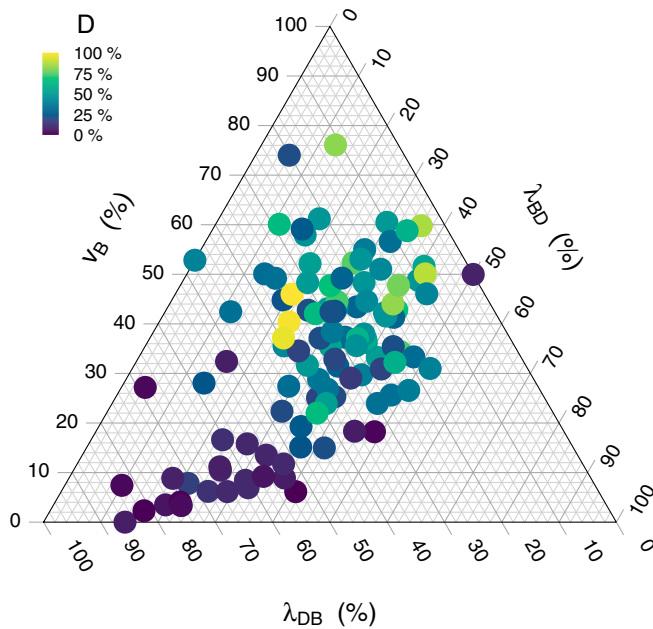


FIG. 5. Ternary heatmap to highlight the joint distribution of all four parameters of the IS when fitting eye-gaze trajectories. Here, all four parameters are normalized to the respective range of fitted values (see text).

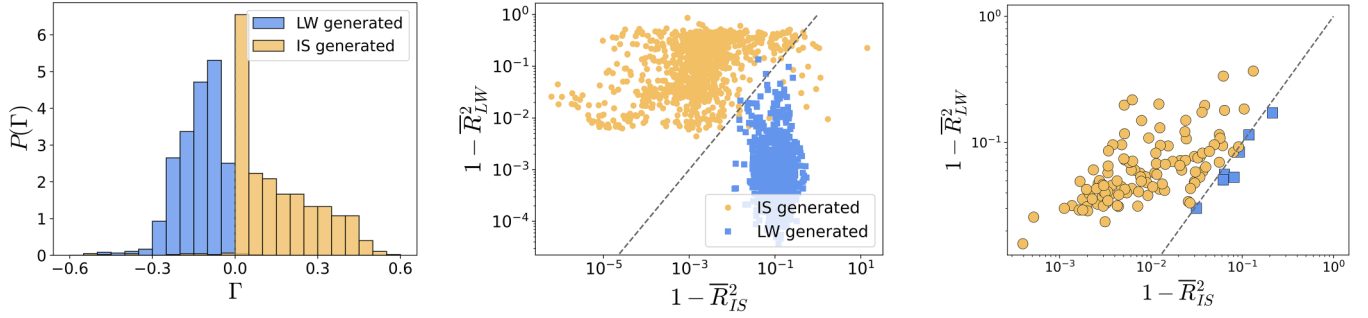


FIG. 6. Benchmark of synthetically generated LW (blue) and IS trajectories (yellow). (Left) Distribution of Γ values for 1000 synthetic realizations of LWs and of IS processes. (Middle) Respective comparison of the coefficient of determination R^2 for both models, when fitting the second and fourth moments. For LW trajectories, the parameter γ [see Eq. (5) of the main text] is less than zero ($\gamma < 0$) for the vast majority of LW trajectories (996 out of 1000). Conversely, for IS trajectories, γ exceeds zero ($\gamma > 0$) in 980 out of 1000 cases. The IS and LW trajectories were simulated using the same estimated parameters and number of data points as the empirical eye-tracking trajectories described in the main text (cf. Fig. 4). (Right) For comparison, we also show the coefficient of determination for the eye-gaze trajectories shown in Fig. 3, here in a different scale.

not be simpler than the ones derived in this paper (cf. the Appendixes).

Another simplification of our models is the assumption of unbounded environments: Theoretically, space is unlimited, although in practice it is confined. However, for the classification of trajectories, the key point is the presence of analytical dependencies at relatively small time values. This is what makes it possible to estimate the parameters in our approach. It will not be possible to obtain such formulas exactly if we introduce boundary restrictions (this can only be done in the asymptotic regime, which does not meet our requirements). The second important point that ensures the adequacy of our unbounded model is that we do not classify an ensemble of trajectories, instead, we focus on each trajectory separately, and analyze it individually, sliding over the data series with a window of a specific time length. These time spans of the windows we consider for our calculations, are typically short enough to disable the eye-gaze to reach the boundaries. Therefore, we claim that at all such distances within the domain, a theoretical model that is unconstrained but has exact moment expressions is a reasonable enough approximation. In fact, such a claim is supported by Figs. 1 and 3, which show the real trajectories of the human eye obtained in our experiment. Within the region, the movement appears as if there are no boundaries, as they are far typically not close to them. However, close to the boundaries, they usually affect the fact that relocations occur with restrictions to certain directions, a fact that could be taken into consideration in future works.

Finally, we also emphasize the idea of score-based model selection, which we believe enables us to transform the ill-posed problem of identifying the “best model” into a well-defined, implementable approach that works on the level of individual trajectories. Several steps in this direction have already been made. For instance, likelihood-based methods have been proposed to distinguish between different diffusion processes [59]. Additionally, machine-learning-based approaches, as highlighted in the recent AnDi Challenge [60,61], may offer powerful tools for score-based model selection at the single-trajectory level. Score-based classification of individual trajectories will allow replacing the task of “macro-

scopic” en masse classification of foraging patterns [12–17] with an analysis of the distributions of scores and optimized parameters, similar to the results presented in Figs. 3 and 4. This microscopic approach could offer deeper insights into the statistical features of the patterns and thus would help to understand the underlying foraging dynamics better.

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APPENDIX A: ANALYTICAL EXPRESSIONS FOR THE MOMENTS OF AN INTERMITTENT PROCESS

In this section, we derive the second and fourth moments of an intermittent process explicitly. Notice that the utilized LW and IS models are isotropic, so all odd moments are identically equal to zero. Therefore, odd moments cannot be used to estimate the parameters of experimental data. The date analysis could be implemented based on cumulants. At the same time, it is known that cumulants are expressed in terms of moments. Particularly, in the isotropic case, the second-order cumulant is equal to the second moment. In principle, one could focus on the fourth cumulant instead of the fourth moment. However, we find the present derivation more straightforward to estimate the parameter values of the intermittent process.

1. Preliminary assumptions and derivations

In the case of the intermittent process, we model eye-gaze trajectories as the planar movement of a random walker (a single particle) characterized by the following properties. Namely, we assume that one has a particle that could be in two phases, either ballistical (B) or diffusive (D) (further, for the convenience of deriving equations, we will denote phases B and D as “+” or “−”, respectively). Next, let $\psi_{\pm}(t)$ be the PDF that the particle is in phase “ $\pm$ ” without switching during time t . After this period of time, the particle switches

to another phase. The PDF that the particle being in phase “ $\pm$ ” has moved by vector $\mathbf{r}$ in time t is denoted by $w_{\pm}(\mathbf{r}, t)$.

We also assume that at the beginning of the wandering, the particle is located at the origin while staying in the phase “ $\pm$ ” with the probability $c_{\pm}$ ($c_+ + c_- = 1$). Note that in the equivalent ensemble interpretation, instead of considering a single particle’s random trajectories, we view an ensemble of identical independent particles, each of which follows its random trajectory. In this context, the quantity $c_{\pm}$ represents the fraction of particles that are initially in the phase “ $\pm$ ”.

We are looking for an equation for the PDF $P(\mathbf{r}, t)$ of finding the particle at a point $\mathbf{r}$ at a given time t . It is important to note that $P(\mathbf{r}, t)$ is the probability density function concerning a coordinate $\mathbf{r}$ only; time t serves as a parameter. We can express it as $P(\mathbf{r}, t) = P_+(\mathbf{r}, t) + P_-(\mathbf{r}, t)$, where $P_{\pm}(\mathbf{r}, t)$ represents the density corresponding to the particle located at a coordinate $\mathbf{r}$ being in the phase “ $\pm$ ” at time t .

For this purpose, we introduce the auxiliary function $v_{\pm}(\mathbf{r}, t)$, which is the density (over both variables $\mathbf{r}$ and t) such that the particle reaches the space-time coordinate $(\mathbf{r}, t)$ and changes the phase from “ $\mp$ ” to “ $\pm$ ”. In ensemble interpretation, the density $v_{\pm}(\mathbf{r}, t)$ can be interpreted as a flux describing the fraction of particles that just have arrived (per unit of time) at position $\mathbf{r}$ at time t while switching from phase “ $\mp$ ” to the opposite phase “ $\pm$ ”. It is important to note that $P_{\pm}(\mathbf{r}, t)$ and $v_{\pm}(\mathbf{r}, t)$ are densities, but they are not probability density functions since they are not normalized to one.

Further, we provide the derivation of the integral equations for $P_{\pm}(\mathbf{r}, t)$ and $v_{\pm}(\mathbf{r}, t)$ by using a similar technique as applied for obtaining the governing equations for continuous-time random walks [6]. For convenience, we give this derivation only for densities $v_+(\mathbf{r}, t)$ and $P_+(\mathbf{r}, t)$. The equations for $v_-(\mathbf{r}, t)$ and $P_-(\mathbf{r}, t)$ will follow from them by simply interchanging the corresponding indices “ $+$ ” and “ $-$ ”.

We denote by $dv_+(\mathbf{r}, t|\mathbf{r}', t')$ the conditional density of finding the particle at the space-time point $(\mathbf{r}, t)$ and in phase “ $+$ ”, given that it arrived at this point from the coordinate $(\mathbf{r}', t')$ and phase “ $-$ ”. Then, the (total) density $v_+(\mathbf{r}, t)$ can be written as the integral of the mentioned conditional density over $\mathbf{r}'$ and t' , namely,

$$v_+(\mathbf{r}, t) = c_- w_-(\mathbf{r}, t) \psi_-(t) + \int dv_+(\mathbf{r}, t|\mathbf{r}', t'). \quad (\text{A1})$$

Here, the integration volume is defined by the condition $\mathbf{r}' \in \mathbb{R}^2$ and $t' \in (0, t]$. The first term in Eq. (A1) captures the scenario where the particle remained in the initial phase “ $-$ ” all the time (respectively, whole movement to the position $\mathbf{r}$ occurred in this phase) and only at the final instant t the particle switched to phase “ $+$ ”.

Since $\psi_-(t')$ is the PDF of being in a phase “ $-$ ” during time t' , then the probability that a transition from “ $-$ ” to “ $+$ ” happened in time interval $(t', t' + dt')$ is equal to $\psi_-(t') dt'$. Similarly, the probability that during time t' the particle (being in the phase “ $-$ ”) moved to coordinate within the interval $(\mathbf{r}', \mathbf{r}' + d\mathbf{r}')$ is written as $w_-(\mathbf{r}', t') d\mathbf{r}'$. As a result, the conditional density $dv_+(\mathbf{r}, t|\mathbf{r}', t')$ can be represented as a product of the density $v_-(\mathbf{r}', t')$ of finding a particle in phase “ $-$ ” and a coordinate $(\mathbf{r}', t')$, and the joint probability that the

remaining time in this phase will be equal exactly to $t - t'$, and the displacement will be exactly by the vector $\mathbf{r} - \mathbf{r}'$ (i.e., at the instant t the particle switches to phase “ $+$ ”). The mentioned joint probability is given by the expression $\psi_-(t - t') dt' w_-(\mathbf{r} - \mathbf{r}', t - t') d\mathbf{r}'$. Thus, we have

$$dv_+(\mathbf{r}, t|\mathbf{r}', t') = v_-(\mathbf{r}', t') w_-(\mathbf{r} - \mathbf{r}', t - t') d\mathbf{r}' \psi_-(t - t') dt'. \quad (\text{A2})$$

By using formulas (A1) and (A2), and the convolution property of functions, we obtain the equation for $v_+(\mathbf{r}, t)$,

$$v_+(\mathbf{r}, t) = c_- \psi_-(t) w_-(\mathbf{r}, t) + \int_0^t dt' \psi_-(t') \int_{\mathbb{R}^2} d\mathbf{r}' w_-(\mathbf{r}', t') v_-(\mathbf{r} - \mathbf{r}', t - t'). \quad (\text{A3})$$

Analogously to Eq. (A1), the expression for the PDF $P_+(\mathbf{r}, t)$ takes the form

$$P_+(\mathbf{r}, t) = c_+ \Psi_+(t) w_+(\mathbf{r}, t) + \int dP_+(\mathbf{r}, t|\mathbf{r}', t'), \quad (\text{A4})$$

with the integration domain defined by the condition $\mathbf{r}' \in \mathbb{R}^2$ and $t' \in (0, t]$. Here, we define the survival probability $\Psi_+(t) = \int_t^\infty \psi_+(t') dt'$, i.e., the probability that a walking particle remains at the initial position up to time t . Also, we introduce the conditional density $dP_+(\mathbf{r}, t|\mathbf{r}', t')$ of finding the particles located at the coordinate $\mathbf{r}$ in phase “ $+$ ” at time t , given that it was at the point $\mathbf{r}'$ at the earlier time t' . Once again, the first term in the expression (A4) accounts for the situation when the particle remains in its initial phase and does not transition to the opposite phase. The second term considers all other possible paths the particle could take.

Similarly to Eq. (A2), one can deduce that

$$dP_+(\mathbf{r}, t|\mathbf{r}', t') = v_+(\mathbf{r}', t') dt' w_+(\mathbf{r} - \mathbf{r}', t - t') d\mathbf{r}' \Psi_+(t - t'). \quad (\text{A5})$$

The expression (A5) includes the quantity $v_+(\mathbf{r}', t') dt'$, which represents the density (with respect to the variable $\mathbf{r}'$) indicating that the particle changes its phase from “ $-$ ” to “ $+$ ” and is positioned at the coordinate $\mathbf{r}'$ at a time point within the interval $(t', t' + dt')$. For the remaining time, $t - t'$, the particle is in phase “ $+$ ” and moves by vector $\mathbf{r} - \mathbf{r}'$. Consequently, we have a multiplier $w_+(\mathbf{r} - \mathbf{r}', t - t') d\mathbf{r}'$ which expresses the conditional probability that over time $t - t'$ the displacement along the spatial coordinate falls within the interval $(\mathbf{r} - \mathbf{r}', \mathbf{r} - \mathbf{r}' + d\mathbf{r}')$. The latter multiplier, $\Psi_+(t - t')$, ensures that during time $t - t'$ the particle remains in phase “ $+$ ”. Therefore, the equation that gives the density $P_+(\mathbf{r}, t)$ is as follows:

$$P_+(\mathbf{r}, t) = c_+ \Psi_+(t) w_+(\mathbf{r}, t) + \int_0^t dt' \Psi_+(t') \int_{\mathbb{R}^2} d\mathbf{r}' w_+(\mathbf{r}', t') v_+(\mathbf{r} - \mathbf{r}', t - t'). \quad (\text{A6})$$

For convenience, let us introduce

$$q_{\pm}(\mathbf{r}, t) = \psi_{\pm}(t) w_{\pm}(\mathbf{r}, t), \quad Q_{\pm}(\mathbf{r}, t) = \Psi_{\pm}(t) w_{\pm}(\mathbf{r}, t). \quad (\text{A7})$$

Then, Eqs. (A3) and (A6) can be rewritten as

$$\begin{aligned} v_+(\mathbf{r}, t) &= c_- q_-(\mathbf{r}, t) \\ &+ \int_0^t dt' \int_{\mathbb{R}^2} d\mathbf{r}' q_-(\mathbf{r}', t') v_-(\mathbf{r} - \mathbf{r}', t - t'), \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} P_+(\mathbf{r}, t) &= c_+ \Psi_+(t) w_+(\mathbf{r}, t) \\ &+ \int_0^t dt' \int_{\mathbb{R}^2} d\mathbf{r}' Q_+(\mathbf{r}', t') v_+(\mathbf{r} - \mathbf{r}', t - t'). \end{aligned} \quad (\text{A9})$$

As previously mentioned, the equations for $v_-(\mathbf{r}, t)$ and $P_-(\mathbf{r}, t)$ follow from Eqs. (A8) and (A9) by changing the corresponding indices to their opposites.

As a consequence, in the Fourier-Laplace space, we obtain

$$v_{\pm}(\mathbf{k}, s) = q_{\mp}(\mathbf{k}, s)[c_{\mp} + v_{\mp}(\mathbf{k}, s)], \quad (\text{A10})$$

$$P_{\pm}(\mathbf{k}, s) = Q_{\pm}(\mathbf{k}, s)[c_{\pm} + v_{\pm}(\mathbf{k}, s)], \quad (\text{A11})$$

where argument $\mathbf{k}$ indicates the Fourier transform of the respective function with argument $\mathbf{r}$, and argument s indicates the Laplace transform of the respective function with argument t , i.e.,

$$\begin{aligned} f(\mathbf{k}) &= \mathcal{F}_{\mathbf{r} \rightarrow \mathbf{k}}\{f(\mathbf{r})\} = \int_{\mathbb{R}^2} e^{i\mathbf{k} \cdot \mathbf{r}} f(\mathbf{r}) d\mathbf{r}, \\ g(s) &= \mathcal{L}_{t \rightarrow s}\{g(t)\} = \int_0^{\infty} e^{-st} g(t) dt. \end{aligned}$$

Solving the system of equations (A10) and (A11) with respect to $v_{\pm}(\mathbf{r}, t)$, we find

$$v_{\pm}(\mathbf{k}, s) = q_{\mp}(\mathbf{k}, s) \frac{c_{\mp} + c_{\pm} q_{\pm}(\mathbf{k}, s)}{1 - q_{\pm}(\mathbf{k}, s) q_{\mp}(\mathbf{k}, s)}. \quad (\text{A12})$$

Next, taking into account Eqs. (A12) and (A11), we have

$$P_{\pm}(\mathbf{k}, s) = Q_{\pm}(\mathbf{k}, s) \frac{c_{\pm} + c_{\mp} q_{\mp}(\mathbf{k}, s)}{1 - q_{\pm}(\mathbf{k}, s) q_{\mp}(\mathbf{k}, s)}. \quad (\text{A13})$$

Therefore, the required propagator in the Fourier-Laplace space is written as

$$\begin{aligned} P(\mathbf{k}, s) &= \frac{Q_+(\mathbf{k}, s)[c_+ + c_- q_-(\mathbf{k}, s)] + Q_-(\mathbf{k}, s)[c_- + c_+ q_+(\mathbf{k}, s)]}{1 - q_+(\mathbf{k}, s) q_-(\mathbf{k}, s)}. \end{aligned} \quad (\text{A14})$$

Now, to derive the final expression for the propagator, we take into account the specification of our model.

We first write down the probability density of the duration of each of the phases, ballistic (B) and diffusive (D), specifically

$$\psi_B(t) = \lambda_B e^{-\lambda_B t}, \quad \psi_D(t) = \lambda_D e^{-\lambda_D t}. \quad (\text{A15})$$

In the main text of the paper, we denote the transition rates from phase B to D and from phase D to B as λ_{BD} and λ_{DB} , respectively. In this section, for compactness of the notation, we denote λ_{BD} as λ_B and λ_{DB} as λ_D . Next, for our isotropic case, the conditional probabilities that the particle being in each phase (B or D) has the position $\mathbf{r}$ at time t are given by

$$w_B(\mathbf{r}, t) = \frac{\delta(r - v_B t)}{2\pi r}, \quad w_D(\mathbf{r}, t) = \frac{1}{4\pi D t} \exp\left(-\frac{r^2}{4Dt}\right), \quad (\text{A16})$$

where $r = |\mathbf{r}|$.

By replacing the indices “ $\pm$ ” with “ B ” and “ D ”, and using the probability densities (A15) and (A16), Eq. (A7) yields the following:

$$\begin{aligned} q_B(\mathbf{k}, s) &= \frac{\lambda_B}{\sqrt{(s + \lambda_B)^2 + v_B^2 k^2}}, & q_D(\mathbf{k}, s) &= \frac{\lambda_D}{s + \lambda_D + Dk^2}, \\ Q_B(\mathbf{k}, s) &= \frac{1}{\sqrt{(s + \lambda_B)^2 + v_B^2 k^2}}, & Q_D(\mathbf{k}, s) &= \frac{1}{s + \lambda_D + Dk^2}, \end{aligned} \quad (\text{A17})$$

where $k = |\mathbf{k}|$. Finally, using these formulas and Eq. (A14), we obtain the propagator

$$P(\mathbf{k}, s) = \frac{c_B(s + \lambda_D + Dk^2) + c_D \sqrt{(s + \lambda_B)^2 + v_B^2 k^2} + c_B \lambda_B + c_D \lambda_D}{(s + \lambda_D + Dk^2) \sqrt{(s + \lambda_B)^2 + v_B^2 k^2} - \lambda_B \lambda_D}. \quad (\text{A18})$$

2. Derivation of the second moment

The second moment of the intermittent process can be written as

$$\begin{aligned} \langle \mathbf{r}^2(t) \rangle &= \int_{\mathbb{R}^2} r^2 P(\mathbf{r}, t) d\mathbf{r} \\ &= \mathcal{L}_{t \rightarrow s}^{-1} \left\{ - \left(\frac{\partial^2}{\partial k_x^2} + \frac{\partial^2}{\partial k_y^2} \right) P(\mathbf{k}, s) \Big|_{\mathbf{k}=0} \right\}. \end{aligned} \quad (\text{A19})$$

Using expressions (A18) and (A19), along with the application of the inverse Laplace transform, we can derive the second moment

$$\langle \mathbf{r}^2(t) \rangle = C_1^{(2)} t + C_2^{(2)} + C_3^{(2)} e^{-\lambda_B t} + C_4^{(2)} e^{-(\lambda_B + \lambda_D) t}, \quad (\text{A20})$$

where constants

$$\begin{aligned} C_1^{(2)} &= 2 \frac{2D\lambda_B^2 + v_B^2 \lambda_D}{\lambda_B(\lambda_B + \lambda_D)}, \\ C_2^{(2)} &= \frac{2}{\lambda_B^2(\lambda_B + \lambda_D)^2} \{ 2D\lambda_B^2(\lambda_D c_D - \lambda_B c_B) \\ &\quad + v_B^2 [\lambda_B^2 c_B - \lambda_B \lambda_D(1 + c_D) - \lambda_D^2] \}, \\ C_3^{(2)} &= 2 \frac{v_B^2(\lambda_D - \lambda_B c_B)}{\lambda_B^2 \lambda_D}, \\ C_4^{(2)} &= 2 \frac{(2D\lambda_D + v_B^2)(\lambda_B c_B - \lambda_D c_D)}{\lambda_D(\lambda_B + \lambda_D)^2}. \end{aligned}$$

Let us remind that c_B and c_D are the initial probabilities to start the process in the ballistic or diffusive phase, respectively. For equilibrated starting conditions, we have

$$c_B = \frac{\lambda_D}{\lambda_D + \lambda_B}, \quad c_D = \frac{\lambda_B}{\lambda_D + \lambda_B}. \quad (\text{A21})$$

In this case, the second moment simplifies to

$$\langle \mathbf{r}^2(t) \rangle = C_1^{(2)}t - C_2^{(2)}(1 - e^{-\beta\alpha t}), \quad (\text{A22})$$

with the values of the constants

$$C_1^{(2)} = 2 \frac{1 - \alpha}{\alpha} D_I + 4\alpha D, \\ C_2^{(2)} = 2 \frac{1 - \alpha}{\alpha^2 \beta} D_I.$$

Here, we introduce the following parameters:

$$\beta = \lambda_B + \lambda_D, \quad \alpha = \frac{\lambda_B}{\beta}, \quad D_I = \frac{v_B^2}{\beta}. \quad (\text{A23})$$

3. Derivation of the fourth moment

The fourth-order moment can be calculated similarly to the second-order moment. At first, we express it as

$$\langle \mathbf{r}^4(t) \rangle = \int_{\mathbb{R}^2} r^4 P(\mathbf{r}, t) d\mathbf{r} \\ = \mathcal{L}_{t \rightarrow s}^{-1} \left\{ \left(\frac{\partial^4}{\partial k_x^4} + 2 \frac{\partial^2}{\partial k_x^2} \frac{\partial^2}{\partial k_y^2} + \frac{\partial^4}{\partial k_y^4} \right) P(\mathbf{k}, s) \Big|_{\mathbf{k}=0} \right\}. \quad (\text{A24})$$

Then, using the expression for $P(\mathbf{k}, s)$, Eq. (A18), and applying the inverse Laplace transform, we obtain

$$\langle \mathbf{r}^4(t) \rangle = C_1^{(4)}t^2 + C_2^{(4)}t + C_3^{(4)} + C_4^{(4)}t^2 e^{-\lambda_B t} + C_5^{(4)}t e^{-\lambda_B t} \\ + C_6^{(4)}t e^{-(\lambda_B + \lambda_D)t} + C_7^{(4)}e^{-\lambda_B t} + C_8^{(4)}e^{-(\lambda_B + \lambda_D)t}, \quad (\text{A25})$$

with

$$C_1^{(4)} = \frac{8}{\lambda_B^2(\lambda_B + \lambda_D)^2} [2D\lambda_B^2 + v_B^2\lambda_D]^2, \\ C_2^{(4)} = \frac{8}{\lambda_B^3(\lambda_B + \lambda_D)^3} \{ -8c_B D^2 \lambda_B^5 + 4[2(1 + c_D)D^2 \lambda_D + c_B v_B^2 D] \lambda_B^4 - 20v_B^2 D \lambda_D \lambda_B^3 \\ + [v_B^2(2c_B + 3) - 4D(c_B + 1)\lambda_D] v_B^2 \lambda_D \lambda_B^2 - 2v_B^4(1 + c_D) \lambda_D^2 \lambda_B - 3\lambda_D^3 v_B^4 \}, \\ C_3^{(4)} = \frac{8}{\lambda_B^4(\lambda_B + \lambda_D)^4} \{ 8c_B D^2 \lambda_B^6 - (16D^2 \lambda_D + 12Dc_B v_B^2) \lambda_B^5 + [3c_B v_B^4 + 4v_B^2(8 + c_D)D\lambda_D + 8c_D D^2 \lambda_D^2] \lambda_B^4 \\ + v_B^2[(12c_B + 16)D\lambda_D - v_B^2(12 + c_B)] \lambda_D \lambda_B^3 + [4(1 + c_B)D\lambda_D - (3 + 7c_B)v_B^2] v_B^2 \lambda_D^2 \lambda_B^2 \\ + 3v_B^4(1 + c_D) \lambda_D^3 \lambda_B + 3v_B^4 \lambda_D^4 \}, \\ C_4^{(4)} = \frac{4}{\lambda_B^2 \lambda_D} v_B^4 (-c_B \lambda_B + \lambda_D), \\ C_5^{(4)} = \frac{8v_B^2}{\lambda_B^2 \lambda_D^2} [-c_B v_B^2 \lambda_B + (1 + c_B)v_B^2 \lambda_D], \\ C_6^{(4)} = \frac{16}{\lambda_D^2(\lambda_B + \lambda_D)^3} [(-2c_B D v_B^2 \lambda_D - c_B v_B^4) \lambda_B^2 + (c_D v_B^4 + 4c_B D^2 \lambda_D^2 + 2v_B^2 D \lambda_D) \lambda_D \lambda_B \\ - 2c_D D(v_B^2 + 2D\lambda_D) \lambda_D^3], \\ C_7^{(4)} = \frac{8}{\lambda_B^4 \lambda_D^3} \{ -(3v_B^2 + 4D^2 \lambda_D) c_B v_B^2 \lambda_B^3 + [(3 + 5c_B)v_B^2 + (1 + c_B)4D\lambda_D] v_B^2 \lambda_D \lambda_B^2 \\ - 3(2 + c_B) v_B^4 \lambda_D^2 \lambda_B + 3v_B^4 \lambda_D^3 \}, \\ C_8^{(4)} = \frac{8}{\lambda_D^3(\lambda_B + \lambda_D)^4} \{ (3v_B^2 + 4D\lambda_D) c_B v_B^2 \lambda_B^3 + [(2c_B - c_D)4v_B^2 D \lambda_D + (4c_B - 3c_D)v_B^4 + 8c_B D^2 \lambda_D^2] \lambda_D \lambda_B^2 \\ - [4v_B^2(4 + c_B)D\lambda_D + 16D^2 \lambda_D^2 + (5 + c_D)v_B^4] \lambda_D^2 \lambda_B + c_D [8D^2 \lambda_D^2 + 3v_B^4 + 12Dv_B^2 \lambda_D] \lambda_D^3 \}.$$

Again, if we assume equilibrated starting conditions, Eq. (A21), the expression (A25) reduces to

$$\langle \mathbf{r}^4(t) \rangle = C_1^{(4)}t^2 + C_2^{(4)}t - C_3^{(4)} - C_4^{(4)}e^{-\beta t} + (C_5^{(4)}t^2 + C_6^{(4)}t - C_7^{(4)})e^{-\beta\alpha t}, \quad (\text{A26})$$

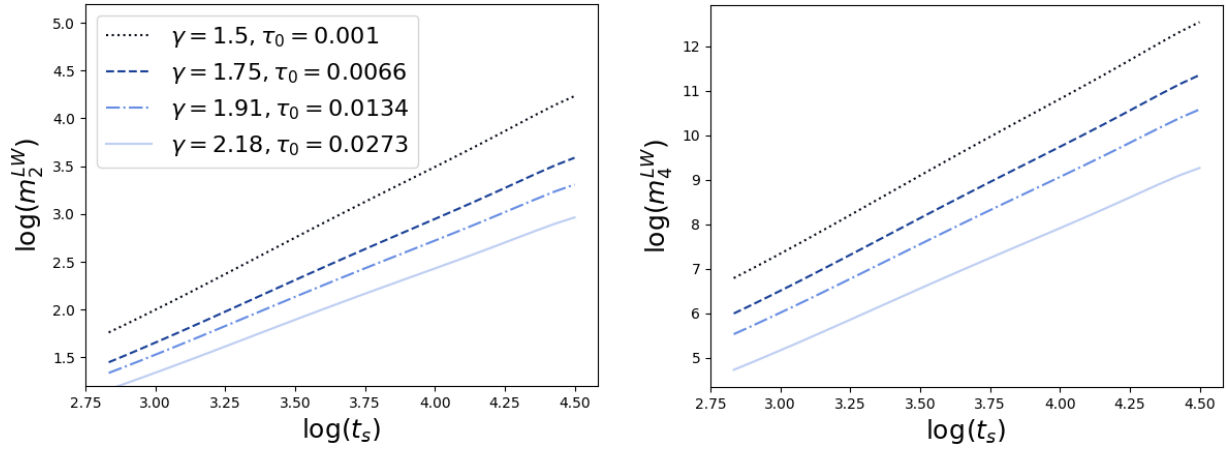


FIG. 7. Sampled values of $m_2^{\text{LW}(1,\gamma,\tau_0)}t$ and $m_4^{\text{LW}(1,\gamma,\tau_0)}(t_s)$ for different values of γ and τ_0 . We observe that, although there is a slight curvature for some of the scaling of moments, the curves are well approximated by a line.

with the coefficients

$$\begin{aligned}
 C_1^{(4)} &= 8\alpha^2 \left(2D + \frac{1-\alpha}{\alpha^2} D_I \right)^2, & C_2^{(4)} &= 8 \frac{1-\alpha}{\alpha^3 \beta^2} \left[\left(D - \frac{\alpha+1}{2\alpha^2} D_I \right)^2 - \frac{5}{8\alpha^4} D_I^2 \right], \\
 C_3^{(4)} &= 4 \frac{1-\alpha}{\alpha^4 \beta^4} \left[\left(2D - \frac{\alpha^2 + \alpha + 1}{\alpha^3} D_I \right)^2 - \frac{6\alpha^3 - 6\alpha^2 - \alpha - 2}{2\alpha^6} \right], & C_4^{(4)} &= 32(1-\alpha)\alpha\beta \left(2D + \frac{1}{1-\alpha} D_I \right)^2, \\
 C_5^{(4)} &= 4 \frac{1-\alpha}{\alpha^2} D_I^2, & C_6^{(4)} &= \frac{16}{\alpha^2 \beta} D_I^2, & C_7^{(4)} &= \frac{8}{\alpha^2 \beta^2} D_I \left(8D + \frac{11\alpha^2 - 12\alpha + 3}{\alpha^2(1-\alpha)} D_I \right)^2.
 \end{aligned}$$

APPENDIX B: A DISTANCE TO COMPARE MOMENTS OF LÉVY WALKS WITH THE ONES OF INTERMITTENT STRATEGIES

In the main text, we have introduced a metric d_k^{mod} to quantify the distance between the values of the moments $m_k(t_s)$, estimated from the experimental data and with each model (“mod”), Lévy walk and intermittent strategies. Here, we demonstrate that this metric is a distance in a mathematical sense.

For this purpose, we represent by x and y the estimated moments, x of the data and y of the model, and we explicitly indicate the distance as a function of these two variables. For simplicity of symbology, we denote our metric as

$$D(x, y) = \left| \log \left(\frac{x}{y} \right) \right|.$$

Clearly, $D(x, y) = d_k^{\text{mod}} = 0$ (for both $k = 2, 4$), when $x = y$. Otherwise, $D(x, y) > 0$. Through the use of the absolute

value of a logarithm, it is also straightforward that the metric is symmetric, $D(x, y) = D(y, x)$.

Finally, the triangle inequality can be proven in the following way. Without loss of generality, we will assume $0 < x \leq y \leq z$, for three different values of the moments, in order we omit the modulus in the formula of the metric due to its symmetric property and the previous assumption. In this way, one has

$$\begin{aligned}
 D(x, y) &= \log \left(\frac{y}{x} \right) = \log \left(\frac{z}{x} \right) + \log \left(\frac{y}{z} \right) \\
 &\quad - 2 \log \left(\frac{z}{y} \right) \leq d(x, z) + d(y, z), \\
 D(x, z) &= \log \left(\frac{z}{x} \right) = \log \left(\frac{y}{x} \right) + \log \left(\frac{z}{y} \right) \\
 &= d(x, y) + d(y, z),
 \end{aligned}$$

TABLE I. Bias and standard deviation of the LW and IS parameter as defined in Eq. (C5) and (C6). We observe a relatively high standard deviation for the LW parameters when compared to the respective bias. For IS this is observed for v_B and D . For all these cases, the smaller bias means that the deviation between simulation and estimated parameter values is within numerical precision. For the transition rates λ_B and λ_D the bias are larger and slightly beyond numerical precision. This can be corroborated by a visual inspection of Figs. 8 and 9.

	v_L	γ	τ_0	v_B	D	λ_B	λ_D
ρ	8.23×10^{-01}	-3.48×10^{-02}	1.04×10^{-03}	1.12	-1.51×10^{-01}	-3.26×10^{-01}	-7.56×10^{-03}
σ	5.28	1.40×10^{-01}	5.96×10^{-03}	8.77	1.85×10^{-01}	2.06×10^{-01}	4.07×10^{-03}

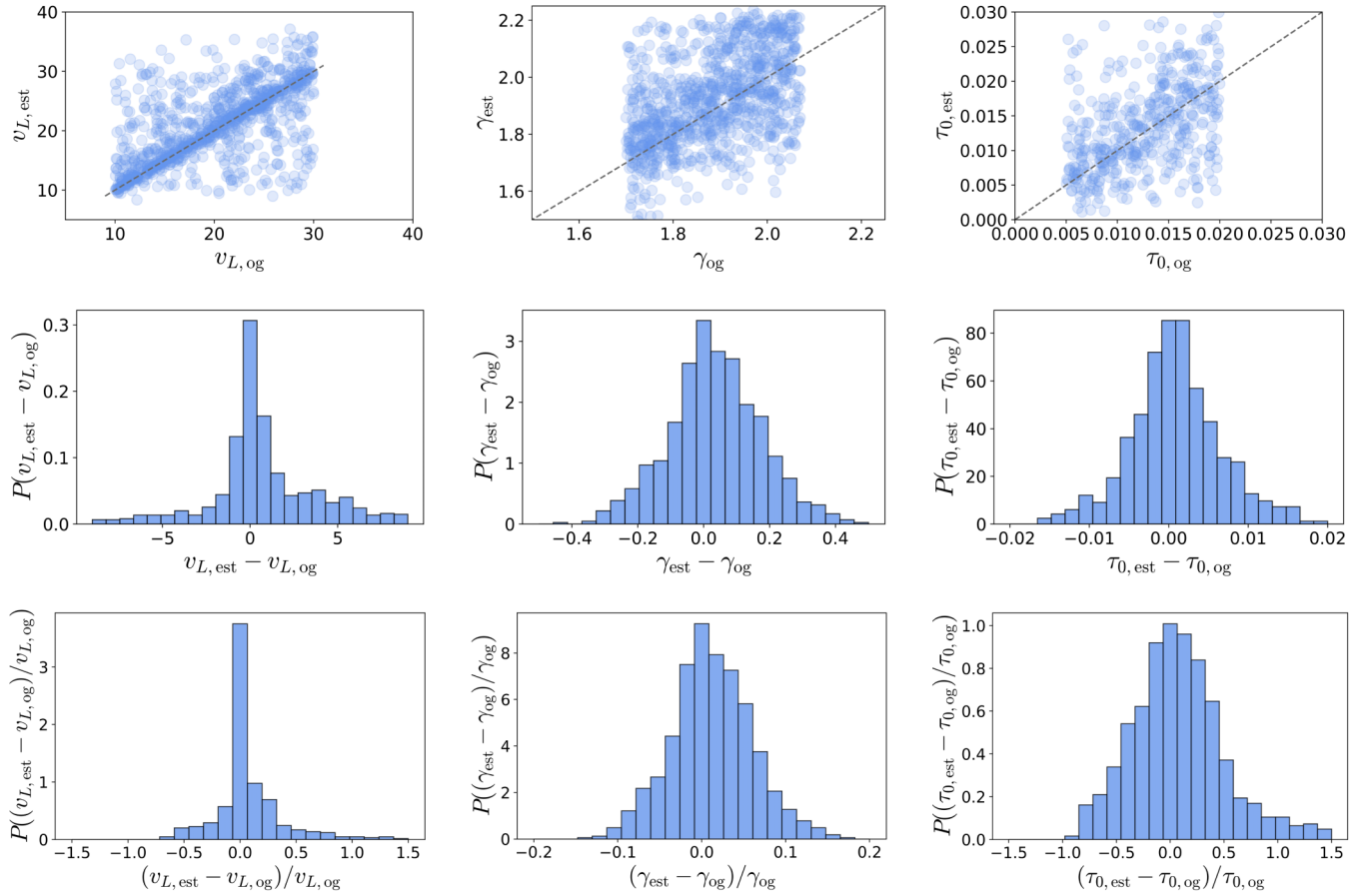


FIG. 8. Benchmark of the estimation accuracy of the simulated LW parameters. We observe that there is no significant bias in the estimation of the parameters. However, also note that there is a significant dispersion in the estimation of γ relative to the range of values that γ can take (cf. Fig. 4), namely that the standard deviation $\sigma(\gamma_{og} - \gamma_{est}) = 0.14$, where γ_{og} represents the original simulated parameter and γ_{est} the estimated value. The same happens for τ_0 , with a range of values between 0.001 and 0.02, we have $\sigma(\tau_{0,og} - \tau_{0,est}) = 0.006$. For better comparison, the last row provides the corresponding distribution of the relative error of each parameter.

$$D(y, z) = \log\left(\frac{z}{y}\right) = \log\left(\frac{y}{x}\right) + \log\left(\frac{z}{x}\right) - 2 \log\left(\frac{y}{x}\right) \leq d(x, y) + d(x, z).$$

So, the triangle inequality holds.

APPENDIX C: CHOOSING THE BEST MODEL AND ESTIMATING ITS PARAMETERS

1. The Lévy walk model

In a Lévy walk, the k moment of the displacements ($m_k^{LW(v_L, \gamma, \tau_0)}(t_s)$) scales with the velocity magnitude as follows:

$$m_k^{LW(v_L, \gamma, \tau_0)}(t_s) = v_L^k m_k^{LW(1, \gamma, \tau_0)}(t_s). \quad (C1)$$

Thus, when fitting the curve $\log(m_k^{LW(v_L, \gamma, \tau_0)}(t_s))$ to $\log(m_k^{emp}(t_s))$, we can just estimate v_L considering only one moment as

$$\log \hat{v}_L = \frac{1}{k} (\langle \log(m_k^{LW(1, \gamma, \tau_0)}(t_s)) \rangle_{t_s} - \langle \log(m_k^{emp}(t_s)) \rangle_{t_s}). \quad (C2)$$

Since we are considering second and fourth moments, we choose the estimate of v_L

$$\hat{v}_L = \exp\left(\frac{1}{2} \sum_{k=2,4} \frac{1}{k} (\langle \log(m_k^{LW(1, \gamma, \tau_0)}(t_s)) \rangle_{t_s} - \langle \log(m_k^{emp}(t_s)) \rangle_{t_s})\right). \quad (C3)$$

The only known expressions for the moments of Lévy walks are asymptotic in the sense that it is assumed that $\frac{\tau_0}{t_s} \sim 0$. In this case, the scaling of the momenta with t_s takes the form

$$m_{2n}^{LW}(t_s) \sim \begin{cases} t_s^{2n}, & \gamma < 1, \\ t_s^{2n+1-\gamma}, & 1 < \gamma < 2, \\ t_s^n, & \gamma > 2. \end{cases} \quad (C4)$$

In this asymptotic regime, some expressions relating the moments and τ_0 and v_L are known for particular parametrizations of the waiting time distribution [see Eq. (1) of the main text] [62]. However, in our case, we observe significant deviations from these expressions, although, as we see in Fig. 7, the moments $m_2^{LW}(t_s)$ and $m_4^{LW}(t_s)$ are well approximated by a power law (a line in a log-log plot).

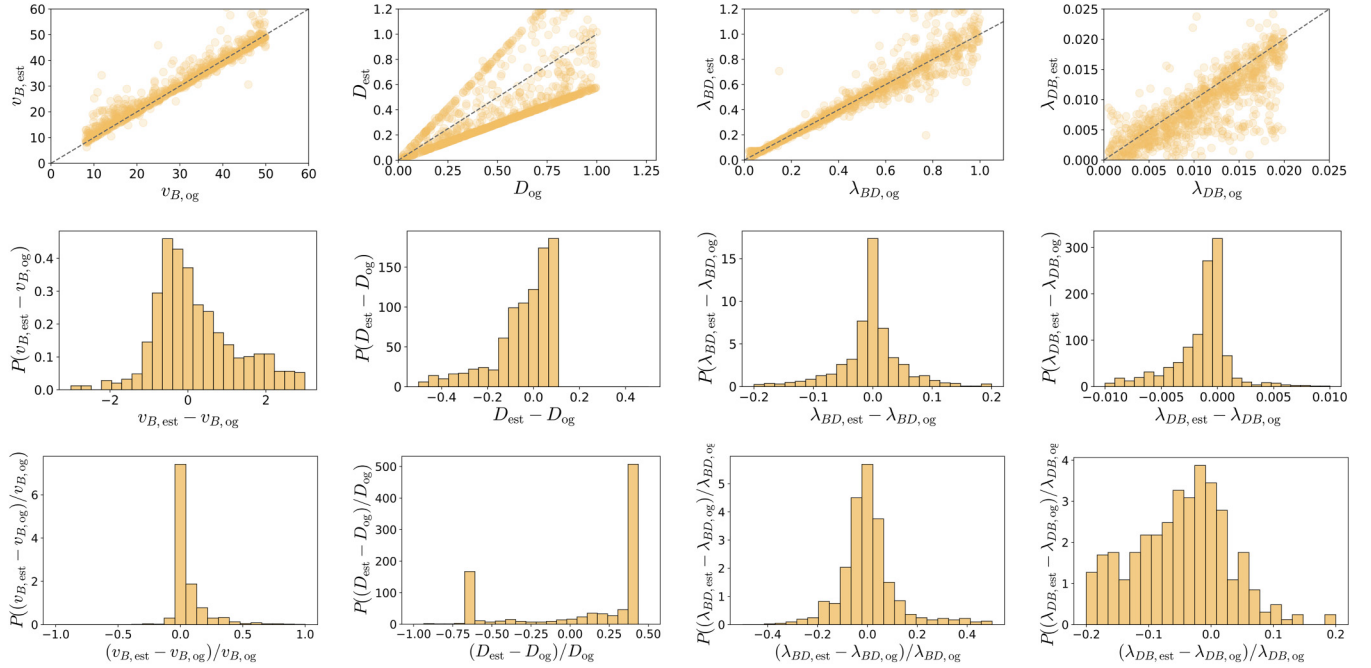


FIG. 9. Benchmark of the estimation accuracy of the simulated IS parameters. Here we observe that the parameters' estimation does not have a significant bias and that the range of fluctuations in the estimation is smaller than the range of values those parameters take in experimental settings. To note the estimation of the parameter D , which has a typical deviation of around 30% of its simulated variable. This happens since there is a trade-off between optimizing the estimation of the parameters and obtaining the highest possible value of R_{IS}^2 (see text). The last row provides the corresponding distribution of the relative error of each parameter.

We thus create a grid of values of $m_2^{\text{LW}(1,\gamma,\tau_0)}(t_s)$ and $m_4^{\text{LW}(1,\gamma,\tau_0)}(t_s)$ which is created by sampling 10^5 Lévy walks with $N = 11\,250$ data points with a specific γ and τ_0 . The number of data points in each trajectory N was chosen in order to match the number of data points we collected. In our study, we sample 1000 γ values between the values of 1.4 to 2.4 and 1000 values of τ_0 , logarithmically distributed from 0.1 to 0.001 to a total of 10^6 possible combinations of values of (γ and τ_0). Given a series of empirical points (X_t, Y_t) with second moment $m_2^{\text{emp}}(t_s)$ and fourth moment $m_4^{\text{emp}}(t_s)$, we find the estimate of γ and τ_0 by finding the set of these values that maximizes $\bar{R}_{\text{LW},k}^2$.

When it comes to estimating the parameters, we simulated 1000 LWs with randomly selected values of $\gamma \in [1.75, 2.05]$, $\tau_0 \in [0.005, 0.02]$, and $v_L \in [10, 30]$. To determine the quality of the estimation of the parameters, we define the bias (ρ_Z) and the standard deviation (σ_Z) of the estimator as

$$\rho_Z = \langle Z_{\text{est}} - Z_{\text{og}} \rangle, \quad (\text{C5})$$

$$\sigma_Z = \sqrt{\langle (Z_{\text{est}} - Z_{\text{og}})^2 \rangle}, \quad (\text{C6})$$

where Z standards for a parameter of either the LW or IS model.

The results can be seen in Table I, where, in conjunction with Fig. 8, we observe different dynamics when it comes to estimating the different LW parameters. In particular, we observe that there is a slight positive bias when it comes to v_L , but a very small or negligible one when it comes to γ and τ_0 ,

while when it comes to σ_Z we observe that v_L has the lowest typical deviation among all parameters.

2. Fitting the IS moments and estimating the parameters

As we mentioned before, the second moment $m_2(t_s)$ is not enough to estimate the parameters of the IS and that is the reason why $m_4(t_s)$ is needed in the first place.

However, the estimation of the IS parameters is more complex than simply optimizing for the fit of $m_4(t_s)$ or optimizing at the same time using as a target function one single function of $m_2(t_s)$ and $m_4(t_s)$. We observe that, with this approach, the estimation of D_{est} typically deviates by several orders of magnitude from the value of D_{og} . We found, instead, that one should estimate the parameters for $m_2(t_s)$ and $m_4(t_s)$ iteratively, as described below.

We also note that, given the presence of finite sampling noise, often the estimation of IS parameters finds local minima. To combat that, we will use a simulated annealing approach throughout our algorithm. Given a planar trajectory (X_t, Y_t) , with a second moment $m_2^{\text{emp}}(t_s)$ and fourth moment $m_4^{\text{emp}}(t_s)$ the algorithm proceeds as follows:

(1) The simulated annealing procedure optimized the parameters based on a pre-determined range of parameters. If this range is not known, as is our case, a pre-estimation of the values is performed. This is done using already established routines to estimate the parameters of hidden Markov models. This is then used to establish a rough acceptable range for the parameters.

(2) After having an acceptable range of parameters, we fix the parameters D and λ_{DB} optimize the parameters v_B and λ_{BD} so that the expression of $\log(m_{4,IS,v_B,D,\lambda_{BD},\lambda_{DB}}(t_s))$ matches $\log(m'_4(t_s))$ as best possible. The optimization is done via simulated annealing and several iterations each with a different starting point, which is selected from a grid of possible values of v_B and λ_{BD} . This results in several parameter values and, from these, we chose the ones that better fit $\log(m_{4,IS,v_B,D,\lambda_{BD},\lambda_{DB}}(t_s))$ to $\log(m'_4(t_s))$ and $\log(m_{2,IS,v_B,D,\lambda_{BD},\lambda_{DB}}(t_s))$ to $\log(m'_2(t_s))$.

(3) Analogously, in this step we fix v_B and λ_{BD} and optimize D and λ_{DB} so that the expression of $\log(m_{2,IS,v_B,D,\lambda_{BD},\lambda_{DB}}(t_s))$ matches $\log(m'_2(t_s))$ as best possible. Similarly, we repeat this procedure for several possible initial values.

(4) Steps 2 and 3 are repeated 20 times. Each iteration is accepted if it improves the fit of the moments. This is done until a final value of the parameters is found.

(5) Even if we have used simulated annealing and taken further precautions to avoid local minima in our estimation, this still happens. For that reason, we repeat Steps 2, 3, and 4 a total of five times. This allows us to detect if some estimation

of parameter values is orders of magnitude different than the other estimations or if the moment fitting is significantly worse for some given set of parameters.

(6) Up until this point, we have optimized $\log(m_2(t_s))$ and $\log(m_4(t_s))$ one at a time, for a different sets of parameters. In this final step, we optimize all parameters and try to approximate $\log(m_{4,IS,v_B,D,\lambda_{BD},\lambda_{DB}}(t_s)) + 2 \times \log(m_{2,IS,v_B,D,\lambda_{BD},\lambda_{DB}}(t_s))$ to $\log(m'_4(t_s)) + 2 \times \log(m'_2(t_s))$. Usually, to have a good fit for both moments, a final adjustment of parameters is needed and, in this step, we allow the parameters to become 1.3 times larger or smaller. Without this final step, usually, the estimation of parameters is a bit better, but the overall fit of the second and fourth moments is worse. This is also the reason the behavior of D_{est} relative to D_{og} seen in Fig. 9.

In Fig. 9, we can see the dependency of the estimated parameters on the originally simulated ones. Besides the case of the diffusion coefficient D , which was already discussed, we see a larger deviation from the original values when it comes to λ_{DB} , while we observe a larger bias in the values of v_B (see ρ_{v_B} in Table I). The remaining values of ρ and σ relative to the quality of the estimation of the IS parameters can be seen in Table I.

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