

Engineering magnon lasing in full quantum regime

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(Received 28 March 2026; accepted 9 July 2026; published 22 July 2026)

We develop an analytical approach for steady-state normal-ordered magnon moments (NOMMs) in a driven-dissipative magnon-qubit system with coherent and dissipative couplings, and further benchmark it against the full quantum master-equation simulations of mean magnon number and second-order intensity correlation function, $g^{(2)}(0)$. Specifically, we first show that strong coherent coupling alone can produce an intermediate-pump magnon-lasing-like emission window, identified by a small-signal stability criterion and characterized by near-Poissonian statistics [$g^{(2)}(0) \simeq 1$] and pronounced linewidth narrowing. For dissipative coupling alone, the magnon emission exhibits nonclassical antibunching [$g^{(2)}(0) < 1$] at the optimal mean magnon number and features a large pump-tunable spectral linewidth. In the hybrid dissipative and weak-coherent-coupling regime, the dissipative channel can substantially enhance the steady-state mean magnon number while simultaneously inducing strong magnon antibunching, with $g^{(2)}(0) \ll 1$. The results obtained here may find potential applications in microwave-magnonic and related hybrid quantum platforms, and the analytical NOMM method could also provide a route to treating other driven open boson-fermion systems.

DOI: [10.1103/1mvk-5n9j](https://doi.org/10.1103/1mvk-5n9j)

I. INTRODUCTION

Hybrid magnon-qubit platforms [1–4] provide a promising setting for quantum magnonics by interfacing long-lived collective spin excitations with the strong nonlinearity and controllability of superconducting qubits [3,5–8]. Recent experiments have demonstrated coherent magnon-qubit coupling [1], qubit-based readout of magnons in the strong dispersive regime [9], and single-shot detection of a single magnon [10], bringing hybrid magnon-qubit systems into the regime of single-magnon quantum control [11,12]. These advances make the steady-state mean magnon number and quantum statistical signatures of hybrid magnon-qubit systems key probes of their ability to generate and control nonclassical magnon states, which are expected to be useful resources for quantum-state engineering, coherent information transfer, and quantum-enhanced sensing [10,13–15].

On the one hand, coherent magnon-qubit coupling allows the magnon mode to inherit the nonlinearity of the qubit, which can strongly influence the steady-state emission properties with drive and loss [11,16]. Under appropriate pumping conditions, such systems can give rise to lasinglike emission, linewidth narrowing, and fluctuations approaching Poissonian statistics [17–19]. On the other hand, recent advances in reservoir engineering have shown that suitably designed dissipation need not be merely detrimental, but can instead serve as a powerful resource for preparing, stabilizing, and controlling

nontrivial steady states in open quantum systems [20–22]. In related cavity-magnonic platforms, coexisting coherent and dissipative magnon-photon couplings have been shown to generate qualitatively distinct dynamics [23–25], including level attraction [23], nonreciprocity, and unidirectional invisibility [24]. However, in a driven-dissipative magnon-qubit system, it remains unclear how coherent and engineered dissipative couplings reshape the steady-state magnon population, intensity correlations, and spectral response, especially in the mixed-coupling regime [26]. Although these observables are, in principle, accessible from full master-equation simulations, numerical results alone do not readily reveal their underlying connections or the distinct roles played by the two coupling channels.

Dissipative coupling has recently attracted considerable attention in cavity magnonics [23,24,26–29], where a common microwave channel can mediate an effective non-Hermitian interaction between two bosonic modes and, with suitable gain compensation, give rise to magnon-polariton lasing [27]. In such semiclassical cavity-magnonic descriptions, the propagation phase of the shared reservoir can be used to tune the relative coherent and dissipative components of the effective linear mode-mode coupling. An important question is how this phase-controlled semiclassical picture is modified in a fully quantum magnon-qubit system, where the gain medium is a finite two-level system and the engineered reservoir enters through a collective quantum-jump channel. In this regime, the dissipative coupling is no longer characterized solely by an effective linear coupling coefficient; higher-order magnon-qubit correlations can also modify its signatures in the steady-state population, linewidth, and intensity statistics. A related issue arises from the perspective of magnon blockade: Previous studies have shown that qubit-induced nonlinearity can generate sub-Poissonian magnon statistics, strong antibunching, and optimized single-magnon

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brightness [16,30]. Here, we pursue a complementary route based on engineered collective dissipation, asking how a reservoir-mediated channel modifies the brightness-statistics trade-off in a pumped magnon-qubit system. This motivates a full quantum master-equation analysis of how coherent magnon-qubit exchange and engineered collective dissipation separately and jointly shape magnon emission in the presence of qubit saturation and incoherent pumping.

In view of these facts, in the present work we investigate a driven-dissipative magnon-qubit system in which coherent exchange and engineered collective dissipation coexist. We develop an approach based on steady-state normal-ordered magnon moments (NOMMs) and benchmark it against full master-equation simulations. We first show that strong coherent coupling produces an intermediate-pump lasinglike magnon-emission window, whose onset is identified by an independent small-signal gain criterion. In contrast, dissipative coupling alone yields weak but antibunched emission with a pump-tunable linewidth. In the hybrid weak-coherent regime, the collective reservoir enhances the mean magnon number while preserving sub-Poissonian statistics, revealing a reservoir-controlled brightness-statistics trade-off. These results clarify the distinct roles of coherent magnon-qubit exchange and reservoir-mediated collective dissipation in quantum magnon emission.

The remainder of this paper is organized as follows. In Sec. II, we introduce the driven-dissipative magnon-qubit model and present the steady-state normal-ordered magnon-moment approach used to calculate the magnon emission properties. In Sec. III, we analyze the strong-coherent-coupling regime and identify the intermediate-pump magnon-lasing-like emission window through both quantum simulations and a small-signal stability criterion. In Sec. IV, we turn to the weak-coherent-coupling regime and examine how engineered collective dissipation modifies the magnon population, intensity statistics, and spectral response. In Sec. V, we quantify the dissipative-coupling-induced enhancement of the mean magnon number and discuss its relation to sub-Poissonian magnon statistics. In Sec. VI, we discuss the experimental feasibility of the proposed scheme. Finally, Sec. VII summarizes the main results and gives an outlook. Additional details on the output-field interpretation, analytical derivations, parameter dependence, cutoff convergence, and small-signal stability analysis are provided in the Appendixes.

II. MODEL AND NOMMS

We consider a driven-dissipative magnon-qubit device in which the bosonic degree of freedom is the Kittel mode of a ferrimagnetic sample, such as a yttrium iron garnet (YIG) sphere or film [28,31–41], coupled to a superconducting qubit [1,9,10,42]. The annihilation (creation) operator $\hat{m}$ ($\hat{m}^\dagger$) describes the nearly uniform collective spin precession of the magnet, whose resonance frequency ω_m can be tuned by an external bias magnetic field [9,31,34]. The qubit of transition frequency ω_σ is described by the lowering (raising) operators $\hat{\sigma}$ ($\hat{\sigma}^\dagger$) and is coupled to the magnon mode through a coherent exchange interaction mediated by magnetic-dipole coupling, with strength g_c . Such an effective magnon-qubit interaction

may be realized either through cavity-mediated coupling or through direct magnetic-dipole coupling mediated by stray fields in superconducting-circuit architectures [1,16,42]. In addition to this coherent channel, the magnon and the qubit are coupled to a common microwave transmission-line reservoir, giving rise to the engineered collective dissipative channel introduced below [23,26,43,44].

The Kittel mode may also possess an intrinsic magnon Kerr nonlinearity arising from the magnetocrystalline anisotropy of YIG. In the angular-frequency convention used here, this nonlinear term can be written as $(\hbar = 1) H_K = \frac{K_m}{2} \hat{m}^{\dagger 2} \hat{m}^2$, where K_m is the single-magnon Kerr coefficient. For usual millimeter-scale YIG samples, however, this intrinsic nonlinearity is extremely weak. Theoretical estimates give Kerr coefficients in the nanohertz (10^{-9} Hz) range; for example, $K_m/2\pi$ increases from about 0.05 to 100 nHz when the diameter of a spherical YIG sample is reduced from 1 to 0.1 mm, consistent with the approximate inverse-volume scaling $K_m \propto V_m^{-1}$, where V_m denotes the magnetic mode volume [15,45–47]. In the parameter regimes considered here, the steady-state mean magnon number $n_m = \langle \hat{m}^\dagger \hat{m} \rangle$ remains at most of order 10. The corresponding Kerr-induced frequency shift, $|K_m|n_m/2\pi$, is therefore at most on the order of 10^{-6} Hz, many orders of magnitude smaller than the megahertz-scale linewidths and magnon-qubit coupling rates considered below. We thus treat the Kittel mode as an effectively harmonic bosonic mode, while the dominant nonlinearity is provided by the two-level nature of the qubit.

The reduced density matrix $\hat{\rho}$ of the magnon-qubit subsystem is described, in the rotating frame, by the Lindblad master equation

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} = & -i[\hat{H}, \hat{\rho}] + \gamma_m \mathcal{D}(\hat{m})\hat{\rho} + \gamma_\sigma \mathcal{D}(\hat{\sigma})\hat{\rho} + P_\sigma \mathcal{D}(\hat{\sigma}^\dagger)\hat{\rho} \\ & + \gamma_\phi \mathcal{D}(\hat{\sigma}^\dagger \hat{\sigma})\hat{\rho} + \tau \mathcal{D}(\mu e^{i\theta} \hat{m} + \nu \hat{\sigma})\hat{\rho}. \end{aligned} \quad (1)$$

Here, $\hat{H} = \Delta \hat{m}^\dagger \hat{m} + g_c (\hat{m}^\dagger \hat{\sigma} + \hat{m} \hat{\sigma}^\dagger)$ is the effective Jaynes-Cummings Hamiltonian, with $\Delta = \omega_m - \omega_\sigma$ the magnon-qubit detuning. The Lindblad dissipator is defined as $\mathcal{D}(\hat{d})\hat{\rho} = \hat{d}\hat{\rho}\hat{d}^\dagger - \frac{1}{2}\hat{d}^\dagger \hat{d}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{d}^\dagger \hat{d}$. The rates γ_m and γ_σ denote the magnon decay rate and the qubit relaxation rate, respectively, not including the collective decay rates from the collective dissipative channel, while P_σ denotes the incoherent qubit pump rate, corresponding to stochastic population transfer from the qubit ground state $|g\rangle$ to its excited state $|e\rangle$ and providing a phase-insensitive gain channel that supplies gain to the magnon mode indirectly through the coherent and dissipative magnon-qubit couplings [17,48–50]. The rate γ_ϕ denotes the qubit pure dephasing, which damps the qubit coherence without changing its population and may arise from fluctuations of the qubit transition frequency, such as flux, charge, magnetic field, or residual environmental noise [51–54]. The last term in Eq. (1) describes the engineered collective dissipation induced by the common waveguide reservoir, with overall rate τ ; here, μ and ν set the relative weights of the magnon and qubit contributions, and θ is their relative phase. The expansion of $\mathcal{D}[\mu e^{i\theta} \hat{m} + \nu \hat{\sigma}]\hat{\rho}$ generates cross-dissipative terms proportional to $\mu\nu^* e^{i\theta}$ and its complex conjugate, reflecting reservoir-mediated interference between the magnon and qubit emission pathways. Together with

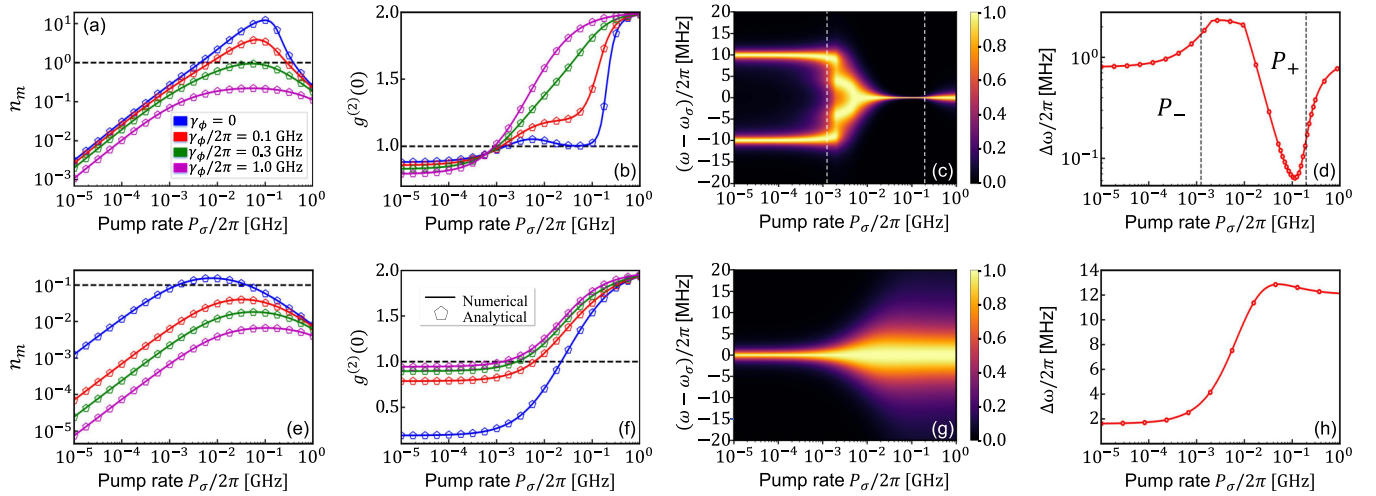


FIG. 1. Strong-coherent-coupling regime. (a), (e) Mean magnon number n_m ; (b), (f) second-order intensity correlation function $g^{(2)}(0)$; (c), (g) magnon emission spectrum with the peak normalized to unity for each pump rate; and (d), (h) linewidth $\Delta\omega/2\pi$ vs pump rate. Panels (a), (b), (e), and (f) are shown for four pure-dephasing rates: $\gamma_\phi = 0$ (blue), $\gamma_\phi/2\pi = 0.1$ GHz (red), $\gamma_\phi/2\pi = 0.3$ GHz (green), and $\gamma_\phi/2\pi = 1.0$ GHz (purple). The spectrum and linewidth, calculated from the master equation, are shown only for $\gamma_\phi = 0$ in panels (c), (d), (g), and (h). In panels (c) and (d), the two vertical dashed lines denote the lower and upper pump boundaries, P_- and P_+ , obtained from the small-signal stability criterion; these boundaries delimit the gain-positive window $P_- < P < P_+$ for the coherent-coupling case. Panels (a)–(d) correspond to the case of coherent coupling alone ($\tau = 0$) with $\Delta = 0$, $g_c/2\pi = 10$ MHz, $\gamma_m/2\pi = 2$ MHz, and $\gamma_\sigma/2\pi = 1.2$ MHz. Panels (e)–(h) correspond to the case of dissipative coupling alone ($g_c = 0$) with $\tau/2\pi = 10$ MHz, $\mu = \nu = 1$, and $\theta = 1.5\pi$, while all other parameters are identical to those in panels (a)–(d). Unless otherwise stated, solid lines are obtained from direct numerical solutions of the master equation, while pentagon markers denote the analytical results.

incoherent qubit pumping, this collective channel provides an irreversible reservoir-assisted pathway by which the pumped qubit affects the magnon dynamics, complementing the reversible coherent exchange governed by g_c .

To characterize the steady-state properties of the system, we compute the mean magnon number n_m and the second-order intensity correlation function $g^{(2)}(0)$. From the master equation, we derive the following recurrence relation for the steady-state NOMMs

$$A_n = \frac{\mathcal{F}_1(n)A_{n-1} - \mathcal{F}_2(n)A_{n+1}}{G(n)}, \quad (2)$$

where $A_n \equiv \langle \hat{m}^{\dagger n} \hat{m}^n \rangle$ denotes the n th order NOMM. The coefficients $\mathcal{F}_1(n)$, $\mathcal{F}_2(n)$, and $G(n)$ depend on order n and are determined by the parameters of the master equation (see Appendix A for their explicit forms). In the steady-state regime of interest, the population of high-lying magnon Fock states is negligibly small [3,30,55]. The magnon state can therefore be accurately described within a truncated Fock subspace spanned by $\{|0\rangle, |1\rangle, \dots, |N-1\rangle\}$, with the contribution from states above the cutoff being negligible. For sufficiently high orders $n > N-1$, the annihilation operator identity $\hat{m}^n|N-1\rangle = 0$ implies that the corresponding NOMMs vanish, i.e., $A_n = 0$. Together with the normalization condition $A_0 = 1$, Eq. (2) therefore admits a natural closure of the moment hierarchy.

Furthermore, the above recurrence relation yields the compact expression

$$A_n = \frac{\mathcal{R}_{n+1}}{\mathcal{F}_1(n+1)} A_{N-1}, \quad n = 0, 1, \dots, N-2, \quad (3)$$

where the boundary value is given by $A_{N-1} = \mathcal{F}_1(1)/\mathcal{R}_1$ obtained from the normalization condition together with Eq. (2). Here, $\mathcal{R}_n$ is an n -dependent auxiliary quantity determined by the parameters of the master equation (see Appendix A for details). The mean magnon number and the second-order intensity correlation function then follow directly from these NOMMs

$$n_m = \langle \hat{m}^\dagger \hat{m} \rangle = A_1 = \frac{\mathcal{R}_2 \mathcal{F}_1(1)}{\mathcal{F}_1(2) \mathcal{R}_1} \quad (4)$$

and

$$g^{(2)}(0) = \frac{\langle \hat{m}^{\dagger 2} \hat{m}^2 \rangle}{\langle \hat{m}^\dagger \hat{m} \rangle^2} = \frac{A_2}{A_1^2} = \frac{\mathcal{R}_1 \mathcal{R}_3 [\mathcal{F}_1(2)]^2}{\mathcal{R}_2^2 \mathcal{F}_1(1) \mathcal{F}_1(3)}. \quad (5)$$

The validity of this approach is benchmarked against direct numerical solutions of the master equation (see Figs. 1–3). More generally, the present approach may provide a route toward treating other driven open boson-fermion hybrid systems. By exploiting the closure of the finite-dimensional fermionic subspace, one can eliminate the fermionic moments from the coupled hierarchy and obtain a closed set of equations for bosonic moments only, thereby enabling an analytical treatment.

Unless otherwise stated, the NOMM calculations employed in the relevant figures are performed with a magnon Fock-space cutoff $N = 25$. We have verified that the plotted observables are converged with respect to this cutoff; details are provided in Appendix B.

Throughout this work, we distinguish between the strong- and weak-coherent-coupling regimes according to the relative magnitude of the coherent exchange interaction g_c and the total dissipation rates of the uncoupled magnon and qubit.

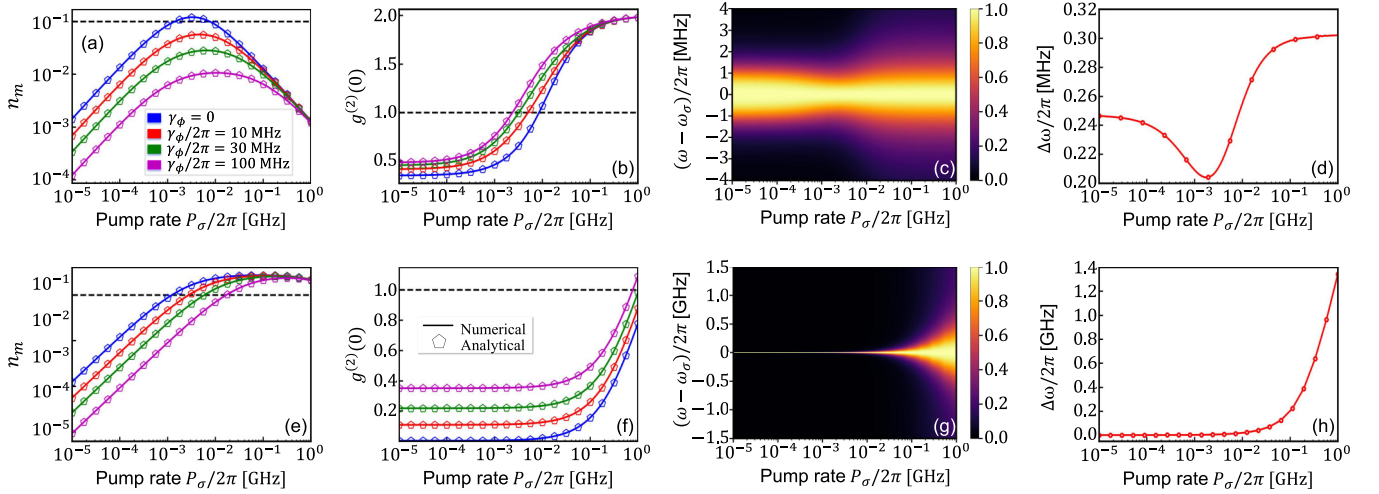


FIG. 2. Weak-coherent-coupling regime. (a), (e) Mean magnon number n_m ; (b), (f) second-order intensity correlation function $g^{(2)}(0)$; (c), (g) magnon emission spectrum with the peak normalized to unity for each pump rate; and (d), (h) linewidth $\Delta\omega/2\pi$ vs pump rate. Panels (a), (b), (e), and (f) are shown for four pure-dephasing rates: $\gamma_\phi = 0$ (blue), $\gamma_\phi/2\pi = 10$ MHz (red), $\gamma_\phi/2\pi = 30$ MHz (green), and $\gamma_\phi/2\pi = 100$ MHz (purple). The spectrum and linewidth, calculated from the master equation, are shown only for $\gamma_\phi = 0$ in panels (c), (d), (g), and (h). Panels (a)–(d) correspond to the case of coherent coupling alone ($\tau = 0$) with $\Delta = 0$, $g_c/2\pi = 1$ MHz, $\gamma_m/2\pi = 3$ MHz, and $\gamma_\sigma/2\pi = 1$ MHz. Panels (e)–(h) correspond to hybrid dissipative and weak-coherent coupling ($g_c/2\pi = 1$ MHz) with $\tau/2\pi = 1$ GHz, $\mu = \nu = 1$, and $\theta = 1.5\pi$, while all other parameters are identical to those in panels (a)–(d). In panel (f), curves of $g^{(2)}(0)$ are vertically offset for clarity by constant amounts 0.1 between successive traces, with the curve for $\gamma_\phi = 0$ serving as the reference (no offset).

In the strong-coherent-coupling regime, coherent excitation exchange dominates over dissipation, leading to pronounced hybridization between the magnon and qubit, namely, the formation of mixed magnon-qubit eigenmodes, and hence resolvable normal-mode splitting in the spectrum [2]. In contrast, in the weak-coherent-coupling regime, dissipation prevails over coherent exchange, such that the hybridization is weak and the spectral splitting is not clearly resolved. As a rough spectral criterion, the strong-coherent-coupling regime is expected when $2g_c \gtrsim (\gamma_m + \gamma_\sigma)/2$, whereas the opposite limit $2g_c \lesssim (\gamma_m + \gamma_\sigma)/2$ corresponds to the weak-coherent-coupling regime [56–58]. To demonstrate the spectral response of the system, we introduce the steady-state magnon emission spectrum, defined as

$$S_m(\omega) = \text{Re} \int_0^\infty dt e^{i\omega t} \langle \hat{m}^\dagger(t) \hat{m}(0) \rangle_{\text{ss}}, \quad (6)$$

where $\langle \hat{m}^\dagger(t) \hat{m}(0) \rangle_{\text{ss}}$ denotes the steady-state two-time magnon correlation function [59,60]. The corresponding linewidth $\Delta\omega/2\pi$ is defined as the full width at half maximum of $S_m(\omega)$ [61]. For completeness, the input-output relations connecting the magnon-mode observables n_m , $g^{(2)}(0)$, and $S_m(\omega)$ used here to the experimentally measurable microwave emission are provided in Appendix C.

III. STRONG-COHERENT COUPLING

In the strong-coherent-coupling regime shown in Fig. 1, we first focus on the scenario with coherent coupling alone. As clearly shown in Fig. 1(a), the mean magnon number exhibits a pronounced intermediate-pump window: It increases rapidly with P_σ , reaches a maximum, and is then suppressed at larger pumping. This nonmonotonic behavior reflects the

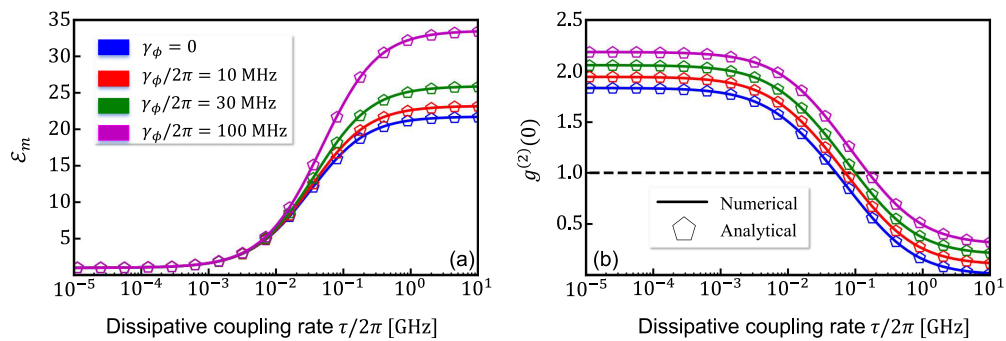


FIG. 3. (a) Enhancement ratio of mean magnon number $\mathcal{E}_m$ and (b) second-order correlation function $g^{(2)}(0)$ for four different pure-dephasing rates: $\gamma_\phi = 0$ (blue), $\gamma_\phi/2\pi = 10$ MHz (red), $\gamma_\phi/2\pi = 30$ MHz (green), and $\gamma_\phi/2\pi = 100$ MHz (purple). Curves of $g^{(2)}(0)$ are vertically offset for clarity by constant amounts 0.1 between successive traces, with the curve for $\gamma_\phi = 0$ serving as the reference (no offset). Panels (a) and (b) correspond to the parameters: $\Delta = 0$, $g_c/2\pi = 1$ MHz, $\mu = \nu = 1$, $\theta = 1.5\pi$, $\gamma_m/2\pi = 3$ MHz, $\gamma_\sigma/2\pi = 1$ MHz, and $P_\sigma/2\pi = 100$ MHz.

competition between gain generation by the incoherently excited qubit and pump-induced destruction of magnon-qubit coherence. In the weak-pump regime, the emission spectrum in Fig. 1(c) displays two well-resolved peaks, characteristic of coherent magnon-qubit Rabi splitting on the order of $2g_c$. As P_σ is increased, the qubit inversion is more efficiently converted into mean magnon number through the coherent exchange channel, and the system enters an intermediate-pump lasinglike regime, where the two Rabi peaks merge into a single peak near resonance, the linewidth $\Delta\omega/2\pi$ narrows markedly, and $g^{(2)}(0)$ remains close to unity [Fig. 1(b)]. In contrast, for stronger pumping, $g^{(2)}(0)$ increases toward the thermal-like value of 2, the linewidth broadens again, and the mean magnon number is quenched. This high-pump behavior is consistent with the self-quenching physics known from single-atom lasers and masers, where excessively strong incoherent pumping suppresses the coherence required for stimulated emission [17,62]. Taken together, the near-Poissonian statistics and linewidth narrowing identify the intermediate-pump regime as a window of coherent magnon emission with magnon-lasing-like character. We also note that the effect of qubit pure dephasing is qualitatively similar throughout the parameter regimes considered in this work. As shown already in Figs. 1(a) and 1(b), increasing γ_ϕ suppresses the mean magnon number buildup enabled by coherent exchange and progressively erodes the near-Poissonian coherence window, causing $g^{(2)}(0)$ to approach the thermal-like value more rapidly. Physically, pure dephasing does not directly remove magnons, but destroys the magnon-qubit coherence required for efficient coherent exchange and stimulated-emission-like magnon buildup.

Linewidth narrowing and near-Poissonian intensity fluctuations, $g^{(2)}(0) \simeq 1$, are useful signatures of coherent emission, but they do not by themselves provide a unique lasing criterion, since similar features can also arise from coherent oscillatory dynamics [63]. We therefore use an independent small-signal stability criterion to identify the magnon-lasing-like regime. In the coherent-coupling limit, $\tau = 0$, the mean-field Maxwell-Bloch equations possess a normal solution with no coherent magnon or qubit polarization,

$$\langle \hat{m} \rangle = \langle \hat{\sigma} \rangle = 0, \quad w_0 = \frac{P_\sigma - \gamma_\sigma}{P_\sigma + \gamma_\sigma}, \quad (7)$$

where w_0 is the unsaturated qubit inversion. The lasing onset is defined as the pump value at which this normal solution becomes unstable against an infinitesimal coherent magnon seed. On resonance and for $\gamma_\phi = 0$, this stability condition gives two pump boundaries,

$$P_\pm = \frac{4g_c^2 \pm \sqrt{16g_c^4 - 32\gamma_m g_c^2 \gamma_\sigma}}{2\gamma_m} - \gamma_\sigma. \quad (8)$$

The lower boundary P_- marks the onset of the small-signal gain-positive emission regime, whereas the upper boundary P_+ gives the high-pump self-quenching boundary. Hence, the gain-positive window is

$$P_- < P_\sigma < P_+. \quad (9)$$

Within this independently determined window, the observed linewidth narrowing and $g^{(2)}(0) \simeq 1$ behavior are interpreted

as supporting quantum signatures of magnon-lasing-like emission. The derivation of the stability condition is given in Appendix D.

We next consider the regime of dissipative coupling alone, with the coherent exchange switched off ($g_c = 0$). In this case, the common-reservoir term induces a cross-dissipative magnon-qubit coupling. In the presence of incoherent qubit pump, this channel provides an irreversible reservoir-assisted pathway through which the pumped qubit dynamics can feed the magnon mode. The dissipative channel does not constitute a purely amplifying channel for the magnon mode, since the reservoir-mediated process that correlates the qubit and magnon also introduces direct dissipative loss for the $\mu e^{i\theta} \hat{m} + \nu \hat{\sigma}$ collective structure of the jump operator, therefore acting simultaneously as a conversion channel and a loss channel. As a result, the steady-state mean magnon number in Fig. 1(e) remains low over the whole pump range and exhibits only a shallow maximum as P_σ increases. At weak pumping, however, the nonlinear two-level nature of the qubit is directly imprinted onto the magnon emission. Since each pump cycle creates at most one qubit excitation, which is then released through the collective dissipative channel, coincident two-magnon events are suppressed, leading to sub-Poissonian statistics with $g^{(2)}(0) < 1$ in Fig. 1(f). With increasing P_σ , this antibunching is progressively washed out by pump-induced incoherent fluctuations. Because no coherent exchange is present to sustain a lasinglike phase-coherent magnon field, the system instead crosses over toward thermal-like bunching, with $g^{(2)}(0) \rightarrow 2$. The same physical picture is reflected in the spectrum and linewidth shown in Figs. 1(g) and 1(h). In the absence of coherent magnon-qubit hybridization, the emission remains centered near resonance and broadens monotonically with pump, indicating pump-induced decoherence and power broadening rather than linewidth narrowing. Therefore, for dissipative coupling alone, the engineered reservoir primarily supports weak but antibunched magnon emission at low pump, while at higher pump it mainly acts as a knob controlling the crossover to thermal-like statistics and the emission linewidth.

IV. WEAK-COHERENT COUPLING

We next turn to the weak-coherent-coupling regime, shown in Figs. 2(a)–2(d). Because the coherent magnon-qubit exchange is weak, the mean magnon number cannot be efficiently accumulated by the coherent channel alone. Accordingly, the steady-state mean magnon number in Fig. 2(a) remains well below unity over the whole pump range and exhibits only a shallow maximum at intermediate pumping. At weak pump, the magnon emission still inherits the two-level nonlinearity of the pumped qubit, so that excitations are transferred essentially one by one and the intensity statistics remain sub-Poissonian, as shown in Fig. 2(b). As P_σ increases, the pump initially enhances the excitation transfer into the magnon mode, giving rise to the observed increase of n_m and a modest linewidth reduction in Fig. 2(d). For stronger pumping, the system enters a self-quenching regime. In this regime, the incoherent qubit pump no longer simply increases the available gain; instead, it repeatedly randomizes the qubit state and acts as an effective dephasing mechanism for the

magnon-qubit coherence. This suppresses the stimulated-emission-like buildup responsible for the intermediate-pump coherent-emission window. Consequently, the mean magnon number decreases, the emission linewidth broadens, and the intensity statistics become strongly bunched. The approach of $g^{(2)}(0)$ toward 2 should therefore be understood as a crossover to thermal-like, chaotic-field statistics, not as equilibration with a finite-temperature reservoir. The corresponding spectrum in Fig. 2(c) remains a single peak throughout, without any resolved mode splitting. Therefore, weak-coherent coupling alone supports only weak sub-Poissonian magnon emission, without producing a bright magnon field.

A qualitatively different behavior emerges in the hybrid regime, where the weak-coherent coupling coexists with the engineered dissipative coupling, as shown in Figs. 2(e)–2(h). In this case, the dissipative channel provides an additional reservoir-mediated transfer path from the pumped qubit to the magnon mode, leading to a substantial enhancement of the mean magnon number at intermediate and high pump [Fig. 2(e)]. At the same time, the second-order intensity correlation function in Fig. 2(f) shows that the hybrid coupling preserves sub-Poissonian statistics over the whole pump range. Although the antibunching becomes less pronounced at high pump, the correlation function remains well below that of the case of weak-coherent coupling alone and stays below unity even when the latter has already crossed into the super-Poissonian regime. This physical picture is further supported by the spectrum and linewidth in Figs. 2(g) and 2(h). The emission remains single peaked but becomes strongly broadened at high pump, with the linewidth reaching the gigahertz scale. The hybrid coupling therefore realizes a markedly brighter and sub-Poissonian nonclassical magnon source.

V. DISSIPATIVE-COUPPLING-INDUCED ENHANCEMENT

To characterize more directly how the engineered dissipative channel reshapes the weak-coherent regime, we define the enhancement ratio

$$\mathcal{E}_m \equiv \frac{n_m(\tau)}{n_m(0)}, \quad (10)$$

where $n_m(\tau) = \langle \hat{n}^\dagger \hat{n} \rangle$ is the steady-state mean magnon number in the presence of dissipative coupling τ , and $n_m(0)$ is the corresponding value in the absence of the collective dissipative channel.

As shown in Fig. 3(a), $\mathcal{E}_m$ remains close to unity for weak dissipative coupling, then rises rapidly with increasing τ , and eventually saturates at large τ . This trend shows that the contribution of the dissipative channel to the steady-state mean magnon number becomes increasingly important as τ increases. For small τ , this channel is negligible and the mean magnon number is still limited by the weak-coherent exchange alone. As τ increases, however, the dissipative transfer becomes increasingly efficient and substantially enhances the steady-state mean magnon number, yielding an order-of-magnitude enhancement. The saturation of $\mathcal{E}_m$ at large τ indicates that this enhancement is ultimately limited by the finite pump, the two-level saturation of the qubit, and the residual losses of the open system, rather than increasing without bound.

The same dissipative transfer channel also drives a pronounced change in the magnon statistics, as shown in Fig. 3(b). For weak τ , the system remains close to the weak-coherent limit, where the mean magnon number is low and the statistics are still super-Poissonian. As τ increases, $g^{(2)}(0)$ decreases monotonically and is driven deep into the sub-Poissonian regime. Physically, increasing τ makes the reservoir-mediated transfer of qubit excitation into the magnon mode more efficient, so that the two-level nonlinearity of the qubit is more effectively imprinted onto the magnon emission and multiparticle fluctuations are correspondingly suppressed. The dissipative channel therefore plays a dual role in this regime: It enhances the mean magnon number while simultaneously promoting antibunched magnon statistics.

We further note that the increase of $\mathcal{E}_m$ with γ_ϕ does not imply that pure dephasing enhances the absolute mean magnon number. Rather, it reflects the fact that dephasing suppresses the reference value $n_m(0)$ in the purely coherent weak-coupling case more strongly than it suppresses the dissipatively assisted mean magnon number $n_m(\tau)$. In this sense, the dissipative enhancement is comparatively robust against qubit dephasing. The dependence of the reservoir-assisted enhancement and the magnon statistics on the relative phase θ , the magnon-qubit detuning Δ , and the dissipative coupling rate τ is further analyzed in Appendix E. In particular, the results explain the choice $\theta = 1.5\pi$ used in the main text and show that an enhanced mean magnon number and sub-Poissonian statistics can coexist over a finite parameter region.

VI. EXPERIMENTAL FEASIBILITY

We discuss the experimental feasibility of the present scheme. High-quality YIG magnons and superconducting qubits can both exhibit dissipation rates at the megahertz level. In particular, in a quantum-magnonics experiment based on a YIG sphere and a superconducting transmon, the Kittel-mode linewidth was reported to be $\gamma_m/2\pi \approx 1.3$ – 1.4 MHz, while the qubit linewidth was $\gamma_q/2\pi \approx 1.2$ MHz [1]. Moreover, state-of-the-art magnetic field resilient superconducting transmons can achieve substantially smaller linewidths, indicating that assuming both magnon and qubit loss rates to be of order 1 MHz is experimentally reasonable [64].

Coherent magnon-qubit interactions can, in principle, be implemented in several ways, such as in the system of the magnon and superconducting qubit, where tunable coupling strengths up to $g_c/2\pi = 10$ MHz have been experimentally demonstrated [1]. Magnon-qubit interactions have also been experimentally explored in spin-qubit platforms based on nitrogen-vacancy centers [65]. In addition, a more direct magnon-transmon architecture has been proposed theoretically, which is particularly close in spirit to the present model [16,42].

Dissipative magnon-qubit coupling can be engineered in waveguide-based architectures, where traveling microwaves induce both coherent and dissipative interactions, and the relative weight of the two can be tuned by the propagation phase set by the coupling geometry [43]. Beyond such a direct waveguide implementation, an alternative multistage route is to couple the magnon and the qubit to a common lossy auxiliary cavity or continuum and then adiabatically

eliminate the intermediary degrees of freedom [66–69]. In the resulting effective description, the dissipative interaction is governed by the coupling of the magnon and the qubit to an engineered common dissipative channel and is correspondingly enhanced as these effective coupling rates increase, with a characteristic scaling as $\tau \propto \sqrt{\gamma_m^{\text{eff}} \gamma_\sigma^{\text{eff}}}$, where γ_m^{eff} and $\gamma_\sigma^{\text{eff}}$ denote the effective coupling rates of the magnon and the qubit to the engineered common dissipative channel after elimination of the intermediary network [43,68–70]. Open-waveguide superconducting qubits have demonstrated external relaxation rates as large as $2\pi \times 11$ MHz [71], while waveguide-controlled radiative damping of YIG magnons has reached $2\pi \times 6.434$ MHz [72]. Along this direction, dissipative magnon-photon coupling as large as 666.4 MHz has been demonstrated in cavity magnonics [29]. Although this experiment does not directly realize the pumped two-level magnon-qubit model considered here, it demonstrates that spin excitations can be coupled to an engineered electromagnetic reservoir with a dissipative rate approaching the gigahertz scale. Moreover, when an auxiliary photonic mode is operated within its lowest two Fock states, or effectively truncated by a strong nonlinearity, it can emulate a two-level degree of freedom. This provides an additional analog route toward realizing the reservoir-mediated magnon-qubit-type dissipative interaction considered in our model. Together with recent progress in ultrastrong coupling between superconducting artificial atoms and one-dimensional electromagnetic continua, where the spontaneous-emission rate can exceed the bare transition frequency [73], these advances suggest that high-megahertz effective dissipative couplings, and possibly stronger couplings in more elaborate architectures, may be achievable as longer-term targets.

As for the qubit incoherent excitation, an effective pumping rate up to $2.0 \times 10^9 \text{ s}^{-1}$ (i.e., $P_\sigma/2\pi \approx 320$ MHz) has already been demonstrated in a superconducting artificial-atom platform [17]. These considerations suggest that the proposed scheme may become experimentally accessible with further progress in engineered dissipative coupling.

VII. CONCLUSION

In conclusion, we have developed an analytical approach based on steady-state normal-ordered magnon moments for a driven-dissipative magnon-qubit system with coherent exchange and engineered collective dissipation. The method exploits the finite-dimensional qubit subspace to close the moment hierarchy and gives results in good agreement with full quantum master-equation simulations. It therefore provides an analytical route for understanding driven open boson-fermion hybrid systems, complementing direct numerical calculations. Our analysis shows that coherent and dissipative couplings play qualitatively different roles in quantum magnon emission. Strong-coherent coupling produces an intermediate-pump magnon-lasing-like regime, characterized by near-Poissonian statistics and pronounced linewidth narrowing. In contrast, dissipative coupling alone supports weak low-pump antibunched emission and leads to a strongly pump-tunable linewidth, reflecting the combined effects of reservoir-mediated transfer and additional dissipation. In the hybrid weak-coherent regime, the collective dissipative

channel substantially enhances the steady-state magnon population while maintaining sub-Poissonian statistics. This demonstrates that engineered dissipation can improve the brightness-statistics trade-off of nonclassical magnon emission. These results clarify how coherent and dissipative channels can be used separately and jointly to control magnon emission, and point toward tunable nonclassical microwave magnon sources in open hybrid quantum platforms.

ACKNOWLEDGMENTS

We thank the two anonymous referees for their constructive comments and valuable suggestions, which helped improve the clarity and presentation of this work. We are grateful to Ying Wu and Xin-You Lü for helpful discussions. The present research is supported in part by the National Natural Science Foundation of China (NSFC) through Grant No. 12275092 and by the National Key Research and Development Program of China under Contract No. 2021YFA1400700. The computation is completed in the HPC Platform of Huazhong University of Science and Technology.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: DERIVATION OF THE NORMAL-ORDERED MAGNON MOMENTS

The system density matrix $\hat{\rho}$ obeys the Markovian master equation

$$\begin{aligned} \frac{d\hat{\rho}}{dt} = & -i[\hat{H}_0, \hat{\rho}] + \gamma_m D(\hat{m})\hat{\rho} + \gamma_\sigma D(\hat{\sigma})\hat{\rho} + P_\sigma D(\hat{\sigma}^\dagger)\hat{\rho} \\ & + \gamma_\phi D(\hat{\sigma}^\dagger \hat{\sigma})\hat{\rho} + \tau D(\mu e^{i\theta} \hat{m} + \nu \hat{\sigma})\hat{\rho}, \end{aligned} \quad (\text{A1})$$

with the Hamiltonian

$$\hat{H}_0 = \omega_m \hat{m}^\dagger \hat{m} + \omega_\sigma \hat{\sigma}^\dagger \hat{\sigma} + g_c (\hat{m}^\dagger \hat{\sigma} + \hat{m} \hat{\sigma}^\dagger), \quad (\text{A2})$$

where $\hat{m}$ ($\hat{m}^\dagger$) is the annihilation (creation) operator of the magnon mode and $\hat{\sigma}$ ($\hat{\sigma}^\dagger$) is the qubit lowering (raising) operator. Here, ω_m is the resonance frequency of the magnon mode, ω_σ is the transition frequency of the qubit, and g_c is the coherent coupling strength. The Hamiltonian in Eq. (A2) is written in the laboratory frame; after transforming to the rotating frame, it reduces to the detuned Hamiltonian used in Eq. (1). The laboratory- and rotating-frame formulations give identical NOMM results. The Lindblad superoperator is defined as $\mathcal{D}(\hat{O})\hat{\rho} = \hat{O}\hat{\rho}\hat{O}^\dagger - \frac{1}{2}(\hat{O}^\dagger \hat{O}\hat{\rho} + \hat{\rho}\hat{O}^\dagger \hat{O})$. In Eq. (A1), γ_m and γ_σ denote the decay rates of magnon and qubit, respectively; P_σ is an incoherent pump rate of the qubit; γ_ϕ is the pure-dephasing rate of the qubit; and the last term describes a correlated dissipative channel with rate τ , relative amplitude ratio (μ, ν) , and phase θ . For later convenience, we define the auxiliary superoperators

$$\begin{aligned} b(\hat{x}, \hat{y})\hat{\rho} & \equiv \hat{y}^\dagger \hat{\rho} \hat{x} - \frac{1}{2}\{\hat{y}^\dagger \hat{x}, \hat{\rho}\}, \\ d(\hat{x}, \hat{y})\hat{\rho} & \equiv \hat{x} \hat{\rho} \hat{y}^\dagger - \frac{1}{2}\{\hat{y}^\dagger \hat{x}, \hat{\rho}\}. \end{aligned} \quad (\text{A3})$$

When the two arguments coincide, these reduce to the superoperators with $B(\hat{x}) \equiv b(\hat{x}, \hat{x})$ and $D(\hat{x}) \equiv d(\hat{x}, \hat{x})$. From the identity $\text{Tr}[\hat{k} d(\hat{x}, \hat{y}) \hat{\rho}] = \langle b(\hat{x}, \hat{y}) \hat{k} \rangle$ valid for arbitrary system operators $\hat{x}$, $\hat{y}$, and $\hat{k}$, we obtain the equation of motion for an arbitrary normal-ordered moment

$$\begin{aligned} \frac{d}{dt} \langle \hat{m}^{\dagger q_1} \hat{m}^{q_2} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{q_4} \rangle \\ = -i \langle [\hat{m}^{\dagger q_1} \hat{m}^{q_2} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{q_4}, \hat{H}_0] \rangle \\ + \sum_j \Gamma_j \langle B(\hat{L}_j) \hat{m}^{\dagger q_1} \hat{m}^{q_2} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{q_4} \rangle, \end{aligned} \quad (\text{A4})$$

where the sum runs over $\hat{L}_j \in \{\hat{m}, \hat{\sigma}, \hat{\sigma}^\dagger, \hat{\sigma}^\dagger \hat{\sigma}, \mu e^{i\theta} \hat{m} + \nu \hat{\sigma}\}$ with corresponding rates $\Gamma_j \in \{\gamma_m, \gamma_\sigma, P_\sigma, \gamma_\phi, \tau\}$ and non-negative integers $q_1, q_2 \in \mathbb{N}_0$ and $q_3, q_4 \in \{0, 1\}$. Here, the qubit operator algebra (in particular, $\hat{\sigma}^2 = (\hat{\sigma}^\dagger)^2 = 0$) ensures that no higher powers of $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ are generated in the hierarchy. For brevity, we define the normal-ordered operator moment

$$\hat{O}_{\mathbf{q}} \equiv \hat{m}^{\dagger q_1} \hat{m}^{q_2} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{q_4}. \quad (\text{A5})$$

The commutator with the coherent coupling in Eq. (A4) is given explicitly by

$$\begin{aligned} [\hat{O}_{\mathbf{q}}, \hat{H}_0] = & [\omega_m(q_2 - q_1) + \omega_\sigma(q_4 - q_3)] \hat{O}_{\mathbf{q}} + g_c [q_4 \hat{m}^{\dagger q_1} \hat{m}^{q_2+1} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{1-q_4} - 2(1 - q_3) q_4 \hat{m}^{\dagger q_1} \hat{m}^{q_2+1} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{q_4} \\ & - q_1(1 - q_3) \hat{m}^{\dagger(q_1-1)} \hat{m}^{q_2} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{q_4}] + g_c [-q_3 \hat{m}^{\dagger(q_1+1)} \hat{m}^{q_2} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{q_4} \\ & + 2q_3(1 - q_4) \hat{m}^{\dagger(q_1+1)} \hat{m}^{q_2} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{1-q_4} + q_2(1 - q_4) \hat{m}^{\dagger q_1} \hat{m}^{q_2-1} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{1-q_4}]. \end{aligned} \quad (\text{A6})$$

The dissipative part in Eq. (A4) can be expressed through the action of the superoperator $B(\hat{L})$ associated with $\mathcal{D}[\hat{L}]$. For the jump operators $\hat{L}_j \in \{\hat{m}, \hat{\sigma}, \hat{\sigma}^\dagger, \hat{\sigma}^\dagger \hat{\sigma}\}$, we obtain

$$\begin{aligned} B(\hat{m}) \hat{O}_{\mathbf{q}} &= -\frac{1}{2}(q_1 + q_2) \hat{O}_{\mathbf{q}}, \quad B(\hat{\sigma}) \hat{O}_{\mathbf{q}} = -\frac{1}{2}(q_3 + q_4) \hat{O}_{\mathbf{q}}, \\ B(\hat{\sigma}^\dagger) \hat{O}_{\mathbf{q}} &= -\frac{1}{2}(q_3 + q_4) \hat{O}_{\mathbf{q}} + q_3 q_4 \hat{m}^{\dagger q_1} \hat{m}^{q_2} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{1-q_4}, \\ B(\hat{\sigma}^\dagger \hat{\sigma}) \hat{O}_{\mathbf{q}} &= [q_3 q_4 - \frac{1}{2}(q_3 + q_4)] \hat{O}_{\mathbf{q}}. \end{aligned} \quad (\text{A7})$$

For the jump operator of the dissipative coupling with $\hat{L}_j = \mu e^{i\theta} \hat{m} + \nu \hat{\sigma}$, we obtain

$$\begin{aligned} B(\mu e^{i\theta} \hat{m} + \nu \hat{\sigma}) \hat{O}_{\mathbf{q}} = & |\mu|^2 B(\hat{m}) \hat{O}_{\mathbf{q}} + |\nu|^2 B(\hat{\sigma}) \hat{O}_{\mathbf{q}} + \mu e^{i\theta} \nu^* \left[-\frac{1}{2} q_4 \hat{m}^{\dagger q_1} \hat{m}^{q_2+1} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{1-q_4} + (1 - q_3) q_4 \hat{m}^{\dagger q_1} \hat{m}^{q_2+1} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{q_4} \right. \\ & \left. - \frac{1}{2} q_1(1 - q_3) \hat{m}^{\dagger(q_1-1)} \hat{m}^{q_2} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{q_4} \right] + \mu^* e^{-i\theta} \nu \left[-\frac{1}{2} q_3 \hat{m}^{\dagger(q_1+1)} \hat{m}^{q_2} \hat{\sigma}^{\dagger(1-q_3)} \hat{\sigma}^{q_4} \right. \\ & \left. + q_3(1 - q_4) \hat{m}^{\dagger(q_1+1)} \hat{m}^{q_2} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{1-q_4} - \frac{1}{2} q_2(1 - q_4) \hat{m}^{\dagger q_1} \hat{m}^{q_2-1} \hat{\sigma}^{\dagger q_3} \hat{\sigma}^{1-q_4} \right]. \end{aligned} \quad (\text{A8})$$

Although Eqs. (A6) and (A8) are structurally similar at the level of moment equations, this similarity is purely formal: The corresponding terms carry different coefficients and are therefore governed by different parameter combinations. Consequently, the two couplings lead to qualitatively different steady-state responses of the magnon mode, as shown in Fig. 1. Substituting the explicit coherent and dissipative contributions derived above [see Eqs. (A6)–(A8)] into the moment equation (A4), and setting the time derivatives of the relevant moments to zero (steady-state condition), we obtain a closed set of coupled algebraic equations for a minimal set of bosonic normal-ordered moments. To close the hierarchy in the bosonic sector, we define

$$\begin{aligned} A_n &= \langle \hat{m}^{\dagger n} \hat{m}^n \rangle, \\ B_n &= \langle \hat{m}^{\dagger(n-1)} \hat{m}^n \hat{\sigma}^\dagger \rangle, \\ C_n &= \langle \hat{m}^{\dagger n} \hat{m}^{n-1} \hat{\sigma} \rangle, \\ S_n &= \langle \hat{m}^{\dagger n} \hat{m}^n \hat{\sigma}^\dagger \hat{\sigma} \rangle. \end{aligned} \quad (\text{A9})$$

In Eq. (A9), A_n and S_n are defined for $n \geq 0$, whereas B_n and C_n are defined for $n \geq 1$. Using the definitions in Eq. (A9), the steady-state moment equations can be written as the following coupled recursive relations:

$$A_n = \frac{(ig_c - \frac{1}{2} \tau \mu e^{i\theta} \nu^*) B_n - (ig_c + \frac{1}{2} \tau \mu^* e^{-i\theta} \nu) C_n}{\gamma_m + \tau |\mu|^2} \quad (\text{A10})$$

for the bosonic moment A_n ,

$$B_n = \frac{[(ig_c - \frac{1}{2} \tau \mu^* e^{-i\theta} \nu) A_n - (2ig_c - \tau \mu^* e^{-i\theta} \nu) S_n - n(ig_c + \frac{1}{2} \tau \mu^* e^{-i\theta} \nu) S_{n-1}]}{[i(\omega_m - \omega_\sigma) + \frac{1}{2}(2n - 1)(\gamma_m + \tau |\mu|^2) + \frac{1}{2}(\gamma_\sigma + \tau |\nu|^2) + \frac{1}{2}P_\sigma + \frac{1}{2}\gamma_\phi]}, \quad (\text{A11})$$

and

$$C_n = \frac{[(-ig_c - \frac{1}{2} \tau \mu e^{i\theta} \nu^*) A_n - (-2ig_c - \tau \mu e^{i\theta} \nu^*) S_n - n(-ig_c + \frac{1}{2} \tau \mu e^{i\theta} \nu^*) S_{n-1}]}{[-i(\omega_m - \omega_\sigma) + \frac{1}{2}(2n - 1)(\gamma_m + \tau |\mu|^2) + \frac{1}{2}(\gamma_\sigma + \tau |\nu|^2) + \frac{1}{2}P_\sigma + \frac{1}{2}\gamma_\phi]}, \quad (\text{A12})$$

for the boson-spin mixed moments B_n and C_n , and

$$S_n = \frac{[P_\sigma A_n - (ig_c + \frac{1}{2}\tau\mu e^{i\theta}\nu^*)B_{n+1} + (ig_c - \frac{1}{2}\tau\mu^* e^{-i\theta}\nu)C_{n+1}]}{n(\gamma_m + \tau|\mu|^2) + (\gamma_\sigma + \tau|\nu|^2) + P_\sigma}, \quad (\text{A13})$$

for the population-correlated moment S_n . Equations (A10)–(A13) constitute a recursion in n once the higher-order moments in Eq. (A13) are expressed in terms of the normal-ordered moments set at order $n+1$ (i.e., B_{n+1} and C_{n+1}), thereby closing the hierarchy into a three-term recursion [Eq. (A31)] in bosonic mode. For clarity, we further define

$$\begin{aligned} \Gamma_m &= \gamma_m + \tau|\mu|^2, \\ \Gamma_\sigma &= \gamma_\sigma + \tau|\nu|^2, \\ \Delta &= \omega_m - \omega_\sigma \end{aligned} \quad (\text{A14})$$

and

$$\chi = \frac{1}{2}\tau\mu e^{i\theta}\nu^*, \quad \tilde{\chi} = \frac{1}{2}\tau\mu^* e^{-i\theta}\nu. \quad (\text{A15})$$

The complex denominators are

$$\begin{aligned} D_n &= i\Delta + \frac{1}{2}(2n-1)\Gamma_m + \frac{1}{2}\Gamma_\sigma + \frac{1}{2}P_\sigma + \frac{1}{2}\gamma_\phi, \\ \bar{D}_n &= -i\Delta + \frac{1}{2}(2n-1)\Gamma_m + \frac{1}{2}\Gamma_\sigma + \frac{1}{2}P_\sigma + \frac{1}{2}\gamma_\phi, \end{aligned} \quad (\text{A16})$$

and

$$E_n = n\Gamma_m + \Gamma_\sigma + P_\sigma. \quad (\text{A17})$$

We introduce the compact coefficients

$$\alpha = ig_c - \chi, \quad \beta = ig_c + \tilde{\chi}, \quad (\text{A18})$$

$$\begin{aligned} b_1 &= ig_c - \tilde{\chi}, \quad b_2 = -2ig_c + 2\tilde{\chi}, \\ b_3^{(n)} &= -n(ig_c + \tilde{\chi}) = -n\beta, \end{aligned} \quad (\text{A19})$$

$$\begin{aligned} c_1 &= -ig_c - \chi, \quad c_2 = 2ig_c + 2\chi, \\ c_3^{(n)} &= n(ig_c - \chi) = n\alpha. \end{aligned} \quad (\text{A20})$$

With these definitions, the steady-state relations read

$$\Gamma_m A_n = \alpha B_n - \beta C_n, \quad (\text{A21})$$

$$D_n B_n = b_1 A_n + b_2 S_n + b_3^{(n)} S_{n-1}, \quad (\text{A22})$$

$$\bar{D}_n C_n = c_1 A_n + c_2 S_n + c_3^{(n)} S_{n-1}, \quad (\text{A23})$$

$$E_n S_n = c_1 B_{n+1} + b_1 C_{n+1} + P_\sigma A_n. \quad (\text{A24})$$

Before proceeding to the elimination of the mixed moments, it is useful to clarify the physical meaning of these coefficients. The different roles of the coherent and dissipative couplings become transparent in the NOMM hierarchy. The coherent exchange enters through the Hamiltonian commutator and appears as the purely imaginary coefficient ig_c , which couples the bosonic moments A_n to the magnon-qubit coherence moments B_n and C_n . It therefore promotes reversible magnon-qubit excitation exchange and coherent mode hybridization. In contrast, the collective dissipative channel enters in two inseparable ways. First, it increases the effective decay rates, $\Gamma_m = \gamma_m + \tau|\mu|^2$ and $\Gamma_\sigma = \gamma_\sigma + \tau|\nu|^2$, which appear in the

denominators of the moment equations and contribute to spectral broadening. Second, it generates phase-sensitive cross coefficients $\chi = \tau\mu e^{i\theta}\nu^*/2$ and $\tilde{\chi} = \tau\mu^* e^{-i\theta}\nu/2$, which enter the transfer coefficients together with ig_c . Consequently, the dissipative channel is not equivalent to replacing g_c by an imaginary coupling; it simultaneously provides reservoir-mediated transfer, additional damping, and phase-dependent interference. This explains why coherent coupling favors Rabi splitting and lasinglike linewidth narrowing, whereas dissipative coupling alone supports low-pump antibunched emission with pump-induced spectral broadening, and why their coexistence can enhance n_m while preserving sub-Poissonian statistics. Eliminating B_n and C_n from Eq. (A21) using Eqs. (A22) and (A23) gives

$$R_n A_n = M_n S_n + N_n S_{n-1}, \quad (\text{A25})$$

with

$$\begin{aligned} R_n &= \Gamma_m - \frac{\alpha}{D_n} b_1 + \frac{\beta}{\bar{D}_n} c_1, \\ M_n &= \frac{\alpha}{D_n} b_2 - \frac{\beta}{\bar{D}_n} c_2, \\ N_n &= \frac{\alpha}{D_n} b_3^{(n)} - \frac{\beta}{\bar{D}_n} c_3^{(n)}. \end{aligned} \quad (\text{A26})$$

Next, expanding Eq. (A24) by substituting B_{n+1} and C_{n+1} from Eqs. (A22) and (A23) with $n \rightarrow n+1$ yields a second linear relation between S_n and S_{n+1} ,

$$Q_n S_n - V_n S_{n+1} = U_n A_{n+1} + P_\sigma A_n, \quad (\text{A27})$$

where

$$\begin{aligned} Q_n &= E_n - c_1 \frac{b_3^{(n+1)}}{D_{n+1}} - b_1 \frac{c_3^{(n+1)}}{\bar{D}_{n+1}}, \\ V_n &= c_1 \frac{b_2}{D_{n+1}} + b_1 \frac{c_2}{\bar{D}_{n+1}}, \\ U_n &= c_1 \frac{b_1}{D_{n+1}} + b_1 \frac{c_1}{\bar{D}_{n+1}}. \end{aligned} \quad (\text{A28})$$

Using Eq. (A25) at $n+1$ to express S_{n+1} and substituting into Eq. (A27) gives

$$S_n = \frac{(U_n + V_n \frac{R_{n+1}}{M_{n+1}}) A_{n+1} + P_\sigma A_n}{(Q_n + V_n \frac{N_{n+1}}{M_{n+1}})}. \quad (\text{A29})$$

Similarly, shifting $n \rightarrow n-1$ in Eq. (A29) gives

$$S_{n-1} = \frac{(U_{n-1} + V_{n-1} \frac{R_n}{M_n}) A_n + P_\sigma A_{n-1}}{(Q_{n-1} + V_{n-1} \frac{N_n}{M_n})}. \quad (\text{A30})$$

Substituting Eqs. (A29) and (A30) into Eq. (A25) yields the desired three-term recursion for A_n

$$A_n = \frac{\mathcal{F}_1(n) A_{n-1} - \mathcal{F}_2(n) A_{n+1}}{\mathcal{G}(n)}, \quad (\text{A31})$$

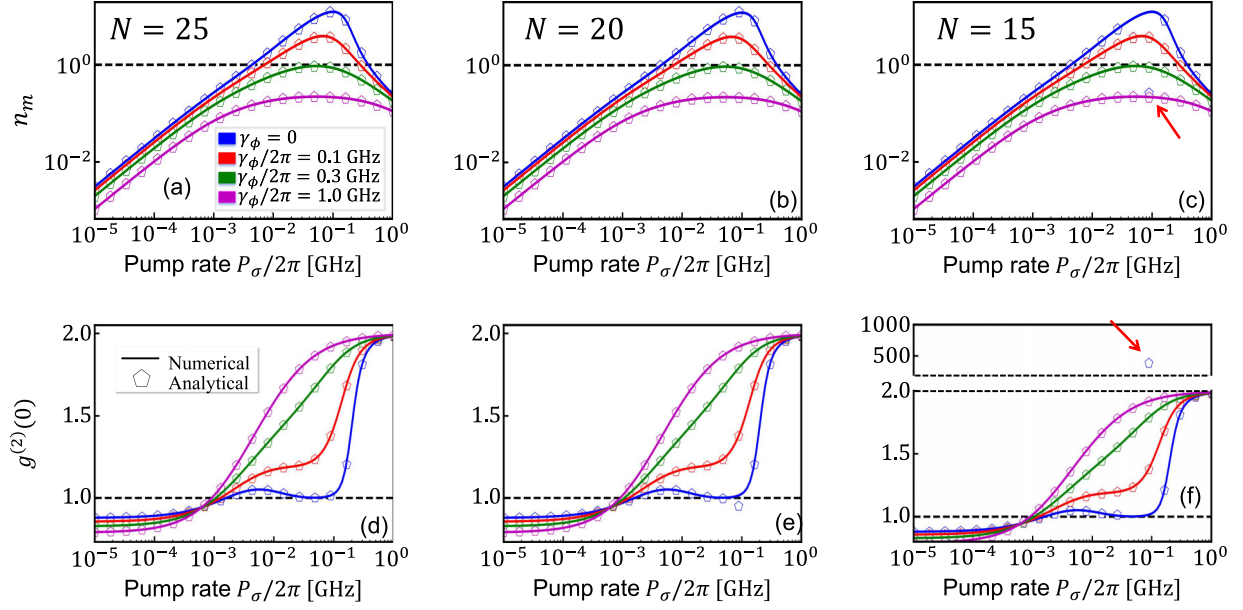


FIG. 4. Cutoff convergence of the analytical NOMM solutions. Panels (a)–(c) show the mean magnon number n_m , and panels (d)–(f) show the second-order correlation function $g^{(2)}(0)$, as functions of the incoherent qubit pump rate $P_\sigma/2\pi$. All parameters are the same as in Figs. 1(a)–1(d). Solid curves are full numerical master-equation results, while open pentagons are analytical NOMM results with bosonic cutoffs $N = 25$ in panels (a) and (d), $N = 20$ in panels (b) and (e), and $N = 15$ in panels (c) and (f). The red arrows highlight the visible cutoff-induced deviations for the smaller cutoff $N = 15$.

where a compact representation is obtained by defining

$$\begin{aligned} Q(n) &= Q_n + V_n \frac{N_{n+1}}{M_{n+1}}, \\ U(n) &= U_n + V_n \frac{R_{n+1}}{M_{n+1}}, \end{aligned} \quad (\text{A32})$$

so that

$$\begin{aligned} \mathcal{F}_1(n) &= \frac{N_n P_\sigma}{Q(n-1)}, \quad \mathcal{F}_2(n) = -\frac{M_n U(n)}{Q(n)}, \\ \mathcal{G}(n) &= R_n - \frac{M_n P_\sigma}{Q(n)} - \frac{N_n U(n-1)}{Q(n-1)}. \end{aligned} \quad (\text{A33})$$

For the numerical evaluation of observables, we truncate the hierarchy at a sufficiently large N with the boundary condition $A_N = 0$, together with the normalization condition $A_0 = 1$. The three-term recursion in Eq. (A31) can then be solved using a continued-fraction procedure. For numerical convenience, we recast the three-term recursion in Eq. (A31) into a backward continued-fraction scheme. This transformation can be established recursively (or, equivalently, by induction). To this end, we introduce the auxiliary quantities

$$\mathcal{R}_{N-1} = \mathcal{G}(N-1), \quad \mathcal{T}_N = 1, \quad (\text{A34})$$

from which the backward recursion

$$\begin{aligned} \mathcal{R}_n &= \mathcal{F}_2(n) \mathcal{T}_{n+2} + \frac{\mathcal{G}(n) \mathcal{R}_{n+1}}{\mathcal{F}_1(n+1)}, \\ n &= N-2, N-3, \dots, 1 \end{aligned} \quad (\text{A35})$$

follows, together with

$$\mathcal{T}_n = \frac{\mathcal{R}_n}{\mathcal{F}_1(n)}, \quad n = 1, 2, \dots, N-1. \quad (\text{A36})$$

The moments A_n are then reconstructed from

$$A_n = \frac{\mathcal{R}_{n+1}}{\mathcal{F}_1(n+1)} A_{N-1}, \quad A_{N-1} = \frac{\mathcal{F}_1(1)}{\mathcal{R}_1}, \quad n = 0, 1, \dots, N-2. \quad (\text{A37})$$

The observables of interest follow directly from the recursive solution. In particular, the mean magnon number is

$$n_m = \langle \hat{m}^\dagger \hat{m} \rangle = A_1 = \frac{\mathcal{R}_2 \mathcal{F}_1(1)}{\mathcal{F}_1(2) \mathcal{R}_1}, \quad (\text{A38})$$

and the second-order intensity correlation function is

$$g^{(2)}(0) = \frac{\langle \hat{m}^{\dagger 2} \hat{m}^2 \rangle}{\langle \hat{m}^\dagger \hat{m} \rangle^2} = \frac{A_2}{A_1^2} = \frac{\mathcal{R}_1 \mathcal{R}_3 [\mathcal{F}_1(2)]^2}{\mathcal{R}_2^2 \mathcal{F}_1(1) \mathcal{F}_1(3)}. \quad (\text{A39})$$

From Eq. (A29), we further obtain the excited-state population of the qubit

$$\langle \hat{\sigma}^\dagger \hat{\sigma} \rangle = S_0 = \frac{(U_0 + V_0 \frac{R_1}{M_1}) A_1 + P_\sigma A_0}{Q_0 + V_0 \frac{N_1}{M_1}} = \frac{U(0) A_1 + P_\sigma}{Q(0)}. \quad (\text{A40})$$

APPENDIX B: CUTOFF CONVERGENCE OF THE NOMM HIERARCHY

To substantiate the cutoff closure of the NOMM hierarchy, we have performed a convergence analysis with respect to the bosonic Fock-space cutoff N . Unless otherwise stated, all analytical data obtained from the NOMM approach shown in the main text are obtained with $N = 25$. Figure 4 compares the analytical results for $N = 25$, $N = 20$, and $N = 15$ with direct numerical solutions of the full Lindblad master equation, using the same parameters as in Figs. 1(a)–1(d). The results for

$N = 25$ and $N = 20$ are essentially indistinguishable over the full pump range for both n_m and $g^{(2)}(0)$, confirming convergence of the hierarchy. When the cutoff is reduced to $N = 15$, the overall trends are still reproduced, but isolated deviations appear in the high-pump regime with large n_m . These deviations are expected because, when the mean magnon number becomes large, higher magnon-number states acquire non-negligible populations, invalidating the assumption used in the derivation that their populations can be neglected. Therefore, $N = 25$ provides a sufficiently converged cutoff for the parameter regimes considered in the main text.

APPENDIX C: OUTPUT FIELD AND MEASURABLE MAGNON EMISSION

For completeness, we relate the magnon-mode observables used in the main text to the measurable microwave output field. In this work, “magnon emission” denotes the microwave radiation generated by the Kittel mode and emitted into a magnon-coupled readout port. It is therefore an output-field measurement of the Kittel-mode dynamics, rather than a direct particle-counting measurement inside the magnetic sample [74,75].

We decompose the magnon decay rate (not including the collective decay rates from the collective dissipative channel) as

$$\gamma_m = \kappa_{\text{out}} + \gamma_m^{\text{int}}, \quad (\text{C1})$$

where κ_{out} is the radiative decay rate into the measured microwave port and γ_m^{int} is the internal loss rate. The input-output relation for this magnon readout channel is [74,76]

$$\hat{b}_{\text{out}}(t) = \hat{b}_{\text{in}}(t) + \sqrt{\kappa_{\text{out}}} \hat{m}(t), \quad (\text{C2})$$

where $\hat{b}_{\text{in}}$ and $\hat{b}_{\text{out}}$ are the incoming and outgoing microwave fields of the readout port, respectively. For vacuum input and normally ordered detection, the output intensity is [76,77]

$$I_{\text{out}} = \kappa_{\text{out}} \langle \hat{m}^\dagger \hat{m} \rangle, \quad (\text{C3})$$

and the second-order intensity correlation function satisfies

$$g_{\text{out}}^{(2)}(0) = \frac{\langle \hat{b}_{\text{out}}^{\dagger 2} \hat{b}_{\text{out}}^2 \rangle}{\langle \hat{b}_{\text{out}}^\dagger \hat{b}_{\text{out}} \rangle^2} = \frac{\langle \hat{m}^{\dagger 2} \hat{m}^2 \rangle}{\langle \hat{m}^\dagger \hat{m} \rangle^2} \equiv g_m^{(2)}(0). \quad (\text{C4})$$

Similarly, the measured output spectrum is proportional to the magnon-mode spectrum [74,75],

$$S_{\text{out}}(\omega) = \kappa_{\text{out}} S_m(\omega), \quad (\text{C5})$$

apart from calibrated detection backgrounds and frequency-independent gain factors.

This magnon-dominated readout channel is distinct from the engineered common reservoir that produces the collective dissipator in the master equation. If the output field of the common reservoir were collected, its input-output relation would involve the collective jump operator [78],

$$\hat{c}_{\text{out}}(t) = \hat{c}_{\text{in}}(t) + \sqrt{\tau} \hat{L}(t), \quad \hat{L}(t) = \mu e^{i\theta} \hat{m}(t) + \nu \hat{\sigma}(t). \quad (\text{C6})$$

Here, $\hat{c}_{\text{in}}$ and $\hat{c}_{\text{out}}$ are the input and output fields of the common reservoir. Under the same vacuum-input and normally ordered detection conditions, the corresponding output intensity

would be

$$I_{\text{col}} = \tau \langle \hat{L}^\dagger \hat{L} \rangle, \quad (\text{C7})$$

with

$$\begin{aligned} \langle \hat{L}^\dagger \hat{L} \rangle &= |\mu|^2 \langle \hat{m}^\dagger \hat{m} \rangle + |\nu|^2 \langle \hat{\sigma}^\dagger \hat{\sigma} \rangle + \mu^* \nu e^{-i\theta} \langle \hat{m}^\dagger \hat{\sigma} \rangle \\ &\quad + \mu \nu^* e^{i\theta} \langle \hat{\sigma}^\dagger \hat{m} \rangle. \end{aligned} \quad (\text{C8})$$

Thus, a measurement of the common-reservoir output would contain magnon, qubit, and magnon-qubit interference contributions. It would characterize the hybrid emission associated with the collective jump channel, whereas the quantities n_m , $g^{(2)}(0)$, and $S_m(\omega)$ discussed in the main text refer to the Kittel-mode emission inferred from the magnon-dominated readout port.

In this convention, magnon lasing refers to a pump-induced coherent-emission regime of the Kittel mode, whose presence is inferred from the microwave output emitted through the magnon readout channel. Experimentally, $g_{\text{out}}^{(2)}(0)$ can be measured by splitting the emitted microwave field with a hybrid coupler and performing two-channel heterodyne correlation measurements, as in microwave Hanbury Brown-Twiss detection [79,80].

APPENDIX D: MEAN-FIELD DERIVATION OF THE MAGNON-LASING ONSET CRITERION

We derive here a mean-field criterion for the onset of the coherent magnon-emission window in the strong-coherent-coupling regime. The criterion is obtained from the linear stability of the zero-coherent-field solution and is therefore independent of the linewidth narrowing or the value of $g^{(2)}(0)$ [81,82]. In this Appendix, we set the engineered collective dissipation to zero, $\tau = 0$, and use

$$\begin{aligned} \frac{d\hat{\rho}}{dt} &= -i[\hat{H}, \hat{\rho}] + \gamma_m \mathcal{D}(\hat{m})\hat{\rho} + \gamma_\sigma \mathcal{D}(\hat{\sigma})\hat{\rho} \\ &\quad + P_\sigma \mathcal{D}(\hat{\sigma}^\dagger)\hat{\rho} + \gamma_\phi \mathcal{D}(\hat{\sigma}^\dagger \hat{\sigma})\hat{\rho}, \end{aligned} \quad (\text{D1})$$

with the Hamiltonian $\hat{H} = \Delta \hat{m}^\dagger \hat{m} + g_c(\hat{m}^\dagger \hat{\sigma} + \hat{m} \hat{\sigma}^\dagger)$. Here, $\Delta = \omega_m - \omega_\sigma$ is the magnon-qubit detuning.

We introduce the mean-field amplitudes

$$x = \langle \hat{m} \rangle, \quad p = \langle \hat{\sigma} \rangle, \quad w = \langle \hat{\sigma}_z \rangle, \quad (\text{D2})$$

with

$$\hat{\sigma}_z = \hat{\sigma}^\dagger \hat{\sigma} - \hat{\sigma} \hat{\sigma}^\dagger. \quad (\text{D3})$$

Thus, $w = 1$ corresponds to an inverted qubit and $w = -1$ to the qubit ground state. Applying the standard factorization [81,82]

$$\langle \hat{m} \hat{\sigma}_z \rangle \simeq xw, \quad \langle \hat{m}^\dagger \hat{\sigma} \rangle \simeq x^* p, \quad \langle \hat{m} \hat{\sigma}^\dagger \rangle \simeq x p^*, \quad (\text{D4})$$

one obtains the Maxwell-Bloch equations via Eq. (A4),

$$\dot{x} = -\left(\frac{\gamma_m}{2} + i\Delta\right)x - ig_c p, \quad (\text{D5a})$$

$$\dot{p} = -\Gamma_\perp p + ig_c x w, \quad (\text{D5b})$$

$$\dot{w} = P_\sigma(1 - w) - \gamma_\sigma(1 + w) + 2ig_c(x^* p - x p^*). \quad (\text{D5c})$$

The transverse decoherence rate of the qubit is

$$\Gamma_\perp = \frac{\gamma_\sigma + P_\sigma + \gamma_\phi}{2} \quad (\text{D6})$$

for the convention used in Eq. (D1).

We now define the zero-coherent-field solution. In the present context, “zero field” means zero coherent magnon amplitude,

$$x = 0, \quad p = 0. \quad (\text{D7})$$

This condition should not be confused with $\langle m^\dagger m \rangle = 0$. In a full quantum treatment, spontaneous-emission processes may still generate a finite incoherent magnon population below onset. The mean-field onset criterion instead probes whether an infinitesimal coherent magnon seed is damped or amplified. For $x = p = 0$, the stationary unsaturated inversion of Eq. (D5c) is

$$w_0 = \frac{P_\sigma - \gamma_\sigma}{P_\sigma + \gamma_\sigma}. \quad (\text{D8})$$

The magnon-lasing onset is identified with the loss of linear stability of the zero-coherent-field solution. To see this explicitly, we add an infinitesimal coherent magnon seed and linearize Eqs. (D5a) and (D5b) around Eq. (D7). Since x and p are infinitesimal at onset, the inversion has not yet been depleted by the field, and we may set $w = w_0$. The linearized equations are then

$$\frac{d}{dt} \begin{pmatrix} x \\ p \end{pmatrix} = M \begin{pmatrix} x \\ p \end{pmatrix}, \quad (\text{D9})$$

where

$$M = \begin{pmatrix} -\frac{\gamma_m}{2} - i\Delta & -ig_c \\ ig_c w_0 & -\Gamma_\perp \end{pmatrix}. \quad (\text{D10})$$

If the eigenvalues of M are denoted by λ_i , the small-signal dynamics contains components proportional to $e^{\lambda_i t}$. A mode with $\text{Re}\lambda_i < 0$ decays in time, whereas a mode with $\text{Re}\lambda_i > 0$ is amplified. Therefore, the zero-coherent-field solution is linearly stable when all perturbations decay,

$$\max_i \text{Re}\lambda_i < 0, \quad (\text{D11})$$

and becomes unstable when at least one perturbation grows,

$$\max_i \text{Re}\lambda_i > 0. \quad (\text{D12})$$

The magnon-lasing onset is identified with the stability boundary

$$\max_i \text{Re}\lambda_i = 0, \quad (\text{D13})$$

where the small-signal gain exactly balances the total damping. Accordingly, we define the magnon-lasing onset by

$$\max_i \text{Re}\lambda_i(M) = 0. \quad (\text{D14})$$

This is the small-signal gain criterion [81,82]. It is independent of the linewidth narrowing and of the value of $g^{(2)}(0)$, which are used only as supporting signatures of the laserlike emission window.

At the stability boundary $\text{Re}\lambda = 0$, the critical eigenvalue satisfies $\lambda = i\Omega$. Substituting this into the characteristic equation gives

$$\left(\Gamma_\perp + \frac{\gamma_m}{2}\right)\Omega + \Gamma_\perp \Delta = 0. \quad (\text{D15})$$

Thus, for the resonant case $\Delta = 0$, one has $\Omega = 0$, and the critical eigenvalue reduces to $\lambda = 0$. The onset condition therefore becomes $\det M = 0$, yielding

$$g_c^2 w_0 = \frac{\gamma_m \Gamma_\perp}{2}. \quad (\text{D16})$$

This condition can be written in the form of a gain-loss balance. Eliminating the qubit polarization from the linearized equations (set $\dot{p} \simeq 0$) gives an effective equation for the magnon amplitude,

$$\dot{x} = \left(\frac{g_c^2 w_0}{\Gamma_\perp} - \frac{\gamma_m}{2}\right)x. \quad (\text{D17})$$

The first term,

$$G_{\text{amp}} = \frac{g_c^2 w_0}{\Gamma_\perp}, \quad (\text{D18})$$

is the small-signal amplitude gain supplied by the inverted qubit, whereas $\gamma_m/2$ is the magnon amplitude decay rate. The onset therefore occurs when $G_{\text{amp}} = \gamma_m/2$.

Substituting Eqs. (D8) and (D6) into Eq. (D16), the resonant onset pump P_{on} is determined by

$$g_c^2 \frac{P_{\text{on}} - \gamma_\sigma}{P_{\text{on}} + \gamma_\sigma} = \frac{\gamma_m}{4}(P_{\text{on}} + \gamma_\sigma + \gamma_\phi). \quad (\text{D19})$$

When $\gamma_\phi = 0$, Eq. (D19) can be solved analytically. The two roots are

$$P_\pm = \frac{4g_c^2 \pm \sqrt{16g_c^4 - 32\gamma_m g_c^2 \gamma_\sigma}}{2\gamma_m} - \gamma_\sigma. \quad (\text{D20})$$

The lower boundary P_- corresponds to the lasing onset, where the zero-coherent-field solution first becomes unstable. The upper boundary P_+ corresponds to a high-pump quenching boundary. The existence of two boundaries reflects the competition between the pump-induced inversion and the pump-induced broadening of the qubit polarization. Increasing P_σ enhances the unsaturated inversion w_0 , but it also increases $\Gamma_\perp$, thereby reducing the effective stimulated-emission gain $G_{\text{amp}} = g_c^2 w_0 / \Gamma_\perp$. Consequently, the small-signal gain can be positive only within a finite pump window,

$$P_- < P_\sigma < P_+. \quad (\text{D21})$$

This finite window is a characteristic feature of a single-emitter laser [17,19,62]: Excessive incoherent pumping broadens the emitter polarization and suppresses the coherent exchange responsible for stimulated-emission-like magnon buildup.

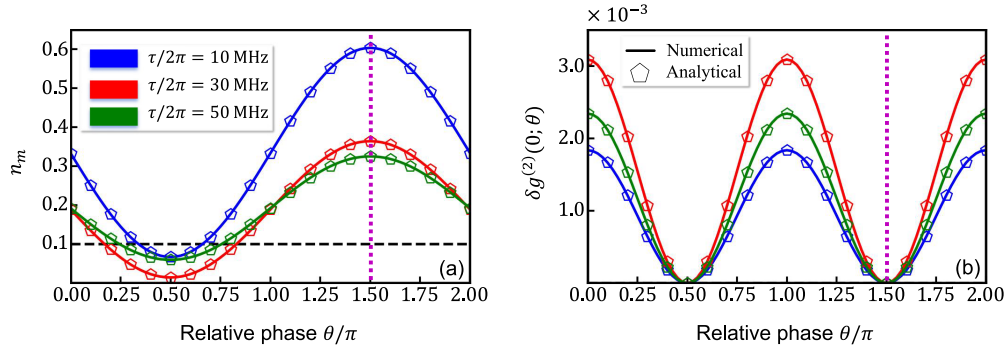


FIG. 5. (a) Mean magnon number n_m and (b) phase-shifted second-order correlation function $\delta g^{(2)}(0; \theta)$ for three different collective dissipation rates: $\tau/2\pi = 10$ MHz (blue), $\tau/2\pi = 30$ MHz (red), and $\tau/2\pi = 50$ MHz (green). Panels (a) and (b) correspond to the parameters: $\Delta = 0$, $g_c/2\pi = 10$ MHz, $\mu = \nu = 1$, $\gamma_m/2\pi = 2$ MHz, $\gamma_\sigma/2\pi = 1.2$ MHz, $\gamma_\phi = 0$, and $P_\sigma/2\pi = 100$ MHz. The vertical dotted line indicates the phase $\theta = 1.5\pi$ used in the main text.

For completeness, we also give the detuned criterion $\Delta \neq 0$. The characteristic equation of Eq. (D10) is

$$\left(\lambda + \frac{\gamma_m}{2} + i\Delta\right)(\lambda + \Gamma_\perp) - g_c^2 w_0 = 0. \quad (\text{D22})$$

At the stability boundary, we set $\lambda = i\Omega$. The imaginary part gives

$$\Omega = -\frac{\Gamma_\perp}{\Gamma_\perp + \gamma_m/2}\Delta, \quad (\text{D23})$$

and the real part gives

$$g_c^2 w_0(P_{\text{on}}) = \frac{\gamma_m \Gamma_\perp(P_{\text{on}})}{2} \left[1 + \frac{\Delta^2}{(\Gamma_\perp(P_{\text{on}}) + \gamma_m/2)^2} \right]. \quad (\text{D24})$$

Thus detuning increases the inversion required to reach the onset.

We emphasize that the above criterion is not a statement about a sharp thermodynamic phase transition [83,84]. The present system contains a single two-level gain element, so the onset should be understood as a finite-system small-signal instability of the zero-coherent-field solution. The linewidth narrowing and the approach of $g^{(2)}(0)$ to unity are not used to define the onset; rather, they provide supporting quantum signatures of the laserlike emission window identified independently by the gain-loss condition.

APPENDIX E: DEPENDENCE ON OTHER PARAMETERS

The collective dissipation channel is described by the jump operator $\hat{L}_j = \mu e^{i\theta} \hat{m} + \nu \hat{\sigma}$, where the relative phase θ controls the interference between the magnon and qubit decay pathways. To examine the phase dependence of the steady-state response, we vary $\theta \in [0, 2\pi)$ and evaluate both the steady-state mean magnon number n_m and the second-order intensity correlation function $g^{(2)}(0; \theta)$, while keeping all other system parameters fixed for each chosen value of τ . To highlight the phase dependence, we define the phase-shifted second-order correlation function

$$\delta g^{(2)}(0; \theta) \equiv g^{(2)}(0; \theta) - g_{\min}^{(2)}, \quad (\text{E1})$$

where $g_{\min}^{(2)} = \min_{\theta \in [0, 2\pi)} g^{(2)}(0; \theta)$ is the minimum value of $g^{(2)}(0; \theta)$ over the scanned phase interval for each fixed parameter set. Figure 5 shows that the mean magnon number n_m depends strongly on the relative phase and reaches its maximum near $\theta = 1.5\pi$. At the same time, $\delta g^{(2)}(0; \theta)$ is minimized at essentially the same phase over the range of τ considered here. This observation motivates our choice of $\theta = 1.5\pi$ in the main text, as it simultaneously enhances the mean magnon number and suppresses intensity fluctuations for the parameter regime studied. The results obtained from the steady-state NOMM approach agree well with direct numerical solutions of the full master equation, supporting the validity of the analytical treatment.

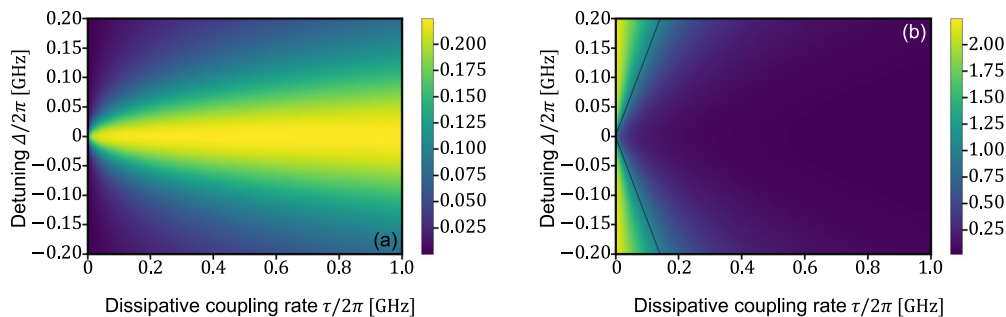


FIG. 6. (a) Mean magnon number n_m and (b) second-order correlation function $g^{(2)}(0)$. Panels (a) and (b) correspond to the parameters: $g_c/2\pi = 1$ MHz, $\mu = \nu = 1$, $\theta = 1.5\pi$, $\gamma_m/2\pi = 3.57$ MHz, $\gamma_\sigma/2\pi = 1.2$ MHz, $P_\sigma/2\pi = 10$ MHz, and $\gamma_\phi = 0$. The black solid line in panel (b) corresponds to $g^{(2)}(0) = 1$.

Finally, we examine the joint dependence of the steady-state response on the magnon-qubit detuning Δ and the dissipative coupling rate τ . Figure 6(a) shows the steady-state mean magnon number n_m in the Δ - τ plane. A clear ridge of enhanced n_m appears near $\Delta \simeq 0$, indicating that the mean magnon number is maximized close to resonance and remains elevated within a finite detuning window as τ increases. Figure 6(b) shows the second-order intensity correlation function $g^{(2)}(0)$. The black solid line marks the

boundary $g^{(2)}(0) = 1$, separating the bunched [$g^{(2)}(0) > 1$] and sub-Poissonian [$g^{(2)}(0) < 1$] regimes. For sufficiently large τ , a sizable sub-Poissonian region appears and overlaps with the high ridge of mean magnon number n_m in Fig. 6(a), indicating that a large magnon population and sub-Poissonian statistics can coexist over a finite detuning range. Taken together, Figs. 6(a) and 6(b) provide a compact parameter map for identifying the regime of interest in the (Δ, τ) plane.

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