

Predicting entanglement entropy from particle tunneling of interacting fermions using Kolmogorov-Arnold networks

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Entanglement entropy is a fundamental measure of quantum correlations and a key resource underpinning advances in quantum information and many-body physics. We uncover a universal relationship between bipartite entanglement entropy and particle number after the barrier in a one-dimensional Fermi-Hubbard system with an external asymmetric potential. Decomposing the von Neumann entropy into number entropy S_n and configurational entropy S_c , we show that in the barrier-dominated tunneling regime both components are individually well-defined functions of the postbarrier particle density n_A , even though S_c encodes off-diagonal coherences that are not directly accessible from density measurements alone. Using Kolmogorov-Arnold networks—a novel machine learning architecture—we learn the relationship for entropy and its components across a broad range of interaction strengths and barrier heights with high predictive accuracy. Furthermore, we propose a simple analytical binary-entropy-like expression that quantitatively captures the observed correlation for fixed parameters. Our findings open avenues for characterizing quantum correlations in transport phenomena and provide a powerful framework for estimating the full von Neumann entropy—including its configurational component—from a single transport observable.

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I. INTRODUCTION

Quantifying entanglement entropy in many-body quantum systems remains one of the most challenging yet fundamental tasks in quantum physics [1,2]. The von Neumann entropy, which captures the quantum correlations between spatially separated regions, requires full knowledge of the many-body wavefunction—a formidable computational and experimental challenge that scales exponentially with system size. Direct measurement of entanglement entropy demands sophisticated quantum state tomography techniques [3] that become prohibitively complex for large systems. While alternative approaches [4–9], including machine-learning-based techniques [10,11], have provided valuable insights, they either offer only partial characterization or remain computationally intensive for large systems, which limits the ability to probe quantum correlations in realistic many-body settings [5,12].

In stark contrast, particle transport measurements have become remarkably accessible in modern quantum simulation platforms. Ultracold atoms trapped in optical lattices [13–17] now allow unprecedented control and monitoring of single-particle dynamics with site-resolved precision [18–22]. These experimental advances enable direct observation of particle tunneling, density distributions, and transport phenomena in real time, providing a natural window into the underlying quantum dynamics [23,24]. The ability to engineer controlled barriers, tune interaction strengths, and track individual particles makes ultracold atoms ideal test beds for exploring many-body systems.

These developments breathe new life into foundational quantum phenomena, bringing well-established concepts like tunneling into dialogue with entropy—a modern theoretical tool. Quantum tunneling stands among the earliest and most fundamental discoveries in quantum physics, dating back to the pioneering work of the 1920s and serving as a cornerstone of our understanding of quantum mechanics [25,26]. In contrast, entanglement entropy has emerged as a central concept only in recent decades, driven by advances in quantum information theory and the growing recognition of its role in characterizing quantum correlations [27,28]. In this broader context, entanglement entropy has previously been related to measurable quantities in various settings: particle-number fluctuations in equilibrium systems [29], current noise and full counting statistics in nonequilibrium quantum point contacts with noninteracting fermions [30], and density correlations

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following quantum quenches in one-dimensional integrable models [31]. These connections, in turn, rest on earlier results establishing universal scaling of entanglement entropy in conformal field theories and its formulation in terms of correlation matrices for Gaussian many-body states [32–34]. However, analytical methods remain unavailable for generic interacting fermions. Building on these foundations, we combine the time-tested physics of barrier tunneling with modern entanglement measures to reveal functional relationships in interacting systems, providing a pathway to probe quantum correlations through readily accessible transport observables.

In this article, we uncover a remarkable relationship that bridges this experimental divide: The bipartite entanglement entropy $S(t)$ exhibits a functional dependence on particle density $n(t)$ in the one-dimensional Fermi-Hubbard model, i.e., $S(t) = f(n(t))$. Crucially, this relationship depends only on the particle density, not on time itself [$S(t) \neq f(t, n(t))$], and reveals a connection independent of the specific dynamical trajectory. By decomposing the entanglement entropy into its number and configurational components, we show that not only S_n but also S_c —which generally encodes information inaccessible from density measurements—is functionally determined by the postbarrier particle density in the barrier-dominated tunneling regime. By employing Kolmogorov-Arnold networks (KANs) [35], we uncover a deeper structure in the entropy-transport correlation, revealing a multivariate dependence $S(t) = f(U, h, n(t))$, where the entanglement and its components can be learned as a function of interaction strength U , barrier height h , and particle density $n(t)$ across diverse parameter regimes with high accuracy. Remarkably, we find that for fixed parameters of the interacting system, this complex relationship reduces to an analytical form resembling binary entropy for all entropy components, providing an unexpected connection between quantum information theory and transport phenomena. In this way, a synthesis of old and new quantum concepts establishes an efficient framework for inferring quantum correlations—including the generally inaccessible configurational entropy—from readily measurable transport observables, opening avenues for characterizing entanglement in quantum many-body systems without the need for complex state reconstruction protocols.

II. MODEL AND METHODS

We study quantum correlations and particle transport in a one-dimensional lattice of the size L with open boundary conditions. The system is described by the single-band Fermi-Hubbard Hamiltonian with an external field forming an additional potential barrier:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{\text{hop}} + \hat{\mathcal{H}}_{\text{int}} + \hat{\mathcal{H}}_{\text{barrier}}, \quad (1)$$

where

$$\begin{aligned} \hat{\mathcal{H}}_{\text{hop}} &= -J \sum_{j=1, \sigma=\uparrow, \downarrow}^{L-1} (\hat{c}_{j, \sigma}^\dagger \hat{c}_{j+1, \sigma} + \hat{c}_{j+1, \sigma}^\dagger \hat{c}_{j, \sigma}), \\ \hat{\mathcal{H}}_{\text{int}} &= U \sum_{j=1}^L \hat{n}_{j, \uparrow} \hat{n}_{j, \downarrow}, \quad \hat{\mathcal{H}}_{\text{barrier}} = \sum_{j=L/2}^{L/2+1} V_j^{\text{ex}} \hat{n}_j. \end{aligned} \quad (2)$$

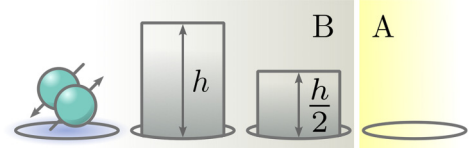


FIG. 1. Graphical representation of the model setup. Initially, N fermions are localized on the very left side. The external potential barrier with the maximum height h acts on the two central sites, dividing the system into subsystem B (containing the sites with the barrier sites and before it, highlighted in gray) and subsystem A (postbarrier region, highlighted in yellow).

Here, $\hat{c}_{j, \sigma}^\dagger$ ($\hat{c}_{j, \sigma}$) is the fermionic creation (annihilation) operator for a particle with spin $\sigma \in \{\uparrow, \downarrow\}$ at site j and $\hat{n}_{j, \sigma} = \hat{c}_{j, \sigma}^\dagger \hat{c}_{j, \sigma}$ is the corresponding number operator, $\hat{n}_j = \hat{n}_{j, \uparrow} + \hat{n}_{j, \downarrow}$. The parameters $J > 0$, U , and V_j^{ex} represent the hopping amplitude, the on-site repulsive interaction, and the site-dependent barrier height, respectively. The external potential acts only on the two central lattice sites, creating an asymmetric barrier that divides the system. The site-dependent barrier heights are defined as

$$V_j^{\text{ex}} = \begin{cases} h, & j = L/2, \\ h/2, & j = L/2 + 1, \end{cases} \quad (3)$$

where h is the maximum barrier height, creating an asymmetric potential configuration as illustrated in Fig. 1.

We partition the lattice into subsystem A (all sites after the barrier) and subsystem B (the remaining sites, including those with the barrier and all sites before it), as illustrated in Fig. 1. At the initial moment of time ($t = 0$), we place N particles on the left side before the barrier.

The key transport observable is the expectation value of the number operator $\hat{n}_A = \sum_{j \in A} \hat{n}_j$, where j runs over all sites of subsystem A . To characterize the quantum correlations in the system described by the state $|\psi\rangle$, we calculate the von Neumann entanglement entropy S_A :

$$S_A = -\text{Tr}(\rho_A \ln \rho_A), \quad (4)$$

where $\rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$ is the reduced density matrix of subsystem A . Because the total particle number N is conserved, ρ_A is block diagonal in sectors of fixed particle number $\tilde{N}$ within subsystem A . Each sector has probability $p_{\tilde{N}} = \text{Tr}(\Pi_{\tilde{N}} \rho_A)$, where $\Pi_{\tilde{N}}$ projects onto states with exactly $\tilde{N}$ particles in A . This structure yields a natural decomposition of the entanglement entropy into two contributions [36]:

$$S_A = S_n + S_c, \quad (5)$$

where the number entropy

$$S_n = - \sum_{\tilde{N}} p_{\tilde{N}} \ln p_{\tilde{N}} \quad (6)$$

captures classical fluctuations in the particle number across the partition, and the configurational entropy

$$S_c = \sum_{\tilde{N}} p_{\tilde{N}} S(\rho_A^{(\tilde{N})} / p_{\tilde{N}}) \quad (7)$$

quantifies quantum correlations within each fixed-particle-number sector, with $\rho_A^{(\tilde{N})} = \Pi_{\tilde{N}} \rho_A \Pi_{\tilde{N}}$. While $S_{\tilde{N}}$ is

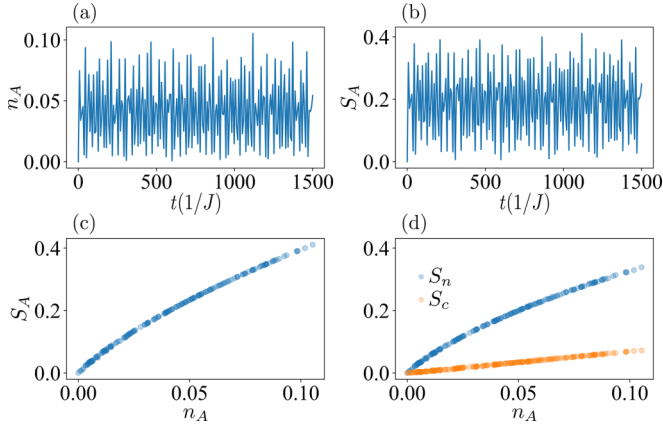


FIG. 2. Entropy-density relationship and its decomposition. (a) Particle density n_A as a function of time. (b) Entanglement entropy S_A as a function of time. (c) von Neumann entanglement entropy S_A plotted against the postbarrier particle density n_A . (d) Corresponding number and configurational entropy contributions, S_n and S_c , plotted against n_A . Each point in panels (c) and (d) represents one time step of the evolution over $tJ \in [0, 1500]$. Parameters are $L = 4$, $N = 2$, $U = 2J$, and $h = 5J$.

experimentally accessible through site-resolved density measurements, the probabilities $p_{\bar{N}}$ can be extracted from repeated snapshots of atom number in the subsystem [36]. The configurational entropy S_c encodes off-diagonal coherences that are not directly observable from density data alone.

Time evolution calculations are performed using the QuSpin package [37,38]. We evolve the initial state $|\psi(t=0)\rangle$ over a sufficiently long time interval ($t \sim 5000/J$ in units of $\hbar = 1$) to resolve the particle tunneling dynamics. Our simulation timescales are consistent with the experimental capabilities of current cold-atom lattice setups [21,23,24]. To extract the correlations between S_A and $\langle \hat{n}_A \rangle$, we employ Kolmogorov-Arnold networks [35], a recently developed and effective neural network architecture based on the Kolmogorov-Arnold representation theorem [39,40]. In KANs, learnable activation functions are placed on network edges and parametrized as weighted combinations of basis functions, typically implemented using B-splines.

III. RESULTS

We study the bipartite entanglement entropy $S_A(t)$, which characterizes quantum correlations between postbarrier sites and the remaining system. To reveal the underlying relationship between entanglement and particle transport, we analyze the dependence $S_A(n_A)$, where $n_A = \langle \hat{n}_A \rangle(t)$. This directly correlates entanglement with particle occupation probability after the barrier.

Figure 2(a) demonstrates the relation between S_A and n_A for $L = 4$, $U = 2J$, and $h = 5J$. While individually n_A [Fig. 2(c)] and S [Fig. 2(d)] exhibit complex temporal fluctuations, the plot $S_A(n_A)$ reveals a clear dependence. At $n_A \approx 0$, corresponding to early times when particles have not yet tunneled through the barrier, S_A remains minimal, reflecting weak correlations between subsystems A and B . As particles tunnel

through the barrier, S_A increases exhibiting a characteristic functional dependence on particle occupation changes.

To understand the origin of this relationship, we decompose von Neumann entropy into number entropy S_n and configurational entropy S_c defined in Eqs. (6) and (7). Figure 2(b) shows that both components $S_n(n_A)$ and $S_c(n_A)$ separately demonstrate a clean relationship with the post-barrier density. The configurational entropy S_c constitutes a non-negligible fraction of the total entropy, yet it is itself functionally determined by n_A in the barrier-dominated tunneling regime.

A. Utilizing KAN

While $S_A(n_A)$ reveals clear correlations for fixed parameters, understanding how interaction strength and barrier height influence this relationship requires exploring the multivariate dependence $S_A(U, h, n_A)$. To this end, we employ Kolmogorov-Arnold networks (for architecture and training details, see Appendix).

The choice of parameter ranges is dictated by the tunneling physics. We restrict $h \in [5J, 10J]$ to ensure that transport occurs via tunneling rather than over-barrier transmission. Simultaneously, the on-site interaction must remain below the barrier, $U < h$, to prevent interaction-assisted over-barrier processes; we scan $U \in [0.5J, 10J]$ and exclude all pairs with $U \geq h$. We further filter the dataset by requiring that the maximum value of $n_A(t) > 0.01$ over the full time evolution, removing configurations in which the barrier suppresses tunneling. In the R^2 heatmaps of Fig. 3, these excluded configurations appear as gray cells.

For each (U, h) pair, we perform time evolution over $tJ \in [0, 1000]$ and collect data $S_A(n_A)$. Combining datasets across multiple (U, h) values creates a unified training set, where U , h , and n_A serve as independent inputs for learning $S_A(U, h, n_A)$. Robustness is verified through fivefold cross-validation with results reported in the Appendix.

We train separate KAN models for S_A and its components, S_n and S_c . Figure 3 summarizes the results for the worst-performing cross-validation fold. For both system sizes, the number entropy S_n remains a well-defined function of n_A across the full parameter space. KAN achieves $R^2 > 0.9$ for almost all included (U, h) pairs, consistent with S_n being determined by the particle-number distribution alone.

The configurational entropy behaves differently as the system grows. For $L = 4$, $N = 2$, S_c stays small and retains a clean functional dependence on n_A , yielding $R^2 > 0.9$ across all parameter pairs. For the larger system ($L = 8$, $N = 6$), S_c grows in magnitude and exhibits a more scattered dependence on n_A : As the number of postbarrier sites increases, tunneled particles have more configurations in which to distribute, and the within-sector correlations encoded by S_c become sensitive to details beyond the mean density. This is reflected in the S_c heatmap, where R^2 degrades notably at low h/U ratios. However, despite this increased scatter, the values of S_c remain bounded, and KAN still captures the overall magnitude and trend of $S_c(n_A)$. Consequently, the total entanglement entropy S_A is predicted with high accuracy throughout the barrier-dominated regime.

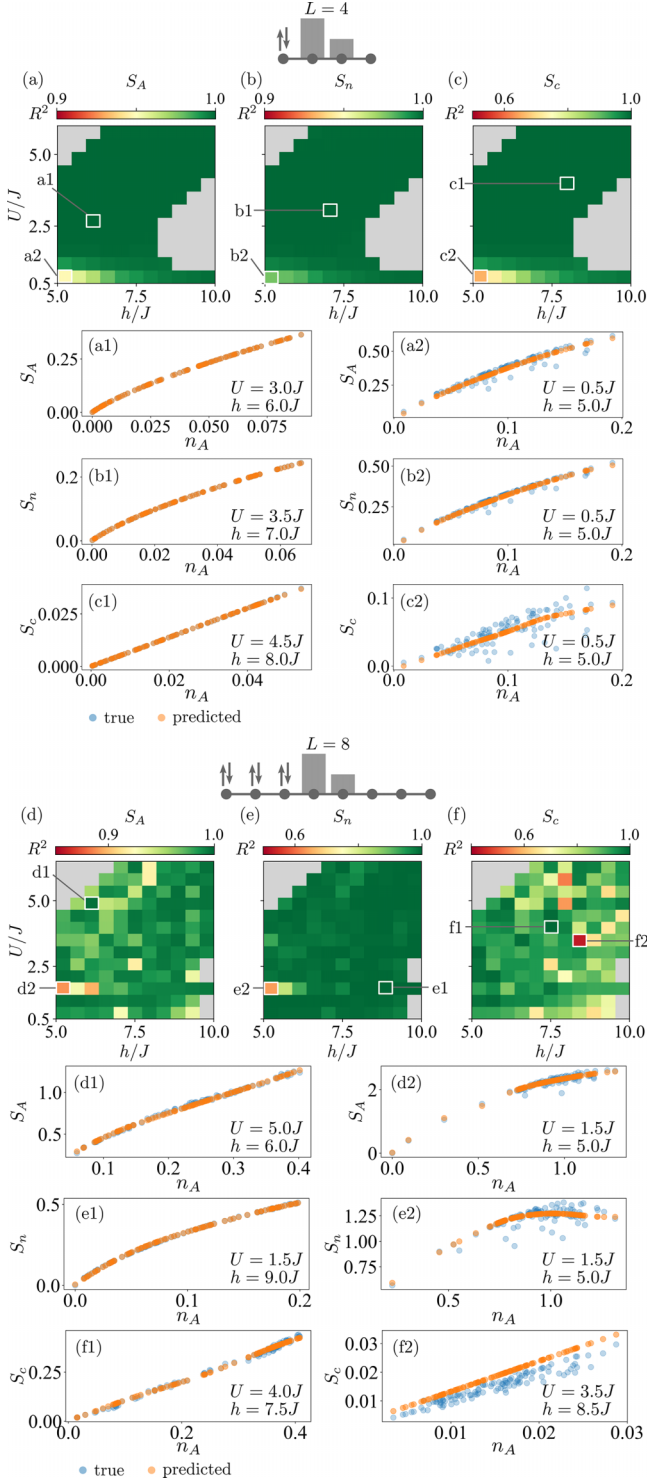


FIG. 3. KAN prediction accuracy for the entropy-density relationship across the (U, h) parameter space, shown for the worst-performing cross-validation fold. R^2 heatmaps for S_A , S_n , and S_c for $L = 4, N = 2$ [panels (a)–(c)] and $L = 8, N = 6$ [panels (d)–(f)]. Gray cells indicate excluded parameter pairs where $U \geq h$ or $\max n_A(t) < 0.01$. (a1)–(c2), (d1)–(f2) Comparison of true (blue) and KAN-predicted (orange) values of $S_\alpha(n_A)$ for the best and worst R^2 parameter pairs. Time evolution is performed over $tJ \in [0, 1000]$. The lattice diagrams show the system setup with fermions initially localized on the left and external potential barriers (gray boxes) acting on two central sites.

B. Analytical description of the entropy-density relation

The KAN analysis confirms the existence of a multi-variate relationship but fails to build a symbolic expression using pruning [35]. To address this limitation, we pursued an alternative approach based on physical intuition. We hypothesized that the entropy-density relationship should reflect the nature of quantum entanglement in bipartite systems. Specifically, since our system involves a binary choice for particle localization (before or after the barrier), we expected the entanglement entropy to follow a form similar to binary entropy. This intuition led us to propose that the $S_\alpha(n_A)$ relationship can be accurately described by

$$S_\alpha(n_A) = c_1^\alpha p \ln p + c_2^\alpha (1 - p) \ln (1 - p), \quad p = \frac{n_A}{N}, \quad (8)$$

where $\alpha \in \{A, n, c\}$ labels the entropy component and c_1^α, c_2^α are fitting coefficients that depend on the interaction strength U and barrier height h . These coefficients exhibit complex parameter dependence without simple analytical scaling relations, especially for larger system sizes. The first term captures the contribution from particles that have successfully tunneled into subsystem A (the postbarrier region). Meanwhile, the second term describes the contribution from the particles that remain localized in subsystem B (the pre-barrier region, including the barrier sites themselves). For the number entropy, this structure follows directly from the particle-number distribution $p_{\tilde{N}}$; for S_c and S_A , it serves as an effective parametrization of the more complex within-sector correlations.

We perform least-squares fitting of Eq. (8) independently for S_A , S_n , and S_c at each (U, h) pair, using the same parameter ranges and filtering criteria as in the KAN analysis. Figure 4 displays the resulting R^2 heatmaps and representative $S_\alpha(n_A)$ plots for the best (worst) R^2 values for both system sizes. For $L = 4, N = 2$, the analytical form achieves $R^2 > 0.9$ for S_A and S_n entropies across the entire parameter space, mirroring the KAN results. The S_c fit quality is comparable, with the lowest R^2 value corresponding to the weakly interacting system of $U = 0.5J$. For $L = 8, N = 6$, the fit quality for S_A and S_n remains high, while S_c again shows reduced R^2 at low h/U —consistent with the increased scatter in $S_c(n_A)$ identified in the KAN analysis. The close agreement between KAN and the analytical fit confirms that the binary entropy structure of Eq. (8) captures the essential functional dependence.

VI. DISCUSSION

A central finding of this work is that in the barrier-dominated tunneling regime, not only the number entropy S_n but also the configurational entropy S_c can be predicted from the postbarrier particle density. This is noteworthy because S_c encodes off-diagonal coherences within each particle-number sector of ρ_A —information that is, in general, not accessible from density measurements alone [36]. In the regime studied here, however, the constrained tunneling dynamics couple the growth of S_c to particle transfer, so that $S_c(n_A)$ remains a well-defined function of n_A for small systems and a bounded, predictable quantity even as the system size grows. Combined with the analytical binary entropy structure of Eq. (8), this provides a practical route to estimating the full von Neumann

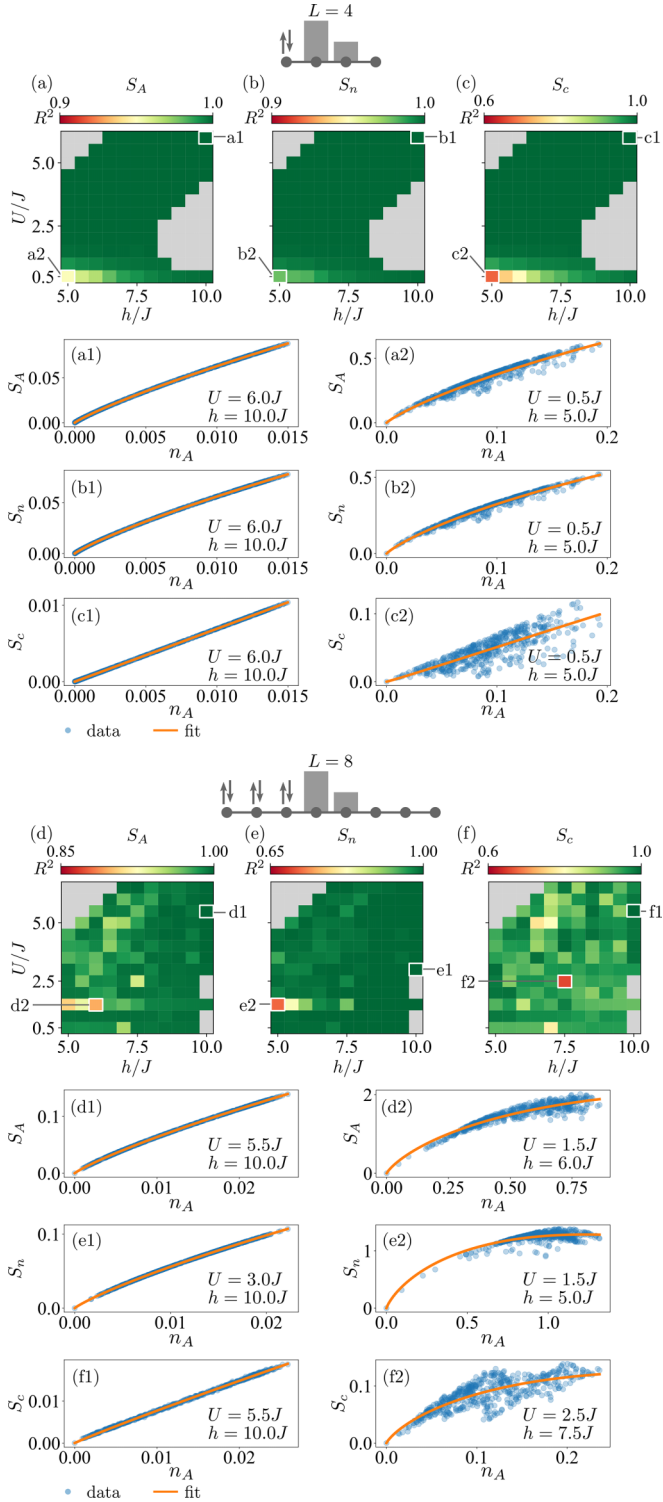


FIG. 4. Validation of the analytical approximation for the entropy-density relationship. R^2 heatmaps for S_A , S_n , and S_c for $L = 4$, $N = 2$ [panels (a)–(c)] and $L = 8$, $N = 6$ [panels (d)–(f)]. Gray cells indicate excluded parameter pairs where $U \geq h$ or $\max n_A(t) < 0.01$. (a1)–(c2), (d1)–(f2) Comparison of true (blue scatter) values and analytical fit (orange curve) of $S_A(n_A)$, $S_n(n_A)$, and $S_c(n_A)$ for the best and worst R^2 parameter pairs. Time evolution is performed over $tJ \in [0, 1000]$. The lattice diagrams show the system setup with fermions initially localized on the left and external potential barriers (gray boxes) acting on two central sites.

entropy—including its configurational component—from a single transport observable.

All detailed calculations and KAN analysis were performed for systems with maximum $L = 8$ lattice sites. However, it is crucial to establish that the entropy-density relationship persists in larger systems. Our preliminary calculations demonstrate the presence of the functional dependence for systems as large as $L = 20$ sites, indicating that the observed correlations are not artifacts of finite-size effects but represent a fundamental feature of quantum transport in interacting lattice systems. The scalability of this relationship suggests potential applications to realistic experimental setups.

The necessary condition for observing the functional dependence is that initially the postbarrier region must be empty, with all particles localized on the opposite side of the potential barrier. This configuration ensures that the entanglement dynamics arise solely from quantum correlations generated during the tunneling process, rather than from preexisting entanglement.

While our analysis primarily employs an asymmetric triangular barrier, the functional relationship $S_A(n_A)$ is not specific to this particular barrier geometry. We have verified that the correlation persists for rectangular (symmetric) barriers as well as different triangular profiles. The asymmetric triangular barrier was chosen because it reveals new fundamental transport phenomena that are absent in systems with symmetric barriers [41]. The universal feature is the binary entropy structure itself, not the specific potential shape. Our findings link quantum correlations and transport, offering a powerful framework for predicting entanglement dynamics in interacting systems, which potentially can be measured by current tunneling mechanism [42].

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DATA AVAILABILITY

All codes used in this study can be found in Ref. [43].

APPENDIX: KAN ARCHITECTURE AND TRAINING

We implement KANs with three input nodes (corresponding to U , h , and n_A) and one output node (S_α). For the smaller system ($L = 4$, $N = 2$), we use three hidden nodes in a single hidden layer, resulting in a $[3, 3, 1]$ architecture with a grid

TABLE I. KAN parameters used to model the $S_\alpha(U, h, n_A)$ functional relationship.

L	Architecture	Spline order	Grid size	Learning rate	Loss
4	[3, 3, 1]	3	10	0.01	10^{-3}
8	[3, 6, 3, 1]	3	20	0.01	10^{-3}

size of 10. For the larger system ($L = 8, N = 6$), we employ a two-hidden-layer architecture [3, 6, 3, 1] with a grid size of 20. The learnable activation functions on each edge are parametrized using cubic B-splines. Training is performed for 50 epochs using the L-BFGS optimizer with a learning rate of 0.01. All model parameters are summarized in Table I.

To ensure the robustness of the model, we employ fivefold cross-validation on the combined dataset for each S_α individually. The data are partitioned such that each fold contains representative samples from all (U, h) pairs. Table II reports the R^2 values for all folds. Across all five validation folds, the KAN achieves consistent performance confirming robust predictive accuracy across different system configurations.

TABLE II. Cross-validation results for KAN model performance. R^2 values across five folds for lattice sizes $L = 4, N = 2$ and $L = 8, N = 6$.

$L = 4$			
Fold	S_A	S_n	S_c
1	0.9982	0.9992	0.9837
2	0.9984	0.9992	0.9840
3	0.9981	0.9991	0.9811
4	0.9981	0.9991	0.9833
5	0.9979	0.9989	0.9809
Mean	0.9981 ± 0.0001	0.9991 ± 0.0001	0.9826 ± 0.0013
$L = 8$			
Fold	S_A	S_n	S_c
1	0.9985	0.9986	0.9964
2	0.9986	0.9989	0.9965
3	0.9985	0.9988	0.9962
4	0.9985	0.9987	0.9963
5	0.9986	0.9989	0.9963
Mean	0.9986 ± 0.0001	0.9988 ± 0.0001	0.9963 ± 0.0001

- [1] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, Entanglement in many-body systems, *Rev. Mod. Phys.* **80**, 517 (2008).
- [2] J. Eisert, M. Cramer, and M. B. Plenio, *Colloquium: Area laws for the entanglement entropy*, *Rev. Mod. Phys.* **82**, 277 (2010).
- [3] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Measurement of qubits, *Phys. Rev. A* **64**, 052312 (2001).
- [4] R. Islam, R. Ma, P. M. Preiss, M. Eric Tai, A. Lukin, M. Rispoli, and M. Greiner, Measuring entanglement entropy in a quantum many-body system, *Nature (London)* **528**, 77 (2015).
- [5] T. Brydges, A. Elben, P. Jurcevic, B. Vermersch, C. Maier, B. P. Lanyon, P. Zoller, R. Blatt, and C. F. Roos, Probing Rényi entanglement entropy via randomized measurements, *Science* **364**, 260 (2019).
- [6] H. Pichler, L. Bonnes, A. J. Daley, A. M. Läuchli, and P. Zoller, Thermal versus entanglement entropy: A measurement protocol for fermionic atoms with a quantum gas microscope, *New J. Phys.* **15**, 063003 (2013).
- [7] A. Elben, B. Vermersch, M. Dalmonte, J. I. Cirac, and P. Zoller, Rényi entropies from random quenches in atomic Hubbard and spin models, *Phys. Rev. Lett.* **120**, 050406 (2018).
- [8] N. Hartman, C. Olsen, S. Lüscher, M. Samani, S. Fallahi, G. C. Gardner, M. Manfra, and J. Folk, Direct entropy measurement in a mesoscopic quantum system, *Nat. Phys.* **14**, 1083 (2018).
- [9] Z.-K. Lin, Y. Zhou, B. Jiang, B.-Q. Wu, L.-M. Chen, X.-Y. Liu, L.-W. Wang, P. Ye, and J.-H. Jiang, Measuring entanglement entropy and its topological signature for phononic systems, *Nat. Commun.* **15**, 1601 (2024).
- [10] D. Koutný, L. Ginés, M. Moczala-Dusanowska, S. Höfling, C. Schneider, A. Predojević, and M. Ježek, Deep learning of quantum entanglement from incomplete measurements, *Sci. Adv.* **9**, eadd7131 (2023).
- [11] Y. Huang, L. Che, C. Wei, F. Xu, X. Nie, J. Li, D. Lu, and T. Xin, Direct entanglement detection of quantum systems using machine learning, *npj Quantum Inf.* **11**, 29 (2025).
- [12] A. Elben, R. Kueng, H.-Y. R. Huang, R. van Bijnen, C. Kokail, M. Dalmonte, P. Calabrese, B. Kraus, J. Preskill, P. Zoller, and B. Vermersch, Mixed-state entanglement from local randomized measurements, *Phys. Rev. Lett.* **125**, 200501 (2020).
- [13] S. Fölling, S. Trotzky, P. Cheinet, M. Feld, R. Saers, A. Widera, T. Müller, and I. Bloch, Direct observation of second-order atom tunnelling, *Nature (London)* **448**, 1029 (2007).
- [14] S. Trotzky, P. Cheinet, S. Fölling, M. Feld, U. Schnorrberger, A. M. Rey, A. Polkovnikov, E. A. Demler, M. D. Lukin, and I. Bloch, Time-resolved observation and control of superexchange interactions with ultracold atoms in optical lattices, *Science* **319**, 295 (2008).
- [15] I. B. Spielman, W. D. Phillips, and J. V. Porto, Mott-insulator transition in a two-dimensional atomic Bose gas, *Phys. Rev. Lett.* **98**, 080404 (2007).
- [16] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Condensation of pairs of fermionic atoms near a Feshbach resonance, *Phys. Rev. Lett.* **92**, 120403 (2004).
- [17] Y. Shin, M. W. Zwierlein, C. H. Schunck, A. Schirotzek, and W. Ketterle, Observation of phase separation in a strongly interacting imbalanced Fermi gas, *Phys. Rev. Lett.* **97**, 030401 (2006).
- [18] M. Miranda, R. Inoue, Y. Okuyama, A. Nakamoto, and M. Kozuma, Site-resolved imaging of ytterbium atoms in a two-dimensional optical lattice, *Phys. Rev. A* **91**, 063414 (2015).
- [19] R. Yamamoto, J. Kobayashi, T. Kuno, K. Kato, and Y. Takahashi, An ytterbium quantum gas microscope with narrow-line laser cooling, *New J. Phys.* **18**, 023016 (2016).
- [20] A. W. Young, W. J. Eckner, N. Schine, A. M. Childs, and A. M. Kaufman, Tweezer-programmable 2D quantum walks in a Hubbard-regime lattice, *Science* **377**, 885 (2022).
- [21] M. F. Parsons, A. Mazurenko, C. S. Chiu, G. Ji, D. Greif, and M. Greiner, Site-resolved measurement of the spin-correlation

- function in the Fermi-Hubbard model, *Science* **353**, 1253 (2016).
- [22] L. W. Cheuk, M. A. Nichols, K. R. Lawrence, M. Okan, H. Zhang, E. Khatami, N. Trivedi, T. Paiva, M. Rigol, and M. W. Zwierlein, Observation of spatial charge and spin correlations in the 2D Fermi-Hubbard model, *Science* **353**, 1260 (2016).
- [23] P. T. Brown, D. Mitra, E. Guardado-Sanchez, R. Nourafkan, A. Reymbaut, C.-D. Hébert, S. Bergeron, A.-M. S. Tremblay, J. Kokalj, D. A. Huse, P. Schauß, and W. S. Bakr, Bad metallic transport in a cold atom Fermi-Hubbard system, *Science* **363**, 379 (2019).
- [24] M. A. Nichols, L. W. Cheuk, M. Okan, T. R. Hartke, E. Mendez, T. Senthil, E. Khatami, H. Zhang, and M. W. Zwierlein, Spin transport in a Mott insulator of ultracold fermions, *Science* **363**, 383 (2019).
- [25] R. W. Gurney and E. U. Condon, Wave mechanics and radioactive disintegration, *Nature (London)* **122**, 439 (1928).
- [26] G. Gamow, Zur quantentheorie des atomkernes, *Z. Phys.* **51**, 204 (1928).
- [27] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Concentrating partial entanglement by local operations, *Phys. Rev. A* **53**, 2046 (1996).
- [28] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, 10th anniversary ed. (Cambridge University Press, Cambridge, UK, 2010).
- [29] I. Klich, G. Refael, and A. Silva, Measuring entanglement entropies in many-body systems, *Phys. Rev. A* **74**, 032306 (2006).
- [30] I. Klich and L. Levitov, Quantum noise as an entanglement meter, *Phys. Rev. Lett.* **102**, 100502 (2009).
- [31] A. Del Maestro, H. Barghathi, and B. Rosenow, Measuring postquench entanglement entropy through density correlations, *Phys. Rev. Res.* **4**, L022023 (2022).
- [32] P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory, *J. Stat. Mech.: Theory Exp.* (2004) P06002.
- [33] P. Calabrese and J. Cardy, Time dependence of correlation functions following a quantum quench, *Phys. Rev. Lett.* **96**, 136801 (2006).
- [34] I. Peschel, Calculation of reduced density matrices from correlation functions, *J. Phys. A: Math. Gen.* **36**, L205 (2003).
- [35] Z. Liu, Y. Wang, S. Vaidya, F. Ruehle, J. Halverson, M. Soljačić, T. Y. Hou, and M. Tegmark, KAN: Kolmogorov-Arnold networks, [arXiv:2404.19756](https://arxiv.org/abs/2404.19756).
- [36] A. Lukin, M. Rispoli, R. Schittko, M. E. Tai, A. M. Kaufman, S. Choi, V. Khemani, J. Léonard, and M. Greiner, Probing entanglement in a many-body-localized system, *Science* **364**, 256 (2019).
- [37] P. Weinberg and M. Bukov, QuSpin: A Python package for dynamics and exact diagonalisation of quantum many body systems. Part I: Spin chains, *SciPost Phys.* **2**, 003 (2017).
- [38] P. Weinberg and M. Bukov, QuSpin: A Python package for dynamics and exact diagonalisation of quantum many body systems. Part II: Bosons, fermions and higher spins, *SciPost Phys.* **7**, 020 (2019).
- [39] A. N. Kolmogorov, On the representation of continuous functions of many variables by superpositions of continuous functions of one variable and addition, *Dokl. Akad. Nauk SSSR* **114**, 953 (1957).
- [40] V. I. Arnold, On the representation of functions of several variables as a superposition of functions of a smaller number of variables, in *Collected Works: Representations of Functions, Celestial Mechanics and KAM Theory, 1957–1965*, edited by A. B. Givental, B. A. Khesin, J. E. Marsden, A. N. Varchenko, V. A. Vassiliev, O. Y. Viro, and V. M. Zakalyukin (Springer, Berlin, 2009), pp. 25–46.
- [41] E. Bilokon, V. Bilokon, D. R. Lindberg, L. Kaplan, A. Sotnikov, and D. I. Bondar, Few-fermion resonant tunneling and under-barrier trapping in asymmetric potentials, *Commun. Phys.* **8**, 259 (2025).
- [42] M. Sennary, J. Shah, M. Yuan, A. Mahjoub, V. Pervak, N. V. Golubev, and M. T. Hassan, Light-induced quantum tunnelling current in graphene, *Nat. Commun.* **16**, 4335 (2025).
- [43] E. Bilokon, Code for “Predicting entanglement entropy from particle tunneling of interacting fermions using Kolmogorov-Arnold networks”, GitHub, 2026, https://github.com/ebilokon/1band_fermi_hubbard.