

Controlling excitation localization in waveguide QED systems

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We theoretically investigate excitation dynamics in one-dimensional arrays of quantum emitters coupled to a waveguide, focusing on localization and long-time population trapping. By combining time-domain simulations with spectral analysis of an effective non-Hermitian Hamiltonian, we identify two distinct mechanisms that give rise to localization: geometry-induced subradiance and disorder-induced Anderson-like confinement. Spatially modulated emitter arrangements—such as single- and double-Gaussian transverse profiles—enable long-lived subradiant modes even in the absence of disorder, with decay rates that can be finely controlled via geometric parameters. In contrast, localization in uniform arrays emerges only when disorder breaks spatial symmetry and suppresses collective emission through interference. We track the crossover between geometric and disorder-induced regimes, finding that double-Gaussian profiles exhibit clear spatial signatures of this transition, while single-Gaussian configurations display more gradual changes. These results establish geometry and disorder as complementary tools for engineering long-lived quantum states in waveguide QED systems, with direct relevance for scalable implementations in photonic platforms.

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I. INTRODUCTION

Waveguide quantum electrodynamics (QED) provides a versatile platform for exploring light-matter interactions in low-dimensional, open quantum systems [1–15]. Arrays of quantum emitters coupled to one-dimensional (1D) photonic reservoirs exhibit rich collective phenomena, including superradiance, subradiance, and photon-mediated long-range interactions [6,10,16–19].

A central challenge in such systems is understanding and controlling excitation localization, i.e., the long-time retention of population within the emitter array. In conventional models, localization typically arises from disorder-induced interference, as in Anderson localization [20]. However, in open systems with non-Hermitian dynamics, engineered geometry can also induce localization via spatially inhomogeneous coupling, leading to the formation of subradiant modes [21–35] that are protected from collective decay. Despite extensive studies on disorder-induced localization in photonic and atomic systems [36–42], the role of emitter geometry in waveguide QED remains underexplored. Existing works often fix emitter positions, treating geometry as a static design

choice, rather than as a tunable parameter for engineering localization.

In this work, we investigate population trapping in 1D arrays of quantum emitters coupled to a waveguide, by performing a systematic comparison of three spatial configurations: linear, single-Gaussian, and double-Gaussian transverse profiles. Disorder is introduced along the propagation axis, and localization behavior is analyzed through time-domain simulations and spectral decomposition of an effective non-Hermitian Hamiltonian [43]. The imaginary parts of the eigenvalues determine the decay rates of collective modes, enabling the identification of dominant long-lived eigenstates responsible for excitation retention.

Our results show that localization originates from two qualitatively distinct mechanisms: geometry-induced subradiance and disorder-induced localization, with a crossover behavior observed as disorder strength increases. The transition is subtle in single-Gaussian structures, but becomes more pronounced in double-Gaussian configurations that exhibit clear spatial signatures of the crossover. We further examine how localization depends on system size and find that structured emitter geometries—such as single- and double-Gaussian profiles—exhibit greater robustness and sustained population trapping than uniform linear arrays, especially in the weak-disorder regime.

These findings provide insights into controllable radiative dynamics in waveguide QED and offer practical design principles for suppressing decoherence in quantum optical networks. The predicted behaviors are accessible using current experimental platforms, including superconducting qubits [44,45], cold atoms coupled to nanophotonic waveguides

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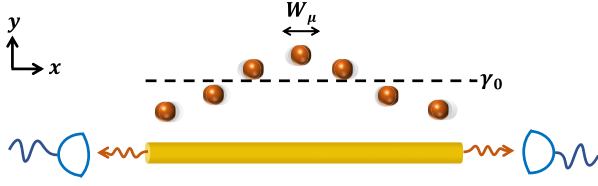


FIG. 1. Schematic of the system under consideration: a one-dimensional array of quantum emitters symmetrically coupled to a waveguide. Different transverse emitter geometries are realized by introducing position-dependent displacements y_μ . The coupling rate γ_0 is defined with respect to a reference position relative to the waveguide, as illustrated.

(including optical nanofibers) [46–53], and quantum dots in photonic crystals [3].

II. THEORETICAL MODEL

We consider a 1D array of N identical two-level quantum emitters, with ground state $|g\rangle$ and excited state $|e\rangle$, separated by a transition frequency ω_{eg} , and coupled to a photonic waveguide, as illustrated in Fig. 1. In this setup, the transverse positions y_μ of the emitters can be varied to produce inhomogeneous waveguide-emitter couplings, allowing control over the collective relaxation dynamics of the system. The system's dynamics in the interaction picture are governed by the Lindblad master equation [54,55]:

$$\frac{d\rho}{dt} = -i[H, \rho] - \frac{\gamma_0}{4} \sum_{\mu, v=1}^N e^{-(y_\mu + y_v)/\zeta_c} e^{-ik_{eg}|x_\mu - x_v|} \times (\sigma_\mu^\dagger \sigma_v \rho + \rho \sigma_\mu^\dagger \sigma_v - 2\sigma_v \rho \sigma_\mu^\dagger), \quad (1)$$

with the coherent interactions described by

$$H = -i \frac{\gamma_0}{4} \sum_{\mu \neq v} \sum_{v=1}^N e^{-(y_\mu + y_v)/\zeta_c} (e^{ik_{eg}|x_\mu - x_v|} \sigma_\mu^\dagger \sigma_v - \text{H.c.}), \quad (2)$$

where $\sigma_\mu^\dagger \equiv |e\rangle_\mu \langle g|$ is the dipole raising operator and γ_0 denotes the emitter-waveguide coupling strength. This master equation is derived under the Born-Markov approximation [56] and applies to 1D photonic reservoirs that mediate long-range dipole-dipole interactions [57]. The parameters ζ_c and k_{eg} represent the characteristic decay length and photon wavevector, respectively, where $k_{eg} \equiv \omega_{eg}/v$ and v is the group velocity of the guided mode. For simplicity, we set $\zeta_c = 1$ throughout this work, so that all lengths and coupling expressions are normalized by the evanescent decay length. The spatially inhomogeneous coupling arises from the evanescent nature of the guided mode, which decays exponentially with transverse displacement from the waveguide. The single-emitter decay into the guided mode is $\gamma_{1D, \mu} = \gamma_0 e^{-2y_\mu}$. We neglect additional loss channels ($\gamma_{ng} = 0$); therefore, the emitter-waveguide coupling efficiency is $\beta_\mu \equiv \gamma_{1D, \mu}/(\gamma_{1D, \mu} + \gamma_{ng}) = 1$, where γ_{ng} denotes the decay rate into nonguided modes.

To characterize the interaction range, we define the dimensionless parameter $\xi \equiv k_{eg}|x_{\mu+1} - x_\mu|$, which corresponds to the normalized interemitter spacing along the propagation

direction. The emitters are positioned uniformly along the propagation axis, such that the interemitter spacing is fixed ($\xi = \text{const}$). However, their transverse positions y_μ are allowed to vary, leading to position-dependent coupling strengths with the photonic waveguide. This geometry preserves the one-dimensional topology of the system, while enabling spatially inhomogeneous interactions through transverse displacement.

In the context of this model, we make several important assumptions. First, we neglect the retardation effects in the light propagation between emitters, assuming that the propagation time between emitters is much shorter than the relevant timescales of the system. This allows us to treat the interactions as effectively instantaneous. Second, we assume that the waveguide supports a linear dispersion relation around the operating frequencies, such that the group velocity of the guided modes is approximately constant. This leads to a simple exponential decay form of the effective coupling between emitters. Finally, we work in the weak-coupling limit, where the emitter-waveguide coupling strength γ_0 is sufficiently small to justify a perturbative treatment. Under this condition, we neglect higher-order terms in the interaction Hamiltonian and any nonlinear effects arising from strong light-matter interactions.

We restrict our analysis to the single-excitation subspace, which fully describes the system dynamics under initial single-excitation conditions. The general state can be written as $|\Psi(t)\rangle = \sum_{\mu=1}^N a_\mu(t) |\psi\rangle_\mu$, where $|\psi\rangle_\mu = |e\rangle_\mu \otimes |g\rangle^{\otimes(N-1)}$ denotes a single excitation localized on the μ th emitter. Within this subspace, the effective non-Hermitian Hamiltonian takes the form [6,58]

$$H_{\text{eff}} = -i \frac{\gamma_0}{4} \sum_{\mu \neq v} \sum_{v=1}^N e^{-(y_\mu + y_v)} e^{ik_{eg}|x_\mu - x_v|} \sigma_\mu^\dagger \sigma_v - i \frac{\gamma_0}{2} \sum_{v=1}^N e^{-2y_v} \sigma_v^\dagger \sigma_v. \quad (3)$$

To study the effect of imperfections, we introduce static disorder $W_\mu \in \pi[-\bar{\omega}, \bar{\omega}]$, modeling onsite phase fluctuations along the waveguide axis. This leads to a Schrödinger-type equation governing the time evolution of the excitation amplitudes:

$$\dot{a}_\mu(t) = -\frac{\gamma_0}{2} \sum_{\mu < v} e^{-(y_\mu + y_v)} e^{i|\mu - v|\xi - i(W_\mu - W_v)} a_v(t) - \frac{\gamma_0}{2} \sum_{\mu > v} e^{-(y_\mu + y_v)} e^{i|\mu - v|\xi - i(W_v - W_\mu)} a_v(t) - \frac{\gamma_0}{2} e^{-2y_\mu} a_\mu(t). \quad (4)$$

In the following sections, we analyze the time evolution of $a_\mu(t)$ under various array configurations and examine how disorder influences localization and excitation transport in geometrically structured one-dimensional emitter arrays.

III. GEOMETRY-INDUCED LOCALIZATION AND MODE ANALYSIS

In this section, we investigate how the geometry of emitter arrays influences localization phenomena and long-time excitation trapping. We consider a system of $N = 101$ emitters with uniform interemitter spacing ξ , initially prepared in the symmetric Dicke state: $|D\rangle = \frac{1}{\sqrt{N}} \sum_{\mu=1}^N |g_1, \dots, e_\mu, \dots, g_N\rangle$, where $|g_\mu\rangle$ and $|e_\mu\rangle$ denote the ground and excited states of the μ th emitter. This state features a single excitation uniformly delocalized across the array.

A. Localization induced by varying interemitter spacing

We first examine how localization depends on the interemitter spacing ξ , keeping the transverse displacement parameters fixed at $\eta = 0.05$ and $\sigma = 0.2$ for the single-Gaussian profile and $\sigma = 0.075$ for the double-Gaussian case. These parameters define the spatial displacement y_μ , which controls the emitter-waveguide coupling via $\gamma_\mu \sim e^{-y_\mu}$. Specifically, η sets the overall transverse shift, while σ determines the profile width.

To ensure fair comparison across geometries, we choose the parameters such that the total effective coupling strength $\sum_\mu e^{-y_\mu}$ remains approximately constant. This allows us to isolate the influence of geometric structure on localization behavior, independent of overall emission strength.

As a baseline, we first consider a linear configuration, where all emitters are placed along the waveguide axis without transverse displacement ($y_\mu = 0$ for all μ). In this case, the coupling strengths are uniform. As shown in Fig. 2(a), the remaining population $P(t)$ exhibits fast and nearly monotonic decay for all values of ξ . The corresponding spatial distributions at $\gamma_0 t = 10^4$ [Fig. 2(b)] are broadly spread with low amplitude, indicating poor localization and efficient radiation loss. This demonstrates that periodic linear arrangements do not sustain long-lived modes and instead promote collective superradiant decay.

For the single-Gaussian configuration, defined as

$$y_\mu = 2\eta \exp\left(-\frac{(\mu/N - 1/2)^2}{2\sigma^2}\right) - \eta, \quad (5)$$

the remaining population $P(t) = \sum_\mu |a_\mu(t)|^2$ exhibits monotonic decay for $\xi = 0.1\pi$, as shown in Fig. 2(c). At $\xi = 0.15\pi$, the decay slows considerably, and a finite fraction of the excitation persists at long times. When ξ is increased to 0.2π , an even larger fraction survives; however, the spatial distribution $P_n(t) = |a_n(t)|^2$ at $\gamma_0 t = 10^4$ deviates significantly from the original Gaussian profile [Fig. 2(d)].

A similar trend is observed in the double-Gaussian configuration, described by

$$y_\mu = 2\eta \exp\left(-\frac{(\mu/N - 1/4)^2}{2\sigma^2}\right) + 2\eta \exp\left(-\frac{(\mu/N - 3/4)^2}{2\sigma^2}\right) - \eta. \quad (6)$$

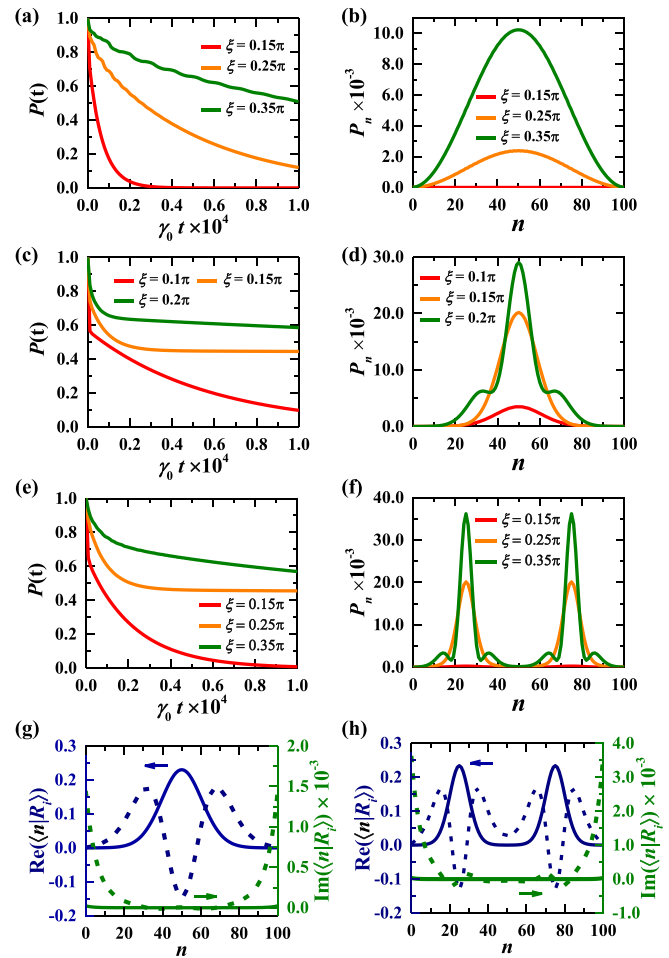


FIG. 2. (a) Time evolution of the remaining population in a linear configuration for $\xi = 0.15\pi$, 0.25π , and 0.35π . (b) Population distribution at time $\gamma_0 t = 10^4$ corresponding to panel (a). (c) Time evolution of the remaining population in a single-Gaussian configuration for $\xi = 0.1\pi$, 0.15π , and 0.2π . (d) Population distribution at time $\gamma_0 t = 10^4$ corresponding to panel (c). (e) Temporal behavior of the remaining population in a double-Gaussian configuration for $\xi = 0.15\pi$, 0.25π , and 0.35π . (f) Population distribution at $\gamma_0 t = 10^4$ for cases in panel (e). (g) Eigenmodes contributing to the dynamics at $\gamma_0 t = 10^4$ for the single-Gaussian case with $\xi = 0.2\pi$; exactly two long-lived modes are involved. Solid and dashed lines represent the first mode and the second mode, respectively, with real and imaginary parts shown on the left and right axes. (h) Same as panel (g), but for the double-Gaussian configuration with $\xi = 0.35\pi$. Note that n labels emitter sites, while i is used throughout the text to label eigenmodes.

At $\xi = 0.25\pi$, the decay becomes markedly slower, and a non-negligible portion of the excitation remains at long times [Fig. 2(e)]. For larger values of ξ , the spatial profile becomes increasingly distorted [Fig. 2(f)].

While the transverse displacements y_μ modify the emitter-waveguide coupling strength, the overall interatomic coupling matrix $J_{\mu\nu}$ is largely shaped by the sinelike dependence on emitter separation along the waveguide. As a result, the coupling structures for different geometries appear qualitatively similar, with geometry mainly entering through an

amplitude modulation $e^{-(\gamma_\mu + \gamma_\nu)}$. To illustrate this, representative coupling matrices for linear, single-Gaussian, and double-Gaussian configurations are shown in Appendix A.

To uncover the mechanism behind the deviations in localization patterns at long times, we analyze the eigenmodes of the effective non-Hermitian Hamiltonian H_{eff} [Eq. (3)]. We compute its complex eigenvalues λ_i and corresponding biorthogonal eigenvectors $\langle L_i |$ and $| R_i \rangle$, and project the initial state onto the eigenbasis as $c_i = \langle L_i | \psi(0) \rangle$. The time-dependent contribution of each mode is given by $w_i(t) = |c_i e^{\lambda_i t}|^2$, allowing us to identify how many modes remain populated at late times.

In the single-Gaussian case, for $\xi = 0.1\pi$ and 0.15π , only one long-lived mode contributes significantly, with decay rates $\text{Im}(\lambda) = -8.8 \times 10^{-5}\gamma_0$ and $-4.9 \times 10^{-7}\gamma_0$, respectively. The corresponding population contributions at $\gamma_0 t = 10^4$ are $w(t) = 0.10$ and 0.45 . At $\xi = 0.2\pi$, the dynamics involve exactly two modes, with $\text{Im}(\lambda_1) = -3.3 \times 10^{-9}\gamma_0$ and $\text{Im}(\lambda_2) = -1.3 \times 10^{-5}\gamma_0$, leading to deformation of the excitation profile [Fig. 2(g)]. The contribution of the first mode is $w_1(t) = 0.38$ and the second mode contributes $w_2(t) = 0.20$.

In the double-Gaussian case, the system is again dominated by a single eigenmode at $\xi = 0.15\pi$ and $\xi = 0.25\pi$, with decay rates $\text{Im}(\lambda) = -2.2 \times 10^{-4}\gamma_0$ and $-6.9 \times 10^{-7}\gamma_0$, respectively. The population contribution at $\gamma_0 t = 10^4$ is $w(t) = 0.0079$ at $\xi = 0.15\pi$ and $w(t) = 0.46$ at $\xi = 0.25\pi$. At $\xi = 0.35\pi$, two long-lived modes contribute, with $\text{Im}(\lambda_1) = -3.2 \times 10^{-9}\gamma_0$ and $\text{Im}(\lambda_2) = -3.1 \times 10^{-5}\gamma_0$, resulting in a visibly deformed spatial profile [Fig. 2(h)]. The contribution of the first mode is $w_1(t) = 0.38$ and the second mode is $w_2(t) = 0.19$.

To clarify the connection between this mode analysis and the observed spatial population distribution, we emphasize that at long times ($\gamma_0 t \gg 1$), the system dynamics are governed by the few eigenmodes with the smallest imaginary parts (i.e., the slowest decay rates). The total wavefunction can be written as a sum over these long-lived modes: $|\psi(t)\rangle = \sum_i c_i e^{\lambda_i t} |R_i\rangle$. The population on the n th emitter is then given by $P_n(t) = |\langle e_n | \psi(t) \rangle|^2$, which reflects both the spatial structure of the eigenmodes and their relative amplitudes. When a single eigenmode dominates (i.e., one coefficient c_i is much larger than the others), the resulting spatial population $P_n(t)$ closely follows the modulus squared of that eigenvector. When multiple long-lived modes contribute significantly, interference between them can lead to visible distortions or modulations in the spatial distribution. This behavior is clearly seen in Figs. 2(g) and 2(h).

These results show that the geometry of the emitter array significantly affects long-time localization by shaping the system's spectral structure. Specifically, subradiant eigenmodes with exceptionally small decay rates can remain populated over extended timescales, enabling robust excitation trapping.

B. Localization control via dominant eigenmode under transverse modulation

Building on the above, we now systematically map how geometry controls localization by focusing on the dominant eigenmode, defined as the one with the largest contribution

$\max(w_i(t))$ at $\gamma_0 t = 10^4$. This mode is primarily responsible for the residual excitation observed at long times.

To isolate geometric effects, we rescale all profiles such that the total effective coupling strength $\sum_\mu e^{-\gamma_\mu}$ is held constant across different (η, σ) values. This ensures that variations in decay rate arise purely from the spatial structure of the coupling.

We examine how the decay rate of this dominant eigenmode, $\gamma_{\text{dom.}} = \text{Im}(\lambda_i)|_{\max(w_i(t))}$, depends on the transverse modulation parameters η and σ that define the single- and double-Gaussian profiles [Eqs. (5) and (6)]. These geometries control the spatial variation of the emitter-waveguide coupling, which in turn influences the formation of subradiant modes.

For the single-Gaussian case at fixed $\xi = 0.15\pi$, increasing η while holding $\sigma = 0.2$ leads to an exponential suppression of the decay rate, as shown in Fig. 3(a), indicating enhanced localization due to reduced collective coupling. When $\eta = 0.05$ is held fixed and σ is varied, the decay rate reaches a minimum near $\sigma = 0.1$, as seen in Fig. 3(b). For larger σ , the smoother profile weakens localization as the geometry approaches a uniform linear array. The full parameter dependence is illustrated in Fig. 3(c), where a distinct region of minimized decay identifies an optimal regime for long-lived excitation trapping.

A similar analysis for the double-Gaussian configuration at $\xi = 0.25\pi$ yields analogous trends. Increasing η at fixed $\sigma = 0.075$ suppresses the decay rate monotonically [Fig. 3(d)]. When $\eta = 0.05$ is fixed and σ is varied, the decay rate exhibits a local minimum around $\sigma = 0.05$, as shown in Fig. 3(e). Although an even smaller decay rate appears around $\sigma = 0.35$, this configuration no longer preserves the double-Gaussian structure; instead, the profile effectively becomes a single broad Gaussian, losing the intended geometric separation. The full parameter dependence is summarized in the colormap shown in Fig. 3(f), which maps the dominant decay rate across the (η, σ) space. Strong localization is achieved within a narrow region where the spatial configuration maintains well-separated peaks and sufficient transverse displacement ($\sigma \sim 0.05$).

These results demonstrate that by tuning the transverse geometry of the emitter array, one can control both the lifetime and spatial structure of the dominant eigenmode. This provides a powerful route to engineering long-lived, spatially localized excitations in waveguide QED systems.

IV. EFFECT OF DISORDER ON DIFFERENT EMITTER ARRANGEMENTS

In this section, we study how the introduction of disorder affects the localization properties of emitter arrays that are originally structured to support geometry-induced localization. We compare three representative spatial configurations: a linear array, a single-Gaussian profile, and a double-Gaussian profile. The system parameters are chosen based on the findings from the previous section: for both the linear and single-Gaussian arrangements, we consider $N = 101$ emitters with an interemitter spacing of $\xi = 0.15\pi$. This choice ensures that, in the absence of disorder, the single-Gaussian configuration supports localization dominated by a single

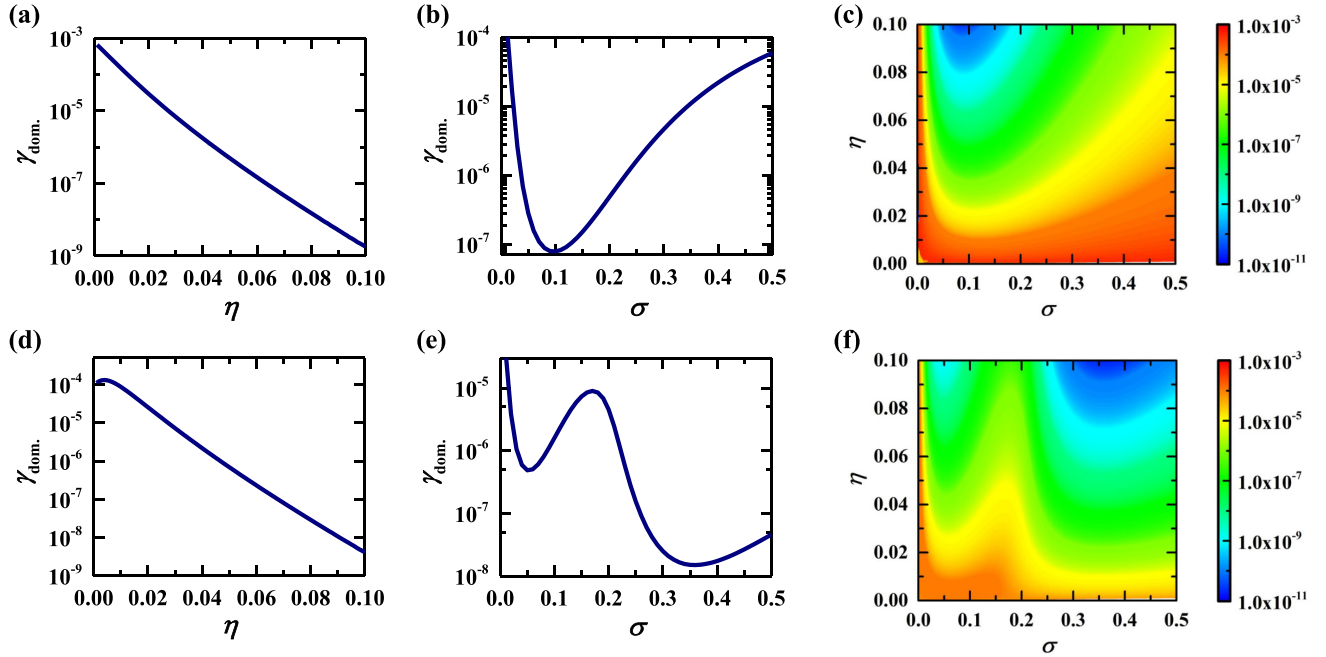


FIG. 3. (a), (b) Decay rate of the dominant eigenmode of the non-Hermitian Hamiltonian H_{eff} for emitters arranged in a single-Gaussian transverse profile [Eq. (5)], with $\xi = 0.15\pi$. In panel (a), $\sigma = 0.2$ is fixed while η is varied, showing an exponential suppression of the decay rate as η increases. In panel (b), $\eta = 0.05$ is fixed and the decay rate exhibits a minimum around $\sigma = 0.1$, suggesting an optimal configuration for minimizing radiative loss. (c) Colormap of the decay rate as a function of both σ and η . (d), (e) Decay rate of the dominant eigenmode for emitters arranged in a double-Gaussian transverse profile [Eq. (6)], with $\xi = 0.25\pi$. In panel (d), $\sigma = 0.075$ is fixed and η is varied, leading to a monotonic reduction in decay rate with increasing η . In panel (e), $\eta = 0.05$ is fixed and the decay rate shows a local minimum near $\sigma = 0.35$. A secondary minimum appears around $\sigma = 0.35$; configuration no longer retains the intended double-Gaussian structure. (f) Colormap of the decay rate across the (σ, η) parameter space, identifying regions that support strongly subradiant dominant modes. In all panels, the total emitter-waveguide coupling strength $\sum_{\mu} e^{-y_{\mu}}$ is kept constant by applying a uniform offset to each transverse profile.

long-lived eigenmode at long times. By restricting the dynamics to a single mode, we avoid complications arising from multimode interference, thereby facilitating a clearer analysis of disorder effects. In contrast, for the double-Gaussian arrangement, the same interemitter spacing yields negligible long-time population; thus, we adopt $N = 151$ emitters to enhance dipole-dipole interactions and maintain sufficient population trapping. These settings allow a consistent and meaningful comparison of disorder-induced modifications across different geometries.

A. Linear arrangement

We begin by analyzing a linear array of two-level quantum emitters initialized in a symmetric Dicke state. In this configuration, the transverse displacement is set to zero for all emitters, such that $y_{\mu} = 0$. The time evolution of the remaining population $P(t)$ under increasing disorder W_{μ} is shown in Fig. 4(a). As the disorder strength increases, a larger fraction of the excitation remains in the system, indicating a progressive suppression of radiative decay.

The corresponding spatial distribution of the excitation at a long time $\gamma_0 t = 10^4$ is shown in Fig. 4(b). For weak disorder, the spatial profile remains approximately Gaussian, while for stronger disorder, it becomes irregular but remains concentrated near the center of the chain. These results suggest that disorder leads to the formation of more localized and longer-lived subradiant modes.

B. Single-Gaussian arrangement

We next consider a single-Gaussian emitter arrangement defined in Eq. (5), with $\eta = 0.05$ and $\sigma = 0.2$. In this configuration, the emitters remain uniformly spaced along the propagation axis, while the transverse displacements create a smoothly varying emitter-waveguide coupling strength across the array. The time evolution of the remaining population $P(t)$ under increasing disorder W_{μ} is shown in Fig. 5(a). Even in the absence of disorder, significant population trapping occurs due to the site-dependent coupling strengths, which suppress collective emission and lead to the formation of a long-lived

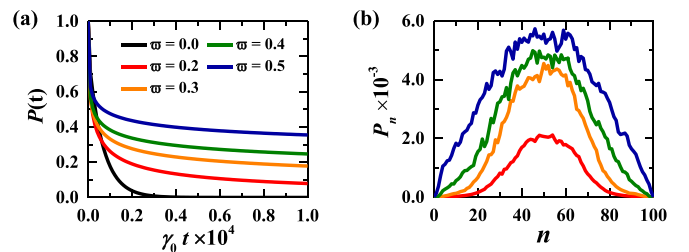


FIG. 4. (a) Time evolution of the total remaining population in a linearly arranged emitter array under increasing disorder. The system comprises $N = 101$ uniformly spaced emitters with interemitter spacing $\xi = 0.15\pi$ and zero transverse displacement. (b) Spatial population distributions at $\gamma_0 t = 10^4$ for selected disorder strengths. Parameter values and color coding match those used in panel (a).

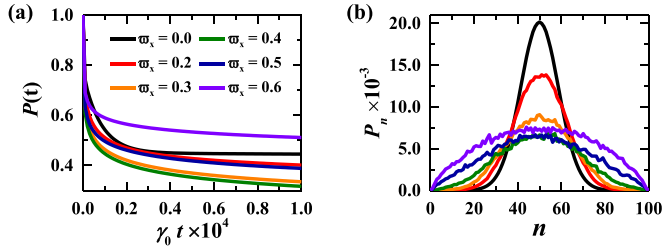


FIG. 5. (a) Time evolution of the total remaining population for emitters arranged in a single-Gaussian transverse profile, with parameters $\eta = 0.05$, $\sigma = 0.2$, $N = 101$, and $\xi = 0.15\pi$. Moderate disorder reduces the effect of geometry-induced localization, while stronger disorder restores trapping via disorder-induced localization. (b) Population distributions at $\gamma_0 t = 10^4$ for selected disorder strengths. In the absence of disorder, spatial modulation of the coupling leads to geometry-induced localization near the center of the array. Parameter values and color coding match those used in panel (a).

localized mode. As disorder is introduced, the trapped population initially decreases, indicating that geometry-induced localization is sensitive to perturbations. Upon further increasing the disorder strength, the population begins to recover and eventually surpasses the clean-case level, suggesting a crossover from geometry-induced to disorder-induced localization.

The spatial excitation distributions at a long time of $\gamma_0 t = 10^4$ provide additional insight into the effect of disorder. At zero disorder, the population profile resembles a smooth Gaussian, reflecting the underlying symmetry of the emitter geometry. As disorder strength increases, the profile becomes increasingly irregular while remaining centered around the middle of the array, as shown in Fig. 5(b).

While a crossover from geometry-induced to disorder-induced localization is also present in the single-Gaussian configuration, this transition is not readily visible in the spatial distributions. Both mechanisms tend to produce centrally localized excitation profiles, making it difficult to infer the dominant localization mechanism based solely on the shape of the distribution. This limitation motivates the introduction of a more structured geometry—namely, the double-Gaussian profile—which enables a clearer visualization of the localization crossover.

C. Double-Gaussian arrangement

We finally examine the double-Gaussian emitter arrangement defined in Eq. (6), with $\eta = 0.05$ and $\sigma = 0.075$. In this setup, the emitters are spatially configured to create two regions of weak coupling to the waveguide, producing distinct localization behavior. The evolution of the remaining population $P(t)$ under varying disorder strength is depicted in Fig. 6(a). Upon the introduction of disorder, the trapped population initially diminishes, indicating that geometry-induced localization is sensitive to perturbations. As the disorder strength increases further, the population begins to recover and eventually exceeds the clean-case value, consistent with a crossover from geometry-driven to disorder-driven localization.

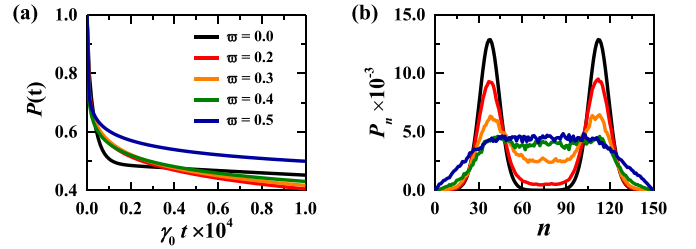


FIG. 6. (a) Time evolution of the total remaining population for emitters arranged in a double-Gaussian transverse profile, with parameters $\eta = 0.05$, $\sigma = 0.075$, $N = 151$, and $\xi = 0.15\pi$. Results are shown under increasing axial disorder. (b) Spatial excitation distributions at $\gamma_0 t = 10^4$ for selected disorder strengths. In the clean limit, excitation is localized in two distinct regions corresponding to weakly coupled emitter clusters. As disorder increases, the peaks become less well defined and partially overlap, indicating a deformation of the original geometry-induced profile.

Further insights into the dynamics are revealed by examining the spatial excitation profile at a long evolution time of $\gamma_0 t = 10^4$. In the clean limit ($\bar{\omega} \approx 0$), the distribution exhibits two clearly resolved Gaussian-like peaks, corresponding to the engineered regions of minimal coupling. As disorder increases, the initially well-separated peaks become less distinct and gradually merge into a single broad distribution centered in the array, as shown in Fig. 6(b). This transformation captures a transition between distinct localization mechanisms: at low disorder, excitation confinement is governed by geometry-induced subradiance, while at higher disorder, random scattering dominates and drives disorder-induced localization.

V. SIZE EFFECTS

To investigate how system size influences population trapping in the presence of disorder, we examine the total excitation population remaining at a long time $\gamma_0 t = 10^4$ as a function of the number of emitters N . Results for three emitter configurations—linear, single-Gaussian, and double-Gaussian—are shown in Fig. 7. All three configurations are based on a fixed interemitter spacing of $\xi = 0.15\pi$. The single-Gaussian profile is defined by transverse modulation parameters $\eta = 0.05$ and $\sigma = 0.2$, while the double-Gaussian configuration uses $\eta = 0.05$ and $\sigma = 0.075$.

In the linear configuration [Fig. 7(a)], population trapping is unexpectedly strong for large N in the clean limit due to subradiant modes supported by the spatial symmetry of the system. These modes originate from destructive interference that inhibits collective decay. However, even weak disorder breaks this symmetry and rapidly suppresses the subradiant behavior. At higher disorder strengths, Anderson-like localization becomes the dominant mechanism, leading to a recovery in population trapping.

To directly confirm that this population trapping corresponds to spatial localization, we provide a complementary analysis based on the inverse participation ratio (IPR) in Appendix B. This analysis quantitatively demonstrates the emergence and robustness of localized modes under disorder.

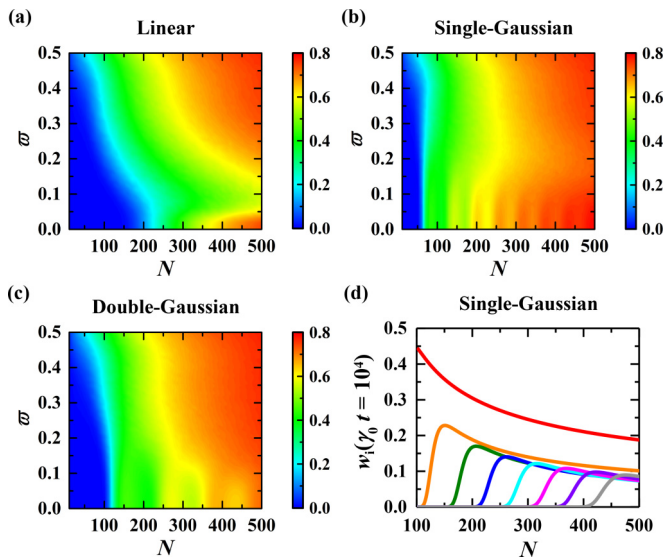


FIG. 7. Total population remaining at time $\gamma_0 t = 10^4$ as a function of emitter number N and disorder strength W_μ for three emitter configurations: (a) linear, (b) single-Gaussian, and (c) double-Gaussian. (d) Contribution at $\gamma_0 t = 10^4$ as the emitter number N changes. The red line represents the contribution of the firstst mode, orange is for the second mode, green for the third mode, blue for the fourth mode, cyan for the fifth mode, magenta for the sixth mode, purple for the seventh mode, and gray for the eighth mode.

The single-Gaussian configuration [Fig. 7(b)] exhibits more stable population retention across a wide range of system sizes, particularly at low to moderate disorder strengths. This robustness arises from geometry-induced spatial variation in emitter-waveguide coupling. In this profile, emitters near the center are transversely displaced furthest from the waveguide and therefore couple only weakly to the photonic reservoir. As the system size increases, emitters are added throughout the array, including both central and peripheral regions. However, the long-time dynamics remain dominated by excitation confined to the weakly coupled central region. Emitters located farther from the center experience stronger coupling and tend to decay more rapidly, contributing little to the population that persists at late times.

The double-Gaussian configuration [Fig. 7(c)] exhibits population trapping that depends sensitively on system size and disorder strength. At low disorder strengths, the system effectively behaves as two spatially separated single-Gaussian segments, and therefore requires a larger number of emitters for significant population trapping. As disorder increases, the distinction between the two lobes becomes less significant, and the system begins to support long-lived modes with enhanced excitation confinement. This transition reflects a shift in the dominant localization mechanism, from geometry-induced subradiance to disorder-induced localization.

In both the single-Gaussian and double-Gaussian configurations, ripples in the remaining population are observed in the clean limit as the number of emitters N increases [Figs. 7(b) and 7(c)]. This behavior can be attributed to two factors: first, the increasing number of subradiant modes as N grows, which leads to a redistribution of the population among these modes; second, some of these subradiant modes

experience a decrease in their decay rates. These two factors combined cause the trapped population to exhibit ripples as the system size increases. For the single-Gaussian configuration, these ripples are most pronounced when N is in the range of 100–350, where the number of remaining long-lived modes at $\gamma_0 t = 10^4$ increases from one to six [Fig. 7(d)]. As more long-lived modes become available, the population becomes more evenly distributed among them, leading to a reduction in the amplitude of the ripples. This explains why ripples are no longer observed for $N \gtrsim 350$, where a sufficiently large number of modes results in a smooth population distribution. Additionally, as N increases, the number of subradiant modes contributing to the dynamics also increases, and the position of the ripple aligns with the peak value of the i th subradiant mode's contribution [Figs. 7(b) and 7(d)], showing that the excitation trapping is directly related to the contribution of these modes. A similar trend is observed in the double-Gaussian configuration. However, as disorder increases, this effect diminishes. Since disorder lacks a specific geometric structure, the population distribution becomes more uniform on average, and the ripple effect vanishes as the system becomes dominated by disorder-induced localization.

VI. DISCUSSION AND CONCLUSION

In this work, we have investigated excitation localization and population trapping in one-dimensional arrays of quantum emitters coupled to a waveguide, focusing on the interplay between engineered emitter geometries and static disorder. Using time-domain simulations combined with biorthogonal eigenspectrum analysis of the non-Hermitian effective Hamiltonian [43], we identified two distinct mechanisms of localization in open quantum systems: geometry-induced subradiance and disorder-induced Anderson-like localization [39,40,42,59].

Our results show that spatially modulated emitter arrangements—such as single- and double-Gaussian transverse profiles—can support long-lived collective modes with significantly reduced decay rates compared to uniform arrays. These subradiant modes arise from inhomogeneous coupling to the waveguide and are highly sensitive to the transverse emitter distribution. Through systematic parameter mapping, we identified geometric regimes that optimize population trapping and coherence retention.

In contrast, localization in uniform linear arrays requires sufficient disorder to break spatial symmetry and enable interference-based confinement. By varying disorder strength, we observed a crossover from geometry-induced to disorder-induced localization. This transition is subtle in single-Gaussian configurations, but becomes pronounced in double-Gaussian arrays, where the spatial excitation pattern evolves from a double-peaked profile to a broad, centrally localized distribution, indicating a transition from geometry-induced to disorder-induced confinement.

We further examined the impact of system size, showing that geometry-induced localization remains effective in large arrays. This suggests a practical advantage over disorder-based or symmetry-protected localization schemes, particularly in the context of scalability and experimental implementation.

These findings provide insights into how spatial geometry and disorder can be used as complementary tools to control decoherence and dissipation in waveguide QED systems. The predicted localization behavior and enhanced excitation lifetimes are directly relevant to ongoing experiments with cold atoms coupled to nanophotonic structures [51,52], superconducting qubits coupled via microwave waveguides [44,45], and solid-state emitters embedded in photonic crystals [3].

Looking ahead, several promising directions remain, including extending the analysis to the multiexcitation regime and incorporating temporal disorder or nonlinear interactions. In addition, introducing chiral coupling—asymmetric emission into left- and right-propagating modes—would provide a promising route to explore nonreciprocal localization dynamics and direction-dependent subradiance. By bridging theoretical predictions with practical implementations, our findings pave the way for designing scalable, coherent quantum networks and quantum memories with enhanced control over dissipation and decoherence.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: GEOMETRY-INDUCED INTERATOMIC COUPLING STRUCTURE

To visualize how transverse emitter geometry influences the interatomic interactions, we plot the real part of the coherent coupling matrix $J_{\mu\nu} \equiv \text{Re}[(H_{\text{eff}})_{\mu\nu}]$ for representative emitter configurations. From Eq. (3), this coupling takes the form $J_{\mu\nu} \propto e^{-(y_\mu+y_\nu)} \sin(k_{eg}|x_\mu - x_\nu|)$, which exhibits a sine-like oscillation along the propagation axis and exponential decay from transverse displacement.

As shown in Fig. 8, all three geometries—linear, single-Gaussian, and double-Gaussian—share a qualitatively similar structure in $J_{\mu\nu}$, dominated by the sine function $\sin(k_{eg}|x_\mu - x_\nu|)$. The role of geometry is to modulate the amplitude of the interaction via the prefactor $e^{-(y_\mu+y_\nu)}$, which introduces only subtle variations in the overall matrix pattern.

While these variations may appear minor in the heatmaps, they have a profound effect on the system's spectral properties. In particular, they alter the support and decay rates of subradiant eigenmodes that govern long-time dynamics. These effects are analyzed in detail in Sec. III.

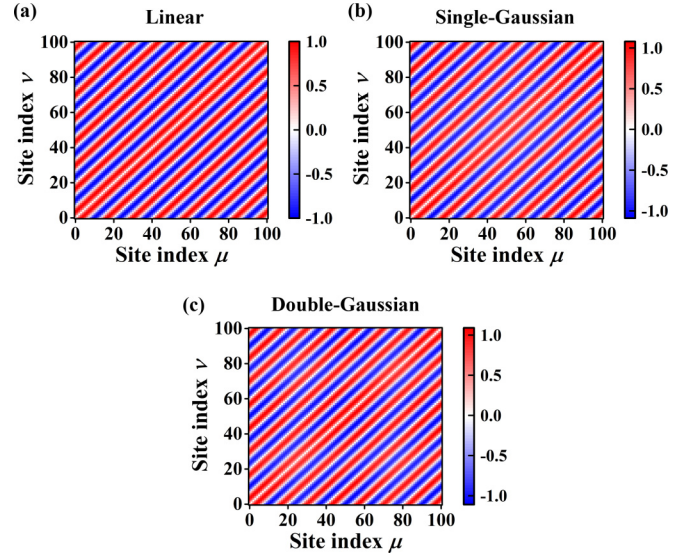


FIG. 8. Heatmaps of the real part of the coherent coupling matrix $J_{\mu\nu} \equiv \text{Re}[(H_{\text{eff}})_{\mu\nu}]$ for (a) a linear array ($y_\mu = 0$), (b) a single-Gaussian profile with $\eta = 0.05$, $\sigma = 0.2$, and (c) a double-Gaussian profile with $\eta = 0.05$, $\sigma = 0.075$. In all cases, $N = 101$ and $\xi = 0.15\pi$. The sinusoidal interaction pattern arises from the $\sin(k_{eg}|x_\mu - x_\nu|)$ term in the coupling, while the geometric modulation enters as an overall envelope that affects the effective strength but not the phase structure of $J_{\mu\nu}$.

APPENDIX B: INVERSE PARTICIPATION RATIO ANALYSIS

To further support our claim that the long-time excitation is spatially localized, we analyze the IPR, which serves as a quantitative indicator of spatial confinement. Specifically, the IPR measures the degree to which an excitation is concentrated on a limited number of sites: higher IPR values signify stronger localization. We consider both spectral and dynamical definitions to provide a comprehensive characterization of localization.

In open systems with dissipation and long-range interactions—such as our waveguide-mediated emitter arrays—IPR values must be interpreted carefully. Nonetheless, the IPR remains a reliable tool for distinguishing localized modes from extended ones in such systems.

We consider three complementary measures.

1. Average IPR versus disorder strength

We first compute the average IPR across all eigenmodes of the effective non-Hermitian Hamiltonian:

$$\langle \text{IPR} \rangle = \frac{1}{N} \sum_{i=1}^N \text{IPR}_i = \frac{1}{N} \sum_{i=1}^N \sum_{n=1}^N |\langle e_n | R_i \rangle|^4,$$

where $|R_i\rangle$ is the right eigenvector of mode i . This measure characterizes how spatially localized the eigenmodes are. This calculation is performed only for the linear configuration, since in the Gaussian-shaped arrays (single or double peak), the contribution from geometry-induced localized modes is

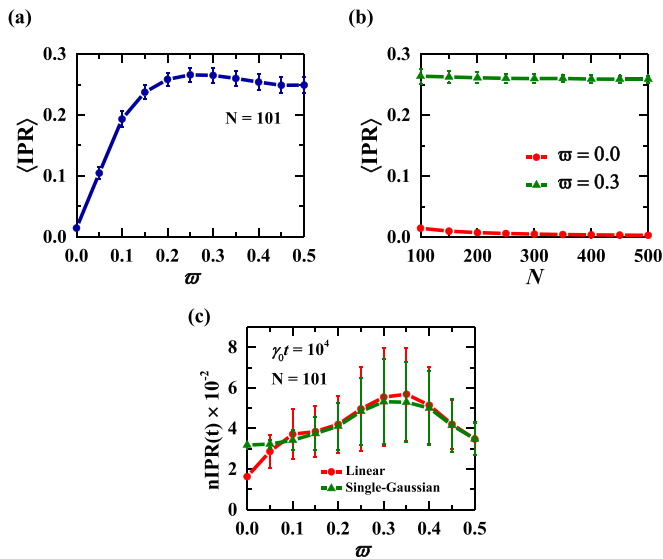


FIG. 9. (a) Average IPR as a function of disorder strength in the linear configuration. The IPR increases from approximately 0.014 in the clean limit to ~ 0.24 under strong disorder, indicating the emergence of spatially localized eigenmodes despite the presence of long-range coupling. (b) Average IPR vs system size N in the linear configuration. The IPR remains roughly constant as N increases, demonstrating the persistence of localized modes in large systems. (c) nIPR at $\gamma_0 t = 10^4$ for the linear and single-Gaussian configurations as a function of disorder strength. The single-Gaussian geometry exhibits higher nIPR in the clean limit due to geometry-induced localization. As disorder increases, the nIPR rises for both geometries, with consistently higher values compared to the clean linear case, indicating enhanced confinement of long-time excitation.

averaged out and does not significantly affect the global spectral average.

A key feature of our system is the presence of all-to-all long-range coupling mediated by a common photonic reservoir. This interaction fundamentally modifies the nature of localization compared to systems with short-range hopping, resulting in spatially localized modes with characteristically lower IPR values.

As shown in Fig. 9(a), the average IPR increases from approximately 0.014 in the clean limit to around 0.24 under strong disorder, clearly indicating the formation of spatially localized eigenmodes despite the delocalizing tendency of long-range coupling.

2. Average IPR versus system size

We also examine the dependence of average IPR on system size N in the linear configuration. We find that the IPR remains roughly constant as N increases, consistent with the persistence of localized modes in large systems. This further supports the robustness of localization under disorder [Fig. 9(b)].

We also examine the dependence of the average IPR on the system size N in the linear configuration. We find that the IPR remains approximately constant as N increases, indicating the persistence of localized modes even in large systems. This observation further supports the robustness of localization under disorder. As shown in Fig. 9(b), the average IPR does not decay with increasing system size, highlighting the non-trivial nature of disorder-induced localization in the presence of long-range coupling.

3. Normalized time-dependent IPR versus disorder strength

To directly probe the localization of excitation at long times, we compute the normalized time-dependent IPR (nIPR) of the wavefunction at $\gamma_0 t = 10^4$:

$$\text{nIPR}(t) = \frac{\sum_{\mu} |a_{\mu}(t)|^4}{(\sum_{\mu} |a_{\mu}(t)|^2)^2}.$$

This normalization eliminates the influence of dissipation, allowing us to isolate the spatial spread of the remaining excitation. Unlike the average IPR, the nIPR captures both the system's dynamics and the projection of the initial state onto localized eigenmodes.

We focus here on systems with fixed size $N = 101$. Although the double-Gaussian configuration also supports localized modes, aligning its localization centers with those of the single-Gaussian case would require adjusting N , which in turn alters the participating modes and affects the clean-limit nIPR. To ensure consistency and comparability, we therefore present results only for the linear and single-Gaussian configurations.

In the clean limit, the single-Gaussian geometry exhibits a higher nIPR than the linear array, reflecting geometry-induced localization. As disorder strength increases, the nIPR rises for both geometries. Notably, across all disorder strengths considered, the nIPR remains substantially higher than that of the clean linear configuration, indicating enhanced spatial confinement of the surviving excitation. This behavior is clearly illustrated in Fig. 9(c).

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