

Microscopic theory of a precessing ferromagnet for ultrasensitive magnetometry

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Levitated systems have great potential in quantum sensing and exploring fundamental physics at the macroscopic scale. Of particular interest are recent works suggesting that a levitated ferromagnet can beat the standard quantum limit of magnetometry, a benchmark for quantum sensors. In this work, we show how a microscopic theory capturing atomic-scale spin-lattice interactions can be used to fully explain the emergence of collective precession dynamics of a levitated ferromagnet and the origin of its enhanced magnetometric sensitivity beyond that of independent spins. Our theory further takes us to two innovative experimental designs of immediate interest: measurement of the celebrated Berry phase with a precessing ferromagnetic needle and the use of its nutation motion to sense a low-frequency oscillating magnetic field. With a microscopic theory established for levitated ferromagnetic needles, future studies of macroscopic quantum effects and the associated quantum-classical transition also become possible.

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I. INTRODUCTION

Observing, controlling, and utilizing quantum mechanics phenomena on scalable platforms is a key direction for future technology development. Quantum sensing is one of the most mature quantum technologies, integrating fundamental physics and advanced applications [1–5]. Relying on the response of a quantum system to external parameters, quantum sensing scales up its precision and sensitivity as we boost the measurement time t and the number of particles involved (denoted N). For example, spin-exchange relaxation-free magnetometers based on alkali-metal atomic vapors can achieve magnetic field sensitivity down to $0.16 \text{ fT}/\sqrt{\text{Hz}}$ [6,7]. Yet, for independent and identically distributed particles, the

uncertainty in frequency estimation is bounded by the standard quantum limit (SQL) [8,9], given by $\Delta\omega \geq 1/\sqrt{NT_2t}$, where T_2 is the characteristic coherence time. Two strategies to enhance sensing sensitivity for a fixed N are to extend T_2 or to utilize quantum entanglement [10]. Along these directions, current state-of-the-art experiments can obtain a coherence time of several minutes [11–14], and researchers have indeed demonstrated the use of maximally entangled states to surpass the SQL [15,16]. Despite these breakthroughs, key challenges remain to advance quantum sensing further. In particular, as N —and hence the system size—increases, the complexity of generating entangled states in a controlled manner grows. Furthermore, larger systems become more susceptible to decoherence, and in a broad class of situations, decoherence suppresses sensing advantages arising from entanglements [17]. These important issues in quantum sensing are therefore closely related to studies of macroscopic quantum phenomena [18–23] and the quantum-to-classical transition in massive systems.

In this paper, we investigate an alternative pathway to surpass the SQL—not through quantum entanglement, but by harnessing the coherent collective dynamics of a macroscopic classical spin system: a levitated ferromagnetic needle. This concept was first introduced by Kimball *et al.* in 2016 [24],

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proposing based on intuitive arguments that due to strong spin-lattice locking in ferromagnets, such a needle could precess as a single macrospin, thereby suppressing quantum fluctuations. However, no first-principle theoretical framework has been established to physically examine spin noise suppression when spin dynamics and lattice rotation evolve on comparable timescales. To digest many different aspects of the system, it also remains to computationally simulate how the collective precession dynamics emerges from the intricate interplay of an enormous number of degrees of freedom. Here, we fill this gap by introducing a microscopic framework that captures all fundamental aspects of levitated ferromagnet-based sensing. Our theoretical investigations advance the previously phenomenological concept of spin-lattice damping and locking, revealing the physical origin of ultrahigh sensitivity. Unlike earlier phenomenological or macrospin models [25,26], our framework allows for parameter-resolved predictions and is extendable to realistic scenarios such as multidomain ferromagnets with spatially varying spin textures. Our study makes it possible to (1) guide the design of ferromagnetic materials for magnetometry, (2) explore the roles of key physical parameters, and (3) propose sensing applications using this platform. Of particular importance, this work shows how to achieve the measurement of the Berry phase by a precessing needle, and the detection of weak oscillating magnetic fields through the needle's nutation dynamics, a technique particularly relevant for searching axionlike dark matter, whose coupling to spins often manifests as an effective oscillating field at a characteristic Compton frequency. Experiments demonstrating the superconducting levitation of micron-scale ferromagnetic particles [27,28] and reporting magnetometry that surpasses the energy resolution limit [29–32] confirm that the proposed levitated ferromagnetic needle platform is rapidly transitioning from theoretical concept to experimental reality.

II. MICROSCOPIC FRAMEWORK OF COHERENT SPIN-LATTICE PRECESSION DYNAMICS

We construct a first-principle Hamiltonian model of a levitated ferromagnetic needle, consisting of interacting atomic spins embedded in a lattice. Beyond the conventional spin-exchange interaction (with strength J), a crucial component identified from the literature is the pseudodipolar interaction [33,34], with coupling strength denoted by C . This interaction, which originates from spin-orbit coupling, generates magnetic anisotropy and couples spin orientations to the relative positions of lattice sites [35]. The pseudodipolar interaction serves as an effective spin-lattice interaction that is essential for the emergence of coherent precession of the entire needle. Incorporating these terms, we arrive at the following total Hamiltonian:

$$H = - \sum_i \gamma \mathbf{B} \cdot \hat{\mathbf{S}}_i - J \sum_i \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_{i+1} - C \sum_{ij} (\hat{\mathbf{S}}_i \cdot \hat{\mathbf{r}}_{ij})(\hat{\mathbf{r}}_{ij} \cdot \hat{\mathbf{S}}_j) + \sum_i \frac{\hat{\mathbf{p}}_i^2}{2m} + \sum_{ij} V(\hat{r}_{ij} - \bar{r}_{ij})^2, \quad (1)$$

where $\gamma \approx 1.76 \times 10^{11} \text{ rad s}^{-1} \text{ T}^{-1}$ is electron gyromagnetic ratio, m is the mass of the atom, $\mathbf{B}$ is an external magnetic field to be detected and measured, $\hat{\mathbf{r}}_{ij}$ denotes the displacement vector between two nearest neighboring atoms i and j , and $\bar{r}_{ij}$ is the corresponding equilibrium distance. The lattice vibrations are modeled by a harmonic potential with a coupling strength V . The timescale of lattice vibration, determined by the amplitude of V , differs from that of the characteristic collective needle dynamics presented below by around 10 orders of magnitude. Though this huge timescale mismatch justifies an option to freeze the lattice vibrational motion altogether, this term is essential for the lattice structure in our system. Due to the levitation, there is no additional external force or other interaction with a substrate. Below we use dimensionless variables where the time t is rescaled by the characteristic Larmor frequency $\omega_L = \gamma |\mathbf{B}|$ and the spin is rescaled by the largest eigenvalue of z component $\mathbf{s}_i = \mathbf{S}_i/S_0$. As a result, the spin-lattice coupling constant is mapped to $C_0 = (S_0^2/m\omega_L^2)C$ (Appendix B). Further, each spin is assumed to be spin-1/2 and hence $S_0 = \hbar/2$, while it can be readily extended to larger spin systems.

Exactly solving the quantum many-body dynamics governed by the Hamiltonian in Eq. (1) is practically impossible. However, as confirmed by our classical trajectory simulations, all the spins remain approximately “coherent” with each other throughout the evolution, indicating negligible entanglement either among the spins or between the spins and the lattice motion [36]. This justifies the use of classical equations of motion: In systems of aligned spins—especially in coherent or nearly separable states—the spin expectation values evolve according to classical dynamics [37]. We therefore use classical equations of motion to investigate the underlying physics, replacing $\hat{\mathbf{S}}$, $\hat{\mathbf{r}}$, and $\hat{\mathbf{p}}$ with three-dimensional classical vectors. In particular, the equation of motion in terms of the spins is given by

$$\frac{d\mathbf{S}_i}{dt} = \gamma \mathbf{S}_i \times \mathbf{B} + J \mathbf{S}_i \times (\mathbf{S}_{i+1} + \mathbf{S}_{i-1}) + 2C \mathbf{S}_i \times \sum_j \mathbf{r}_{ij}(\mathbf{r}_{ij} \cdot \mathbf{S}_j). \quad (2)$$

The equations of motion for the lattice coordinate $\mathbf{r}_i$ and momentum $\mathbf{p}_i$ are given in Appendix A.

Figure 1 shows computational results based on Hamiltonian dynamics governed by the equations of motion mentioned above. Simulations shown in Fig. 1 are for a one-dimensional (1D) atomic chain comprising 50 cobalt atoms. Initially, all the atoms are arranged along the x axis according to the lattice constant of cobalt. An external magnetic field is applied along the z direction with an amplitude of 1 nT, as illustrated in Fig. 1(a). The exchange interaction energy J is chosen to be 10^4 times the Zeeman interaction energy. The choice of the spin-lattice constant C_0 is 1.2×10^5 to match the experimental Gilbert damping coefficient (Appendix C), but the actual value is not essential to the results presented here. Note that we use a one-dimensional model to illustrate the locked spin-lattice precession dynamics. In a real ferromagnet with short-range interactions, true long-range order is only stable in three or higher spatial dimensions at finite temperature. This presents

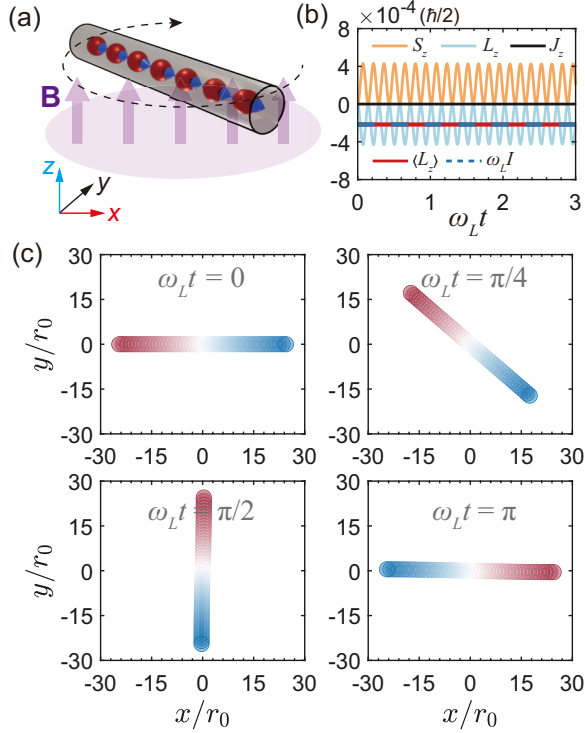


FIG. 1. The precession dynamics of a levitated ferromagnetic needle, as predicted by the microscopic Hamiltonian in Eq. (1). (a) Schematic of a levitated macroscopic ferromagnetic needle over a superconductor. (b) Einstein-de Haas effect shows the transfer between spin angular momentum S_z and mechanical angular momentum L_z . (c) The configuration of an atomic chain with $N = 50$ atoms at different times. r_0 is the lattice constant. For cobalt, r_0 is 250.71 pm. Colors denote atoms with different initial positions.

no concern here because a similarly successful demonstration using a three-dimensional needle is presented in Appendix E.

Figure 1(b) verifies the Einstein-de Haas effect [38–41], where S_z , the total spin angular momentum along z axis, and L_z , the lattice mechanical angular momentum along z axis, keep exchanging, but with their sum conserved over time. That is, during the dynamical evolution, $J_z = S_z + L_z$ is a conserved quantity. Remarkably, the ferromagnetic needle as a whole is seen to acquire a mechanical angular momentum and start to precess collectively around the z axis. This precession behavior is much like that of a single spin initially polarized in the x - y plane. If there were one single spin, the applied magnetic field would lead to the Larmor frequency ω_L . For a 1 nT magnetic field, the frequency is 28 Hz. Now with spin-lattice coupling, the spins are seen to drive the entire lattice to precess around the z axis. Indeed, Fig. 1(c) shows the explicit configuration of the 1D atomic chain, obtained from the dynamics simulations, at 0, $1/8$, $1/4$, and $1/2$ of the precession period $2\pi/\omega_L$. A continuous animation is also provided in the Supplemental Material [42]. Interestingly, the small oscillation in L_z [as depicted in Fig. 1(b)] does indicate that the following of the needle orientation with the spin precession is not instantaneous—there are small oscillations upon the collective precession. The frequency of this oscillation, governed by the spin-lattice interaction strength C , is found numerically

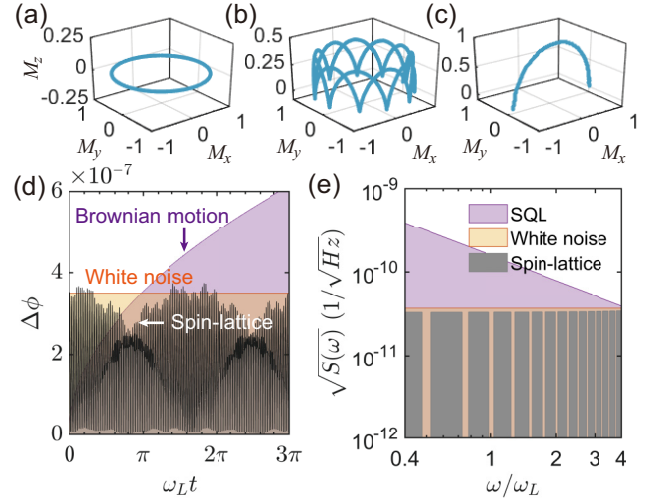


FIG. 2. (a)–(c) Three dominating dynamical modes of needle collective motion, including (a) precession, (b) nutation, and (c) libration, over two periods $4\pi/\omega_L$. (d) Standard deviations in the precession angle ϕ as a function of time, under different scenarios. (e) Bar plot of noise power spectral density due to spin-lattice interaction, as compared with that of white noise (which is similar) and that in the case of a collection of independent spins illustrating the SQL.

to be approximately $\omega_{\text{osc}} \approx 1.2$ kHz in our case. Moreover, we observe a slight deviation in the effective precession frequency compared to the spin-only case, due to an internal field induced by the lattice, leading to a small correction to the precession frequency $\omega = \gamma(B_{\text{external}} + B_{\text{effective}})$, where $B_{\text{effective}}$ stems from lattice backaction. Similar effects have been discussed in Ref. [26].

III. DIFFERENT MODES OF NEEDLE COLLECTIVE MOTION

The precession motion seen above is modified at stronger magnetic fields. Indeed, assuming pure precession dynamics, we have $|S_z| = |L_z| \approx \omega_L I$, where I is the moment of inertia of the needle, with $I \approx \frac{1}{12} m r_0^2 N^3$. Because the spin angular momentum S_z is on the order of $N\hbar/2$, for collective lattice precession to dominate we must have $\omega_L I \ll N\hbar/2$, corresponding to a magnetic field of $B \ll B_c \approx 6\hbar/(\gamma m r_0^2 N^2)$. For instance, B_c is approximately 230 μ T given 50 cobalt atoms. It is then interesting to observe, from our first-principle simulations, the dynamics of the needle if the magnetic field strength increases. As shown in Figs. 2(a)–2(c), the collective motional modes of the needle gradually change from precession to nutation and eventually to libration around the magnetic field direction. Specifically, Figs. 2(a)–2(c) depict the time evolution of the normalized magnetization $\mathbf{M}$, defined as $\frac{1}{N} \sum_i \mathbf{s}_i$, in three-dimensional space under magnetic fields of 1 nT, 50 μ T, and 5 mT, respectively. Under weak magnetic fields, the collective precession of the lattice dominates, and the trajectory of the magnetization forms a unit circle in the x - y plane over time [Fig. 2(a)]. As the magnetic field strength increases, nutation becomes increasingly significant [Fig. 2(b)]. When the magnetic field exceeds B_c , all spins tend to align parallel to

the field direction [Fig. 2(c)]. In this regime, the magnetization oscillates in the z - x plane, where z (x) is the direction of the magnetic field (initial lattice orientation). Over a long time of evolution, the plane of the libration will also rotate at an intrinsic frequency (Appendix G). Using a microscopic theory, our results confirm and strengthen an earlier study that predicts the three different dynamical regimes with a macrospin model [25]. Significantly, in all three dynamical regimes, we always have $M_x^2 + M_y^2 + M_z^2 = 1$, indicating that the macrospin (M_x, M_y, M_z) stays on the Bloch sphere and there is no observable dephasing. That is, all individual spins (upon fast averaging around the needle axis) have the same polarization. As mentioned earlier, this is one strong justification for a classical treatment of the spin degree of freedom.

IV. BEATING THE SQL: A MICROSCOPIC DYNAMICS PERSPECTIVE

Next we computationally investigate how a levitated ferromagnetic needle can sense a magnetic field with a precision surpassing the SQL. To that end, we first elaborate on the SQL using an ensemble of independent spins to sense a magnetic field $\mathbf{B}$. The noise independently experienced by the spins necessarily causes each spin's precession motion to fluctuate with time. This process can be modeled by a stochastic noise term so that the total energy is given by $\sum_i (\mathbf{B} + \xi_i) \cdot \mathbf{S}_i$, where ξ_i represents the independent white noise experienced by each spin. In this picture, the ensemble of the spins undergoes a random walk. Let $\Delta\phi = \sqrt{\frac{1}{N} \sum_i (\phi_i - \langle\phi\rangle)^2}$ be the uncertainty in precession angle ϕ . Then, it grows as $\Delta\phi \propto \sqrt{t}$ [Fig. 2(d)] and the corresponding noise power spectral density (PSD) $S(\omega)$ has a typical $1/\omega^2$ scaling [Fig. 2(e)] [43] (Appendix H). Consequently, quantum sensing using independent spins is constrained by the SQL, where uncertainty in the precession frequency estimation follows $\Delta\omega = \Delta\phi/t \propto 1/\sqrt{t}$.

In contrast, for a levitated ferromagnetic needle, despite the noise due to the intrinsic spin-lattice coupling, the uncertainty $\Delta\phi$ during the precession dynamics saturates instead of the characteristic diffusive behavior. Microscopically, this verifies the qualitative insight by Kimball *et al.* [24]. Due to the saturation of the precession angle uncertainty, all the spins remain “in phase,” thus explaining the observation above that the macrospin does stay on the Bloch sphere. Indeed, the spin-spin exchange interaction and the spin-lattice interaction jointly lead to a neat dynamical decoupling effect [44,45]: Any noise causing the spins to drift away from the lattice axis is canceled by fast rotation around the lattice axis. Interestingly, microscopic dynamics here also allows us to track the explicit time dependence of the uncertainty in $\Delta\phi$: For the parameters chosen in the shown simulations, it exhibits oscillations more than two orders of magnitude faster than the precession frequency itself. The rate of this important self-averaging is found to be directly determined by the strength of the spin-lattice coupling. We further examine the PSD of the intrinsic noise due to spin-lattice coupling and spin-spin interaction. It is seen to have a characteristic flat spectrum [Fig. 2(e)], hence close to that of white noise. Using the white-noise limit and the associated Cramér-Rao lower bound (CRLB), the scaling of uncertainty in frequency estimation is

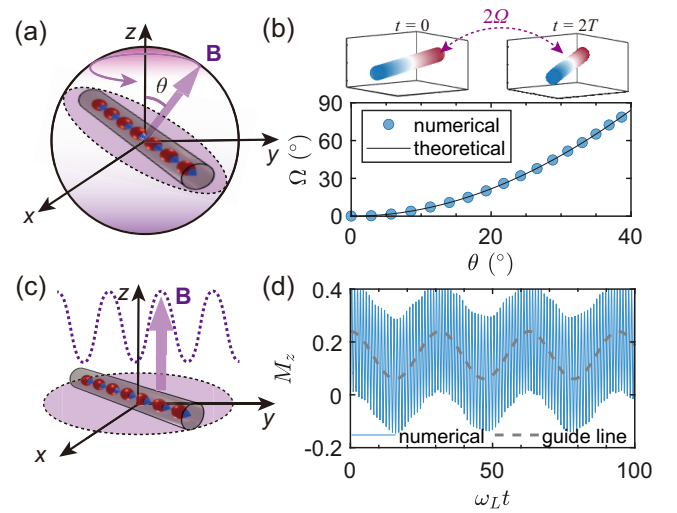


FIG. 3. (a) Schematic of a ferromagnetic needle in the presence of a slowly rotating magnetic field with a fixed polar angle θ . (b) The initial and final configurations of the needle differ by an angle $2\Omega = 4\pi(1 - \cos\theta)$, manifesting the Berry phase associated with the adiabatic protocol. (c) Schematic of a needle undergoing nutation in an oscillating magnetic field. (d) Results from the microscopic dynamics, indicating that the oscillation in the magnetic field is mapped to the time dependence of the nutation amplitude.

given by $\Delta\omega \geq \sqrt{\frac{12}{\text{SNR} f_{\text{BW}} t^3}}$, where SNR means signal-to-noise ratio and $f_{\text{BW}} = 1/\text{dt}$ is the frequency bandwidth [46]. As the measurement time increases, $\Delta\omega$ scales as $t^{-3/2}$, thus beating the SQL. Unlike the realization of cooperative spins using feedback control [12], here the spin-spin correlation inherent in a ferromagnetic needle offers an intrinsic approach to surpassing the SQL.

V. BERRY PHASE MANIFESTED BY A LEVITATED NEEDLE

Armed with our microscopic theory, we next propose to detect the celebrated Berry phase using the precession dynamics of a levitated ferromagnetic needle. This will be of fundamental interest to our understanding of quantum-classical correspondence and to quantum metrology. We first describe the protocol using a quantum mechanics language for a collection of N isolated spins. If all the spins are initially polarized perpendicular to the external field, the total spin state is a product state, i.e., $[\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)]^{\otimes N}$, where $|\uparrow\rangle$ and $|\downarrow\rangle$ are spin-up and spin-down states with respect to the field direction. If the applied magnetic field $\mathbf{B}$ rotates slowly (as compared with the precession frequency) around the z axis [Fig. 3(a)] and completes one cycle, the two spin components $|\uparrow\rangle$ and $|\downarrow\rangle$ accumulate Berry phases $\gamma_{\uparrow}^B$ and $\gamma_{\downarrow}^B = -\gamma_{\uparrow}^B$, respectively, on top of their dynamical phases. Similar to how experiments in NMR, Nitrogen-Vacancy Center, and atomic magnetometer systems [47–49] extracted the Berry phase, in the second cycle we repeat the same process but reverse the applied magnetic field, thus canceling the dynamical phases. This two-cycle protocol then leads to the final spin state $[\frac{1}{\sqrt{2}}(e^{2i\gamma_{\uparrow}^B}|\uparrow\rangle + e^{2i\gamma_{\downarrow}^B}|\downarrow\rangle)]^{\otimes N}$, a state

evidently misaligned from the initial orientation by $2\Omega = 2\gamma_{\uparrow}^B - 2\gamma_{\downarrow}^B$, where Ω is the solid angle traced out by the rotating magnetic field. Our dynamical simulation shows that this misalignment arising from a geometrical effect is indeed passed to the final configuration of the needle. In particular, we assume that $\mathbf{B}(t)$ is rotating around the z axis at a fixed polar angle θ . As shown in Fig. 3(b), upon reading out the final orientation of the needle, the misalignment from its initial configuration can be found and compared with the theoretical result 2Ω , with $\Omega = 2\pi(1 - \cos\theta)$. The results retrieved from our microscopic dynamics simulations of the needle are in excellent agreement with the theoretical Berry phase results. Berry phase enables stroboscopic measurements, enhancing robustness against external noise and quantum backaction [50]. Furthermore, we again demonstrate from microscopic dynamics that a ferromagnetic needle can behave like a macrospin, even in terms of highly subtle geometrical aspects of the spin dynamics.

VI. FREQUENCY SENSING OF AN OSCILLATING MAGNETIC FIELD

Finally, exploiting our microscopic theory, we propose to use the dynamical regime of nutation to determine the frequency of a magnetic field with a slowly oscillating amplitude, as illustrated in Figs. 3(c) and 3(d). In the presence of an oscillating field, the precession, manifested by sinusoidal M_x or M_y , becomes chirped due to its frequency dependence on the field amplitude, thus making the signal analysis more challenging. Interestingly, nutation motion offers a simpler alternative for sensing because its amplitude M_z is found to be directly proportional to the amplitude of the oscillating field. As an example, the nutation dynamics under an oscillating magnetic field $B_z = B_0[1 + 0.5 \cos(0.2\omega_L t)]$ is shown in Fig. 3(d). There, the blue line represents numerical results from the dynamics of the needle, whereas the dashed gray line serves as a guide of our eyes to show that the temporal profile of B_z and M_z coalesce. Using the Fourier analysis, we can then determine the oscillation frequency of the applied magnetic field. Already supported by first-principle simulations here, the nutation motion of a levitated needle is expected to be useful for the sensing of axionlike dark matter [51–53] that is often connected with an oscillating field. Using some analysis similar to the case of precession dynamics, it is straightforward to find that here the precision limit in determining the oscillation frequency of the field also surpasses the SQL. More details of this sensing protocol are elaborated in Appendix I.

VII. CONCLUSION

With an explicit spin-lattice interaction accounted for, we are able to fully investigate the atomic-scale dynamics of a levitated ferromagnetic needle as a sensing platform. The phenomenological damping due to spin-lattice coupling, including its quantum analog [54–57], can now be investigated from a Hamiltonian theory. Both conceptually and computationally, we have demonstrated the great potential of the levitated ferromagnet platform in ultrasensitive magnetometry. The demonstration of how the Berry phase can

be measured by a precessing needle shall motivate future studies, especially on possibilities in connecting single-spin dynamical behaviors with those of a macroscopic ferromagnetic needle. As illustrated in the Appendixes, our framework can also be extended to model spin-lattice relaxation in multidomain ferromagnets, further reinforcing its versatility as a theoretical toolbox for guiding future experiments.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [58].

APPENDIX A: DYNAMIC EQUATIONS OF MOTION FOR THE SPIN-LATTICE HAMILTONIAN

In the main text, we highlight the equations of motion for the spin degree of freedom, showing how the spin-lattice interaction enables the angular momentum transfer between $\hat{\mathbf{S}}$ and $\hat{\mathbf{L}}$. Here, we explicitly give the equations of motion with respect to other degrees of freedom:

$$\frac{d\hat{\mathbf{r}}_i}{dt} = \frac{d\hat{\mathbf{p}}_i}{m}, \quad (\text{A1})$$

$$\begin{aligned} \frac{d\hat{\mathbf{p}}_i}{dt} = & 2C \sum_j [\hat{\mathbf{S}}_i(\hat{\mathbf{r}}_{ij} \cdot \hat{\mathbf{S}}_j) + \hat{\mathbf{S}}_j(\hat{\mathbf{S}}_i \cdot \hat{\mathbf{r}}_{ij})] \\ & - 4V \sum_j \frac{\hat{r}_{ij} - \bar{r}_{ij}}{\hat{r}_{ij}} \hat{\mathbf{r}}_{ij}. \end{aligned} \quad (\text{A2})$$

These equations at the operator level, derived from the Hamiltonian in Eq. (1), are based on the Heisenberg equation of motion:

$$\frac{d\hat{A}}{dt} = \frac{i}{\hbar} [\hat{H}, \hat{A}], \quad (\text{A3})$$

along with the commutation relations $[\hat{S}_i, \hat{S}_j] = i\hbar\epsilon_{ijk}\hat{S}_k$ and $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$.

TABLE I. Dimensionless variables used in the calculations.

Physical quantity	Dimensionless variable
Time t	$t_0 = \omega_L t$
Spin angular momentum $\mathbf{S}$	$\mathbf{s} = \mathbf{S}/S_0$
Magnetic field $\mathbf{B}$	$\mathbf{b} = \mathbf{B}/ \mathbf{B} $
Position $\mathbf{r}$	$\mathbf{r}' = \sqrt{\frac{\gamma \mathbf{B} m}{S_0}} \mathbf{r}$
Momentum $\mathbf{p}$	$\mathbf{p}' = \sqrt{\frac{1}{\gamma \mathbf{B} S_0 m}} \mathbf{p}$
Spin-spin interaction J	$J_0 = \frac{S_0}{\gamma \mathbf{B} } J$
Spin-lattice coupling C	$C_0 = \frac{S_0^2}{m\gamma^2 \mathbf{B} ^2} C$
Harmonic potential V	$V_0 = \frac{1}{m\gamma^2 \mathbf{B} ^2} V$

For a system with N spins, the total Hilbert space is as large as $S^N \times [d(H_r)]^N$, where S is the number of states for each spin and $d(H_r)$ is the dimension of the position operator $\hat{\mathbf{r}}$ in position spaces. This makes an exact solution of the quantum many-body dynamics computationally impossible. To address this issue, we adopt a classical approach such that the above Heisenberg equations of motion are used to write down the corresponding classical Hamilton's equations of motion. Our Hamiltonian-based theory here helps to offer first-principle insights and serves as a foundation for exploring interesting physics in the future through approximations such as the macrospin model, mean-field methods, or Lindblad master equations (treating the lattice as an external environment).

APPENDIX B: DIMENSIONLESS PARAMETERS FOR NUMERICAL CALCULATIONS

To facilitate numerical calculations of the dynamics, it is necessary to consider the characteristic timescale of the system. In our Hamiltonian model, the spin precession, spin-spin interaction, and lattice vibration give rise to drastically different timescales. We consider spin precession to be the main feature of the system. In the numerical calculations, we use the following dimensionless variables, where the unit of time becomes the period of the Larmor precession, as listed in Table I.

By using the dimensionless variables, the dynamics equations of motion become

$$\frac{d\mathbf{s}_i}{dt_0} = \mathbf{s}_i \times \mathbf{b} + J_0 \mathbf{s}_i \times (\mathbf{s}_{i+1} + \mathbf{s}_{i-1}) + 2C_0 \mathbf{s}_i \times \sum_j \mathbf{r}'_{ij} (\mathbf{r}'_{ij} \cdot \mathbf{s}_j), \quad (\text{B1})$$

$$\frac{d\mathbf{r}'_i}{dt_0} = \mathbf{p}'_i, \quad (\text{B2})$$

$$\frac{d\mathbf{p}'_i}{dt_0} = 2C_0 \sum_j [\mathbf{s}_i (\mathbf{r}'_{ij} \cdot \mathbf{s}_j) + \mathbf{s}_j (\mathbf{s}_i \cdot \mathbf{r}'_{ij})] - 4V_0 \sum_j \frac{r'_{ij} - \bar{r}'_{ij}}{r'_{ij}} \mathbf{r}'_{ij}. \quad (\text{B3})$$

We solve the equations of motion using the fourth-order Runge-Kutta (RK4) method with a time step of 0.0001 in

matlab. The results are consistent with those obtained using higher-order methods (e.g., RK8) and stiff solvers like Rodas4P (a fourth-order A-stable Rosenbrock method with stiff-aware interpolation) implemented in Julia.

APPENDIX C: CONNECTION TO THE LANDAU-LIFSHITZ-GILBERT EQUATION

The pioneering proposal of needle magnetometry largely used the phenomenological Landau-Lifshitz-Gilbert (LLG) damping for physical reasoning. The LLG equation is widely used in micromagnetic simulations for spintronics applications, but it is fair to say that its explicit physical origin has not yet been fully understood [59,60]. To connect actual material parameters and physical properties of a system with LLG damping to facilitate experimental designs, we exploit our microscopic theory to dynamically estimate the LLG damping coefficient in our one-dimensional spin-lattice model. In particular, the LLG equation is given by

$$\frac{d\mathbf{s}_i}{dt} = \gamma \mathbf{s}_i \times \mathbf{H}_{\text{eff}} + \gamma \eta \mathbf{s}_i \times (\mathbf{s}_i \times \mathbf{H}_{\text{eff}}), \quad (\text{C1})$$

where $\mathbf{H}_{\text{eff}}$ is an effective magnetic field including spin-spin exchange interaction $\mathbf{H}_{\text{eff}} = \mathbf{B} + J/\gamma(\mathbf{S}_{i+1} + \mathbf{S}_{i-1})$ and η is the Gilbert damping coefficient that quantifies the energy dissipation rate from the spins to the lattice. The first term $\gamma \mathbf{s}_i \times \mathbf{H}_{\text{eff}}$ results in a collective precession, while the second term $\gamma \eta \mathbf{s}_i \times (\mathbf{s}_i \times \mathbf{H}_{\text{eff}})$ contributes to the damping. In terms of the microscopic dynamics derived from our model, the energy damping arises intrinsically from the pseudodipolar spin-lattice interaction $2C\mathbf{S}_i \times \sum_j \mathbf{r}_{ij}(\mathbf{r}_{ij} \cdot \mathbf{S}_j)$. Therefore, by comparing Eq. (C1) with the microscopic dynamics equations of motion as given by Eq. (2), the effective damping coefficient in our model can be evaluated by performing a simple time average, namely,

$$\langle \eta \rangle = \frac{\langle 2C_0 \sum_j \mathbf{r}'_{ij} (\mathbf{r}'_{ij} \cdot \mathbf{s}_j) \rangle}{\langle \mathbf{s}_i \times \mathbf{H}_{\text{eff}} / |\mathbf{B}| \rangle}. \quad (\text{C2})$$

From Eq. (C2), it is evident that $\langle \eta \rangle$ is proportional to the spin-lattice coupling strength C_0 . In our simulations for a given magnetic field of 1 nT, we have set C_0 to 1.2×10^5 to match the experimental value of $\eta \approx 0.01$. The damping coefficient versus the spin-lattice coupling strength C_0 is shown in Fig. 4(a). Though spin-exchange interaction contributes a strong internal magnetic field, it hardly changes the Gilbert damping because all the spins are essentially “in phase” in our simulation, meaning $\mathbf{S}_i \parallel \mathbf{S}_{i\pm 1}$ [Fig. 4(b)]. Besides, our microscopic model incorporating the lattice vibration also allows us to explicitly study how the lattice harmonic potential strength V_0 influences the effective LLG damping coefficient. As Fig. 4(c) shows, the damping coefficient monotonously and slowly decreases as V_0 increases, at least until $V_0 = 10^8$, where we are forced to take a cut due to the huge timescale mismatch between the lattice vibration and the spin precession dynamics. In Fig. 4(d), we also investigate the temperature dependence of the effective Gilbert damping coefficient by changing the lattice temperature T_L up to 0.5 mK. In levitated experiments, the temperature of nanoparticles has been cooled down to around 12 μK using feedback cooling [61]. Similarly, cooling down a levitated ferromagnetic needle would

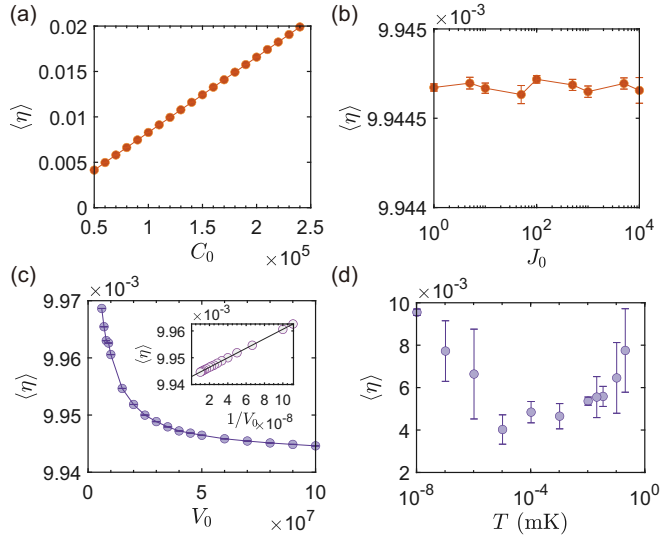


FIG. 4. Effective Gilbert damping coefficients when varying (a) spin-lattice coupling strength C_0 , (b) spin-spin exchange interaction strength J_0 , (c) lattice vibration strength V_0 , and (d) the effective temperature of the lattice.

help suppress thermal noise and enable the exploration of quantum superposition. It should also be noted that the $t^{-3/2}$ sensitivity scaling remains unaffected at higher temperatures, provided that the coherence between spins is preserved by inherent interactions. For pure ferromagnetic materials Fe, Co, and Ni, ferromagnetic resonance measurements have observed that the Gilbert damping will increase as temperature decreases [62,63]. Our microscopic simulations do agree with this observed trend in the low-temperature regime accessible by our computational methods.

APPENDIX D: RESULTS WITH VARYING NUMBERS OF ATOMS

The actual number of atoms involved in the calculation will not introduce qualitative differences from the results in the main text. In other words, there are no significant finite-size effects in our calculations. In this Appendix, we elaborate on this point. First, we calculate the Gilbert damping coefficients similar to the last Appendix, with varying numbers of atoms. It is shown in Fig. 5(a) that when N changes by one order of magnitude (from 10 to 100), there is only a negligible change in the Gilbert damping coefficients. As such, our computer simulations confirm that Gilbert damping is a material property intrinsically determined by the coupling strengths J , C , V , and temperature T and has little to do with the size of the material.

Besides the Gilbert damping coefficients, in Fig. 5(b), we compare the explicit configuration of the lattice at the same time with $N = 10$ and $N = 100$ atoms under 1 nT magnetic field. They both present essentially perfect precession in the x - y plane at the same precessing frequency. Therefore, though computational ability limits the total number of atoms that can be simulated, the main features are truly robust with respect to the number of atoms.

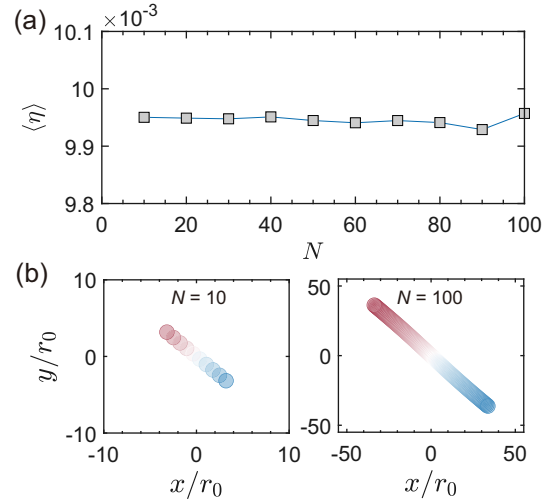


FIG. 5. (a) Gilbert damping coefficient with varying numbers of atoms in the simulation. (b) Lattice configuration at $1/8$ of a precession period for $N = 10$ and $N = 100$ atoms.

An important consideration due to an increase in the number of atoms is the maximum amplitude of the magnetic field to observe precession dominantly. As discussed in the main text, this value scales as $1/N^2$ for a one-dimensional ferromagnet. For $N = 10^6$ atoms, the corresponding magnetic field B_c decreases to 580 fT. Experiments must therefore balance the size of the ferromagnetic material with the amplitude of the magnetic fields, as larger structures can only precess at lower frequencies. Fortunately, in real experiments we always have three-dimensional materials. For a given number of atoms N , B_c will be much higher compared with the one-dimensional case illustrated in the main text. That is, because three-dimensional materials have a more compact structure, we expect to have a smaller moment of inertia as compared with the 1D case with the same number of atoms. Furthermore, the material's shape affects its moment of inertia and thus alters B_c . For a three-dimensional cylinder with radius r and length L , the moment of inertia is given by $I = \frac{1}{12} \rho \pi r^2 L^3$ and the total number of spins is $N = \rho \pi r^2 L / m$, where m is the mass of an atom and ρ is the density. Comparing $\gamma B I$ and $N \hbar / 2$, the corresponding magnetic field is now $B_c \approx 6 \hbar / m \gamma L^2$. For a microscale needle, for instance, $L = 1 \mu\text{m}$, B_c is around 40 nT, a value achievable in experiments by proper magnetic shielding [7].

APPENDIX E: RESULTS FOR A THREE-DIMENSIONAL LATTICE

As mentioned in the main text, true ferromagnetic order is only stable in three or higher dimensions when interactions are short-ranged. In this Appendix, we present results for a three-dimensional cuboidal lattice with nearest-neighbor spin-exchange and spin-lattice interactions, as illustrated in Fig. 6(a).

We solve the coupled dynamics of both the spin and lattice degrees of freedom based on our Hamiltonian mechanics framework. Figure 6(b) shows the time evolution of the normalized magnetization, defined as $\mathbf{M} = \frac{1}{N} \sum_i \mathbf{S}_i$, which

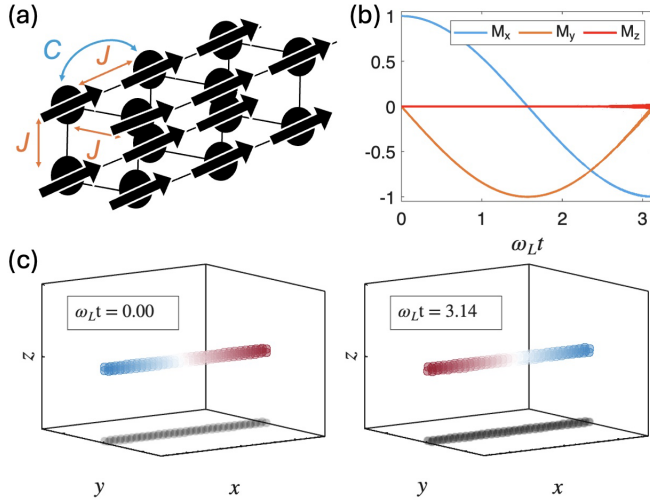


FIG. 6. (a) Illustration of a three-dimensional lattice with short-range interactions, where J denotes the spin-spin exchange interaction and C denotes the spin-lattice interaction. (b) Corresponding dynamics of normalized magnetization under static magnetic field. (c) Lattice configurations at two different times.

exhibits the characteristic collective Larmor precession of the spin ensemble. For the lattice dynamics, Fig. 6(c) displays the lattice configurations at $t = 0$ and $t = \pi/\omega_L$, clearly showing a 180° rotation, consistent with the spin-driven rigid-body rotational motion.

This three-dimensional simulation presented here confirms that our theoretical framework is not restricted to the one-dimensional case studied in detail in the main text and can be readily extended to more realistic geometries relevant for experimental systems.

APPENDIX F: EXTENSION TO MULTIDOMAIN FERROMAGNETS

One important experimental challenge is the fabrication and stabilization of a macroscopic single-domain ferromagnet, where all spins are initially aligned in the same direction. As noted in prior work, the estimated critical size below which a ferromagnetic particle remains single domain is approximately $2\mu\text{m}$ [64]. Reaching or maintaining this condition experimentally can be difficult, especially for larger samples or in samples with structural imperfections.

In this Appendix, we demonstrate that even in the presence of a multidomain spin texture, a ferromagnet can still exhibit collective precession, albeit with enhanced fluctuations. This finding underscores the practical relevance of our model in interpreting upcoming experimental data where perfect single-domain preparation is not guaranteed.

We begin by examining a one-dimensional ferromagnetic chain initialized with a domain wall at its center, as shown in Fig. 7(a). The domain-wall dynamics are studied under both exchange interaction and spin-lattice interaction. Figure 7(b) shows the time evolution of the S_y component, the sign of which effectively tracks the domain structure. While the center of the domain wall remains relatively stationary, both left and right domains undergo diffusive transport,

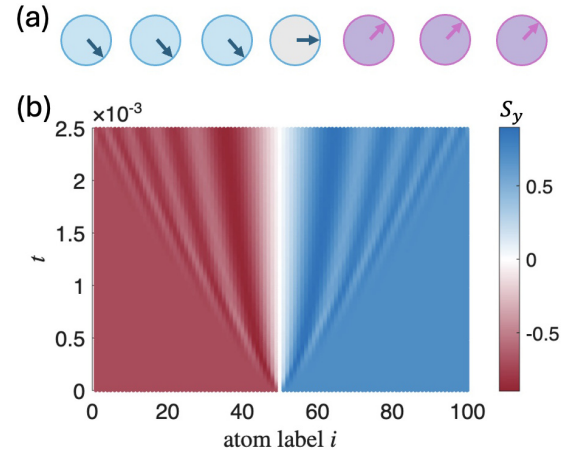


FIG. 7. (a) Illustration of a domain wall in a multidomain spin chain: For the left part of the chain, spins are rotated clockwise relative to the center spin, while for the right part, spins are rotated counterclockwise with respect to the center spin in the S_y - S_x plane. (b) Dynamics of S_y , showing that the domain wall will transport diffusively in our system.

resembling behavior observed in near-isotropic XXZ Heisenberg chains [65].

We then simulate the collective precession of a multidomain ferromagnetic needle under a 1 nT static magnetic field. As shown in Fig. 8(a), the lattice configuration after half a Larmor period ($t = \pi/\omega_L$) reveals a 180° rotation, consistent with collective precession. However, compared to the single-domain case, the multidomain configuration exhibits larger fluctuations around the needle axis, due to spin waves excited by internal domain-wall motion. This is quantified in Fig. 8(b), where the uncertainty in the precession angle $\Delta\phi$ is substantially larger.

Importantly, despite the enhanced fluctuations, the noise spectrum retains a white-noise character, as shown in Fig. 8(c). This means that the coherent collective motion of the needle is preserved, even in the presence of internal spin

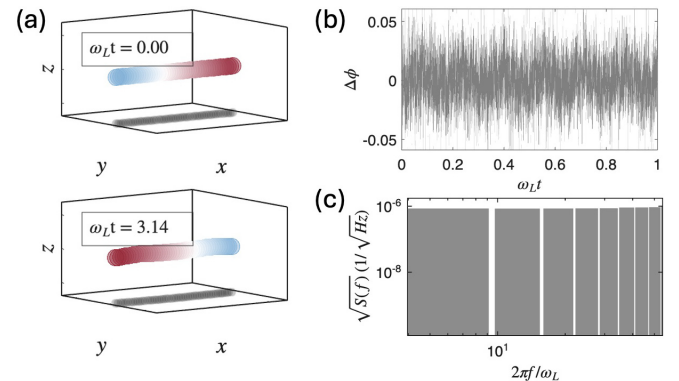


FIG. 8. (a) Dynamics of a needle with a centered domain wall similar to Fig. 7. (b) Fluctuations of the precession angle ϕ among spins within the needle, showing significantly larger noise compared to the single-domain case in Fig. 2(d). (c) Corresponding noise power spectral density, indicating that the characteristic white-noise behavior is preserved.

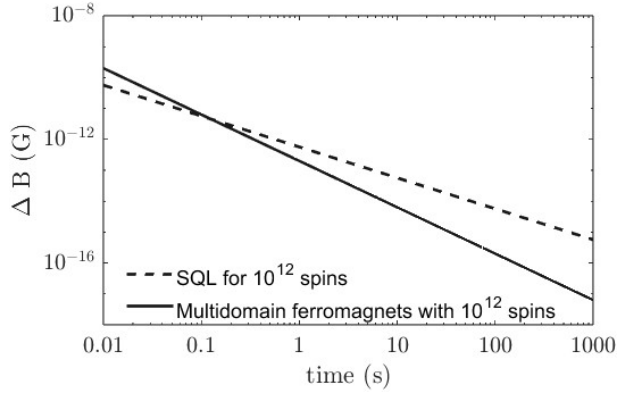


FIG. 9. Comparison of magnetic field uncertainty between SQL and multidomain ferromagnets.

texture. Thus, our results show that observable precession dynamics are still possible in multidomain ferromagnets. While we expect increased measurement noise in such systems, the favorable sensitivity scaling $\Delta B_z \propto 1/t^{3/2}$ —a hallmark of coherent needle precession—is still maintained. To quantify the impact of multidomain noise on sensitivity, we analyze the spin precession uncertainty from simulations involving 50 spins [Fig. 8(b)], where the phase uncertainty is approximately $\Delta\phi_m \approx 0.05$. This allows us to estimate the corresponding magnetic field uncertainty as

$$\Delta B_m \approx \frac{0.05\sqrt{50}}{\sqrt{N}t^{3/2}},$$

where N is the total number of spins and t is the measurement time. For comparison, the SQL yields a phase uncertainty of

$$\Delta\phi_s \approx \sqrt{\frac{\Gamma t}{N}},$$

which reduces to $\Delta\phi_s \approx \sqrt{1/N}$ in the ideal limit of infinite coherence time ($\Gamma \approx 1/t$), leading to a magnetic field sensitivity of

$$\Delta B_{\text{SQL}} \approx \frac{1}{\gamma\sqrt{N}t}.$$

In Fig. 9, we compare the performance of multidomain ferromagnets with the SQL by scaling the total number of spins to 10^{12} —a realistic experimental scale. The results show that while multidomain systems initially exhibit higher noise, their sensitivity improves more rapidly with time, and eventually outperforms SQL as t increases. These findings demonstrate that even in imperfect, multidomain configurations, the precision of magnetic field sensing goes beyond the SQL, highlighting the robustness of our framework and its relevance to realistic experimental implementations.

APPENDIX G: ROTATION OF THE LIBRATION PLANE

In this Appendix, we provide more insight into the three different dynamical regimes. For the precession regime, the precession frequency of M_x or M_y is proportional to the external magnetic field $\omega_L = \gamma|\mathbf{B}|$. Therefore, we can use the precession motion to infer the amplitude of an external magnetic field. When the magnetic field increases, the transitions

TABLE II. Classification of three dynamical regimes.

	Precession	Nutation	Libration
$\max[S_z]$	$\ll N\hbar/2$	$< N\hbar/2$	$= N\hbar/2$

to nutation and libration are smooth. We can classify the three dynamical regimes, where each dynamic mode dominates, according to the maximum value of the total spin- z component (Table II).

For libration, which is the usual case of a heavier ferromagnet or higher external magnetic field, L_z and S_z saturate to $N\hbar/2$. Note that the plane of libration will still rotate over a long time of evolution since L_z is nonzero. Figure 10(a) shows the slight rotation of the libration plane over 10 periods $20\pi/\omega_L$. Here, L_z is an intrinsic value that only depends on the number of spins in the material. For the $N = 50$ cobalt atoms in the main text, the frequency of the intrinsic rotation will be around 0.23 MHz, which is one magnitude lower than the frequency of the libration [Fig. 10(b)].

APPENDIX H: METHOD FOR CALCULATING THE POWER SPECTRAL DENSITY

In this Appendix, we outline the method for calculating the PSD from a time series $\phi(t)$ obtained via dynamical simulations. According to the Wiener-Khinchin theorem [66,67], the PSD is given by the Fourier transform of the autocorrelation function of $\phi(t)$:

$$S(\omega) = \int_{-\infty}^{\infty} \langle \phi(t)\phi(t+\tau) \rangle e^{-i\omega\tau} d\tau, \quad (\text{H1})$$

where $\langle \phi(t)\phi(t+\tau) \rangle$ is the time autocorrelation function, defined as

$$\langle \phi(t)\phi(t+\tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \phi^*(t)\phi(t+\tau) dt. \quad (\text{H2})$$

We evaluate $S(\omega)$ numerically using the fast Fourier transform (FFT). The relation between $S(\omega)$ and the squared magnitude of the Fourier transform $F(\omega)$ follows from

$$\begin{aligned} |F(\omega)|^2 &= \lim_{T \rightarrow \infty} \left| \int_{-T/2}^{T/2} \phi(t) e^{-i\omega t} dt \right|^2 \\ &= \lim_{T \rightarrow \infty} \left(\int_{-T/2}^{T/2} \phi^*(t) e^{i\omega t} dt \right) \left(\int_{-T/2}^{T/2} \phi(t') e^{-i\omega t'} dt' \right) \end{aligned}$$

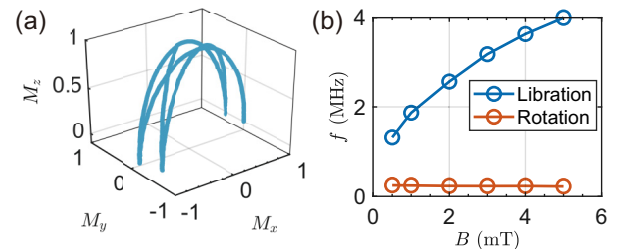


FIG. 10. (a) Slowly rotation of the libration plane over ten periods. (b) Frequency of libration and the slow rotation with 50 atoms. With higher magnetic field, the libration frequency increases, while the rotation frequency remains approximately the same.

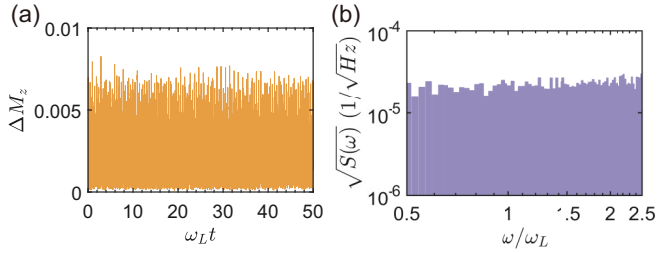


FIG. 11. (a) Uncertainty of nutational motion and (b) the corresponding power spectral density.

$$\begin{aligned}
 &= \lim_{T \rightarrow \infty} T \int_{-T/2}^{T/2} \left[\frac{1}{T} \int_{-T/2}^{T/2} \phi^*(t) \phi(t + \tau) dt \right] e^{-i\omega\tau} d\tau \\
 &= \lim_{T \rightarrow \infty} T S(\omega). \quad (\text{H3})
 \end{aligned}$$

Since time series are finite in practical computations, the factor $\lim_{T \rightarrow \infty} T$ is effectively replaced by the inverse of the frequency resolution (or bandwidth) to ensure a well-defined estimate of the PSD.

For white noise $w(t)$, the power spectral density is flat, meaning $S_w(\omega) = \text{const}$. For Brownian noise, namely, the noise related to Brownian motion, the corresponding time series $b(t)$ is the integral of white noise:

$$\frac{db(t)}{dt} = w(t). \quad (\text{H4})$$

By taking the Fourier transform of the Brownian noise $b(t)$ and using the integration property of the Fourier transform, we obtain

$$F_b(\omega) = \frac{F_w(\omega)}{i\omega}, \quad (\text{H5})$$

where $F_b(\omega)$ and $F_w(\omega)$ are the FFT of Brownian noise and white noise, respectively. Therefore, the power spectral density of the Brownian noise linearly depends on $1/\omega^2$:

$$S_b(\omega) \propto |F_b(\omega)|^2 = \frac{\text{const}}{\omega^2}. \quad (\text{H6})$$

APPENDIX I: SENSITIVITY OF MEASURING AN OSCILLATING FIELD

In the main text, we proposed using nutational motion to determine the frequency of a slowly oscillating magnetic field. Here, we analyze the noise in nutation caused by intrinsic spin-lattice relaxation and evaluate the sensing sensitivity. The uncertainty in nutation over an ensemble of spins in the ferromagnetic needle is calculated as $\Delta M_z = \sqrt{\sum_i (s_{i,z} - \bar{s}_z)^2 / N}$, as Fig. 11(a) shows. The noise in nutational motion exhibits nondiffusive behavior. Its power spectral density, shown in Fig. 11(b), resembles that of white noise. This is similar to the precession case discussed in the main text.

Having understood the typical noise power spectrum of nutation, we demonstrate here why the uncertainty of frequency estimation decays according to $\Delta\omega \propto 1/t^{3/2}$ based on the CRLB. First, the nutational signal can be expressed as

$$M_z(t_n) = A \cos(\omega t_n + \phi) \cos(\omega_0 t_n + \phi_0) + w[n], \quad (\text{I1})$$

where $w[n]$ denotes Gaussian white noise with zero mean and uncertainty σ . Here, ω is the frequency of the oscillating magnetic field to be measured, ω_0 is the typical oscillation frequency of nutation under a static magnetic field, ϕ and ϕ_0 are arbitrary initial phase factors, and A is the amplitude proportional to the moment of inertia of the needle. We consider a discrete time step, where $t_n = n\Delta t$, and $n = 0, 1, \dots, N-1$. For each t_n , $M_z(t_n)$ is a random variable that satisfies the following likelihood function:

$$\begin{aligned}
 p = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{n=0}^{N-1} [M_z(t_n) \right. \\
 \left. - A \cos(\omega t_n + \phi) \cos(\omega_0 t_n + \phi_0)]^2 \right\}. \quad (\text{I2})
 \end{aligned}$$

Different from typical projective measurements that collapse quantum states, the measurement of a macroscopic ferromagnetic needle in the classical regime is weak and of nondemolition type, so that the interaction with the measurement apparatus will not destroy the state of the system. Due to this reason, we obtain a time series with a size of N over the total measurement time t_N .

Next, we calculate the Fisher information $I(\omega)$ for the time series $M_z(t_n)$:

$$\begin{aligned}
 I(\omega) &= E \left[-\frac{\partial^2 \ln p}{\partial \omega^2} \right] \\
 &= \frac{1}{\sigma^2} \sum_{n=0}^{N-1} \left[\frac{\partial A \cos(\omega t_n + \phi) \cos(\omega_0 t_n + \phi_0)}{\partial \omega} \right]^2 \\
 &\approx \frac{1}{4\sigma^2} \sum_{n=0}^{N-1} A^2 n^2 (\Delta t)^2 \\
 &\approx \frac{A^2 f_{\text{BW}} t_N^3}{12\sigma^2}, \quad (\text{I3})
 \end{aligned}$$

where we have used the summation $\sum_{n=0}^{N-1} n^2 = (N-1)N(2N-1)/6$, and $f_{\text{BW}} = 1/\Delta t$ denotes frequency bandwidth. According to the Cramér-Rao lower bound [46], the variance of ω must satisfy

$$\text{var}(\omega) \geq \frac{1}{I(\omega)} \approx \frac{12\sigma^2}{A^2 f_{\text{BW}} t_N^3}. \quad (\text{I4})$$

Consequently, the standard deviation in estimating the frequency ω follows

$$\Delta\omega \geq \sqrt{\frac{6}{\text{SNR} f_{\text{BW}}^2 t_N^3}}, \quad (\text{I5})$$

where $\text{SNR} = A^2/2\sigma^2$ is the signal-to-noise ratio. Thus, the sensitivity for measuring the frequency of an oscillating magnetic field using a ferromagnetic needle scales as $t^{-3/2}$ with the total time of measurement, surpassing the standard quantum limit.

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