

Calibrating magnetic flux control in superconducting circuits by compensating distortions on timescales from nanoseconds up to tens of microseconds

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Fast tuning of the transition frequency of superconducting qubits using magnetic flux is essential, for example, for realizing high-fidelity two-qubit gates with low leakage or for reducing errors in dispersive qubit readout. To apply accurately shaped flux pulses, signal distortions induced by the flux control lines need to be carefully compensated for. This requires their *in situ* characterization at the reference plane of the qubit. However, many existing approaches are limited in time resolution or in pulse duration. Here, we overcome these limitations and demonstrate accurate flux control with submilli residual frequency errors on timescales ranging from nanoseconds to tens of microseconds. We achieve this by combining two complementary methods to characterize and compensate for pulse distortions. We have deployed this approach successfully in a quantum error correction experiment calibrating 24 flux-activated two-qubit gates. Reliable calibration methods, as the ones presented here, are essential in experiments scaling up superconducting quantum processors.

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I. INTRODUCTION

The Josephson energy of a superconducting quantum interference device (SQUID) can be tuned *in situ* by controlling the magnetic flux through the SQUID loop. For a transmon qubit [1,2], flux tuning of the Josephson energy gives control over its transition frequency. This not only enables a variety of control and measurement protocols for individual transmon qubits, such as measuring the frequency dependence of the relaxation time T_1 [3], tuning the parameters of dispersive qubit readout [4], resetting qubits to their ground state [5,6], or mitigating leakage out of the computational subspace [7], but also provides the basis for implementing fast entangling gates between pairs of qubits. Flux-activated two-qubit gates include ones based on resonant interactions [8–11], gates parametrically activated by modulating the qubit frequency [12,13], and gates based on flux-tunable couplers [14–19]. As

two-qubit gates frequently limit the performance of superconducting quantum processors, for example, in the context of implementing quantum error correction [20–24], accurate flux control is of great importance.

Flux control waveforms created by an arbitrary waveform generator (AWG) are subject to distortions in the flux control line, stemming from the AWG output stage, the control wiring leading up to the superconducting quantum device, its package, and the on-chip part of the flux line. For implementing fast two-qubit gates with high fidelity and low leakage [10,11,20], we aim at compensating distortions on timescales ranging from nanoseconds to tens of microseconds. Compensation on short timescales avoids ringing and overshoots for rapidly changing flux profiles, while compensation on long timescales enables deep sequences of gates to be executed faithfully.

In situ characterization methods [25] account for all flux control line components [26] as well as their operation at cryogenic temperatures [25]. While characterization of the room-temperature part of the flux control line and its electronics could proceed using conventional methods [25–29] using oscilloscopes, spectrum analyzers, or network analyzers, any change in the load impedance when connecting a measurement instrument instead of the cryogenic flux line would lead to inaccuracies (see Appendix A). Such inaccuracies are avoided by performing *in situ* characterization as an end-to-end approach for the whole flux control line, including the AWG output stage and the room-temperature wiring.

In prior work, *in situ* characterization was performed by measuring the instantaneous qubit frequency as a function of the time after the rising or falling edge of a flux pulse using

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pulsed spectroscopy [27,29–31] or a combination of an $X_{\pi/2}$ and a $Y_{\pi/2}$ pulse applied to the qubit [28,29]. For this class of methods, the time resolution is limited by the drive pulse duration [27,30]. An alternative method is based on characterizing the dynamic phase acquired by a qubit during flux pulses of various durations. This approach was implemented using quantum state tomography [16,32–34] or Ramsey-type experiments [25]. Measuring dynamic phases is inherently limited by the coherence time of the qubit and, thus, not well suited for long flux pulses. A similar limitation applies to the method from Ref. [35], which measures the transfer function of the flux control line in the frequency domain by characterizing the qubit with a microwave drive applied to the line. Other methods require a dedicated on-chip circuit for flux measurements [26], are designed for tunable couplers [36], or are only focused on repeatability of two-qubit gates [37].

Here, we present an end-to-end framework for *in situ* characterization and compensation of flux pulse distortions occurring on timescales of tens of microseconds (Sec. III) down to nanoseconds (Sec. IV). We combine and extend known methods, which are either limited in their time resolution or by the coherence times of the qubits employed. By characterizing flux waveforms up to a duration of 100 μs , we go far beyond the flux pulse durations of up to a few microseconds in previous demonstrations of the characterization of flux pulse distortions with flux-tunable qubits [16,25,27–33,37]. The considered time span of 100 μs exceeds the time constants of the dominant distortions in a typical flux line configuration, and only the longest gate sequences implemented on superconducting qubits reach durations on this order of magnitude, e.g., in randomized benchmarking or in quantum error correction experiments. In our procedure, we calibrate a higher-order infinite impulse response (IIR) filter based on a single series of measurements, and finite impulse response (FIR) filters using robust regularized optimization at the sampling rate of the arbitrary waveform generator. Using these methods, we demonstrate accurate flux control with subpermille residual qubit frequency errors (Sec. V). In contrast to previous studies in Refs. [25–29], our approach does not require any prior room-temperature measurements of components. The presented methods are applicable in settings in which the flux control line is in good approximation linear.

II. FLUX TUNING OF TRANSMON QUBITS

We implement flux control of a transmon qubit with a dedicated flux line, which couples inductively to the SQUID loop of the transmon [see Fig. 1(a)]. A voltage applied at room temperature yields a current through the on-chip flux line, inducing magnetic flux through the SQUID loop, which tunes the transition frequency of the transmon [1], see Appendix B for a detailed model. By combining a direct current (dc) bias with flux pulses, we control both the idle frequency and time-dependent frequency excursions of the qubit. Since flux noise has a detrimental effect on the coherence time of the qubit [38], we reduce the noise from the room-temperature electronics by employing filters in the flux control line. For a detailed description of the experimental setup, see Appendix C.

In Fig. 1(b), we show the excursion of the transition frequency $f_{01}(t)$ of the qubit (solid purple line) from its idle

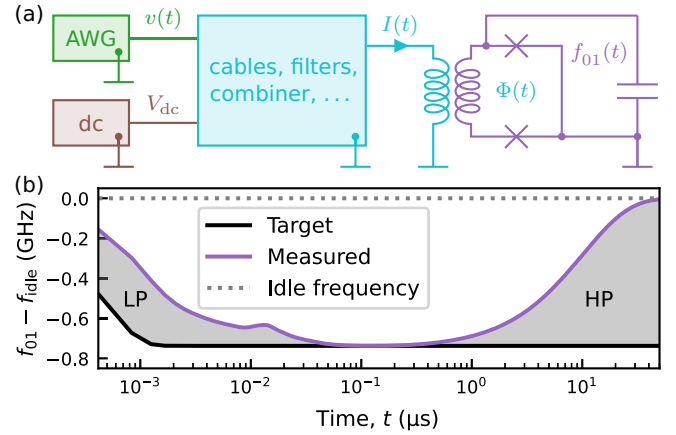


FIG. 1. (a) Schematic of the experimental setup (details in Appendix C) with a superconducting transmon qubit (purple) with transition frequency $f_{01}(t)$, which is tuned by a magnetic flux $\Phi(t) = \Phi_{dc} + \phi(t)$, consisting of a static bias Φ_{dc} and a time-varying component $\phi(t)$. The magnetic flux is induced from an inductively coupled flux line (cyan), to which we apply a dc bias voltage V_{dc} (brown) and pulses $v(t)$ generated by an arbitrary waveform generator (green). (b) Simulated transition frequency $\Delta f_{01}(t)$ relative to the idle frequency f_{idle} in response to a Gaussian-filtered step function at the input of the flux control line (simulation based on the measured response of the flux control line; see Appendix E), showing distortions with low-pass and high-pass characteristic (labeled as LP and HP, respectively).

frequency f_{idle} (dotted gray line) on a logarithmic timescale in response to a Gaussian-filtered step function programmed into the AWG. The solid black line indicates the target qubit frequency that would result from the same AWG waveform in the absence of distortions in the flux control line. The dominant distortion at long timescales (beyond approximately 100 ns) is an exponential decay of the flux waveform due to the high-pass (HP) characteristic of a bias tee (time constant $\tau_{HP} = 19.2 \mu\text{s}$; see Appendix D), which we employ to combine the constant bias voltage with the AWG signal. The bias tee also suppresses $1/f$ noise from the AWG. Further contributions at long timescales can originate from other filters, either intentionally included in the flux control line (not the case in our setup) or formed by parasitic capacitances or inductances together with resistive elements (attenuators). At short timescales, relevant distortions are caused by low-pass (LP) filters in the output stage of the AWG and at the base plate of the cryogenic setup (see Appendix C). Due to the uncompensated low-pass characteristic, the qubit reaches the target frequency only after approximately 50 ns. Further possible causes for distortions at short timescales are the skin effect and reflections due to impedance mismatches [25].

We model the distortions by the impulse response $h(t)$ of a linear time-invariant (LTI) system, which represents the entire flux control line. The linear model is justified as we have carefully designed and realized the experimental setup such that nonlinearities in the flux control line are negligible. For a detailed discussion, see Appendix D. The flux change $\phi(t)$ with respect to the idle flux is given by the output of the LTI system h ; see the signal flow diagram in

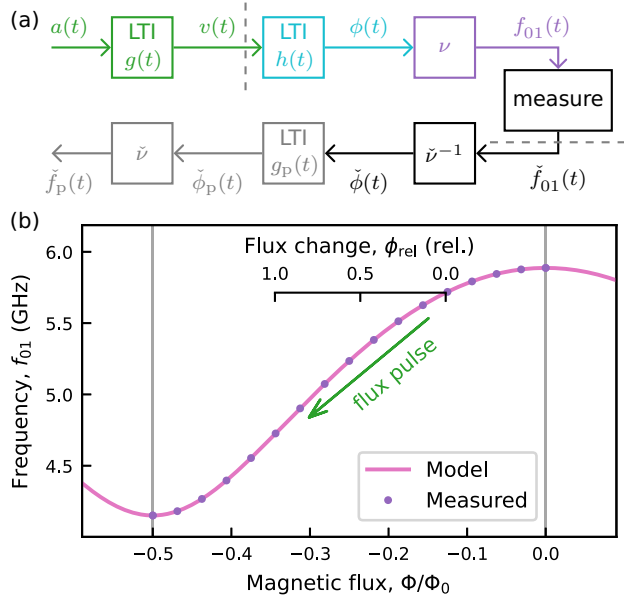


FIG. 2. (a) Signal flow diagram (details in the main text), showing the flux control (green, cyan) of the transition frequency (purple) of a transmon, the characterization measurement (black), and optional analysis steps (gray; see Sec. III). The signal flow between (outside) the dashed lines is in hardware (software). (b) Measured qubit transition frequency (purple points) and fitted model (pink solid line) plotted vs a static magnetic flux bias $\Phi = \Phi_{dc}$ normalized by the flux quantum Φ_0 . The inset axis illustrates the flux tuning (relative to the maximal flux change) with a flux pulse starting from a given dc bias (origin of the inset axis) during a measurement for characterizing flux pulse distortions (see Sec. III).

Fig. 2(a) and the analytical description in Appendix A. We introduce a nonlinear function ν to describe the functional dependence of the transition frequency $f_{01}(t)$ on the flux change as $f_{01}(t) = \nu(\phi(t))$.

To characterize the flux dependence of the qubit transition frequency $f_{01}(t) = \nu(\phi(t))$, we first measure the transition frequency from the ground state to the first excited state of the transmon in a series of Ramsey experiments as a function of a static flux bias [see Fig. 2(b)]. The asymmetric SQUID loop of the transmon leads to two first-order flux-insensitive points; see the first and the last data point. At the upper first-order flux-insensitive point, we also measure the transition frequency from the first to the second excited state. We then fit the model to the data as described in Appendix B [see Fig. 2(b)].

The distortions of flux control pulses can be characterized by measuring the qubit transition frequency in response to a known control waveform $v(t)$ and converting the measurement result $\tilde{f}_{01}(t)$ to a flux change $\tilde{\phi}(t)$ by means of the inverse $\tilde{\nu}^{-1}$ of the qubit frequency model ν [see Fig. 2(a)]. Note that we use the symbol $\tilde{}$ to indicate quantities inferred from measured data, in contrast to their respective unknown true values, which appear in the mathematical model without this symbol.

To compensate for the characterized distortions, we employ digital prefiltering (also called predistortion, e.g., [25,39]). That is, we program an intentionally distorted

version $v(t)$ of the desired waveform $a(t)$ into the AWG, in a way that distortions in the flux line wiring and electronics are compensated for. The calibration of this predistortion consists in designing an LTI system with impulse response $g(t)$ such that the cascade of the predistortion filter $g(t)$ and the physical system $h(t)$ form a scaled identity operation. This means that we aim at making $\phi(t)$ equal to the input waveform $a(t)$ up to a scaling factor.

III. DISTORTIONS AT LONG TIMESCALES

For characterizing the distortions in the flux control line, we avoid measuring close to the first-order flux-insensitive points, where the dependence of the qubit transition frequency $\tilde{\nu}$ on the magnetic flux cannot be inverted. Thus, we first adjust the dc bias such that the qubit is tuned to a flux-sensitive point; see the origin of the relative flux change axis in Fig. 2(b). At the selected flux bias $-0.127 \Phi_0$, the slope of the flux-frequency curve of the considered qubit is approximately at half its maximal value. We then use an AWG to apply a step function, smoothed by a Gaussian filter of width $\sigma_{FP} = 0.5$ ns [see Fig. 3(a)]. The step height of $0.217 \Phi_0$ is chosen to be similar to typical pulse amplitudes for implementing two-qubit gates on this device. We do not apply a predistortion filter at this point, i.e., $v(t) = a(t)$ in Fig. 2(a).

We measure the resulting time dependence of the transition frequency $\tilde{f}_{01}(t)$ by probing in a two-dimensional sweep at which combinations of drive time t and drive frequency f_{drive} a qubit transition is resonantly driven. The drive pulse has a Gaussian envelope of width $\sigma_{drive} = 10$ ns, and we adapt the pulse amplitude as a function of f_{drive} to implement a π pulse at any f_{drive} . Details of the drive pulse generation are presented in Appendix F.

To achieve consistent qubit readout despite large frequency excursions of the qubit, we tune the qubit back to its initial bias point ($\phi = 0$ at the qubit) by applying a compensation pulse with negative amplitude to account for the long-time distortions (see Appendix F). This compensation starts at the time $t + t_{fl}$ with $t_{fl} = 110$ ns, before the readout starts at $t + t_{ro}$ with $t_{ro} = 220$ ns. We restrict the sweep to drive frequencies f_{drive} in a range of ± 100 MHz around the transition frequency calculated based on a theoretical model of the dominant distortion [see Fig. 3(b)]. To characterize distortions across several orders of magnitude in time, we measure the excited state population for 70 logarithmically spaced values of the drive time t from $t = 10$ ns up to $t = 100 \mu s \approx 5 \tau_{HP}$ and average the results over 2048 experimental runs.

Unlike a continuous drive tone, a drive pulse combined with a subsequent measurement only leads to a weak power dependence of the measured transition linewidth of the qubit [40]. In this case, the effective linewidth observed in a spectroscopic measurement is determined by the spectral width of the drive pulse. This is observed in the data for a single time t shown in the inset of Fig. 3(b), which is well approximated with a Gaussian, whose width is in good agreement with the spectral width $\sigma_{drive,f} = (2\pi\sigma_{drive})^{-1} = 16$ MHz of the Gaussian drive pulse. The center of the Gaussian fit yields the measured qubit transition frequency $\tilde{f}_{01}(t)$; see right axis of Fig. 4(a). Standard error estimates calculated from the variance of the fit residuals are on average 0.2 MHz in the

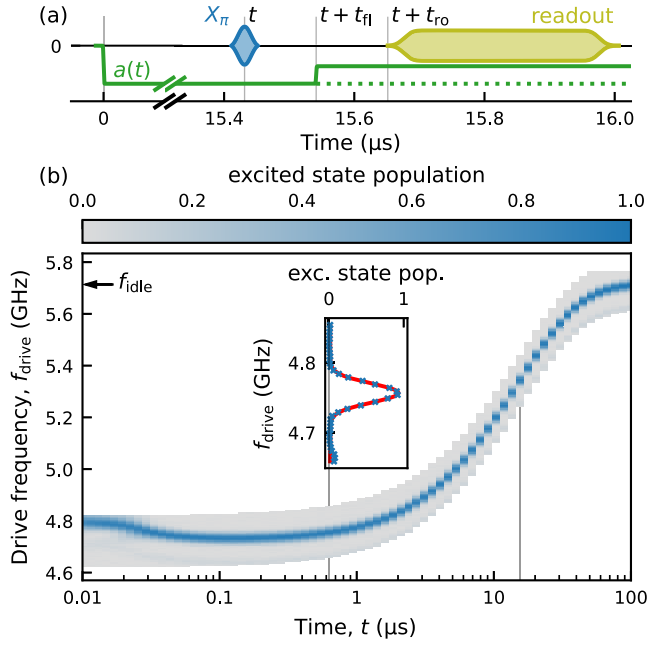


FIG. 3. *In situ* characterization of the flux line step response. (a) Pulse sequence showing envelopes of the qubit drive pulse (blue) and the readout pulse (olive), as well as the flux waveform (solid green), which corresponds to a Gaussian-filtered step function (dotted green) with an adjusted output voltage of the AWG to achieve consistent readout (see main text for details). The times correspond to the measurement indicated by the right gray vertical line in panel (b). (b) Measured excited state population in a two-dimensional sweep of drive pulse time t and drive frequency f_{drive} . The inset shows the data (markers) for a fixed drive pulse time indicated by the left gray vertical line, along with a Gaussian fit (solid line) of the qubit spectral line.

presented data. With the inverse qubit frequency model $\check{\nu}^{-1}$ from Sec. II, we calculate the flux change $\check{\phi}(t)$ and plot it relative to the largest flux excursion observed in the measurement; see left axis of Fig. 4(a). This curve corresponds to a normalized estimate of the step response of the flux control line, i.e., of the integral over the impulse response $h(t)$ (see Appendix A).

We model this step response as

$$s(t) = \left(\alpha_0 + \sum_{i=1}^N \alpha_i e^{-\frac{t}{\tau_i}} \right) u(t) \quad (1)$$

with the unit step function $u(t)$. This physically motivated model accounts for distortions from an overdamped N th-order linear dynamic circuit [41], with N linearly independent reactive elements, i.e., capacitors and inductors, and an arbitrary number of linear resistive elements (see Appendix A). This covers the types of circuit elements contained in the bias tee, the attenuators, and the low-pass filters as well as cable losses and parasitic capacitances and inductances. Similar models have previously been used in Refs. [16,26,34]. We choose $\alpha_0 = 0$ (accounting for the high-pass filter in the bias tee) and fit the remaining parameters $\alpha_1, \dots, \alpha_N$ and $\tau_1, \dots, \tau_N$; see Appendix G for details on the fitting procedure and on the choice of the model order N . Figure 4(d) shows the parameters

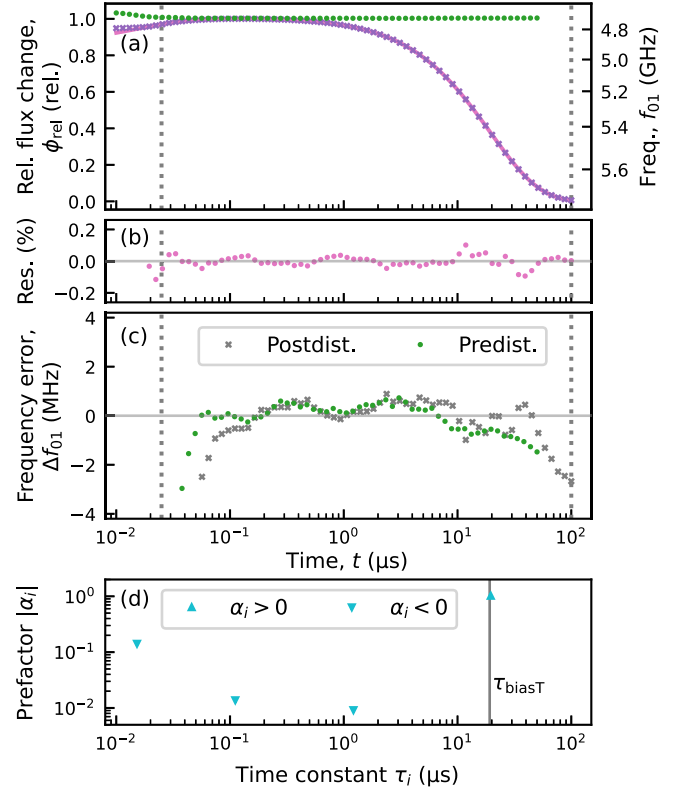


FIG. 4. (a) Flux change at the qubit (relative to the maximum change) and corresponding transition frequency when programming a step function into the AWG (blue crosses), and fit of a sum of exponentials (pink solid line) to the data points between the dotted gray lines. The green dots result from repeating the measurement after calibrating an infinite impulse response filter as a predistortion filter. (b) Residuals of the fit in panel (a). (c) Difference between the measured transition frequency and its targeted value ($\Delta f_{01} = f_{01} - f_{\text{target}}$) when compensating the distortions in the flux control line with an inverse IIR filter based on the fit in panel (a): expected performance (gray crosses) calculated via postdistortion (details in the main text) and verification measurement (green points) when applying the IIR filter for predistortion. (d) Parameters of the sum-of-exponentials model fitted in panel (a).

obtained with $N = 4$, yielding the fit in panel (a) with the fit residuals in panel (b). The fitted time constant with the largest prefactor is in good agreement with a room-temperature measurement of the time constant of the high-pass filter in the bias tee (gray line; see Appendix D).

Note that we have restricted the fit to the time range between the gray dotted lines in Fig. 4 because the spectroscopic approach is not able to track changes of the qubit transition frequency on timescales significantly below the duration of the drive pulse. In the measurement presented here, the drive pulse duration was 40 ns, resulting from the width $\sigma_{\text{drive}} = 10$ ns of the Gaussian pulse shape and a truncation of the pulse to $4\sigma_{\text{drive}}$.

In addition to being physically motivated (see Appendix A) and reaching low fit residuals with only few fit parameters (see Fig. 4), the sum-of-exponentials model from Eq. (1) has the advantage that it can be inverted analytically. As discussed in Appendix G, we use the Laplace transform to derive a filter

with impulse response $h_{\text{inv}}(t)$, which inverts the fitted step response up to a scaling factor. Using a root finding method, we identify the poles and zeros of the Laplace representation. We then translate the poles and zeros into the domain of the z transform to obtain an implementation of $h_{\text{inv}}(t)$ as a digital IIR filter (see Appendix G). To ensure numerical stability, we choose an implementation as a cascade of second-order infinite impulse response filters, so-called second-order sections [42], each of which is implemented digitally as a recursive equation.

Applying h_{inv} as a predistortion filter, we remeasure the qubit transition frequency with the spectroscopic method for waveform durations up to 50 μs ; see the green dots in Fig. 4(a). For most of the time interval that was taken into account for the fit (between dotted gray lines), we observe residual frequency errors of less than 2 MHz [see Fig. 4(c)], with larger deviations at times smaller than 40 ns.

As a qualitative indicator of whether this dataset suffers from significant systematic errors of the measurement method, we compare the results to postprocessed data from the initial characterization measurement, where we apply h_{inv} as a postdistortion filter $g_p(t) = h_{\text{inv}}(t)$ as indicated in Fig. 2(a). We convert the resulting flux $\check{\phi}_p(t)$ to the postprocessed qubit transition frequency $\check{f}_p(t)$; see the crosses in Fig. 4(c). Under ideal conditions, the commutativity of linear filtering implies that the effect of the postdistortion is equivalent to applying $g(t) = h_{\text{inv}}(t)$ as a predistortion filter and remeasuring the transition frequency as done above. However, when errors propagate through the postprocessing, which includes two nonlinear transformations $\check{v}^{-1}$ and $\check{v}$ as shown in Fig. 2(a), this equivalence breaks down. We thus expect measurement errors or model mismatches to manifest themselves in the postprocessed $\check{f}_p(t)$ in a different way than in the directly measured $\check{f}_{01}(t)$ with predistortion. The good agreement of most of the data points in Fig. 4(c) hints at small measurement errors, indicating that $\check{\phi}(t) \approx \phi(t)$, and at accurate model assumptions, including the assumption that distortions are linear and the assumption that the qubit frequency $\nu(\phi)$ is well described by the model $\check{\nu}(\phi)$. Residual errors in the qubit frequency model $\check{\nu}$ can explain discrepancies at long timescales, where the measurements with postdistortion and with predistortion are performed at significantly different transition frequencies [see Fig. 4(a)]. Furthermore, we observe a slight discrepancy at short times, which we attribute to errors due to the limited time resolution of the measurement. These errors, which affect $\check{f}_{01}(t)$ at times that are smaller than the drive pulse duration, propagate to later points in time in $\check{f}_p(t)$ due to error propagation in the postdistortion filter g_p .

To further reduce the residual frequency errors of approximately 2 MHz observed in Fig. 4(c), we treat the cascade of digital predistortion and physical flux control line as an effective linear time-invariant system. By computing the flux change from the measured frequencies with predistortion, we obtain the step response of this effective LTI system. From this step response, which describes the residual distortions that remain despite the predistortion, we calculate another inverse filter and apply it as an additional predistortion filter, i.e., cascaded with the initially calibrated predistortion filter.

This additional predistortion step reduces the residual errors to approximately 0.5 MHz, as we discuss in Sec. V.

Unlike methods based on state tomography or Ramsey experiments, the spectroscopic method presented in this section characterizes the step response of the flux control line at timescales longer than the coherence time of the qubit. However, the time resolution is inherently limited by the drive pulse duration as discussed above. We thus characterize distortions on the nanosecond timescale in a separate measurement described in Sec. IV.

IV. DISTORTIONS AT SHORT TIMESCALES

After calibrating IIR filters for timescales from 100 μs down to 25 ns (see dashed vertical lines in Fig. 4), we employ the cryoscope method [25] to characterize distortions at timescales below 50 ns. To this end, we model the cascade of the IIR predistortion filters and the physical flux control line as an effective linear time-invariant system with impulse response $h(t)$ and aim at further reducing residual distortions with an additional predistortion filter $g(t)$. As the residual distortions are limited to short timescales, we choose $g(t)$ to be an FIR filter.

We first characterize the response of the qubit transition frequency to a rectangular flux pulse $a(t)$ of duration 100 ns without any FIR filter applied to the input signal, i.e., $v(t) = a(t)$. Following Ref. [25], we measure the dynamic phase accumulated by the qubit during a flux pulse by enclosing the flux pulse between two $\pi/2$ pulses and sweeping the phase of the second $\pi/2$ pulse. The pulse sequence of this Ramsey-type measurement is illustrated in Fig. 5(a). By truncating the flux pulse first at $t - \Delta t/2$ and then, in a second experiment, at $t + \Delta t/2$, yielding the dynamic phase $\theta_{t-\Delta t/2}$ and $\theta_{t+\Delta t/2}$, respectively, we approximate the excursion of the qubit from its idle transition frequency as

$$\check{f}_{01}(t) - f_{\text{idle}} = \frac{\theta_{t+\Delta t/2} - \theta_{t-\Delta t/2}}{2\pi \Delta t}; \quad (2)$$

see the purple markers in Fig. 5(b).

As long as there are uncompensated distortions, the method leads to systematic errors if the difference between $f_{01}(t - \Delta t/2)$ and $f_{01}(t + \Delta t/2)$ is large, i.e., in case of steep rising or falling edges [25]. This is because any difference in the turn-off transients when truncating the flux pulse at times $t - \Delta t/2$ and $t + \Delta t/2$ affects the difference of measured dynamic phases in Eq. (2). We partially mitigate these errors by choosing the idle frequency f_{idle} to be at a first-order flux-insensitive bias point, which weakens the effect of flux variations due to turn-off transients on the measured dynamic phase [25].

The time resolution of this method to determine the qubit transition frequency is given by the step size Δt , which can be as small as the AWG sampling interval T_s , i.e., $\Delta t \geq T_s = 1/(2.4 \text{ GHz}) = 0.42 \text{ ns}$ in our setup (see Appendix C). On the one hand, the difference between $f_{01}(t - \Delta t/2)$ and $f_{01}(t + \Delta t/2)$ during edges of the pulse, which causes the systematic errors discussed above, becomes smaller when reducing the step size Δt . On the other hand, increasing Δt reduces the number of data points to be measured and reduces

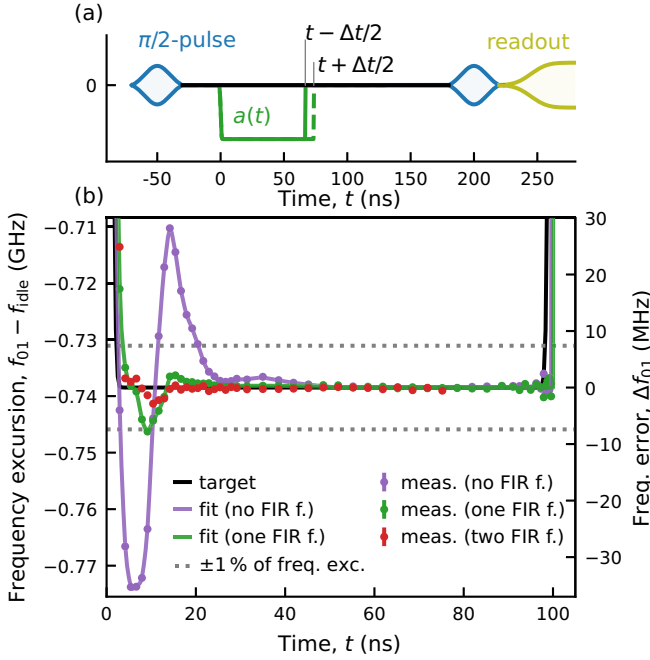


FIG. 5. Characterization and correction of distortions at timescales below 50 ns. (a) Pulse sequence showing envelopes of the $\pi/2$ drive pulses (blue), the readout pulse (olive), and two flux pulses differing in duration by Δt (solid and dashed green); see main text for details. (b) Measured (dots) and fitted qubit frequency excursion (solid lines) with respect to the idle frequency f_{idle} when applying a rectangular flux pulse without an FIR filter (purple), with a single FIR filter (green), or with two FIR filters (red).

statistical errors as it acts as an effective low-pass filter (see Appendix H).

We obtained the purple data points in Fig. 5(b) by applying the described procedure with 16 different phases of the second $\pi/2$ pulse and averaged over 65 536 repetitions, while varying Δt as function of t in the range between one and eight times the AWG sampling period T_s ; see details in Appendix H. This leads to standard errors between 0.25 MHz (for $\Delta t = 8T_s = 3.3$ ns) and 0.73 MHz (for $\Delta t = T_s = 0.42$ ns).

We observe distortions not compensated for by the long-timescale methods in the initial part of the flux pulse, which are most pronounced in the first 25 ns; see purple data points in Fig. 5(b). This is the interval that was excluded from the fit in the IIR calibration (see dashed line in Fig. 4) due to the limited time resolution of the spectroscopic measurement. In fact, the IIR filters for compensating distortions at longer timescales can contribute to additional distortions at timescales that cannot be resolved with the method used in Sec. III, such as the large overshoot after the initial falling edge; see the purple markers in Fig. 5(b). The FIR filter calibration described below compensates for short-time distortions independently of whether they have a physical origin or whether they are artifacts caused by the previously applied filters.

Since the distortions at short timescales have multiple origins (see Sec. II), we describe the transmission through the cascade of IIR predistortion filters and the physical flux

control line by a generic FIR model

$$h(t) = \sum_{\ell=0}^{L-1} h_{\ell} \delta(t - \ell T_s), \quad (3)$$

with $L = 120$ free parameters h_{ℓ} spaced by the AWG sampling period T_s , where δ is the Dirac impulse. To fit this model to the acquired dataset of measured frequencies $\check{f}_{01}(t)$, we use a regularized optimization [43,44] (see Appendix I) to overcome the following challenges. First, unlike for the sum-of-exponentials model used in Sec. III, the high number of coefficients in Eq. (3) makes the model susceptible to overfitting to noise. Second, as the qubit idles at a first-order flux-insensitive bias point to mitigate errors due to turn-off transients as discussed above, the function v mapping flux change ϕ to qubit transition frequency f_{01} is not invertible at f_{idle} . This makes the inverse problem of inferring $\check{\phi}$ from the measured frequencies $\check{f}_{01}$ ill-conditioned for frequencies close to the idle point. For these frequencies, even small errors in $\check{f}_{01}$ would lead to severe errors in the inferred flux change $\check{\phi}$ when explicitly using the inverse qubit frequency model $\check{v}^{-1}$. The regularized optimization overcomes both problems because it avoids an explicit inversion and provides robustness to overfitting (see Appendix I).

The purple line in Fig. 5(b) shows the result of filtering the rectangular input pulse with the fitted impulse response of the residual distortions, $h(t)$, and applying the qubit frequency model $\check{v}$. Further, we compute an inverse FIR filter h_{inv} via a second regularized optimization (see Appendix I). An alternative approach based on a Savitzky-Golay filter and the CMA-ES algorithm is presented in Ref. [25] with extensions in Ref. [45].

With the inverse filter applied as a predistortion filter $g(t) = h_{\text{inv}}(t)$, we remeasure $\check{f}_{01}(t)$ [green markers in Fig. 5(b)] and perform the same fitting procedure as before (solid green line), revealing residual frequency deviations of up to approximately 8 MHz from the target, corresponding to 1% of the total frequency excursion. We attribute the deviations mostly to the fact that we calibrated the FIR predistortion filter based on a characterization dataset that suffers from systematic errors of the method from Ref. [25].

As the systematic errors in the method from Ref. [25] are caused by turn-off transients, they are expected to be reduced by the calibrated FIR filter, which reduces these transients. We thus used the dataset obtained with this filter to calibrate a second FIR filter in order to reduce residual distortions; see the red markers in Fig. 5(b), which show the response to a Gaussian-filtered step function with all predistortion filters (two IIR and two FIR filters) applied.

V. PERFORMANCE VERIFICATION AND APPLICATION FOR TWO-QUBIT GATES

After two iterations each of calibrating infinite impulse response filters and finite impulse response filters, we evaluate the compensation of distortions at long and short timescales using the methods from Secs. III and IV, respectively. For both measurements, when the qubit idles at the upper first-order flux-insensitive bias point we apply a flux pulse with the same amplitude of approximately $\Phi_0/4$ as for the calibration of the

FIR filters, yielding a frequency change of -738 MHz. This is a representative pulse amplitude for flux pulses implementing two-qubit gates on the device used for the present study [20]. From approximately 15 ns after the rising edge, the deviation from the target frequency is below 0.1% of the total frequency excursion (see Fig. 6). This corresponds to a relative flux error of approximately 5×10^{-4} , which is similar to the value of 10^{-3} reported in Ref. [25]. We attribute the slightly larger deviations at times before 15 ns to miscalibrations resulting from systematic errors of the characterization method at steep edges (see Sec. IV). For times at which data from both characterization methods are available, the good agreement between the two methods hints at small systematic errors since we would expect the two conceptually different methods to display different systematic errors.

To study a particularly relevant use case of flux control, we have implemented controlled phase (CZ) gates via resonant $|11\rangle \leftrightarrow |20\rangle$ interactions [9,11] between the 24 pairs of neighboring qubits on a 17-qubit device [20] (see Appendix J). We perform interleaved randomized benchmarking [28,46,47] with extensions to quantify leakage [10,48], yielding gate errors on individual qubit pairs between 0.006 and 0.054 with a mean of 0.015(10) across all 24 pairs [20] and a mean leakage per gate of 0.0011(10). Dominant contributions to the gate errors originate from crossing defect modes and from increased dephasing rates at the interaction frequencies due to flux noise (see Ref. [20]).

The compensation of distortions on short timescales enables steep rising and falling edges, which is crucial to mitigate interactions when crossing defect modes [20] or other transitions accessible in a pair of transmon qubits (e.g., in a case where the $|11\rangle \leftrightarrow |02\rangle$ and $|01\rangle \leftrightarrow |10\rangle$ resonances are crossed to reach the $|11\rangle \leftrightarrow |20\rangle$ resonance [11]). The overshoot of 3 MHz, which we observe around $t = 10$ ns in Fig. 6, is nearly two orders of magnitude below the anharmonicity of 174 MHz, which is a typical anharmonicity for transmon qubits. Thus, this overshoot does not cause undesired $|10\rangle \leftrightarrow |01\rangle$ interactions. As a reference value to judge residual errors at intermediate timescales, we determine the coupling strength of the $|11\rangle \leftrightarrow |20\rangle$ interaction for the considered transmon and one of its neighbors as $J_2/2\pi = 7.4$ MHz (see Appendix J). This is a representative value for a qubit-qubit coupling that allows two-qubit gates with a duration below 100 ns. The residual errors after the overshoot in Fig. 6 are clearly small enough to avoid significant off-resonant interactions.

Finally, the above interleaved randomized benchmarking results obtained with gate sequences lasting up to $50 \mu\text{s}$ also demonstrate the successful mitigation of long-time distortions, which would otherwise affect two-qubit gate performance. This mitigation is achieved by the long-time IIR filters and by making use of net-zero pulses [10,11,49], consisting of two shorter pulses with opposite polarity such that the time average of the control signal vanishes [see Appendix J]. The vanishing time average leads to robustness against distortions on timescales much longer than the duration of an individual two-qubit gate. This can be intuitively understood in the Fourier domain by noting that the exact values of the transfer function of the flux control line at frequencies close to zero have little relevance for control signals that do not have a significant spectral content in that frequency range.

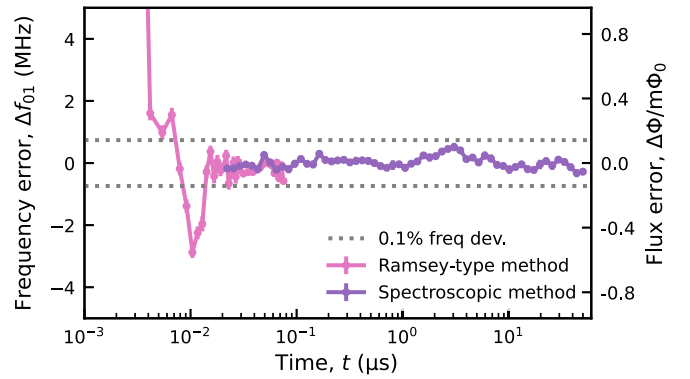


FIG. 6. Performance verification with predistortion filters: transition frequency over time in response to a Gaussian-filtered step function at the input of the predistortion filters. The dashed lines indicate a deviation of 0.1% of the intended frequency change of -738 MHz (flux change $\approx \Phi_0/4$).

Accurate compensation of long-time distortions by means of filtering as demonstrated in this paper can remain relevant for some types of net-zero waveforms [50] and is indispensable when unipolar pulses are applied to quantum devices, e.g., in scenarios in which the idle point is not a symmetry point. Such scenarios occur, for example, when defect modes (two-level systems [51]) restrict the choice of the idle frequency [16,22]. They also result from design choices, for example, for fluxonium qubits that implement a Λ system [52], for tunable couplers idling at the point of minimal coupling [17–19], or for tunable resonators used as flux-sensitive devices [53].

We have demonstrated the compensation of flux distortions on long and short timescales by combining two methods for characterizing the qubit frequency as a function of time while applying flux control pulses. At long timescales, we have extended a spectroscopic method [27,30] which enables the characterization of long flux waveforms of up to $100 \mu\text{s}$ with large excursions of the qubit frequency up to 1 GHz. Our approach allows us to calibrate compensations for multiple timescales at once by directly characterizing the step response and by applying a numerically robust implementation of an inverse, higher-order IIR filter. To compensate for short-time distortions, we have combined the characterization method from Ref. [25] with a method for fitting FIR filter coefficients based on regularized optimization. Our combination of methods achieves accurate flux control with subper mille residual frequency errors on timescales from nanoseconds to tens of microseconds.

Unlike optimal control methods, which optimize the fidelity of quantum logic gates (e.g., [37,54]), the compensation of distortions presented in this paper is independent of the concrete control pulse shapes and needs to be calibrated only once per qubit. This makes our approach applicable to large-scale quantum devices, where multiple flux-activated operations might be executed on each qubit. Such operations include two-qubit gates with multiple neighboring qubits, reset operations [5,6], leakage reduction units [7], and flux-pulse-assisted readout [4]. Moreover, our experience indicates that the distortions stemming from the conventional electronics that are part of the system do not show significant

variations, even on timescales of several months. For reducing the calibration overhead on large-scale quantum devices, it is thus beneficial to calibrate out the distortions independently of the recurring task of optimizing control pulse parameters, which are typically subject to drifts over time [55]. An independent calibration of the compensation of distortions might also be achievable by combining optimal control with model learning [54], but the performance of model learning for mitigating flux pulse distortions remains to be demonstrated in experiments. Finally, our methods avoid closed-loop optimization [37,54] with repeated measurements and recurring overhead of AWG programming [54] within an optimization loop.

The methods presented in this paper allow us to implement fast flux-activated two-qubit gates with high fidelity and low leakage, which are essential, for example, in quantum error correction experiments such as the one we presented in Ref. [20]. Furthermore, compensation of flux distortions is a key ingredient for achieving high fidelity in experiments that require long or repeated unipolar flux pulses.

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C.H., N.L., A.R., R.B., and C.K.A. developed the methodology, C.H. and N.L. planned and performed the measurements, and C.H., N.L., and A.R. analyzed the data. C.H., N.L., A.R., and S.L. developed control and calibration software routines, and C.H. and R.B. performed numerical simulations. J.H., A.R., R.B., C.H., S.K., and F.S. designed and built elements of the experimental setup. F.S., A.R., and C.K.A. designed the device, and S.K. and A.R. fabricated the device. C.H. and N.L. prepared the figures and wrote the manuscript with input from all coauthors. C.E. and A.W. supervised the work.

DATA AVAILABILITY

The data that support the findings of this article are openly available [56].

APPENDIX A: LINEAR TIME-INVARIANT SYSTEMS AND LINEAR DYNAMIC CIRCUITS

The output $y(t)$ of an LTI system with impulse response $h(t)$ can be calculated from the convolution

$$y(t) = (h * v)(t) = \int_{-\infty}^{\infty} v(t')h(t-t')dt', \quad (\text{A1})$$

where $v(t)$ is the input signal. When using this model to describe dynamic distortions in a flux control line, the input $v(t)$ is the voltage programmed to the AWG, and we interpret the change of the flux through the SQUID loop $\phi(t)$ as the output, i.e., $y(t) = \phi(t)$. An LTI system can be equivalently described by its step response $s(t)$ via

$$h(t) = \frac{d}{dt}s(t). \quad (\text{A2})$$

When cascading LTI systems, the impulse response of the combined system is the convolution of the individual impulse responses, e.g., $(h * g)(t)$ for the cascade of predistortion filter and flux control line in Fig. 2(a). An identity operation $y(t) = v(t)$ is obtained with the Dirac impulse $\delta(t)$ as impulse response. The aim of the calibration of the predistortion in this paper is to achieve $(h * g)(t) \approx \alpha\delta(t)$, corresponding to inversion up to a scaling factor α .

An important origin of linear distortions is reactive (capacitive or inductive) circuit elements forming linear dynamic circuits [41]. Consider a circuit that consists of only linear resistors, capacitors, and inductors, as well as of voltage sources to model input signals and constant biases. The dynamics of an N th-order circuit with N linearly independent reactive elements and an arbitrary number of resistive elements are described by [41]

$$\dot{x}(t) = Ax(t) + b(t), \quad (\text{A3})$$

with a system matrix $A \in \mathbb{R}^{N \times N}$ and an input vector $b(t) \in \mathbb{R}^N$, both of which depend on the circuit elements, and with the state vector $x(t) \in \mathbb{R}^N$, which contains the voltages of all (linearly independent) capacitors and the currents through all (linearly independent) inductors.

Since constant bias voltages can be accounted for by a shift of the coordinate system, we consider a unit step function with step height V_{step} as the only input to the system. Solving Eq. (A3) for $t \geq 0$ with a constant excitation $b(t) = b$, $t \geq 0$ yields

$$x(t) = e^{At}(x(0) - x_{\infty}) + x_{\infty} \quad (\text{A4})$$

$$= Q \text{diag}_i(e^{\lambda_i t}) Q^{-1}(x(0) - x_{\infty}) + x_{\infty}, \quad (\text{A5})$$

with $x_{\infty} = -A^{-1}b$. The diagonalization $A = Q \text{diag}_i(\lambda_i) Q^{-1}$ with the modal matrix Q and negative real eigenvalues λ_i (so-called natural frequencies [41]) is possible in any stable, overdamped system.

Due to linearity, every voltage or current in the system can be expressed by a linear combination of the components of $x(t)$ and b , and thus in the form

$$y(t) = \alpha_0 V_{\text{step}} + \sum_{i=1}^N \alpha_i V_{\text{step}} e^{-\frac{t}{\tau_i}}, \quad t \geq 0, \quad (\text{A6})$$

with the time constants $\tau_i = -1/\lambda_i$ and the scalar factors α_i for $i = 0, \dots, N$. For the initial state $x(0) = 0$, we obtain the step response

$$s(t) = \frac{y(t)}{V_{\text{step}}} = \left(\alpha_0 + \sum_{i=1}^N \alpha_i e^{-\frac{t}{\tau_i}} \right) u(t) \quad (\text{A7})$$

with the unit step function $u(t)$. This model is used in Sec. III to describe the long-time distortions [see Eq. (1)]. In case of an underdamped or critically damped system, the model would need to be extended by accounting for complex eigenvalues or a nondiagonalizable system matrix, respectively [41].

While each matrix element in A usually depends only on few circuit elements (with details depending on the structure of the circuit), the time constants in Eq. (A7) depend on the eigenvalues of A and can, thus, be influenced by the device parameters of all circuit elements. Thus, when characterizing the room-temperature flux line with a measurement instrument that nominally has the same load resistance as the cryogenic flux line (in our setup approximately 50Ω due to the attenuator at the 4 K stage), differences of the resistance due to device tolerances can propagate to all time constants τ_i . Even though some τ_i might be influenced only weakly, the dependence can be linear in extreme cases (such as in a first-order RC circuit with time constant $\tau = RC$), meaning that resistance variations on the percent level can directly translate to errors on the percent level in the characterization of a time constant. If *in situ* characterization is applied only for short timescales in a setup where distortions on long timescales are expected from room-temperature components [25–29], inaccuracies as described in this paragraph can stay undetected. Our end-to-end *in situ* characterization on all relevant timescales avoids this problem.

The description of a linear dynamic circuit in Eq. (A7) corresponds to an infinite impulse response, as can be verified by taking the derivative of the step response $s(t)$, treating $\delta(t)$ as the derivative of $u(t)$. When modeling other kinds of distortions, $h(t)$ in the LTI model in Eq. (A1) can be a finite impulse response. For example, a single discrete echo of a signal after a delay T_e can be described by $h(t) = \delta(t) + \alpha_e \delta(t - T_e)$ with a scaling factor α_e . On the other hand, the model from Eq. (A1) and the presented compensation methods do not account for any nonlinear distortions. We have, thus, carefully designed the experimental setup in a way that nonlinearities in the flux control line are negligible (see Appendix D).

APPENDIX B: QUBIT FREQUENCY MODEL

To calculate the frequency of a transmon qubit with charge operator $\hat{n}$ and phase operator $\hat{\phi}$, we consider the Hamiltonian [1,57]

$$\hat{H}_0 = 4E_C \hat{n}^2 - E_J(\Phi) \cos(\hat{\phi}), \quad (\text{B1})$$

with charging energy E_C and total Josephson energy $E_J(\Phi)$. The latter depends on the external flux Φ through the SQUID loop via

$$E_J(\Phi) = E_{J\Sigma} \left| \cos\left(\pi \frac{\Phi}{\Phi_0}\right) \right| \sqrt{1 + d^2 \tan^2\left(\pi \frac{\Phi}{\Phi_0}\right)}, \quad (\text{B2})$$

where $E_{J\Sigma} = E_{J,1} + E_{J,2}$ is the sum of the Josephson energies of the two junctions and $d = (E_{J,2} - E_{J,1})/E_{J\Sigma}$ the junction asymmetry. We can write the Josephson potential in the charge basis as

$$\cos(\hat{\phi}) \stackrel{N \rightarrow \infty}{=} \frac{1}{2} \sum_{n=-N}^N (|n\rangle \langle n+1| + |n+1\rangle \langle n|), \quad (\text{B3})$$

where $|n\rangle$ are the eigenstates of $\hat{n}$.

We model the coupled system of the transmon and the readout resonator with the Hamiltonian [1]

$$\hat{H} = \hat{H}_0 + \hat{H}_R, \quad \hat{H}_R = \hbar\omega_R \hat{a}^\dagger \hat{a} + \hbar g \hat{n}(\hat{a}^\dagger + \hat{a}), \quad (\text{B4})$$

where ω_R is the bare resonance frequency of the readout resonator, $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator for the resonator mode, and g characterizes the coupling strength between the transmon and the resonator. We solve the total Hamiltonian $\hat{H}$ numerically in the charge basis, truncated to $N = 15$ and to three photon number states in the resonator, and we identify the dressed qubit frequencies by finding the eigenstates with the largest overlap with the bare eigenstates of $\hat{H}_0$.

Finally, we convert between magnetic flux Φ and dc bias voltage V_{dc} applied to the flux control line using

$$\Phi(t) \big|_{\phi(t)=0} = \alpha_\Phi (V_{\text{dc}} - V_0), \quad (\text{B5})$$

where the factor α_Φ depends on the total resistance of the flux control line as well as on its mutual inductance with the SQUID loop, the term $\alpha_\Phi V_0$ models a flux offset, and the time-dependent flux change $\phi(t)$ due to flux pulses $v(t)$ from an AWG is zero during the calibration of the qubit frequency model. Note that using Eq. (B5) requires a high degree of linearity between the output voltage requested from the dc source and the true output voltage.

The transmon model as presented here has seven free parameters: E_C , $E_{J\Sigma}$, d , ω_R , g , α_Φ , and V_0 . We fix ω_R to the resonator frequency known from readout tune-up. To find the remaining parameters, we set the time-dependent part $\phi(t)$ to zero by choosing $v(t) = 0$, and we perform a series of Ramsey experiments to characterize the transition frequency $f_{01}(V_{\text{dc}})$ as a function of the dc bias V_{dc} .

First, we need to determine the bias voltages corresponding to the first-order flux-insensitive points $\Phi = 0$ and $\Phi = -\Phi_0/2$, so that α_Φ and V_0 can be obtained by solving Eq. (B5). To this end, we sweep V_{dc} in close vicinity of $\Phi \in \{0, -\Phi_0/2\}$, and we fit a second-order polynomial model to the measured value of $f_{01}(V_{\text{dc}})$ in each of the two datasets. The polynomial fits correspond to second-order Taylor approximations of $f_{01}(V_{\text{dc}})$ around the bias voltages that yield $\Phi \in \{0, -\Phi_0/2\}$.

In order to find the remaining four parameters E_C , $E_{J\Sigma}$, d , and g , we obtain four data points by measuring f_{01} at $\Phi \in \{0, -\Phi_0/4, -\Phi_0/2\}$ and f_{12} at $\Phi = 0$. The fit is then performed by iteratively solving the Hamiltonian from Eq. (B4) inside the loop of a nonlinear least-squares minimization.

We perform further measurements of f_{01} with $\Phi \in [-\Phi_0/2, 0]$ to verify the model, and we plot the residual errors $\Delta f_{\text{model}}(\Phi) = f_{\text{model}}(\Phi) - f_{\text{meas}}(\Phi)$ in Fig. 7. While the error level in principle allows to employ the Hamiltonian model in the presented calibration of flux distortions, it is insufficient

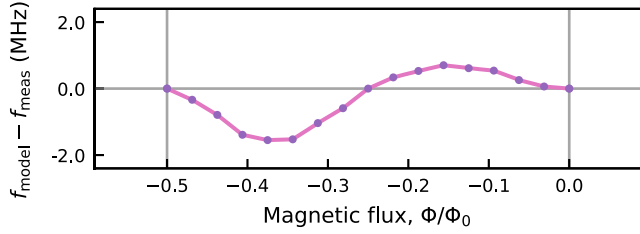


FIG. 7. Difference between the Hamiltonian model and the measured values (markers), and interpolation $\Delta f_{\text{model}}(\Phi)$.

for reaching the high accuracy of flux control that we report in Sec. V. We thus improve the accuracy of the model by using the interpolation $\Delta f_{\text{model}}(\Phi)$ shown in Fig. 7 as an empirical correction that we apply after calculating frequencies with the Hamiltonian model $f_{\text{model}}(\Phi)$ or before applying the inverse model to measured frequencies.

The qubit frequency model $\check{\nu}$ used in the main text can now be summarized as

$$\check{\nu}(\phi) = f_{\text{model}}(\Phi(\phi)) - \Delta f_{\text{model}}(\Phi(\phi)), \quad (\text{B6})$$

with

$$\Phi(\phi) = \alpha_{\Phi} V_{\text{dc}} + \phi - \alpha_{\Phi} V_0 \quad (\text{B7})$$

for a fixed dc bias V_{dc} . For the case of applying the inverse model $\check{\nu}^{-1}$ to spectroscopic data as in Sec. III, we calibrate an additional minor correction as described below in order to reduce systematic errors in the spectroscopy data, which are caused by the influence of the second excited state of the transmon.

With the qubit tuned to the same idle point as in Fig. 3(b), we measure the idle frequency (no flux pulse applied) with both the spectroscopic method and a Ramsey experiment. We observe a frequency shift of $\Delta f_{\text{spec}} = f_{\text{spec}} - f_{\text{Ramsey}} = 1.1$ MHz, which is in good agreement with the following simulation study. We perform a unitary evolution simulation to obtain the excited state population of a transmon with transition frequency f_{01} when applying a drive pulse, whose frequency f_{drive} is swept as in the spectroscopic measurement. We then fit a Gaussian curve to the simulated excited state population in the same manner as in the inset of Fig. 3(b), and we observe a similar shift of the center of the Gaussian compared to the nominal qubit transition frequency f_{01} used in the simulation. The simulated shift decreases with increasing anharmonicity $f_{12} - f_{01}$ and vanishes when restricting the simulation to the computational subspace, indicating that the effect can be attributed to a partial excitation of the second excited state.

Since Δf_{spec} depends only weakly on the flux bias via the flux dependence of the anharmonicity, we interpret Δf_{spec} as a flux-independent offset and subtract it from all frequencies $\check{f}_{01,\text{spec}}(t)$ extracted with the spectroscopic method; i.e., we convert to a flux change via

$$\check{\phi} = \check{\nu}^{-1}(\check{f}_{01,\text{spec}}(t) - \Delta f_{\text{spec}}). \quad (\text{B8})$$

APPENDIX C: EXPERIMENTAL SETUP

All experiments described in this manuscript are performed on the 17-qubit quantum device introduced in Ref. [20]. The device is installed in a sample holder mounted at the base plate

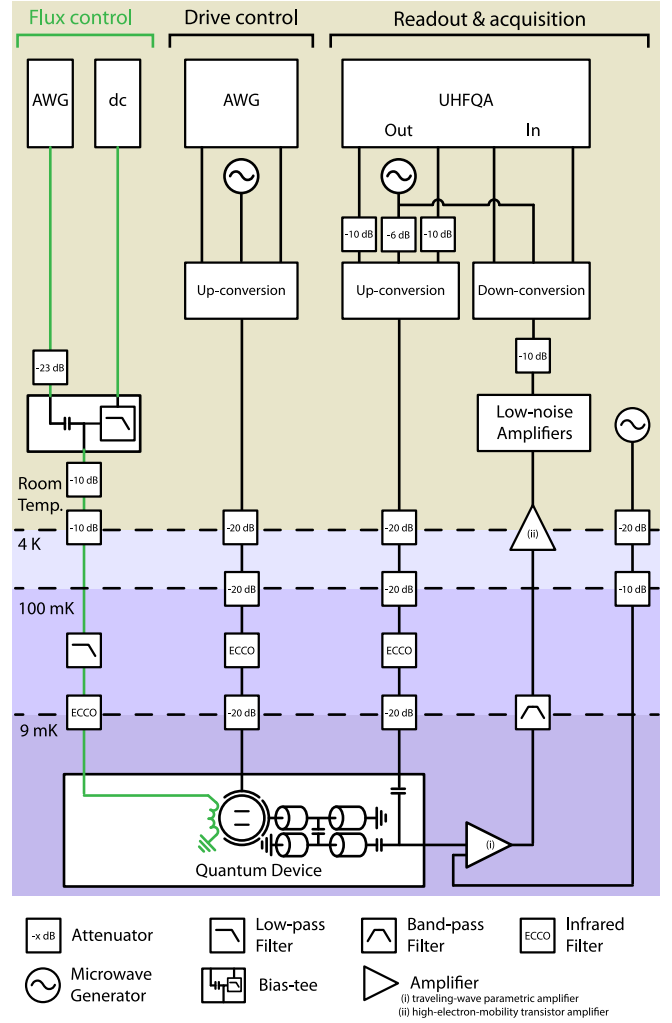


FIG. 8. Simplified schematics of the experimental setup. The flux control (green), drive control, and readout lines connect signal generators to the quantum device (see text for details).

of a dilution refrigerator [58] and connected to the control electronics setup located at room temperature as summarized in Fig. 8.

To control the transmon qubit frequency, a voltage source generates a dc current, which is combined with voltage pulses generated by an AWG using a custom bias tee (see Appendix D). The dominant time constant $\tau_{\text{HP}} = 19.2 \mu\text{s}$ of the high-pass part of the bias tee corresponds to a 3 dB cutoff frequency of approximately 8.3 kHz. The output amplifier of the AWG has a frequency cutoff of 300 MHz, and the low-pass filter in the flux line located at the base plate has a frequency cutoff of 780 MHz. The predistortion of the voltage pulses sent through the flux line is performed in software before uploading the waveforms to the AWG (as, e.g., in Ref. [45]).

To realize single-qubit gates, drive pulses are generated at an intermediate frequency in the range 0–700 MHz by an AWG and subsequently up-converted to microwave frequencies with an analog inphase-quadrature (IQ) mixer using the continuous-wave signal provided by a microwave generator. We operate the drive AWG in a direct output mode bypassing the output amplifier, yielding a relatively weak

TABLE I. Parameters and coherence properties of the qubit used for the flux line characterization measurements.

Parameter	Value
Qubit idle frequency, $\omega_Q/2\pi$ (GHz)	5.887
Minimum qubit frequency, $\omega_{Q,\min}/2\pi$ (GHz)	4.151
Qubit anharmonicity, $\alpha/2\pi$ (MHz)	-174
Relaxation time, T_1 (μ s)	16.2
Ramsey decay time, T_2^* (μ s)	31.4
Echo decay time, T_2^e (μ s)	31.8
Readout frequency, $\omega_{RO}/2\pi$ (GHz)	7.556
Three-state readout error, $\epsilon_{RO}^{(3)}$ (%)	2.5

dependence of the π pulse amplitude on the drive frequency [see Appendix F].

We use an ultrahigh frequency quantum analyzer (UH-FQA) to generate readout pulses. The readout signal at each output port of the device is amplified using a wideband near-quantum-limited traveling-wave parametric amplifier [59], a high-electron-mobility transistor amplifier, and low-noise amplifiers operating at room temperature [20]. Thereafter, the amplified signal is down-converted with an IQ mixer and then both demodulated and integrated by the acquisition unit of the UHFQA.

Using standard spectroscopy and time-domain methods, we measure the coherence properties of the qubit used in the main text for the flux line characterization (see Table I).

APPENDIX D: LINEARITY OF COMPONENTS IN THE FLUX CONTROL LINE

In this Appendix, we describe how we circumvent nonlinearities of the components in the flux control line, including the AWG output stage, a custom bias tee, the cryogenic low-pass filter, and the on-chip circuit. For the output stage of the AWG, potential nonlinearities due to compression in the output amplifier are avoided by keeping the signal amplitudes significantly below the maximum output voltage. The amplitudes of pulses and step functions in the waveform $a(t)$ [see the signal flow in Fig. 2(a)] used in the presented experiments are up to 0.6 V in absolute value while operating the AWG with an output range of ± 2.5 V. Moreover, by filtering the waveform $a(t)$ with a Gaussian filter of width $\sigma = 0.5$ ns, we program 20%–80% rise times longer than 0.84 ns, which is larger than the value of 0.8 ns given for steps of 1 V in the data sheet of the AWG. In this way, we avoid nonlinear effects stemming from the limited slew rate of the AWG output.

We use a custom bias tee, which we designed based on a low number of reactive components to keep the number of terms in the step response in Eq. (A7) low; see circuit schematic in Fig. 9(a). We have built the bias tee from components that were chosen with particular attention to ensuring a linear transfer characteristic.

In Fig. 9(b), we show the transmission $|S_{\text{out},\text{rf}}|$ (blue) and $|S_{\text{out},\text{dc}}|$ (green) of a typical instance of the custom bias tee, measured with a lock-in amplifier up to 0.5 MHz (dark markers) and with a vector network analyzer at the higher frequencies (shaded markers). For the measurement of $|S_{\text{out},\text{rf}}|$ with an impedance of $50\ \Omega$ at the radio frequency (rf) and

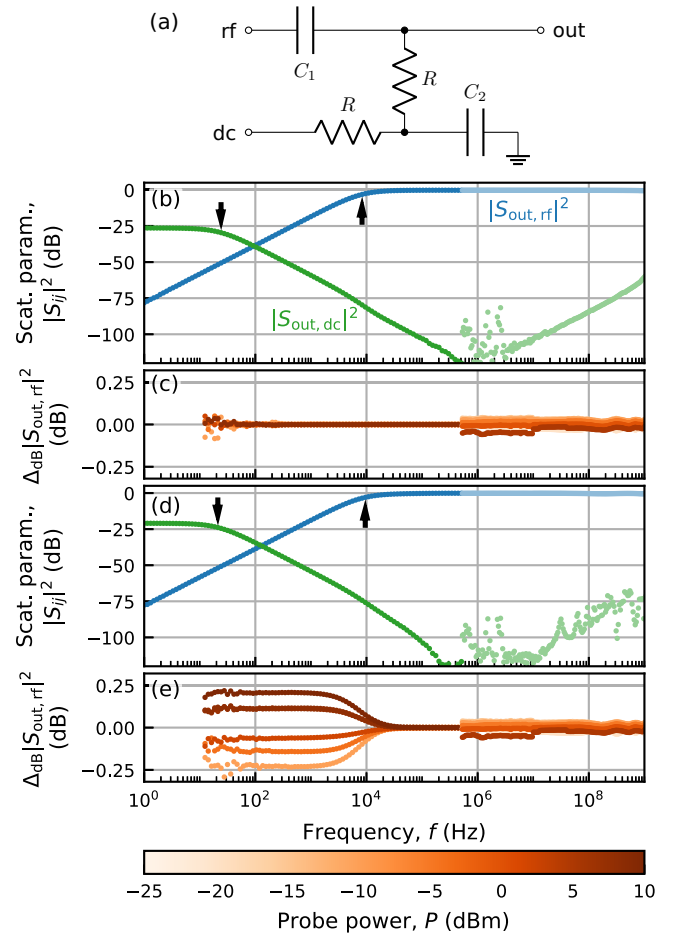


FIG. 9. Custom bias tee. (a) Circuit schematic, where $C_1 = 200$ nF, $C_2 = 13.6$ μ F, and $R = 1$ k Ω . (b) Transmission $|S_{\text{out},\text{rf}}|$ (blue) and $|S_{\text{out},\text{dc}}|$ (green) measured with a lock-in amplifier (dark markers) and a vector network analyzer (shaded markers). Black arrows indicate 3 dB cutoff frequencies. (c) Transmission $|S_{\text{out},\text{rf}}|$ for different input powers, plotted relative to the mean (in the decibel domain) of the measured transmission curves. (d), (e) Same as panels (b) and (c), but for a commercial bias tee (MC ZFBT-4R2GW).

output (out) ports, the dc input was shorted. To verify the linearity of the transfer characteristic, we perform the measurement with different input powers and subtract the mean of the curves computed in the decibel domain [see Fig. 9(c)]. The small nonlinearity in the data from the vector network analyzer is attributed to imperfections in the calibration of this measurement instrument. The reduced signal-to-noise ratio at low frequencies comes from the reduction of signal power in the stop band.

In Figs. 9(d) and 9(e), we repeat the same series of measurements for a commercial bias tee, revealing a power-dependent residual transmission in the stop band. Such a nonlinearity in the stop band might be irrelevant for many applications that only rely on the bias tee to suppress frequency components in the stop band. However, we operate the bias tee in the transient regime, and a linear transfer characteristic of frequency components in the stop band is crucial to allow equalizing the transient response.

For the bias tee used in the experiments in this paper, we have determined a 3 dB cutoff frequency of 8.3 kHz by intersecting low-frequency and high-frequency asymptotes to the $|S_{\text{out,rf}}|$ spectrum (not shown here). This corresponds to a time constant of $\tau_{\text{HP}} = 19.2 \mu\text{s}$ for the high-pass part of the bias tee.

In analogy to the above characterization of the bias tee, we have also measured the transmission through a low-pass filter of the type used at the base temperature stage with a lock-in amplifier and a vector network analyzer. While sweeping the input power applied during these measurements, we have not observed any relevant nonlinearity in the pass band of the filter.

On the quantum device, a nonlinearity is introduced by the Josephson junctions in the SQUID loop, but since the mutual inductance between the on-chip flux control line and the SQUID loop is orders of magnitude lower than the Josephson inductance (pH compared to nH), this has no relevant effect on the transfer characteristics of the flux control line.

APPENDIX E: ILLUSTRATION OF THE TIME DEPENDENCE OF THE QUBIT TRANSITION FREQUENCY

To obtain the time dependence of the transition frequency in Fig. 1, we first measure the long-time impulse response $h_{\text{long}}(t) = ds(t)/dt$ by fitting $s(t)$ from Eq. (1) as described in Sec. III, and we subsequently characterize the residual short-time distortions by determining the impulse response $h_{\text{short}}(t)$ from Eq. (3) as described in Sec. IV while already compensating for distortions from h_{long} . The overall impulse response of the flux control line is then given by $h_{\text{total}}(t) = (h_{\text{long}} * h_{\text{short}})(t)$, where $*$ denotes convolution. We can then compute the flux at the qubit by means of a convolution of h_{total} and the waveform programmed into the AWG, which is a step function with a height corresponding to approximately $\Phi_0/4$, smoothed by a Gaussian filter of width $\sigma_{\text{FP}} = 0.5 \text{ ns}$. We convert the resulting magnetic flux to the time dependence of the transition frequency using the model $\check{\nu}$ from Fig. 2.

To obtain the target curve in Fig. 1, we consider the same Gaussian-filtered step function as above, but we assume an ideal flux control line that only multiplies the input signal by a scalar factor. We choose this scalar in the idealized model such that the maximum flux excursion is the same as in the measurement-based model. The resulting time dependence of the transition frequency is again obtained by applying the model $\check{\nu}$ from Fig. 2.

APPENDIX F: DRIVE PULSE GENERATION, READOUT, AND HIGH-PASS COMPENSATION WHILE MEASURING LONG-TIME DISTORTIONS

To perform the spectroscopic measurement described in Sec. III, we generate a drive pulse at an intermediate frequency f_{IF} and then perform a frequency up-conversion to the desired drive frequency $f_{\text{drive}} = f_{\text{LO}} \pm f_{\text{IF}}$ by an analog IQ mixer with a microwave generator at frequency f_{LO} used as local oscillator. To avoid impairments from driving the $|0\rangle \leftrightarrow |1\rangle$ or $|1\rangle \leftrightarrow |2\rangle$ transition of the transmon with local oscillator leakage [60], we restrict f_{LO} to be at one of the solid orange lines in Fig. 10(a) and sweep f_{IF} . In addition, we

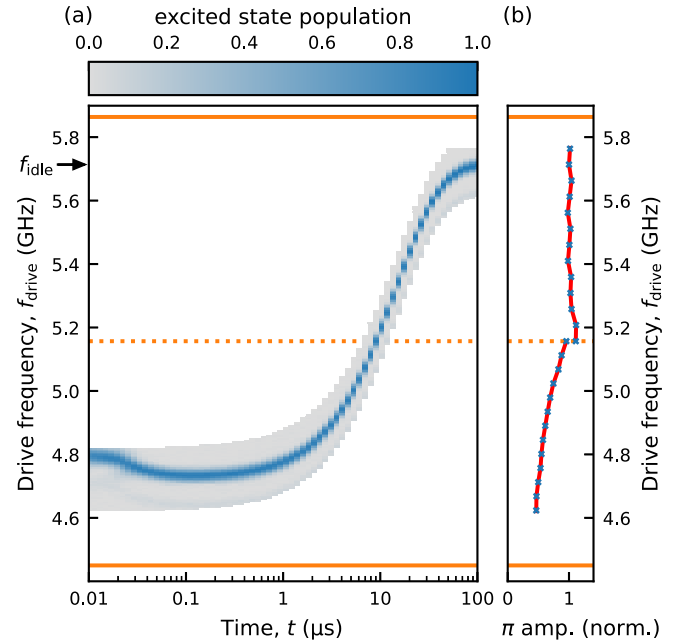


FIG. 10. Drive pulse generation for the *in situ* characterization of the flux line step response in Fig. 3. (a) Local oscillator frequencies f_{LO} (solid orange lines) shown along with the measured excited state population reproduced from Fig. 3(b). The dotted orange line indicates the boundary between using the lower or upper side band with f_{LO} at the higher or lower solid orange line, respectively. (b) π pulse amplitudes normalized to the value at the idle frequency, plotted for the drive frequency range and local oscillator configuration used in panel (a): calibration data (markers) and interpolation (solid lines).

calibrate a suppression of local oscillator leakage individually for both choices of f_{LO} , using a similar method as described in Ref. [61]. For the sake of a simple measurement protocol, we do not suppress sideband leakage [62], which would need to be calibrated individually for each drive frequency f_{drive} in the sweep shown in Fig. 10. An alternative to calibrating sideband leakage suppression would be to use a double frequency conversion scheme as described in Ref. [61] as such a scheme provides a higher spurious-free dynamic range without the need for calibrating out mixer imperfections. According to the discussion in Sec. III, the presented compensation of flux pulse distortions does not appear to be limited by systematic errors in the spectroscopic characterization measurement, including potential systematic errors from mixer imperfections.

To implement a π pulse for each f_{drive} , the drive amplitude is adjusted by interpolating between calibration data, which were obtained in a series of Rabi experiments with the qubit tuned to the desired frequencies using a dc bias [see Fig. 10(b)].

To infer the excited state population in the spectroscopic measurement in Sec. III, we project the measured integrated transmission through the feed line to a line in the complex plane, which is defined by calibration points for the $|0\rangle$ and $|1\rangle$ states of the qubit. The calibration points are measured at the idle frequency (no flux pulse applied) and using an accurately calibrated π pulse amplitude given by the respective data point in Fig. 3(b).

When applying this method during the first characterization of the step response of the flux control line, the uncompensated long-time distortions need to be taken into account since accurate readout is only achieved when the qubit is tuned to the frequency for which the calibration points have been measured. To bring the qubit back to the idle frequency in the presence of flux distortions, we output an appropriate amplitude $a_{\text{during_RO}}$ from the AWG after truncating the flux waveform [see Fig. 3(a)]. Since the required amplitude $a_{\text{during_RO}}$ depends on the impulse response of the flux control line, which is yet to be characterized, we resort to the model-based choice

$$a_{\text{during_RO}} = a_{\text{step}}(1 - e^{-(t+t_{\text{fl}})/\tau}), \quad (\text{F1})$$

where a_{step} is the height of the step function applied to the flux control line, $t + t_{\text{fl}}$ is the moment at which the step function is truncated, and τ is a time constant. By choosing $\tau = 20 \mu\text{s}$, we approximately compensate for the dominant distortion in the setup, which is the high-pass filter with time constant $\tau_{\text{HP}} \approx 20 \mu\text{s}$ in the bias tee. This choice brings the qubit to the idle point only approximately, but since the readout parameters depend on the flux bias only indirectly via the coupling between qubit and resonator, this is sufficient to reach consistent readout throughout the sweep shown in Fig. 3(b). Note that this modeled-based adaptation of the flux waveform only aims at ensuring high readout quality and does not affect the qubit frequency during the drive pulse; i.e., it does not influence the quantity that is to be determined by the measurement. When repeating the spectroscopic measurement from Sec. III after having calibrated a first infinite impulse response filter that compensates for the dominant distortion, we can instead choose $a_{\text{during_RO}} = 0$.

We switch the additional flux amplitude $a_{\text{during_RO}}$ off after the readout, to allow the capacitor of the high-pass filter in the bias tee to discharge before the next measurement. Due to the linearity of the system, the idle time relevant for this discharging is the time between the end of the flux waveform and the start of the drive pulse of the following measurement, even though the flux waveform of the following measurement might already start during this idle time. By triggering measurements with a period of $300 \mu\text{s}$, we achieve an idle time of at least $15 \tau_{\text{HP}}$. When recording data in a cyclic averaging mode in a way that measurements for different values of the time t are performed consecutively, sufficient idle time can be ensured by arranging the measurements in the order of increasing t and interleaving the calibration points (without flux pulses) between the measurements with highest and lowest t .

A method to discharge the capacitor of the high-pass filter rapidly consists in appending a flux pulse after the final readout of each experiment, with amplitude and duration chosen in a way that the integral over the whole flux waveform vanishes. Such a discharging pulse is known to be effective if the waveform is short compared to the high-pass time constant [27]. Since this condition is not fulfilled for the measurements in Sec. III, we study the effectiveness of this approach for long waveforms in the presence of distortions.

To this end, we consider the simplified high-pass circuit with time constant $\tau = RC$ in Fig. 11(a). The inverse filter that compensates for the distortions between the input $v(t)$ and the

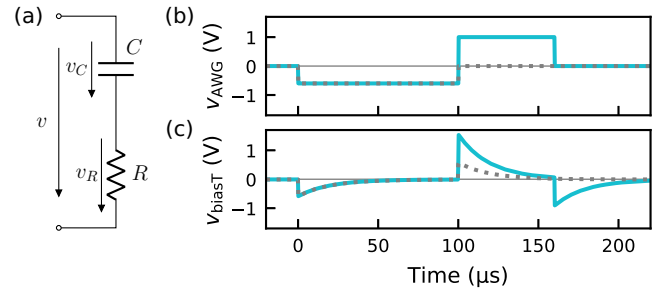


FIG. 11. Study of discharging pulses in the presence of distortions. (a) Simplified high-pass circuit formed of a capacitor C and a load resistor R to illustrate the discussion in Appendix F. (b) Waveform consisting of a long pulse with or without a discharging pulse, programmed to an AWG that is connected to the rf input of a bias tee. (c) Measured output voltage of the bias tee for the two cases in panel (b), showing that a net-zero waveform that is long compared to the high-pass time constant does not achieve the desired discharging (see Appendix F).

output $v_R(t)$ has the impulse response $h_{\text{inv}}(t) = \delta(t) + u(t)/\tau$ with the Dirac impulse $\delta(t)$ and the unit step function $u(t)$. This can be verified by taking the inverse Laplace transform of Eq. (G5) in the special case of $N = 1$ and $\alpha_0 = 0$.

For a waveform $a(t)$ with vanishing integral in $t \in [0, t_1]$ and final amplitude $a(t_1) = 0$, the predistortion with $g = h_{\text{inv}}$ yields

$$v(t_1) = (g * a)(t_1) = a(t_1) + \frac{1}{\tau} \int_0^{t_1} a(t') dt' = 0. \quad (\text{F2})$$

In case of ideally compensated distortions, where $v_R(t)$ is proportional to $a(t)$, this implies $v_R(t_1) = 0$ and, by Kirchhoff's voltage law, $v_C(t_1) = v(t_1) - v_R(t_1) = 0$. This demonstrates that a net-zero target waveform $a(t)$ is effective in discharging the capacitor in the high-pass filter at the end of the waveform if distortions are properly compensated for, and we thus employ this approach in all measurements with predistortion.

However, the above argument does not hold when distortions are not compensated yet, i.e., when $v(t) = a(t)$ and $v_R(t)$ is not proportional to $a(t)$. For the initial characterization measurement, we indeed observe that a discharging pulse leads to a higher charge in the capacitor than letting the circuit idle as described earlier in this section; see the comparison in Figs. 11(b) and 11(c).

APPENDIX G: DETAILS OF THE IIR FILTER CALIBRATION

After measuring the transition frequency $\check{f}_{01}(t)$ with the spectroscopic method described in Sec. III and converting the result to the flux change $\check{\phi}(t)$, we obtain samples of the step response

$$\check{s}(t) = \frac{\check{\phi}(t)}{\check{\phi}(t_{\text{max}})}, \quad \text{with } t_{\text{max}} = \text{argmax}_t |\check{\phi}(t)|. \quad (\text{G1})$$

Expressing the flux change relative to the maximum flux excursion is a pragmatic choice and is valid because the aim is to compensate for distortions up to a scaling factor (see Sec. II).

The low number of coefficients in the sum of N exponentials from Eq. (1) makes this model inherently robust to noise in the following sense. While a model with many free parameters might get adjusted too closely to a noisy dataset to which it is fitted and then fail to capture the true structure of the underlying data (so-called *overfitting*), the sum-of-exponentials model used here has much less parameters than the 70 data points we measure, effectively preventing overfitting. On the other hand, we have observed that the nonlinear least-squares problem of fitting the sum of N exponentials from Eq. (1) to $\tilde{s}(t)$ is very sensitive to the choice of initial parameter values. We initialize the offset α_0 with $\alpha_0 = 0$ when fitting to the first characterization data to account for the high-pass filter in the bias tee, and with $\alpha_0 = 1$ when fitting to data from a measurement with predistortion, where the dominant high-pass characteristic has already been corrected. For the other parameters, we increase the model order N successively by considering only data points above a threshold $t > t_{\min,N}$ in the N th iteration and using the parameters $\alpha_1, \dots, \alpha_{N-1}$ and $\tau_1, \dots, \tau_{N-1}$ fitted in the previous iteration as initial values. Using a fixed initial value $\alpha_N = 1$ in each iteration, the procedure is configured with the threshold $t_{\min,N}$ and the initial time constants τ_N for each iteration, which we choose as decreasing sequences. Best results have been obtained by choosing $t_{\min,N}$ and the initial τ_N manually based on visual inspection of the data. For the data presented in this paper, the order N of the model was chosen manually by observing that a further increase of the model order did not lead to a significant reduction of the fit residuals. Further automation of the procedure, e.g., by applying an information criterion rule for model-order selection [63], would facilitate the calibration of large-scale quantum devices and is a topic for future research. However, it is noteworthy that the current iteration based on human input is only during data analysis without requiring new measurements inside the loop. The procedure is terminated when significant residual errors above the noise floor remain only on timescales below the time resolution of the spectroscopic method.

Having obtained a fit of the step response model from Eq. (1), the inverse filter can be calculated via the Laplace transform $\mathcal{L}$. Using Eq. (A2), we transform the step response $s(t)$ to the impulse response $h(t)$ and further to the transfer function

$$H(p) = \mathcal{L}\{h\}(p) = \frac{\mathcal{L}\{\phi\}(p)}{\mathcal{L}\{v\}(p)} \quad (\text{G2})$$

with the complex variable p of the Laplace transform. We obtain

$$H(p) = \sum_{i=0}^N H_i(p), \quad (\text{G3})$$

with

$$H_i(p) = \begin{cases} \mathcal{L}\left\{\frac{d}{dt}\alpha_0 u(t)\right\}(p) = \alpha_0, & i = 0, \\ \mathcal{L}\left\{\frac{d}{dt}\alpha_i e^{-\frac{t}{\tau_i}} u(t)\right\}(p) = \frac{\alpha_i \tau_i p}{\tau_i p + 1}, & i > 0. \end{cases} \quad (\text{G4})$$

We have used Laplace transform pairs from Ref. [64] and the property that a derivative in the time domain corresponds to a multiplication by p in the frequency domain. Such a

description of the transfer function $H(p)$ of the flux control line was also used in Ref. [33] for the special case $N = 1$ and in Ref. [32].

The transfer function of the inverse filter of Eq. (G3),

$$\frac{1}{H(p)} = \frac{\prod_{i=1}^N (\tau_i p + 1)}{\alpha_0 \prod_{i=1}^N (\tau_i p + 1) + \sum_{j=1}^N \alpha_j \tau_j p \prod_{i \neq j} (\tau_i p + 1)}, \quad (\text{G5})$$

is fully characterized by its zeros $p_{0,i}$, poles $p_{\infty,i}$, and gain factor κ . For the zeros and gain of $1/H(p)$, we have

$$p_{0,i} = -1/\tau_i, \quad \kappa = \left(\sum_{i=0}^N \alpha_i \right)^{-1}, \quad (\text{G6})$$

and we find the poles $p_{\infty,i}$ of $1/H(p)$ by applying the root finding method from Ref. [65] to the denominator polynomial of Eq. (G5). Since we aim at inverting distortions only up to a scaling factor, we opt for a normalized inverse filter $H_{\text{inv}}(p) = (1/H(p))/\kappa$.

For a digital implementation of the filter, we transform the poles and zeros into the domain of the z transform [66] using

$$z_{0,i} = e^{p_{0,i} T_s}, \quad z_{\infty,i} = e^{p_{\infty,i} T_s}, \quad (\text{G7})$$

where $T_s = 1/(2.4 \text{ GHz}) = 0.4167 \text{ ns}$ is the sampling interval of the used AWG. To avoid numerical instabilities that are known to occur in digital implementations of higher-order IIR filters [42], we follow the established practice of splitting the IIR filter into a cascade of second-order IIR filters, so-called second-order sections [42]. To this end, the poles and zeros are divided into groups of two poles and two zeros by pairing poles with the nearest zeros, starting with the poles closest to the unit circle. Letting $z_{0,1}$, $z_{0,2}$, $z_{\infty,1}$, and $z_{\infty,2}$ denote the zeros and poles in such a group, the transfer function $V_{\text{out}}(z)/V_{\text{in}}(z)$ (z transform of the discrete-time impulse response) of the corresponding second-order section is given by

$$\frac{V_{\text{out}}(z)}{V_{\text{in}}(z)} = \frac{(z - z_{0,1})(z - z_{0,2})}{(z - z_{\infty,1})(z - z_{\infty,2})} = \frac{1 + B_1 z^{-1} + B_2 z^{-2}}{1 + A_1 z^{-1} + A_2 z^{-2}}, \quad (\text{G8})$$

with

$$A_1 = -z_{\infty,1} - z_{\infty,2}, \quad A_2 = z_{\infty,1} z_{\infty,2}, \quad (\text{G9})$$

$$B_1 = -z_{0,1} - z_{0,2}, \quad B_2 = z_{0,1} z_{0,2}. \quad (\text{G10})$$

We realize each second-order section in a so-called transposed direct form II structure, which is a canonical and efficient way of implementing discrete-time filters [42]. This structure is described by the recursive equation

$$\begin{aligned} v_{\text{out}}(n) &= v_{\text{in}}(n) + B_1 v_{\text{in}}(n-1) + B_2 v_{\text{in}}(n-2) \\ &\quad - A_1 v_{\text{out}}(n-1) - A_2 v_{\text{out}}(n-2), \end{aligned} \quad (\text{G11})$$

with the discrete time variable n . The digital implementation of $h_{\text{inv}} = \mathcal{L}^{-1}\{H_{\text{inv}}(p)\}(t)$ is then obtained by cascading these sections, i.e., by using the output $v_{\text{out}}(n)$ of one section as the input $v_{\text{in}}(n)$ of the next.

APPENDIX H: RESOLUTION, ACCURACY, AND PRECISION OF THE CRYOSCOPE METHOD

To correct flux pulse distortions at short timescales as described in Sec. IV, it is crucial to obtain a precise estimate of the qubit frequency during the pulse. The uncertainty of the phase σ_θ in the Ramsey measurement from which the qubit frequency is inferred scales approximately as

$$\sigma_\theta \propto \frac{1}{\sqrt{N_\theta N_R}}, \quad (\text{H1})$$

where N_θ is the number of phases for the second $\pi/2$ pulse of the Ramsey sequence (see Sec. IV for details) and N_R is the number of experimental runs over which we average each data point. To motivate Eq. (H1), note that Eq. (22) of Ref. [67] yields that this proportionality becomes exact in the simplified scenario of fitting θ as the only unknown parameter (assuming known frequency, amplitude, and mean value of the fitted cosine) to a dataset with N_θ phase values that come in pairs offset by $\pi/2$, assuming independent and identically distributed noise. By propagating the uncertainty through Eq. (2), the uncertainty of the qubit frequency excursion σ_f yields

$$\sigma_f = \frac{\sqrt{2}\sigma_\theta}{2\pi\Delta t} \propto \frac{1}{\Delta t \sqrt{N_\theta N_R}} \equiv \frac{1}{t_{\text{int}}}, \quad (\text{H2})$$

where we have defined t_{int} as an effective integration time.

We estimate σ_f in practice by calculating the standard deviation σ of 25 repetitions of the cryoscope measurement for a single time point $t = 50$ ns and for different values of Δt and N_R . We observe an inverse linear scaling of σ_f with respect to t_{int} (linear scaling with a slope of -1 in a double-logarithmic plot; see Fig. 12). For large t_{int} , the scaling approaches a plateau. This indicates further disturbances not captured by the model of Eq. (H2), which should be investigated in future work.

Note that we have to quadratically increase the measurement time to linearly decrease σ_f when adjusting N_R or N_θ . However, the same effect can also be achieved by linearly increasing Δt , which is independent of the measurement duration for a single time value t . On the other hand, since the estimate of the qubit frequency excursion is averaged over Δt [see Eq. (2)], a larger Δt leads to a lower time resolution of the inferred qubit frequency. In addition, larger step sizes during rapidly falling or rising edges lead to larger systematic errors caused by the larger phase difference accumulated during the low-pass-like turn-off transients; see Ref. [25] for details.

To accommodate for this trade-off, we adapt Δt in different parts of the measurement. Near the flux pulse edges (± 2.4 ns), which is where we expect the largest systematic errors, we use $\Delta t = T_s$ with the AWG sampling interval $T_s \approx 0.42$ ns. From 2.4 to 30 ns after the rising edge, i.e., the time interval in which residual distortions are typically visible after the tune-up of the infinite impulse response filter, we use $\Delta t = 3T_s$, which is sufficient to resolve features on the order of ~ 1.2 ns while decreasing the measurement noise in this interval. Finally, in the time interval in which we neither expect strong distortions nor large systematic errors (from 30 ns until the falling edge of the pulse), we use $\Delta t = 8T_s$ to reduce the total measurement time.

To ensure low errors for all t in the cryoscope measurements presented in the main text, we use $N_\theta = 16$ and

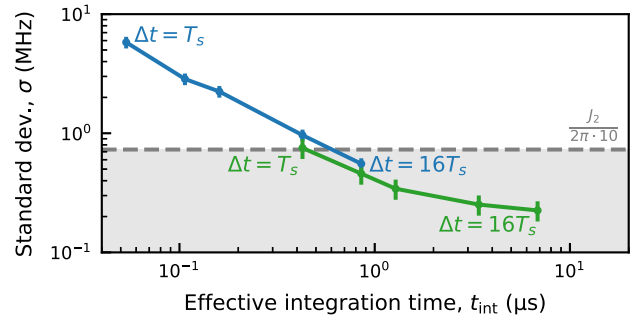


FIG. 12. Qubit frequency standard deviation, σ_f , estimated from 25 cryoscope measurements as a function of the effective integration time, t_{int} . The measurements are performed for $N_R = 1024$ (blue line) and $N_R = 65536$ (green line) while Δt is varied from T_s to $16T_s$. $N_\theta = 16$ is kept constant for all measurements. The gray region indicates deviations smaller than $1/10$ of the coupling strength of the $|11\rangle \leftrightarrow |20\rangle$ interaction $J_2/2\pi$. The error bars correspond to the standard error for 25 measurements.

$N_R = 65536$. This guarantees that $\sigma \lesssim 0.75$ MHz is an order of magnitude smaller than the coupling strength of the $|11\rangle \leftrightarrow |20\rangle$ interaction between neighboring qubits on this device (see the typical value of $J_2/2\pi = 7.4$ MHz determined in Appendix J), such that residual distortions below the noise floor have a limited effect on the two-qubit gate fidelity.

Finally, to further mitigate the systematic errors, we operate the asymmetric transmon at the upper first-order flux-insensitive bias point ($\Phi_{\text{dc}} = 0$) during the cryoscope measurement, which has a lower second-order flux sensitivity than the lower first-order flux-insensitive bias point ($\Phi_{\text{dc}} = 0.5$), thereby reducing the phase accumulated during the low-pass-like turn-off transients.

When using the parameters given above and triggering measurements with a period of hundreds of microseconds ($\approx 10T_1$) to wait for qubit relaxation, the total measurement time of the cryoscope method is several hours. This time can be reduced by an order of magnitude by using active qubit reset [68] to allow for a shorter triggering period.

APPENDIX I: DETAILS OF THE FIR FILTER CALIBRATION

Compensating the distortions of the flux control line at short timescales (see Sec. IV) involves deconvolution operations, which are known to be ill-posed problems in practice [69], combined with the inversion of the nonlinear qubit frequency model. In this Appendix, we detail how we apply regularized optimization, a method that has previously been used for deconvolution [43,44]. Choosing regularization terms that yield a good balance between accuracy and robustness can be seen as an *a priori* modeling decision justified by insights about the specific inverse problem [43,70]. Accordingly, the regularization terms described below are a possible choice that worked well on our experimental setup, but future research might find alternative choices that potentially further improve the compensation of flux pulse distortions.

We decompose the computation of the filter that inverts the impulse response of the flux control line into two steps.

The first optimization step is formulated based on the forward qubit frequency model $\check{v}$ to avoid an explicit inversion $\check{v}^{-1}$. In particular, we find the coefficients $h_0, \dots, h_L$ in Eq. (3) from

$$\min_{h_0, \dots, h_L} (w(t) \| \check{f}_{01}(t) - \check{v}[(h * a)(t)] \|_2^2 + R_1 + R_2), \quad (11)$$

with regularization terms R_1 and R_2 , where $w(t)$ is a weight function proportional to t_{int} (see Appendix H) of each sample such that samples with low statistical uncertainty are trusted more than samples with higher statistical uncertainty. As a first regularization term, we choose a Tikhonov regularizer [71] $R_1 = \lambda_1 \|h(t)\|_2^2$ with regularization parameter λ_1 , which limits the total energy of the impulse response. In addition, we choose $R_2 = \lambda_2 x(t) \|h(t)\|_2^2$ with regularization parameter λ_2 and prefactor $x(t)$, which scales exponentially with t . This additional regularizer smoothly forces the trailing filter coefficients to vanish, motivated by the physical intuition that the impulse response of a physical system has finite support. We implement the optimization in discrete time by resampling $\check{v}[(h * a)(t)]$ at the time values t at which $\check{f}_{01}(t)$ is measured and we compute the Euclidean norm $\|\bullet\|_2$ at these sampling points only.

Note that the linear time-invariant model used in this optimization cannot capture the residual systematic errors of the cryoscope measurement caused by the turn-off transients (see Appendix H and Ref. [25] for more details) because they are different for a step from f_{idle} to a flux-sensitive f_{target} and for a step from f_{target} to the flux-insensitive f_{idle} . To mitigate this effect, we choose a rectangular flux pulse $a(t)$ (see Fig. 5), which encourages the optimizer to find a trade-off between the systematic errors of rising and falling edges.

Having obtained the coefficients h_ℓ of the FIR model in Eq. (3), we compute an inverse FIR filter by performing a second regularized optimization

$$\min_{h_{\text{inv},0}, \dots, h_{\text{inv},L'}} (\|d(t) - (h * h_{\text{inv}})(t)\|_2^2 + R'_1), \quad (12)$$

where $h_{\text{inv}}(t)$ is defined in analogy to Eq. (3) and the target shape $d(t)$ is a realistic approximation of a Dirac impulse, which we choose to be a Gaussian pulse of width $\sigma = 0.75$ ns. The limit case of a Dirac impulse would correspond to ideal inversion. Here, we use a Sobolev regularizer [72] $R'_1 = \lambda'_1 \|\nabla h_{\text{inv}}(t)\|_2^2$ with hyperparameter λ'_1 and numerical derivative operator ∇ , to limit rapid changes in $h_{\text{inv}}(t)$.

APPENDIX J: TWO-QUBIT GATE CALIBRATION

With flux pulse predistortion applied, we calibrate a controlled phase (CZ) gate between a pair of coupled transmons idling at a high and a low frequency (here 5.89 and 3.88 GHz), respectively, as described in detail in Ref. [20] and summarized in the following. We apply rectangular flux pulses (smoothed by a Gaussian filter of width $\sigma_{\text{FP}} = 0.5$ ns) to both transmons in order to tune the $|1\rangle \leftrightarrow |2\rangle$ transition of the high-frequency transmon Q_{high} and the $|0\rangle \leftrightarrow |1\rangle$ transition of the low-frequency transmon Q_{low} to a common, intermediate interaction frequency (here 4.4 GHz). The duration of the resonant interaction is chosen to be half a period of the vacuum Rabi oscillation between the $|11\rangle$ state and the noncomputational $|20\rangle$ state, and a second pulse with opposite polarity is appended for each qubit to implement the second half of the

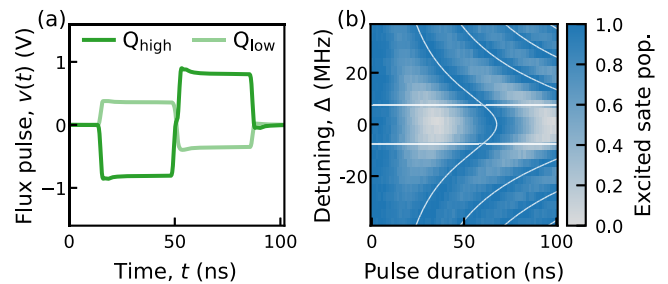


FIG. 13. (a) Waveforms (with predistortion applied) to implement a CZ gate between a high-frequency transmon Q_{high} and a low-frequency transmon Q_{low} . (b) Excited state population of Q_{high} after a $|11\rangle \leftrightarrow |20\rangle$ interaction. The curves indicate maximum recovery of the population to the $|11\rangle$ state, and the horizontal lines indicate the fitted coupling strength J_2 .

oscillation. To calibrate the conditional phase acquired by the $|11\rangle$ state to the target value of π , we tune the flux amplitude of Q_{low} during a short transition phase interleaved between the two halves of the net-zero waveform [11,20]. Figure 13(a) shows the predistorted waveforms for a gate between the qubit studied in the main text (Q_{high}) and one of its neighbors (Q_{low}).

The net-zero pulses employed for implementing the CZ gate make the execution of long gate sequences robust against residual distortions on timescales that are long compared to the duration of a gate. Unlike for residual distortions on short and intermediate timescales, it is not feasible to account for long-time distortions during gate calibration as they depend on the history, i.e., on previously applied flux pulses. Thus, when not using net-zero pulses, even the small frequency errors observed for long timescales in Fig. 6 could lead to undesirably large gate errors in long gate sequences. For instance, the dynamic phase error of a qubit due to a frequency error of 0.5 MHz during a 100 ns flux pulse would be 18° . This reveals that the presented methods should be further refined in future research in order to further reduce residual long-term distortions for use cases where net-zero pulses are not applicable, as discussed in Sec. V.

As a basis for the discussion of residual short-time distortions in Sec. V, we characterize the coupling strength between the $|11\rangle$ and $|20\rangle$ states as follows. After preparing a $|11\rangle$ state, we apply unipolar pulses, similar to the first half of the waveforms in Fig. 13(a), to both transmons and measure the $|1\rangle$ state population of Q_{high} . We sweep the amplitude of the pulse for Q_{low} as well as the duration t of both pulses. To the resulting dataset [see Fig. 13(b)], we fit the model

$$P_{|11\rangle}(\Delta, t) = \frac{\Delta^2 + 2J_2^2(1 + \cos(t\sqrt{4J_2^2 + \Delta^2}))}{4J_2^2 + \Delta^2}, \quad (J1)$$

where we approximate the detuning Δ between the $|11\rangle$ and $|20\rangle$ states as an affine function of the applied pulse amplitude (first-order Taylor approximation around the point of zero detuning) and fit the parameters of this affine model to the data. We obtain $J_2/2\pi = 7.4$ MHz for the coupling strength of the $|11\rangle \leftrightarrow |20\rangle$ interaction, as illustrated by the horizontal lines in Fig. 13(b).

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