Letter

Monogamy of nonlocal games

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Bell monogamy relations characterize the trade-offs in Bell inequality violations among pairs of players in multiplayer settings. In this work, we introduce a method for extending monogamy relations from a distinguished set of configurations to monogamy relations in all possible multiplayer settings. Applying this approach, we show that nonlocality in the Clauser, Horne, Shimony, and Holt game arises in only two cases: the original two-player scenario and the four-player scenario on a line. While the bound for this four-player scenario follows from known quadratic monogamy constraints, we also establish two six-party numerical monogamy relations that cannot be derived from existing results. In particular, we show there are points in the intersection of consecutive quadratic Bell monogamy relations which are not quantum realizable. Finally, we present a nonlocal game in which a single player can simultaneously saturate the quantum value with two other parties. This is an example of a nonlocal game unaffected by the monogamous nature of quantum entanglement.

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Introduction. Entanglement is one of the defining features of quantum mechanics. This resource can lead to correlations that no local hidden-variable theory can reproduce. The phenomenon associated with the existence of such correlations is known as *nonlocality*. On the other hand, entanglement cannot be freely shared among multiple parties. This fundamental limitation, established by Coffman, Kundu, and Wootters [1] and known as the *monogamy of entanglement (MOE)* [2–4], places constraints on how entanglement is distributed in multipartite scenarios. Understanding how MOE interacts with nonlocality is important both from a foundational perspective [5–7] and for a range of applications [8,9].

In 1964, Bell showed that quantum mechanics is incompatible with any local hidden-variable theory [10]. A few years later, Clauser, Horne, Shimony, and Holt [11] introduced a two-party test—now known as the CHSH nonlocal game—that demonstrates outcome statistics impossible to explain with classical means. Since then, nonlocality in the two-party setting has been extensively studied using various nonlocal games [12–14]. Monogamy can also be investigated using nonlocal games. Toner [15] considered a three-party extension of the CHSH game with Alice, Bob, and Charlie, where the test randomly involves either Alice and Bob or Bob and Charlie. Crucially, the players do not know ahead of time which player pair will be selected. In this configuration, monogamy implies that entanglement provides no advantage over classical correlations. Toner and Verstraete [16] then extended these results, characterizing the trade-off in nonlocality between the Alice–Bob and Bob–Charlie pairs.

The three-player scenario described above serves as a prototypical *Bell monogamy relation*, quantifying how MOE restricts simultaneous violations of multiple Bell inequalities. However, its optimal quantum-winning probability is $\frac{3}{4}$, achievable without any use of entanglement [15]. This bound gives rise to the following Bell monogamy relation: $\mathcal{B}_{AB} + \mathcal{B}_{BC} \leqslant 4$.

In this work, we formalize these multipartite scenarios by extending symmetric two-player games $\mathcal{G}$ to n-player games $\mathcal{G}^H$, where players occupy the vertices of a graph $H = (V, E)$ (see Fig. 1). A referee selects an edge uniformly at random and plays $\mathcal{G}$ with the players on the vertices. The resulting Bell monogamy relation is given by the average of the two-party Bell violations across all edges.

We introduce a technique which allows one to extend Bell monogamy relations on smaller fundamental graphs, to obtain new Bell monogamy relations on all possible configurations with any number of players. In particular this allows us to give general sufficient conditions under which no quantum strategy can outperform the optimal classical strategy for any graph with more than two players. We call a two-player game satisfying this property *monogamous*.

We then apply these criteria to a range of well-known two-player games, fully determining their behavior on all graphs H. Surprisingly, for the CHSH game, average case nonlocality appears only in two scenarios: the original two-player setting and the four-player setting played on the path graph P_4 (see Fig. 2). It can be shown that for the cases of P_3 and P_4, the set of quantum realizable correlations is determined exactly by studying the intersection of consecutive quadratic Bell monogamy relations, $\mathcal{B}_{XY}^2 + \mathcal{B}_{YZ}^2 \leqslant 8$. Numerically, we confirm that there is no quantum advantage on two six-party arrangements but that studying consecutive quadratic Bell monogamy relations is not strong enough to determine this.

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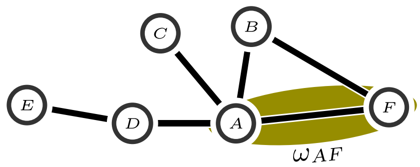

FIG. 1. Several players who share a global state are represented as the vertices of a graph H. A pair of players are chosen to play a symmetric two-player game $\mathcal{G}$, by sampling an edge from H. The edge (A, F) is chosen and the pair of players win the game $\mathcal{G}$ with probability ω_{AF}.

This shows that taking the intersection of the quadratic Bell monogamy relations does not exactly characterize the set of quantum correlations on six parties as it does on P_3 and P_4 [16,17].

In contrast, we establish that other well-studied games, such as the odd cycle game [18], are monogamous. Finally, we give an example of a two-player nonlocal game for which it is not possible to obtain a Bell monogamy relation for the three-party configuration on a line. In particular, Bob can saturate the quantum value with both Alice and Charlie simultaneously. This is an example of a nonmonogamous Bell inequality.

We note that our techniques are applicable in a broader context. In particular, they can be used to determine when no advantage exists over classical strategies for commuting-operator correlations [19,20] or even more general nonsignaling correlations [21]. The latter are especially important for understanding the boundary between physically realizable and nonphysical theories [22–24].

In previous work, Bell monogamy relations have been studied in the context of correlation complementarity, where n observers each perform two measurements yielding dichotomic ± 1 outcomes [25,26]. Tran *et al.* [17] investigated Bell monogamy relations for observers arranged in a network, as illustrated in Fig. 1, and derived a family of tight monogamy relations for an arbitrary number of observers. Unlike our approach, their method selects edges of the graph H according to some nonuniform distribution determined by the line graph $L(H)$. These monogamy relations follow directly from combining a three-party Bell monogamy relation with the handshaking lemma from graph theory. By contrast, the monogamy relations we establish cannot, in general, be obtained solely from such three-party relations.

Of particular relevance is Tran *et al.*'s [17] analysis of general Bell inequalities on P_4, which characterizes the quantum trade-offs imposed by three-party Bell monogamy relations. Our optimal P_4 strategy for CHSH falls within this characterization.

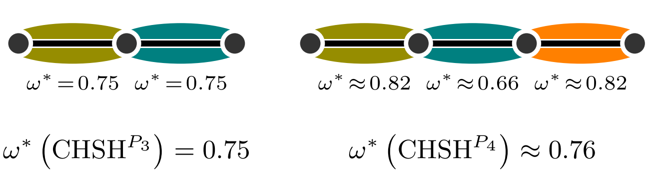

FIG. 2. Optimal quantum-winning probabilities of the CHSH game on the path graphs P_3 (left) and P_4 (right).

FIG. 3. The strategy graph of the CHSH game. The vertices on the extreme left and right are the constant strategies, while the two intermediate vertices correspond to the identity and flip strategies.

Nonlocal games over graphs. In a two-player *nonlocal game* $\mathcal{G}$ (or just *game* in this Letter), the players Alice and Bob are given questions $x_1, x_2 \in \mathcal{I}$, sampled independently according to a uniform distribution, and provide answers $a_1, a_2 \in \mathcal{O}$ according to a quantum correlation $p(a_1, a_2|x_1, x_2)$. The winning condition is determined by a referee predicate $v(x_1, x_2, a_1, a_2)$. A game is *symmetric* if the predicate satisfies the symmetric condition: $v(x_1, x_2, a_1, a_2) = v(x_2, x_1, a_2, a_1)$. In this Letter all games considered are symmetric nonlocal games. We write $\omega(\mathcal{G}; S)$ for the winning probabilities of a game $\mathcal{G}$ given a strategy S. The maximal classical and quantum winning probabilities are denoted $\omega(\mathcal{G})$ and $\omega^*(\mathcal{G})$, respectively. We say that the game $\mathcal{G}$ has a *quantum advantage* if $\omega(\mathcal{G}) < \omega^*(\mathcal{G})$.

In this Letter, a *graph* $H = (V, E)$ is a no multiple-edge undirected graph, where V, E are finite vertex and edge sets, respectively. We denote the path graph on n vertices by P_n. The family of graphs $\mathcal{T}_k$ denotes the collection of trees of size $2k - 1$ formed by joining k copies of P_2 using $k - 1$ edges (see as illustrated in Fig. 5).

A *graph homomorphism* between a graph H_1 and a graph H_2 is a function that maps the vertices of H_1 to the vertices of H_2 that preserves adjacency: adjacent vertices in H_1 are mapped to adjacent vertices in H_2. We write $H_1 \to H_2$ if there exists a homomorphism from H_1 to H_2. For example, bipartite graphs are precisely those that admit a graph homomorphism to P_2. Moreover, if the strategy graph contains a loop, then any graph admits a graph homomorphism to it.

Definition 1. The *strategy graph* $\mathcal{S}_\mathcal{G} = (V, E)$ of a game $\mathcal{G}$ is a (not necessarily simple or connected) graph, with vertices consisting of functions $f : \mathcal{I} \to \mathcal{O}$, and with edges consisting of the pairs of functions (f_A, f_B) that achieve the game's optimal classical winning probability $\omega(\mathcal{G})$ (see for example Fig. 3).

Definition 2. Given a simple connected graph $H = (V, E)$, with $|V| = n$ and a game $\mathcal{G}$, with referee predicate $v(x_1, x_2, a_1, a_2)$, we define an n-player game $\mathcal{G}^H$ with referee predicate

$$v^H(x_1, \dots, x_n, a_1, \dots, a_n) := \frac{1}{|E|} \sum_{(i,j)\in E} v(x_i, x_j, a_i, a_j).$$

A straightforward computation shows that for given strategy S

$$\omega(\mathcal{G}^H; S) = \frac{1}{|E|} \sum_{e\in E} \omega(\mathcal{G}; S|_e),$$

where $\omega(\mathcal{G}, S|_e)$ denotes the winning probability of the marginal strategy of S on edge e, as shown in Fig. 1. A game $\mathcal{G}$ is said to have *quantum advantage on a graph H* if $\omega(\mathcal{G}^H) < \omega^*(\mathcal{G}^H)$.

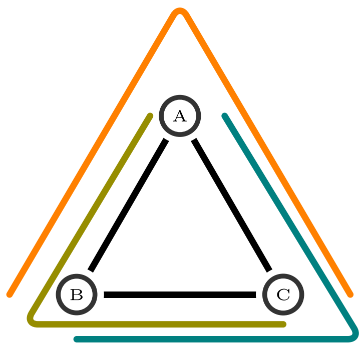

FIG. 4. An example of a covering of the three-cycle graph C_3 by weighted copies of the path graph P_3. Each P_3 subgraph in C_3 (surrounding lines) has the 1/2 weight. Each edge in C_3 is covered by exactly two P_3 subgraphs.

Our main theorem gives a method to determine for which graphs H the quantum value collapses to the classical two-player value, $\omega(\mathcal{G}) = \omega(\mathcal{G}^H) = \omega^*(\mathcal{G}^H)$.

It should be noted that the classical value of a game is not generally the same in all graphs. For example, consider a two-player binary-output game with referee predicate $v(x_1, x_2, a_1, a_2) = 1 - \delta_{a_1 a_2}$ (i.e., the outputs of the two players are required to be different). Then the optimal classical value is 1, but it is not possible for three players in the C_3 (the three-cycle) configuration to obtain this value. In contrast, for games where the strategy graph contains a loop, the classical value of a game is the same in all graphs.

Below, we give a characterization of when the classical value does not decrease on a graph H.

Lemma 1. The classical value of a game $\mathcal{G}$ on a graph H is the same as the classical value of the game if and only if there is a graph homomorphism $H \to \mathcal{S}_{\mathcal{G}}$.

Proof. If the classical value of $\mathcal{G}^H$ equals the classical value of $\mathcal{G}$, then any optimal classical deterministic strategy for $\mathcal{G}^H$ induces a graph homomorphism from H to $H^{\mathcal{G}}$. Specifically, each vertex of H is assigned a classical deterministic strategy for $\mathcal{G}$. For each edge in H, the paired strategies achieve exactly the winning probability $\omega(\mathcal{G})$; if this were not the case, we would have $\omega(\mathcal{G}) < \omega(\mathcal{G}^H)$, contradicting our assumption. Consequently, this pair forms an edge in $H^{\mathcal{G}}$. Since any optimal classical strategy can be taken to be deterministic by convexity, this completes the first direction of the proof.

For the converse, suppose there exists a graph homomorphism from H to the strategy graph $H^{\mathcal{G}}$. This graph homomorphism defines an optimal classical deterministic strategy for $\mathcal{G}^H$ by assigning a deterministic strategy of $\mathcal{G}$ to each vertex in H and ensuring that for any edge in H, the winning probability is precisely $\omega(\mathcal{G})$. ■

In particular, the classical value of a game on a bipartite graph is always equal to the classical value of the corresponding two-player game.

We then give a technique for establishing quantum upper bounds for families of graphs. Before stating this theorem, we provide an illustrative example of how, for certain graphs H, the quantum value $\omega^*(\mathcal{G}^H)$ can be upper bounded by the quantum value on P_3. In Fig. 4 we consider playing the game on the three-cycle graph, C_3, with vertices A, B, and C. One can uniformly cover C_3 with three weighted copies, P_{ABC}, P_{BCA}, and P_{CAB} of the graph P_3. Since each edge in the original graph is covered by exactly two copies of P_3, it can be shown that $\omega(\mathcal{G}^{C_3}; S) \leqslant \omega^*(\mathcal{G}^{P_3})$ for any strategy S:

$$
\begin{aligned}
&\omega^*(\mathcal{G}^{C_3}; S) \\
&\quad = \tfrac{1}{3}(\omega^*(\mathcal{G}_{AB}; S|_{AB}) + \omega^*(\mathcal{G}_{BC}; S|_{BC}) + \omega^*(\mathcal{G}_{CA}; S|_{CA})) \\
&\quad \leqslant \tfrac{2}{3}\big(\tfrac{1}{2}\omega^*(\mathcal{G}^{P_{ABC}}) + \tfrac{1}{2}\omega^*(\mathcal{G}^{P_{BCA}}) + \tfrac{1}{2}\omega^*(\mathcal{G}^{P_{CAB}})\big) \\
&\quad = \omega^*(\mathcal{G}^{P_3}).
\end{aligned}
$$

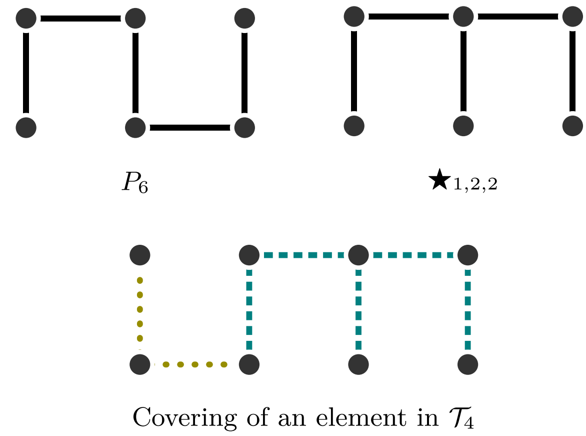

FIG. 5. The graph family $\mathcal{T}_3$ contains two graphs: the path graph P_6 and the tree $\bigstar_{1,2,2}$. Below, a covering of graph in $\mathcal{T}_4$ using one P_3 subgraph, formed using the three nodes on the left, and a subgraph from $\mathcal{T}_3$ on the right.

Hence, we see that if graph H can be suitably decomposed using weighted copies of P_3, then we can readily conclude $\omega^*(\mathcal{G}^H) \leqslant \omega^*(\mathcal{G}^{P_3})$.

Lemma 2. If the quantum value of a game on P_3 is bounded by v, then the quantum value is bounded above by v for all graphs not in $\mathcal{T}_k$, for all $k \geqslant 2$.

Our proof, which is given in Supplemental Material [27], establishes a correspondence between P_3 decompositions of a graph and fractional perfect matchings of its line graph. We then use a well-known characterization of fractional perfect matchings to derive exactly when a graph has a P_3 decomposition. The full proof is contained in the Supplemental Material.

Combining Lemmas 1 and 2, we obtain our main result.

Theorem 1. If there is no quantum advantage of a game $\mathcal{G}$ on P_3, then the game does not have quantum advantage on any graph H such that $H \to \mathcal{S}_{\mathcal{G}}$ and H is not in $\mathcal{T}_k$, for all $k \geqslant 2$.

Next, we show that we can further restrict our attention using the inductive nature of the families of graphs $\mathcal{T}_k$. As shown in Fig. 5, understanding the behavior on P_3 and graphs from $\mathcal{T}_k$ for a fixed value of k is sufficient to conclude the behavior on $\mathcal{T}_{k+1}$. The details are given in the Supplemental Material.

Theorem 2. Suppose a game $\mathcal{G}$ does not have quantum advantage on P_3 and all graphs in $\mathcal{T}_k$, for some k; then, the game has no quantum advantage on all graphs $\mathcal{T}_{k+1}$.

Noticing that $\mathcal{T}_2 = \{P_4\}$ and applying Theorems 1 and 2, we obtain the following corollary which characterizes monogamous games, i.e., games for which any extra connectivity is sufficient to completely inhibit the expression of nonlocality.

Corollary 1. A game has no quantum advantage on P_3 and P_4 if and only if there is no advantage on any graph H such that $H \to \mathcal{S}_{\mathcal{G}}$.

We note that the results of this section also hold if one were to replace quantum advantage with advantage from any given family of correlations, such as nonsignaling correlations.

Next, we will exhibit that numerous well-studied nonlocal games satisfy the above criteria and that the CHSH game does not.

Illustrative examples

The CHSH nonlocal game. The Bell operator for the CHSH game is given by

$$\mathcal{B} = A_0B_0 + A_0B_1 + A_1B_0 - A_1B_1.$$

The classical and quantum bounds for this operator are 2 and $2\sqrt{2}$, respectively. The corresponding CHSH game has classical winning probability of 3/4 and the quantum value is given by the Tsirelson bound of $\frac{1}{2} + \frac{1}{2\sqrt{2}}$ [28]. Since the strategy graph of this game is a loop (see Fig. 3), the classical value of 3/4 holds when playing CHSH on all graphs.

Here, we provide the sums-of-squares proof [29] that the CHSH game has no quantum advantage when played on the graph P_3:

$$2 - \tfrac{1}{2}(\mathcal{B}_{AB} + \mathcal{B}_{BC}) = Q_1^2 + Q_2^2, \tag{1}$$

where Q_1 and Q_2 are as follows:

$$Q_1 := \frac{1}{2\sqrt{2}}(A_0(B_0 - B_1) + C_1(B_0 + B_1) - 2A_0C_1)$$
$$Q_2 := \frac{1}{2\sqrt{2}}(A_1(B_0 + B_1) + C_0(B_0 - B_1) - 2A_1C_0).$$

We show below that there exists a quantum strategy which wins with probability $\frac{1}{2} + \frac{\sqrt{10}}{12} \approx 0.76 > 3/4$ on the P_4 graph. Furthermore, this strategy is optimal. We, once again, show this upper bound via a sum-of-squares decomposition,

$$\frac{2\sqrt{10}}{3} - \frac{1}{3}(\mathcal{B}_{AB} + \mathcal{B}_{BC} + \mathcal{B}_{CD}) = R_1^2 + R_2^2 + R_3^2 + R_4^2, \tag{2}$$

where $R_1, \ldots, R_4$ are given in the Supplemental Material [27] and $\frac{2\sqrt{10}}{3}$ is the bias corresponding to the optimal winning probability.

The quantum state ρ and measurements which saturates this bound are given in the Supplemental Material. The positive-partial-transpose (PPT) criterion [30,31] reveals that the pairs (ρ_{AB}, ρ_{CD}), (ρ_{AC}, ρ_{BD}) and (ρ_{AD}, ρ_{BC}) are all entangled. The local values of the CHSH game on P_4 are given in Fig. 2.

As a consequence of the quantum advantage on P_4, Corollary 1 cannot be applied for the CHSH game. Instead, we will use Theorems 1 and 2 with $k = 3$ to conclude the behavior on all other graphs, since for all graph H, we have $H \to \mathcal{S}_{\mathcal{G}}$. The graph family $\mathcal{T}_3$ contains only two graphs (up to isomorphism) as shown in Fig. 5.

Using the semidefinite hierarchy due to Navascués *et al.* [32,33], we confirm numerically no quantum advantage for the CHSH game on either graph. Thus, CHSH exhibits nonlocality only on the graphs P_2 and P_4.

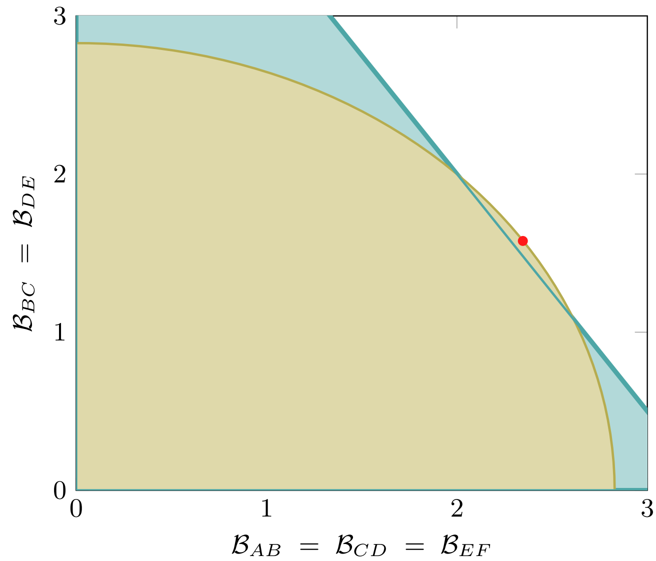

FIG. 6. A slice of the correlation set for the CHSH game with six players on the path graph P_6 is shown, corresponding to the subspace defined by $AB = CD = EF$ and $BC = DE$. The disk of radius $2\sqrt{2}$ represents the intersection of the regions imposed by the quadratic monogamy relations, while the linear region is defined by our monogamy relation. Note that some points, although lying within the intersection of the quadratic relations, do not satisfy our monogamy relation, for instance the point (61/26, 41/26).

We would like to note that the monogamy relation provided above of $\mathcal{B}_{AB} + \mathcal{B}_{BC} + \mathcal{B}_{CD} \leqslant 2\sqrt{10}$ is implied by the previously known quadratic Bell monogamy relation of $\mathcal{B}_{AB}^2 + \mathcal{B}_{BC}^2 \leqslant 8$ given by Toner and Verstraete [16]. Indeed, by the Cauchy-Schwarz inequality,

$$\begin{aligned}\mathcal{B}_{AB} + \mathcal{B}_{BC} + \mathcal{B}_{CD} &\leqslant \sqrt{\mathcal{B}_{AB}^2 + 2\mathcal{B}_{BC}^2 + \mathcal{B}_{CD}^2}\sqrt{2 + \frac{1}{2}} \\ &\leqslant \sqrt{16}\sqrt{2 + \frac{1}{2}} \\ &= 2\sqrt{10}.\end{aligned}$$

As such, this monogamy relation does not imply any more information on the feasibility of certain quantum correlations more than what is implied by the quadratic Bell monogamy relations. In contrast to this, our numerical monogamy relations of $\mathcal{B}_{AB} + \mathcal{B}_{BC} + \mathcal{B}_{CD} + \mathcal{B}_{DE} + \mathcal{B}_{EF} \leqslant 10$ and $\mathcal{B}_{AB} + \mathcal{B}_{BC} + \mathcal{B}_{CD} + \mathcal{B}_{CF} + \mathcal{B}_{EF} \leqslant 10$ (cf. Fig. 5) do actually provide Bell monogamy relations. Consider, for example, setting $\mathcal{B}_{AB} = \mathcal{B}_{CD} = \mathcal{B}_{EF} = \frac{61}{26}$, and $\mathcal{B}_{BC} = \mathcal{B}_{DE} = \frac{41}{26}$. This point will be in the feasible region implied by $\mathcal{B}_{XY}^2 + \mathcal{B}_{YZ}^2 \leqslant 8$ for any three consecutive X, Y, Z. On the other hand, $\mathcal{B}_{AB} + \mathcal{B}_{BC} + \mathcal{B}_{CD} + \mathcal{B}_{DE} + \mathcal{B}_{EF} > 10$ (cf. Fig. 6).

The odd-cycle nonlocal game. We also remark that for the well-known n-vertex odd-cycle game [18], such average case advantage is not possible on any graph other than P_2. First, note that the optimal classical strategy, achieving $1 - \frac{1}{2n}$, is symmetric and thus by Lemma 1 the classical value is preserved on all graphs. Using the Navascués, Pironio, and Acín (NPA) hierarchy, one can establish that this value upper bounds the quantum value on both P_3 and P_4 graphs and thus by Corollary 1, this bound holds for all graphs.

Polygamous nonlocality. In this section, we resolve the question about the existence of a nonlocal game (with a quantum advantage) whose quantum winning probability remains equal when extending the standard two-player configuration to three players distributed along a path graph, i.e., $\omega^*(\mathcal{G}) = \omega^*(\mathcal{G}^{P_3})$.

We provide a positive solution to this question by introducing the following construction. Let $\mathcal{G}$ denote any two-player nonlocal game satisfying $\omega^*(\mathcal{G}) = 1$ and $\omega(\mathcal{G}) \leqslant \frac{1}{4}$. We then define a nonlocal game, $\mathcal{G}^{\vee 2}$, as two simultaneous instances of $\mathcal{G}$. The referee's input is a pair of questions for each instance of $\mathcal{G}$, and the players' outputs are a corresponding pair of answers. The winning condition is satisfied if at least one of the two instances is won. Formally, the referee's predicate is

$$\begin{aligned} v((x_1, x_2), (y_1, y_2), (a_1, a_2), (b_1, b_2)) \\ = v_{\mathcal{G}}(x_1, y_1, a_1, b_1) \vee v_{\mathcal{G}}(x_2, y_2, a_2, b_2), \end{aligned}$$

where $v_{\mathcal{G}}(\cdot)$ is the referee predicate for the original $\mathcal{G}$ game.

The quantum winning probability for $\mathcal{G}$ with two players is 1, and in the case of three players arranged as P_3 the quantum value remains 1. To see this, let $|\mathcal{G}\rangle$ be a bipartite state that wins the game $\mathcal{G}$ with probability 1. The global state shared by the three players A, B, C is given by

$$\underbrace{(|\mathcal{G}\rangle_{AB} \otimes |0\rangle_C)}_{=|\psi\rangle} \otimes \underbrace{(|0\rangle_A \otimes |\mathcal{G}\rangle_{BC})}_{=|\phi\rangle}.$$

The measurement strategy is as follows: for the first instance, the players use the state $|\psi\rangle$: thus, A and B use $|\mathcal{G}\rangle_{AB}$ and win with probability 1; for the second instance, the players use the state $|\phi\rangle$: thus, B and C use $|\mathcal{G}\rangle_{BC}$, also winning with probability 1. Consequently, on each edge of P_3 at least one instance of $\mathcal{G}$ is always won with certainty, giving $\omega^*(\mathcal{G}^{P_3}) = 1$.

To see that this is not possible for any classical strategy note that for two players

$$\begin{aligned} \omega(\mathcal{G}^{\vee 2}) &= \Pr(\mathrm{W}_1 \wedge \mathrm{W}_2) + \Pr(\mathrm{W}_1 \wedge \mathrm{L}_2) + \Pr(\mathrm{W}_2 \wedge \mathrm{L}_1) \\ &\leqslant 2\Pr(\mathrm{W}_1) + \Pr(\mathrm{W}_2) \leqslant \tfrac{3}{4}, \end{aligned}$$

where W_i and L_i are the events that the ith instance of the game $\mathcal{G}$ is won or lost, respectively. Since P_3 is a bipartite graph, there is a graph homomorphism from P_3 to the strategy graph for $\mathcal{G}$. Using Lemma 1 it follows that $\omega(\mathcal{G}^{P_3}) = \omega(\mathcal{G}) \leqslant \frac{3}{4}$.

To show that such a game $\mathcal{G}$ exists, let MS denote the symmetric magic square game [12,13,34], which has a classical winning probability $\omega(\mathrm{MS}) = \frac{35}{36}$ and a quantum winning probability $\omega^*(\mathrm{MS}) = 1$. Then take $\mathcal{G} = \mathrm{MS}^n$, to denote the nth-fold parallel repetition of MS. By Raz's parallel repetition theorem [35] we have that $\omega(\mathcal{G}) \leqslant \frac{1}{4}$ for large enough n. We note that it is possible for a game to have $\omega(\mathcal{G}) < 1$, whereas $\omega(\mathcal{G}^{\vee 2}) = 1$. This can be shown to be the case for the magic square game. Consequently the use of parallel repetition above is required.

Discussion. In conclusion, in this Letter we provide a general graph-theoretic method which allows us to extend Bell monogamy relations on certain smaller configurations to an infinite family of configurations. We apply this result to the well-known games CHSH and the odd-cycle games. For CHSH, we find Bell monogamy relations between six parties which cannot directly be inferred from known monogamy relations on P_3, and demonstrate that this relation is sometimes more informative. Finally, we give the example of a game where the quantum value on P_3 is 1 for both pairs of players simultaneously.

We conclude this Letter with future directions:

(1) The quadratic Bell monogamy relation of Toner and Verstraete has the property that any numerical values that satisfy the inequality are physically realizable [16]. Is there an analogous Bell monogamy relation for six parties?

(2) We can achieve quantum advantage on any degree-bounded graph. To see this, given a degree bound d, consider the analogous "OR parallel repetition game" $\mathcal{G}^{\vee 2d-1}$ for some $\mathcal{G}$ with sufficiently small $\omega(\mathcal{G})$. Is it possible to have a game which has advantage on *all* graphs?

(3) One can consider extending instead k-player nonlocal games to k-uniform hypergraphs as considered in Tran *et al.* [17]. Can we extend our graph decomposition techniques to this more general setting?

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