

Smart navigation of a gravity-driven glider with adjustable center of mass

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Artificial gliders are designed to disperse as they settle through a fluid, requiring precise navigation to reach target locations. We show that a compact glider settling in a viscous fluid can navigate by dynamically adjusting its center of mass. Using fully resolved direct numerical simulations and reinforcement learning, we find two optimal navigation strategies that allow the glider to reach its target location accurately. These strategies depend sensitively on how the glider interacts with the surrounding fluid. The nature of this interaction changes as the particle Reynolds number Re_p changes. Our results explain how the optimal strategy depends on Re_p . At large Re_p , the glider learns to tumble rapidly by moving its center of mass as its orientation changes. This generates a large horizontal inertial lift force, which allows the glider to travel far. At small Re_p , by contrast, high viscosity hinders tumbling. In this case, the glider learns to adjust its center of mass so that it settles with a steady, inclined orientation that results in a horizontal viscous force. The horizontal range is much smaller than for large Re_p , because this viscous force is much smaller than the inertial lift force at large Re_p .

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I. INTRODUCTION

Seeds and small organisms evolved the ability to glide through air without propulsion, allowing navigation and dispersion with high efficiency [1–3]. These examples have inspired the concepts of small artificial gliders, which could form intelligent networks for environmental measurements in air or water [4–7]. These tiny gliders, unlike conventional ones [7,8], are designed with a focus on miniaturization, low cost, and low energy consumption.

One important question is how such gliders can maneuver to reach a prescribed target location. This requires deep understanding of glider-fluid interactions, which vary significantly with the particle settling Reynolds number, $Re_p = v_s a / \nu$. Here, v_s is the settling speed of the glider, a its size, and ν is the kinematic viscosity of the fluid. As Re_p increases from zero, fluid inertia amplifies hydrodynamic forces and torques [9,10]. Nonspherical particles experience an additional fluid-inertia torque compared to spherical ones. At small Re_p , this torque can be computed perturbatively and is proportional to Re_p [11–13]. In addition, a particle rotating relative to the fluid experiences a lift force perpendicular to its slip velocity [14]. At large Re_p , calculations of forces and torques rely on empirical models that assume a quasisteady disturbance flow [15]. Using such models, Refs. [16,17] analyzed gliders settling at $Re_p \sim 10^3$. When the wake of the settling particle

becomes unstable, complex falling patterns emerge, including oscillations, tumbling, and even chaotic dynamics [18].

A pioneering study [19] investigated the navigation of a two-dimensional elliptical glider using an empirical model at $Re_p \sim 10^3$, representing a centimeter-scale glider in air. The authors showed that external control torque enables navigation through two strategies: tumbling and inclined settling, both generating a horizontal force via fluid-solid interaction.

Recent advances using reinforcement learning [20,21] have significantly improved the finding and understanding of optimal navigation strategies for smart particles that can sense their environment and adapt their behavior in viscous flow [22–25]. Reference [23] applied reinforcement learning to the model in Ref. [19], showing that a smart glider, capable of dynamically adjusting its control torque, learns to navigate by either tumbling or inclined settling. The optimal strategy depends on the mass density and upon the shape of the glider. Tumbling outperforms inclined settling for heavier or less elongated gliders [23].

These studies highlight the potential of designing smart artificial gliders, but two key challenges remain. First, how can the control torque be applied? Gliders used in typical applications [4–6] are too small for propellers, so controlling such gliders like a drone is not an option. While magnetic or electric fields can control smart particles [26], they require external fields along the glider trajectory, limiting autonomy. Second, how do the navigation strategies change with Re_p ? Re_p varies greatly in applications, and laboratory experiments are usually easier to perform when the glider settles slowly in a highly viscous fluid [12,27]. This yields a much smaller Re_p than the values $Re_p \sim 10^3$ studied previously [19,23].

Here, we address these two questions. First, we explore the possibility to control the glider using an adjustable center of mass (Fig. 1), inspired by particles with asymmetric mass

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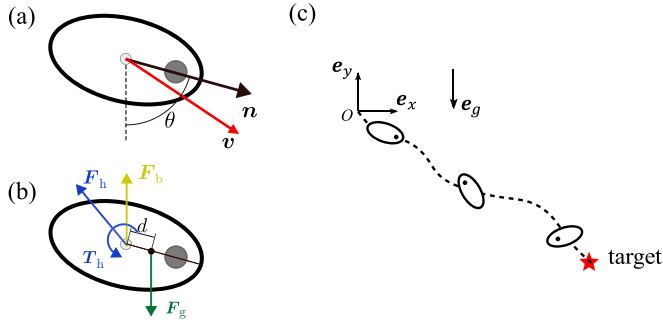


FIG. 1. (a) Glider with a movable mass ($\bullet$). Here, $\mathbf{n}$ is the unit vector along the major axis of the glider, θ is the tilt angle, and $\mathbf{v}$ is the velocity of the geometric center. (b) Illustration of the forces and torques on a glider. The torques are evaluated with respect to the geometric center. Buoyancy $\mathbf{F}_b$ ($\circ$) induces no torque, while gravity $\mathbf{F}_g$ ($\bullet$) causes the torque $T_g = -d \sin \theta$ because the center of mass is located at distance d along $\mathbf{n}$ from the geometric center. Hydrodynamic forces on the glider surface result in a net force $\mathbf{F}_h$ and torque T_h with respect to the geometric center. (c) Schematic of active control of a settling glider in a quiescent fluid by adjusting its center of mass. The red star represents the target point.

distributions that exhibit complex settling dynamics in a viscous fluid [27,28]. Second, we employ direct numerical simulations (DNSs) to fully resolve particle-fluid interactions, in order to determine how the optimal strategy depends on Re_p . As an exemplary task, we consider a gravity-driven glider released in a quiescent fluid at different distances from the target. Assuming the smart glider can measure its current phase-space configuration, we use reinforcement learning to determine the optimal strategy for adjusting its center of mass to reach the target.

We find that the glider can exploit particle-fluid interactions to successfully navigate by changing its center of mass. The optimal navigation strategy depends on Re_p : at small Re_p , the glider settles with steady inclination, while it learns to tumble at larger values of Re_p . Previous studies using empirical models for gliders at large particle Reynolds numbers ($\text{Re}_p \sim 10^3$) identified the same two settling strategies [19,23]. Our work differs in three respects: we use DNS to investigate how the optimal strategy varies with Re_p ; we implement an explicit control mechanism (moving center of mass) rather than a prescribed torque; and we focus on gliders in viscous fluids, which are more practical for laboratory experiments.

II. METHODOLOGY

A. Model

Since the optimization problem using DNS of particle-fluid interactions is computationally demanding, we follow Refs. [19,23] and consider the two-dimensional problem. We model the glider as a two-dimensional elliptical shell with aspect ratio λ and a mass that can move along the direction $\mathbf{n}$ of its major axis [Fig. 1(a)]. Shifting this mass changes the center of mass of the glider, enabling control. Both the shell and the movable mass have equal linear mass density $\frac{1}{2}m_p$, where m_p is the total mass per unit length. We nondimensionalize

the problem using the semimajor axis length of the glider, a , the velocity scale $\sqrt{ag}$, and mass per unit length m_p . Here, g is the gravitational acceleration. The nondimensional form of the governing equations of the glider reads

$$\frac{d}{dt}\mathbf{r} = \mathbf{v}, \quad (1a)$$

$$\frac{d^2}{dt^2}(\mathbf{r} + \mathbf{n}d) = \mathbf{F}_h + \mathbf{F}_g + \mathbf{F}_b, \quad (1b)$$

$$\left[\frac{1}{2} \frac{d^2}{dt^2}(\mathbf{r} + 2\mathbf{n}d) \right] \cdot \mathbf{n} = \frac{1}{2} \mathbf{F}_g \cdot \mathbf{n} + \mathbf{F}_s \cdot \mathbf{n}, \quad (1c)$$

$$\frac{d}{dt}\theta = \omega, \quad \frac{d}{dt}(\omega J) = T_h + T_g - d(\mathbf{n} \wedge \frac{d}{dt}\mathbf{v}) \cdot \mathbf{e}_z. \quad (1d)$$

Here, $\mathbf{r}$ and $\mathbf{v}$ are, respectively, the position and velocity of the geometric center of the glider. In order to account for the moving mass, it is most convenient to write Newton's second law for the center of mass [Eq. (1b)]. The left-hand side of Eq. (1b) is the acceleration of the center of mass in terms of the position of the geometric center $\mathbf{r}$, and the displacement $\mathbf{n}d$ between the geometric and mass centers. The external forces on the glider are the hydrodynamic force $\mathbf{F}_h$ exerted by the fluid upon the glider, gravity $\mathbf{F}_g = \mathbf{e}_g$, and buoyancy, $\mathbf{F}_b = -\beta^{-1}\mathbf{e}_g$, where $\mathbf{e}_g$ is the direction of gravity and $\beta = \rho_p/\rho_f$ is the mass-density ratio between the glider and fluid [Fig. 1(b)].

When the glider shifts its center of mass, momentum is conserved but redistributed between the movable mass, shell, and fluid via hydrodynamic interactions, generating transient forces and torques. To avoid infinite forces from instantaneous displacement, we model the connection between the movable mass and the shell as a damped spring, enabling continuous momentum transfer.

The moving center of mass adds an additional dynamical degree of freedom: the center-of-mass displacement d . Its dynamics is given by Eq. (1c). The left-hand side is dimensionless mass times acceleration of the moving mass, located at $2\mathbf{n}d$ from the glider geometric center, in the direction of $\mathbf{n}$. The right-hand side are the forces on the mass: gravity, $\frac{1}{2}\mathbf{F}_g$, and the damped spring force $\mathbf{F}_s = [2k(d_e - d) - 2c\dot{d}]\mathbf{n}$. The spring stiffness $k = 63.7$ and damping $c = 9.0$ are chosen large to rapidly relax the center of mass to its equilibrium distance d_e . The dynamics of the glider is controlled by adjusting d_e .

Equation (1d) describes the angular dynamics of the glider around its geometric center, with tilt angle θ , and angular velocity ω around $\mathbf{e}_z$, the unit vector normal to the x - y plane. Equation (1d) is written in a comoving frame that follows the translational motion of the geometric center. The left-hand side of Eq. (1d) is the time derivative of angular momentum around the geometric center, with moment of inertia $J = J_{\text{shell}} + J_{\text{mass}} + 2d^2$. Here, J_{shell} and J_{mass} are the moments of inertia of the elliptical shell and the circular movable mass around their respective centers, and the $2d^2$ contribution arises from the parallel axis theorem. The hydrodynamic torque around the geometric center is denoted by T_h . The torque due to gravity, $T_g = -d \sin \theta$, arises due to the mass-center displacement [see Fig. 1(b)]. The last term, $-d(\mathbf{n} \wedge \dot{\mathbf{v}}) \cdot \mathbf{e}_z$, is torque due to acceleration of the comoving frame.

The dynamics in Eq. (1) additionally depends on the aspect ratio λ and mass-density ratio β , both introduced above,

TABLE I. Dimensional and nondimensional parameters. The length scale of the glider is denoted by a , and the kinematic fluid viscosity by ν . Gliding ant in air: parameters from Ref. [1]. Ocean glider prototype [8]. We take a to be the half-length of its cylindrical body. The aspect ratio is defined as the ratio between the length and the diameter of the cylindrical body. The density is determined based on the reported mass and the volume of the cylinder body. Glider in silicon oil (this paper) with mass density slightly larger than that of the fluid.

Case	a (cm)	ν (m ² s ⁻¹)	Glider density (kg m ⁻³)	Fluid density (kg m ⁻³)	λ	β	Ga
Gliding ant in air [1]	0.6	1.8×10^{-5}	290	1.2	2	240	1200
Ocean glider prototype [8]	25	1.0×10^{-6}	950–1500	1000	5	0.95–1.5	$0\text{--}2.8 \times 10^4$
Glider in silicon oil	5	2×10^{-3} to 1×10^{-4}	950–1500	764	>1	1.2–2.0	8–340

as well as the Galileo number $Ga = \sqrt{(\beta - 1)ga^3}/\nu$, which characterizes the fluid-solid interactions governing $\mathbf{F}_h$ and $\mathbf{T}_h$. These three parameters are equivalent to the nondimensional parameters used in Ref. [29] to describe the motion for a spheroid settling in a quiescent fluid. We parametrize the dynamics using Ga since it is uniquely determined by intrinsic particle and flow properties, while Re_p , depending on the actual settling speed, must be evaluated *a posteriori*. For small Ga , Re_p scales as Ga^2 , but this relation overestimates Re_p for larger Ga . Table I presents example parameters [1,8] and a suggestion for future glider experiments. Table I shows that Ga varies over several orders of magnitude, but only navigation strategies for 10^3 were addressed using the large- Ga empirical model in earlier studies [19,23]. Our fully resolved simulations enable exploration of small and moderate Ga , while high Ga remains too computationally costly. We therefore focus on this unexplored regime, using $Ga = 4.4, 35.4$, and 283 , which can be realized experimentally in viscous liquids such as silicon oil. The corresponding values of the particle Reynolds number are $Re_p \approx 3, 31$, and 240 . We use $\beta = 2$ in our simulations to consider settling gliders. The numerical method used in the DNS restricts the possible aspect ratios we can study. Here, we report results for $\lambda = 2$.

B. Numerical method

To solve Eq. (1), we calculate the hydrodynamic force $\mathbf{F}_h$ and torque $\mathbf{T}_h$ using DNSs that fully resolve particle-fluid interactions via the immersed boundary method [30]. The fluid velocity $\mathbf{u}$ and pressure p are obtained by solving the incompressible Navier-Stokes equations on an Eulerian grid (in dimensional units):

$$\nabla \cdot \mathbf{u} = 0, \quad \rho_f \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho_f \mathbf{f}. \quad (2)$$

Here, μ is the dynamic viscosity of the fluid, and the immersed boundary forcing term $\mathbf{f}$ accounts for the disturbance caused by the motion of the glider. It is obtained by enforcing no-slip condition at the fluid-solid interface using the direct-forcing immersed boundary method [31,32]. In this approach, fluid velocity is interpolated from the Eulerian grid to uniformly distributed Lagrangian marker points on the glider surface. Immersed boundary forces are computed from the difference between interpolated and rigid-body velocities, and then spread back onto the Eulerian grid to yield $\mathbf{f}$ in Eq. (2).

See Ref. [32] for details. In this framework, $\mathbf{F}_h$ and $\mathbf{T}_h$ are calculated as [32]

$$\mathbf{F}_h = -\rho_f \int_{\mathcal{V}_p} \mathbf{f} dV + \rho_f \frac{d}{dt} \int_{\mathcal{V}_p} \mathbf{u} dV, \quad (3)$$

$$\mathbf{T}_h = -\rho_f \int_{\mathcal{V}_p} \mathbf{r}_c \wedge \mathbf{f} dV + \rho_f \frac{d}{dt} \int_{\mathcal{V}_p} \mathbf{r}_c \wedge \mathbf{u} dV. \quad (4)$$

Here, $\mathcal{V}_p$ is the volume of the glider and $\mathbf{r}_c$ is the vector from the geometric center to the point of integration.

We solve Eq. (2) using a second-order central difference method [33] on a uniform Eulerian grid of size $24a \times 32a$. The grid resolution Δx depends on the Galileo number: $\Delta x = a/16$ for $Ga = 4.4$ and 35.4 , and $\Delta x = a/32$ for $Ga = 283$. The time step size is $\Delta t = 0.014\sqrt{a/g}$. Simulations are converged with respect to domain size, grid spacing, and time step. Periodic boundary conditions are applied horizontally. Vertically, we impose Neumann conditions at the top and Dirichlet conditions at the bottom to enforce zero fluid velocity, modeling quiescent fluid. This bottom condition does not represent a solid wall because the domain follows the glider downward, preventing it to reach the lower boundary. See Ref. [34] for further details on the numerical method.

C. Reinforcement learning

We use double deep Q -learning [20,21,35,36] to find optimal control strategies for navigating a glider. Unlike the previous study [23] that targets a fixed position, we train a single strategy for reaching targets at different positions, demonstrating robustness.

As illustrated in Fig. 1(c), the glider is released from rest at $\mathbf{r}_0 = (x_0, y_0)$ with random orientation and is tasked to reach a target at $\mathbf{r}_T = (x_T, y_T)$, where $y_T - y_0 = -40$ and $x_T - x_0$ is drawn uniformly from $[-30, 30]$. At regular intervals $\tau_u = 1.41$, the glider selects one of five center-of-mass equilibrium positions ($d_e = 0, \pm 0.2, \pm 0.4$) based on its relative position to the target, velocity, tilt angle, and angular velocity. The update interval τ_u is long enough for the mass to reach its new position but short enough to allow over 30 decisions per trial.

To optimize navigation, we choose the reward:

$$r = -h_1(x - x_T)^2 \left(\frac{y - y_0}{y_T - y_0} \right)^2 + h_2 \Theta(y_T - y) e^{-|x - x_T|/2}, \quad (5)$$

with coefficients $h_1 = 0.125$ and $h_2 = 100$. The first term in Eq. (5) penalizes the horizontal distance to the target, scaled by $[(y - y_0)/(y_T - y_0)]^2$ to reduce the penalty when the glider is far above the target, promoting exploration. The second term is a one-time bonus based on horizontal distance to the

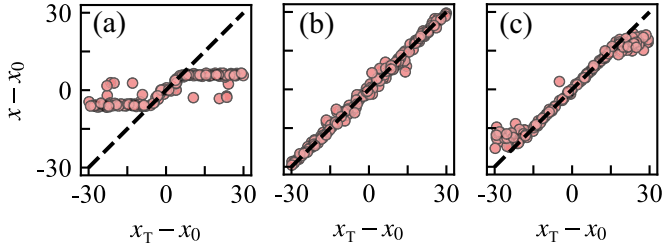


FIG. 2. Landing position x vs randomly prescribed target points x_T . The dashed line represents perfect navigation: $x = x_T$. Parameters: (a) $Ga = 4.4$; (b) $Ga = 35.4$; and (c) $Ga = 283$.

target when the glider reaches the target height ($y = y_T$), at which point the episode terminates. Here, $\Theta(\cdot)$ is the Heaviside function.

Training details and hyperparameters are provided in Appendix A.

III. RESULTS

Figure 2 documents the success of reinforcement learning for three values of Ga . Landing positions x under the learned strategy are shown for 200 different targets x_T . For the smallest Ga , only targets within a narrow range are reachable [Fig. 2(a)]. For the intermediate value of Ga , the glider can reach any target within the entire range of $|x_T - x_0|$ [Fig. 2(b)]. For the largest Ga , the range narrows again [Fig. 2(c)]. The optimal strategies differ for different Ga . This becomes clear from Fig. 3, which shows optimal strategies for a far target at $x_T = 32$. At the lowest value of Ga , the glider learns to maintain an inclined orientation by adjusting its center of mass in a periodic fashion [Fig. 3(a)], so that it glides at an approximately constant angle. At larger values of Ga , the glider learns to move its center of mass to tumble anticlockwise (clockwise) with an approximately constant angular velocity to reach a target to its right [Figs. 3(b) and 3(c)] (left, respectively, not shown). The corresponding trajectories are shown in Fig. 4. In short, by adjusting its center of mass,

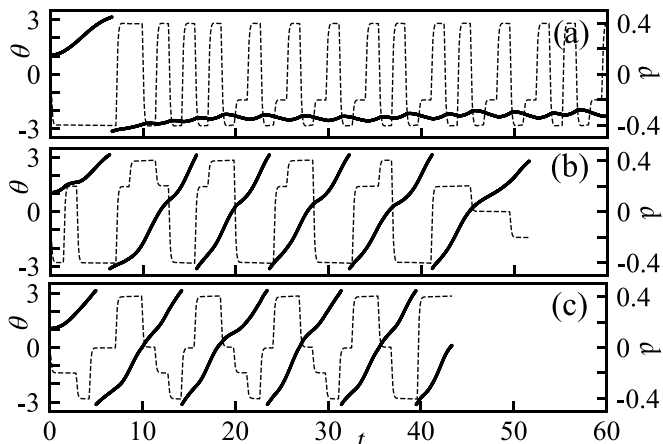


FIG. 3. Glider orientation for optimal strategies at different Ga . The control $d(t)$ (dashed lines) and the tilt angle $\theta(t)$ (solid lines) are shown against time. (a) $Ga = 4.4$; (b) $Ga = 35.4$; and (c) $Ga = 283$.

the glider manages to navigate by two different strategies, either inclined settling or tumbling.

The same two strategies were found using an empirical model designed for gliders settling at large particle Reynolds numbers [19,23], on the order of 10^3 . Novati *et al.* [23] found that tumbling outperforms inclined settling at these Reynolds numbers for small enough aspect ratio λ or large enough density ratio β . Paoletti and Mahadevan [19] used a model similar to Eq. (6) for the torque and a corresponding model for the force acting on the glider. There are three main differences to these works. First, DNS allows us to investigate how the optimal strategy changes as the particle Reynolds number changes (at fixed λ and β). Second, we use an explicit form of control (the moving center of mass), instead of an arbitrary control torque. Third, we consider a glider in a highly viscous fluid (not heavy gliders settling in air). The reason is that laboratory experiments are likely more feasible for more viscous fluids, simply because the glider tends to settle more slowly.

In order to understand how different strategies emerge at different Reynolds numbers, we analyze the torques in Eq. (1d). In this equation, both T_g and the last term depend on the control through $d \sim d_c$. However, the last term is small because v approaches the steady settling velocity after an initial transient. The hydrodynamical torque T_h is modeled using an empirical torque model [16]:

$$T_h = -c_1\omega - c_2|\omega|\omega - c_3|v|^2 \sin\phi \cos\phi - c_4\dot{\omega}. \quad (6)$$

We note that this model applies to both two- and three-dimensional objects that have three mutually orthogonal symmetry planes and move in quiescent fluids [15,17], although the coefficients are quantitatively different. The first term in Eq. (6) is Stokes torque [9] and the second term describes a second-order correction to the Stokes torque [10], similar to the Oseen correction to the Stokes drag. The first two terms together represent the so-called dissipative torque [16,17]. The third term describes a fluid-inertia torque, consistent with the form obtained in small Re_p -perturbation theory [10–13,29], where ϕ is the angle between the orientation $\mathbf{n}$ and velocity $\mathbf{v}$ of the glider. The angle ϕ differs from the tilt angle θ in Eq. (1d). We note that Refs. [13,29] considered the case where $\phi \approx \theta$. The fourth term is added moment of inertia resulting from acceleration of the surrounding fluid [37]. Related models were first used for passively settling particles [16,17], with coefficients chosen to generate qualitatively accurate trajectories at $Re_p \sim 10^3$.

As explained above, we use the Galileo number to parametrize particle-fluid interactions in our model, instead of the Re_p . The two numbers are in a one-to-one correspondence, but Ga has the advantage that it is uniquely determined by intrinsic particle and flow properties, while Re_p must be evaluated *a posteriori*. To study how T_h depends on Ga , we fix the coefficient c_4 in Eq. (6) to the known value for an elliptical plate in the potential flow limit [38],

$$c_4 = \frac{1}{16\beta} \lambda (1 - \lambda^{-2})^2. \quad (7)$$

The remaining coefficients c_1 , c_2 , and c_3 are fitted for each Ga to two sets of DNS trajectories. The first set consists of trajectories with $d = 0$ and different initial conditions, typically resulting in small but nonzero angular velocities.

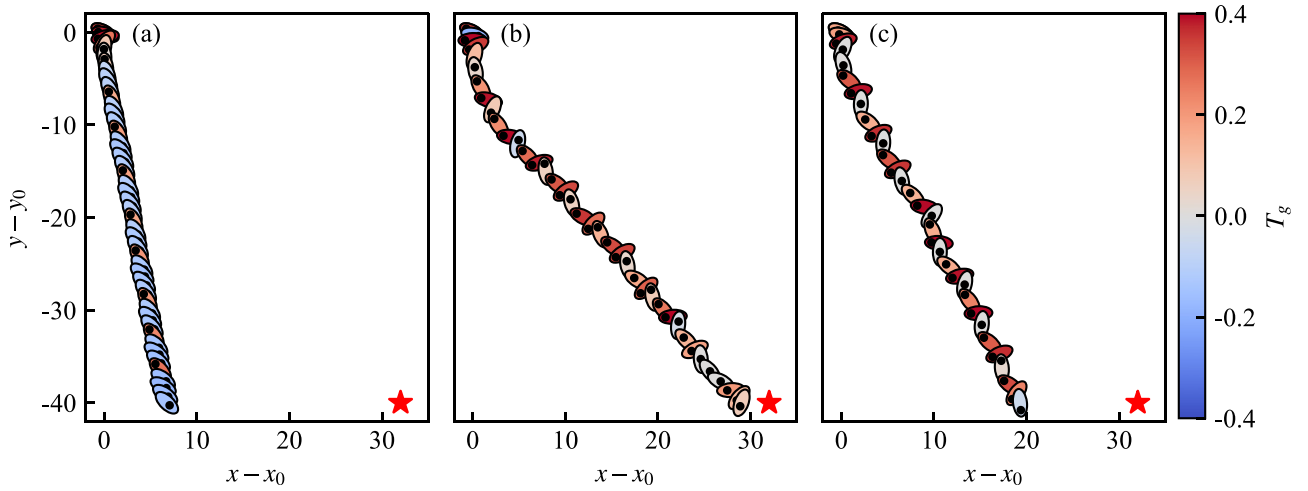


FIG. 4. Glider trajectories corresponding to Figs. 3(a)–3(c). Ellipses outline the glider at time intervals of action update, $\tau_u = 1.41$, colored by instantaneous gravity torque T_g . Black dots mark the center of mass, and the red star indicates the target. The glider size is enlarged by a factor 1.5 for better visualization. (a) $Ga = 4.4$; (b) $Ga = 35.4$; and (c) $Ga = 283$.

The second set is a single tumbling trajectory following the heuristic strategy [Eq. (8)] (explained later). We weigh the two sets by their number of trajectories during fitting, yielding accurate torque estimates for both gliding and tumbling (see Appendix B, Fig. 7).

The resulting coefficients are shown in Table II. Andersen *et al.* [17] suggested that the coefficients c_1 and c_2 decrease as Ga increases. Our results in Table II are broadly consistent with that expectation, except c_1 that increases slightly from $Ga = 35.4$ to 283. Table II shows that c_3 decreases as Ga increases. Our values of c_3 are quantitatively consistent with the coefficients reported by Ref. [39] that relies on DNSs of rotating elliptical cylinders in a uniform flow, with errors less than 15%.

At our largest Galileo number, $Ga = 283$ ($Re_p \approx 240$), the glider learns to tumble, just like the gliders from Refs. [19,23], with $Re_p \sim 10^3$. At high Re_p , the rapidly tumbling glider generates a lift force, similar to the Magnus effect observed for a rotating sphere [40], where rotation breaks the flow symmetry and produces a lift force $\mathbf{F}_L \sim \boldsymbol{\omega} \wedge \mathbf{v}$. Here, the lift force on the glider provides horizontal momentum for navigation, and a vertical component that balances part of the gravity force, slowing down the settling. Because the direction of the lift force depends on the direction of angular velocity, the glider learns to tumble anticlockwise to reach any target to its right, and tumble clockwise for the target to its left. Our DNS shows that the lift force becomes larger if the glider rotates with larger $|\boldsymbol{\omega}|$, similar to a sphere [40]. Therefore, it is important to understand how the gravitational (control) torque

T_g and the hydrodynamical torque T_h from Eq. (6) interact in Eq. (1d) to generate large $|\boldsymbol{\omega}|$. Paoletti and Mahadevan [19] identified a mechanism for this using a model with a constant control torque, where tumbling emerges from balancing this torque with the average dissipative torque. In our case, by contrast, the control torque changes as a function of time, simply because the tilt angle and the center-of-mass displacement change with time. How does the glider learn to tumble under these circumstances? Figure 3(c) shows that the glider shifts its center of mass when the angle θ reaches $\theta = 0$ or π , where the sign of $T_g = -d \sin \theta$ is about to change. This strategy generates a gravity torque that is mostly positive [Fig. 4(a)], a prerequisite for persistent anticlockwise tumbling.

How can this mechanism be understood from Eq. (6)? For the tumbling strategy, Fig. 5(a) shows the two largest contributions to the hydrodynamic torque T_h : the dissipative and the fluid-inertia torques. We see that the latter is significantly smaller than the former, although $|\mathbf{v}|$ and $|\boldsymbol{\omega}|$ are of the same order for the simulated trajectories (not shown). The difference in magnitude is explained by the fact that $\sin \phi \cos \phi$ fluctuates around zero and that the prefactor c_3 is smaller than c_2 by a factor of 2 (Table II). In conclusion, the fluid-inertia

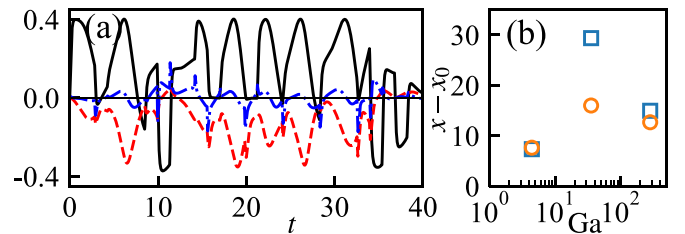


FIG. 5. (a) Torques for $Ga = 283$, for the trajectory shown in Fig. 4(c). Solid line: T_g ; dashed line: dissipative torque $-c_1\boldsymbol{\omega} - c_2|\boldsymbol{\omega}|\boldsymbol{\omega}$; dash-dotted line: fluid-inertia torque, $-c_3|\mathbf{v}|^2 \sin \phi \cos \phi$. The added moment of inertia torque, $-c_4\dot{\boldsymbol{\omega}}$, is negligible (not shown). (b) Maximal horizontal distance covered by a glider when it uses the tumbling strategy [Eq. (8)] ($\square$), or settling with a certain fixed orientation ($\circ$).

TABLE II. Numerical values of the coefficients in Eq. (6), obtained by fitting the DNS torque to Eq. (6) for $d = 0$.

Ga	c_1	c_2	c_3	c_4
4.4	0.192	0.911	0.151	0.070
35.4	0.024	0.203	0.143	0.070
283	0.066	0.183	0.096	0.070

torque has only a minor effect on the dynamics. Figure 5(a) indicates that the control torque T_g is approximately balanced by the dissipative torque on average (the same conclusion holds for intermediate Galileo number, $Ga = 35.4$). From this balance, we obtain an estimate of ω that differs by 20%–30% from our DNS results. This shows that the model is broadly consistent with our DNS, even though the control torque depends on time, unlike Ref. [19] that assumed constant control.

At small Ga , the glider navigates by maintaining an approximately constant angle θ , resulting in a horizontal force that allows steering. The horizontal force originates from the anisotropic drag coefficient of an elliptical glider. For a glider with $\lambda = 2$ at $Ga = 4.4$, the maximal drag coefficient (achieved when $\mathbf{n} \perp \mathbf{v}$, or equivalently when $\phi = \pi$) is only about 30% larger than the minimal drag coefficient (achieved when $\mathbf{n} \parallel \mathbf{v}$, or $\phi = 0$), which constrains the horizontal travel distance even if the glider settles with an optimal orientation. For a constant control T_g [19], settling with optimal orientation can be achieved by balancing the fluid-inertia torque [10–13,29], the third term on the right-hand side of Eq. (6), against T_g [27]. However, this balance may be hard to achieve because it requires very small yet precise values of T_g at small Ga . In practice, it is easier to have a control torque T_g that assumes only discrete values, as in our model. Our DNSs for $Ga = 4.4$ show that T_g is one order of magnitude larger than fluid-inertia torque for the smallest nonzero offset ($d = 0.2$). Figure 3(a) shows that the glider learns to adjust its center of mass dynamically to stabilize its tilt angle, and we see that the fluctuations around the average are small. This average angle is the optimum that maximizes the horizontal traveling distance, verified by measuring the distance in DNS of a glider that settles with fixed θ at different values of θ .

Now we discuss how the reachable horizontal range depends on Ga (Fig. 2). To this end, we consider a heuristic tumbling strategy,

$$d(t) = \text{sgn}[\theta(t)]d_{\max}, \quad (8)$$

which allows the glider to maximize its rotation speed by changing its center of mass whenever the sign of θ changes, inspired by the strategy learned by reinforcement learning shown in Figs. 2(b) and 2(c). Figure 5(b) shows DNS results for the reachable range for the heuristic tumbling strategy (8) as a function of Ga ($\square$). Also shown are DNS results for the reachable range obtained by gliding at a fixed optimal angle θ that maximizes the horizontal distance traveled by the glider. At large Ga , tumbling is better than gliding at a fixed θ , because the glider can achieve large angular velocities and the dissipative torque is smaller compared to the case of smaller Ga [Eq. (6) and Table II]. This explains why the glider learns to tumble at large Ga . However, why does the reachable range for the tumbling strategy decrease as Ga increases from 35.4 to 283? Our DNSs show that the glider rotates at almost the same speed at both values of Ga because of similar dissipative torque coefficients, but the lift force at $Ga = 283$ is 40% smaller than at $Ga = 35.4$. We verified by DNS that the main reason for the decreased lift force is a decrease in the pressure contribution to this force. Finally, at the smallest Ga , gliding and tumbling have approximately equal reachable ranges. Nevertheless, our reinforcement-learning runs show that the gliding strategy always wins at small Ga (even when gliding

is slightly suboptimal). Tumbling is hard to learn, because the glider must manage to rotate in the correct direction, and to stop tumbling when it approaches the target.

Here, we consider a two-dimensional glider. We found this necessary because our direct numerical simulations combined with reinforcement learning are too computationally expensive in three spatial dimensions. We expect that the navigation strategies remain valid for a three-dimensional glider, because the empirical torque model (6) extends to a three-dimensional compact particle [15,17]. Since the coefficients are different in three dimensions, the crossover between the two strategies (inclined settling versus tumbling) may occur at a different Reynolds number, compared with two dimensions.

How can our results be checked by laboratory experiments? For a glider of size $a = 5$ cm—large enough to carry sensors, motors, and batteries [41]—our range of Ga can be realized using silicone oil with viscosity ranging from $2 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$ to $1 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ (Table I). This yields a settling speed of about 1 m s^{-1} , angular velocity 10 rad s^{-1} , and total fall height 2 m. We expect that small changes in β (from 1.2 to 2; see Table I) do not make a qualitative difference to the settling dynamics, because the main effect of changing β is on the buoyancy force F_b . Decreasing β increases buoyancy. This reduces the settling speed of the glider, leading to longer traveling times. Moreover, although the added mass torque is also proportional to β^{-1} [Eq. (7)], its magnitude is smaller than that of the other torques. So, the fact that the added-mass torque changes with β is less important.

Here, we assume an infinitely large domain. How hydrodynamic forces and torques change near a wall or the floor of the container can be addressed by implementing the correct boundary conditions in the simulations. Moreover, we assume that the glider senses its position relative to the target, its orientation, velocity, and angular velocity. For practical applications, it is beneficial to reduce the number of signals. We expect that $x(t) - x_T(t)$, $\theta(t)$, and $\omega(t)$ are the most important signals (Appendix C), because x indicates the horizontal direction to travel, while θ and ω are needed to control the rotational dynamics. These signals can be measured with a small motion processing unit [42].

Controlling an autonomous glider by its moving center of mass has distinct advantages over other control methods. First, it generates a control torque without energy-demanding propulsion. Moving the center of mass within a small particle only requires a small amount of energy. Second, the glider exploits gravity to generate its control torque, which requires no other external field to be applied on the glider for navigation, such as commonly used magnetic or electric fields [26]. A drawback is that the magnitude of the gravity torque is constrained by the geometry of the glider and its instantaneous orientation, which impedes the maximal travel distance. This limitation may be eased by adding shape control of the glider, which has been shown to increase stability in natural gliders such as birds [43].

IV. CONCLUSIONS

In this paper, we investigated how a compact glider navigates with an adjustable center of mass as it settles in a viscous fluid at different Reynolds numbers. Using direct numerical

simulations, we found that the glider learns two distinct strategies to move large horizontal distances: it either settles with an inclined orientation optimized for horizontal displacement or it tumbles. Which of the two strategies is optimal depends sensitively on Re_p for given shape and mass density of the glider. For small Re_p , inclined settling is better, while the glider learns to tumble at larger Re_p , generating a horizontal lift force that extends its travel range.

Both strategies were previously found using an empirical model designed for gliders settling at large particle Reynolds numbers [19,23]. Besides the fact that our direct numerical simulations allowed us to study how the optimal strategy depends on the particle Reynolds number, there are two more important differences to Refs. [19,23]. First, we used an explicit control torque (the moving center of mass) rather than an arbitrary one, and we could show how the glider learns to execute these strategies by adjusting its center of mass. It settles with an inclined orientation by moving the center of mass back and forth or tumbles by adjusting its center of mass as a function of its orientation [Eq. (8)]. It was not *a priori* clear that this works, because the amplitude of the explicit control is constrained by the maximal distance the center of mass can move, and the control mechanism has a delay. Nevertheless, the glider can land precisely on any target within its maximal horizontal travel range by first approaching the target using the appropriate strategy, and then relying on viscous drag to slow down once it is above the target. Second, with laboratory experiments in mind, and because large aspect ratios λ are hard to simulate, we considered a compact glider in a viscous fluid with mass-density ratio β of order unity, rather than a glider with large λ and β .

Like earlier studies [19,23], we considered a two-dimensional elliptical glider. In our case, this is necessary because direct numerical simulations in three dimensions in combination with the reinforcement-learning algorithm were too computationally expensive. We expect that the tumbling and gliding mechanisms survive for a three-dimensional glider, because the empirical torque model (6) extends to three-dimensional particles [15,17], albeit with different coefficients. We cannot exclude that the extra degrees of freedom in three dimensions may allow for additional possibly more complicated strategies.

In nature, settling particles with more complicated shapes are common [44], even chiral [4,45]. Such particles experience translation-rotation coupling even in the Stokes limit, causing the glider to rotate as it settles. This could allow strategies with increased navigation range, as well as additional control possibilities. Moreover, one should also consider shape changes as a control mechanism, because this changes the hydrodynamic torques [27,44,46–49] the glider can use for control. Examples in nature are gliding ants that change posture [1] and birds that control the morphology of their wings [43].

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DATA AVAILABILITY

The data that support the findings of this article are available from the authors upon reasonable request.

APPENDIX A: DETAILS IN THE REINFORCEMENT LEARNING

We implement Double Deep Q -learning [35] to increase the stability of training. In the implementation of deep Q -learning, the feedforward network is constructed with two hidden layers with 16 and 32 neurons, with leaky ReLU functions as activation functions. State transitions are stored in a replay memory. A random minibatch of state transitions is sampled for updating the neuron network at a frequency of every four state transitions.

During training, the learning rate, $\alpha = \max(0.999^{N_t} \alpha_0, \alpha_{\min})$, decays from its initial value α_0 to the minimal value $\alpha_{\min}$, where N_t is the number of updates in the deep Q -network $Q(s, \mathbf{a}; \theta_t)$, which is a function to evaluate the action $\mathbf{a}$ given a state s , and θ_t is the network parameter. To encourage the exploration in different actions, an ϵ -greedy policy is used during training. The glider chooses random actions with a small probability ϵ , and chooses the action that maximizes $Q(s, \mathbf{a}; \theta_t)$ otherwise. The exploration rate, $\epsilon = \max(\epsilon_0 \frac{500-N}{500}, \epsilon_{\min})$, decays in every episode from ϵ_0 to the minimal value, $\epsilon_{\min}$, where N is the number of episodes. Hyperparameters for training are listed in Table III.

Figure 6 shows that the training converges at around 4000 episodes. Here, the training is considered converged when

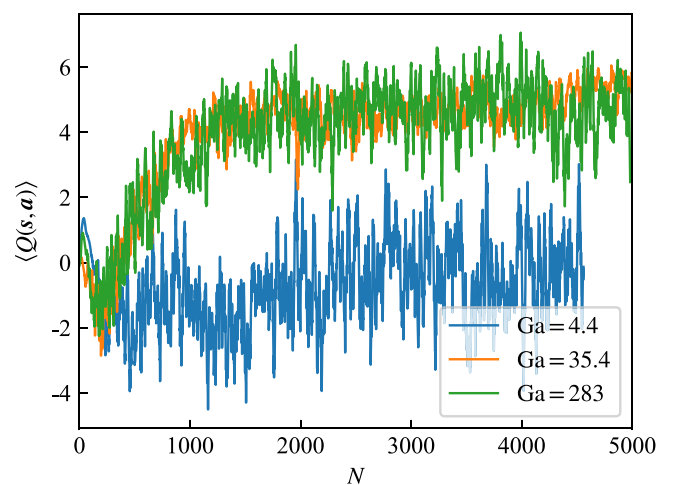


FIG. 6. Average of value function $\langle Q(s, \mathbf{a}) \rangle$ vs the number of episodes N . The data are smoothed with a moving average over 20 episodes.

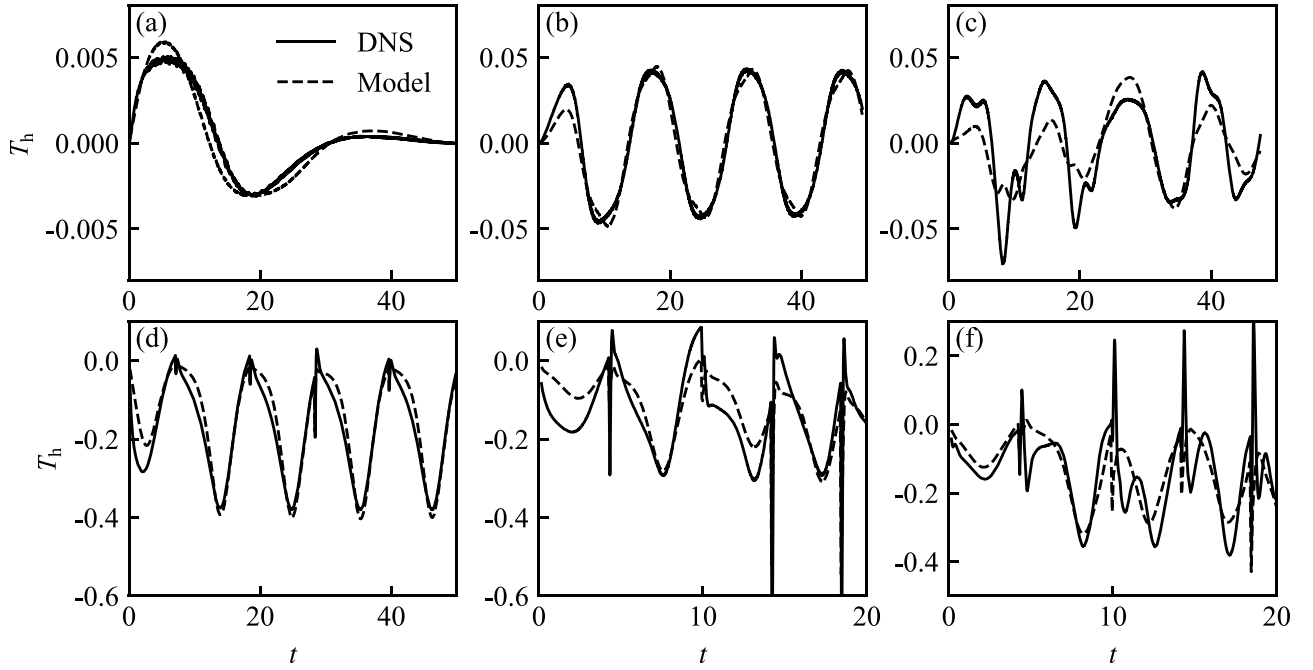


FIG. 7. Comparison of the hydrodynamic torques obtained from DNS and predicted by the empirical model. (a)–(c) Glider settling with $d = 0$ and an initial angle $\theta = \pi/3$. (d)–(f) Glider following the strategy $d(t) = \text{sgn}[\theta(t)]d_{\text{max}}$. (a), (d) $\text{Ga} = 4.4$; (b), (e) $\text{Ga} = 35.4$; and (c), (f) $\text{Ga} = 283$.

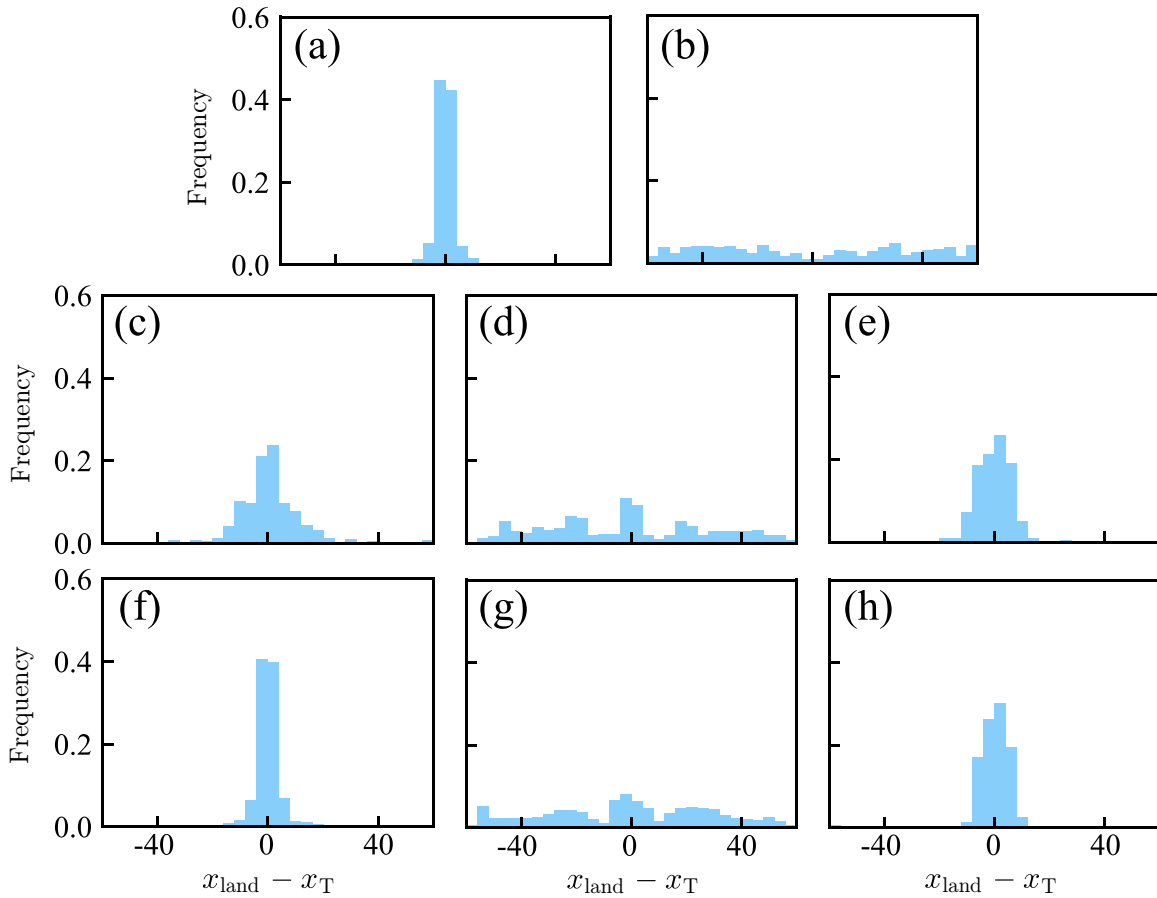


FIG. 8. Frequency histograms of errors in landing positions at $\text{Ga} = 35.4$. Each panel represents a signal combination. (a) All six signals measured; (b) $x - x_T$ masked; (c) $y - y_T$ masked; (d) θ masked; (e) v_x masked; (f) v_y masked; (g) ω masked; and (h) v_x and v_y masked.

TABLE III. Hyperparameters for the deep Q -learning algorithm.

Hyperparameter	Value
Minibatch size	256
Optimizer	Adam
Replay memory size	8192
γ	1.0
α_0	0.1
$\alpha_{\min}$	0.002
ϵ_0	1
$\epsilon_{\min}$	0.2

$Q(s, \mathbf{a}; \theta_t)$ does not substantially change with θ_t . This is indicated by that the average of $\langle Q(s, \mathbf{a}) \rangle = N_t^{-1} \sum_{i=1}^{N_t} Q(s_i, \mathbf{a}_i)$ remains steady as the number of episodes increases. Upon convergence, the learned strategy is tested without ϵ -greedy exploration, i.e., $\epsilon = 0$.

APPENDIX B: EMPIRICAL MODEL FOR ROTATION DYNAMICS

Figure 7 compares the torques predicted by the model in Eq. (6) to the torques obtained by our DNS. Figures 7(a)–7(c)

show that the torque on a settling glider with $d = 0$ is well predicted by the torque model. The torque model is less accurate when the glider can change its center of mass according to the heuristic tumbling [Eq. (8)], as shown in Figs. 7(d)–7(f). The inaccuracy occurs when an impulsive torque is generated by the movement of the center of mass, as a result of the exchange of angular momentum between fluid and the glider. However, these torque impulses last for a short time, and the torque model predicts the torque well the rest of the time.

APPENDIX C: IMPORTANCE OF SIGNALS

The signals perceived by the glider are $s = \{x - x_T, y - y_T, v_x, v_y, \theta, \omega\}$. To examine their importance, we mask part of them, and run trainings at $\text{Ga} = 35.4$ to see how the performance changes. For instance, when $x - x_T$ is masked, the state consists of the remaining five variables: $s = \{y - y_T, v_x, v_y, \theta, \omega\}$. The performances of strategies with different signals masked are evaluated by the distribution of the errors in landing position, $x_{\text{land}} - x_T$, as shown in Fig. 8. In the reference case, where none of the signals are masked [Fig. 8(a)], the error is concentrated around zero. However, the error spreads over a wide range when $x - x_T$, θ , or ω is masked, as shown in Figs. 8(b), 8(d), and 8(g), respectively.

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