

Atmospheric transfer of kinetic energy across planetary-scale circulation and weather

Pejman Hadi Sichani  and Benjamin A. Storer 

Department of Mechanical Engineering, [University of Rochester](#), Rochester, New York, USA

Hussein Aluie *

Department of Mechanical Engineering, [University of Rochester](#), Rochester, New York, USA;

Department of Mathematics, [University of Rochester](#), Rochester, New York, USA;

and Laboratory for Laser Energetics, [University of Rochester](#), Rochester, New York, USA



(Received 4 December 2025; accepted 6 July 2026; published 27 August 2026)

Earth's atmospheric circulation spans more than 10 orders of magnitude in spatial scale, and nonlinear interactions across this vast range remain a major barrier to long-term forecasting. Cross-scale atmospheric coupling is often diagnosed using scale-analysis approaches that are inherently global and obscure geographic inhomogeneity. Here, we apply a recently developed spherical coarse-graining method to map the kinetic energy (KE) cascade across scales from the planetary circulation (>3000 km) down to the synoptic (3000–1000 km) and mesoscale (1000–100 km) ranges that underpin most weather systems. While the cascade maps generally complement current understanding, including the energization of polar jet streams by sub-planetary-scale motions, they also reveal unexpected energy transfers. In the troposphere, we find direct KE exchanges between the planetary-scale circulation and smaller scales, including mesoscale motions. The exchanges occur in latitudinal bands of alternating upscale and downscale KE transfer driven by the Hadley, Ferrel, and polar cells, mirroring recent results in the ocean. The physics behind such transfer is not due to a classical turbulence cascade, but is due to the action of planetary-scale strain against anisotropic effective pressure arising from sub-planetary-scale motions. This anisotropic pressure-strain mechanism is analogous to a piston acting against the pressure of a gas, except that the effective pressure here is tensorial and highly anisotropic: It resists compression or rarefaction primarily along horizontal directions where synoptic and mesoscale motions are strongest. The mechanism remains significant after vertical integration because the sub-planetary-scale motions, and hence their effective stress, are vertically inhomogeneous. These findings offer a unique perspective for exploring the interplay between weather and climate.

DOI: [10.1103/ljfl-rm75](https://doi.org/10.1103/ljfl-rm75)

I. INTRODUCTION

Earth's atmospheric flow occurs on a vast range of scales, from planetary-scale circulation patterns that regulate climate to smaller, regional circulation that influence daily weather. The link between planetary-scale atmospheric circulation and smaller weather patterns is fundamental in meteorology and climate science. In fact, the oft-mentioned two-week limit on our ability to forecast weather [1–3] is primarily due to the nonlinear coupling across this vast range of scales [4].

Present understanding of atmospheric scale interactions is shaped largely by idealized turbulence theories in two dimensions (2D) [5–7] and three dimensions (3D) [8–10]. Central to these theories is the concept of kinetic energy (KE) cascade [11]. In quasi-2D flow that is often assumed to

characterize the rotating, stratified atmosphere at scales larger than $O(1000)$ km [12], KE is transferred upscale, for example, through vortex mergers that generate larger and more coherent structures [13,14]. In contrast, in 3D flows, characteristic of atmospheric motions at scales smaller than $O(100)$ km [12], KE is transferred downscale as vortices break down into smaller, faster eddies [15,16]. These idealized theories, however, assume statistical homogeneity and, therefore, neglect spatially varying geographic conditions and the planetary-scale background circulation—including jet streams and the Hadley, Ferrel, and polar cells—that dominate the real atmosphere.

Oftentimes in atmospheric studies, the cascade direction is inferred only indirectly from the scaling of the KE wave number spectrum $E(k)$ (e.g., Refs. [17–21]). The inference is qualitative at best, leaving both the magnitude and direction of energy scale transfer ambiguous. While phenomenological inertial range scaling, e.g., $E(k) = C_K \epsilon^{2/3} k^{-5/3}$, is associated with a constant energy flux, inferring ϵ from the spectrum $E(k)$ requires additional assumptions: a prescribed direction of transfer (upscale or downscale), a scale-local cascade, the existence of a true inertial range with a constant flux across scales, negligible forcing or damping within that range, a

*Contact author: hussein@rochester.edu

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](#) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

value for the Kolmogorov constant C_K , and isotropy. These assumptions are seldom satisfied in the atmosphere.

Of the studies that analyze the cascade directly, a few relied on so-called structure functions (e.g., Refs. [22–24]) that assume homogeneity and do not yield spatial maps of the transfer. The most common approach for analyzing the cascade directly (e.g., Refs. [25–37]) employs spectral methods [5,38]. By using global basis functions, spectral analysis is inherently global and cannot easily reveal the geographic variability of scale transfer. In fact, spectral decompositions are energy preserving (Parseval’s theorem) only under global averaging and not locally in space (see Sec. IV). Geographical information regarding the KE cascade has the potential to provide insights into how the planetary-scale circulation is coupled to smaller-scale weather patterns. The only previous attempt to map the atmospheric cascade [39] was limited to mesoscale (≈ 200 km) motions because their method’s KE definition is not locally positive definite, and the incidence of nonpositive values increases at larger scales.

Below, we present the first geographic maps of the atmospheric KE transfer between the planetary-scale circulation (>3000 km) and smaller scales. The maps provide clear evidence of a two-way mechanical coupling between subplanetary scales (<3000 km) that underpin most weather systems and the planetary-scale atmospheric cells, i.e., Hadley, Ferrel, and polar cells. The work here relies on a recent commutative coarse-graining methodology [40–44] that can decompose length scales and probe their energy exchange locally in space. The approach is deterministic and does not rely on statistical (ensemble) averaging that underlies traditional mean-eddy approaches [12,45,46]. Employing this framework, we are able to obtain time-dependent geographical maps of the KE cascade across the entire range of resolved spatial scales.

II. RESULTS AND DISCUSSION

Our methodology is applied to velocity fields from the latest global atmospheric reanalysis [47]. We probe a range of length scales, ℓ , spanning 30 km to the equatorial circumference of the Earth ($\approx 40 \times 10^3$ km). Details on coarse graining, the dataset, and the presented diagnostics are provided in Sec. IV. Our results revolve around two key quantities. First is the KE scale transfer (or cascade), Π_ℓ , which measures the amount of KE transferred from scales larger than ℓ to scales smaller than ℓ at any geographic location, and is signed so that a positive/negative value indicates a downscale/upscale energy transfer [40]. Second is the filtering spectrum [44,48], $\bar{E}(k_\ell) = dKE^{>\ell}/dk_\ell$, which measures spectral KE density as a function of length scale, where $k_\ell = \ell^{-1}$ is filtering wave number and $KE^{>\ell}$ is the KE contained at all scales larger than ℓ .

A. Mapping the cascade

Figure 1 presents global maps of KE transfer, Π_ℓ , between the planetary circulation at scales larger than $\ell = 3000$ km and motions at the subplanetary scales (<3000 km), which include synoptic (3000–1000 km) and mesoscale (1000–100 km) ranges that underpin most weather systems. These geographic maps highlight a key advantage of the coarse-

graining methodology over traditional approaches such as spherical harmonics, which are inherently global and do not easily yield spatial information. KE transfer between the planetary-scale circulation and smaller scales is shown in Fig. 1, which presents the two-year averaged Π maps at three different altitudes. Π is computed from hourly fields sampled 4 times per day (see Sec. IV for details) before time averaging its geographic maps. It is important that Π , which is nonlinear, is computed from individual hourly fields prior to time averaging, so that the dynamical contributions from weather scales are retained. In addition to these two-year averaged results, we provide maps of Π for individual hourly fields (Fig. 5) and seasonal averages (Figs. 6 and 7) in Appendix B.

(a) *Planetary circulation cells.* Figure 1(c) (surface) reveals an imprint of the Hadley, Ferrel, and polar circulation cells. This is seen from alternating zonal (east-west) bands of upscale (blue) and downscale (red) transfer, which are reminiscent of recent results in the ocean [43]. The overlaid streamlines (derived from the laterally divergent component of the flow) in Fig. 1 are calculated self-consistently using our coarse-graining framework and show the planetary-scale circulation cells (see Sec. IV). The planetary-scale circulation cells that emerge from our framework should be interpreted as Eulerian components of the flow, which participate in scale-to-scale KE transfer but do not represent Lagrangian-mean circulations; therefore, they do not directly imply material transport pathways. Equatorial convergence of the Hadley cells at the surface is associated with a downscale transfer (red) zonal band seen just north of the equator in Fig. 1(c). Figure 1(d) makes this clear from the zonally averaged Π : At 1000 hPa, there is a positive peak around 8°N . Indeed, this band is strongly correlated with the two-year averaged precipitation belt of the Intertropical Convergence Zone [ITCZ [49]; see also Fig. 8(b)].

The physics behind such scale transfer in Fig. 1(c) is due to anisotropic pressure-strain interaction, which was described in Ref. [43] as a piston mechanism. At planetary scales (>3000 km), the flow experiences an effective stress from sub-planetary-scale motions [Eq. (6)], which is mainly due to the synoptic scales and mesoscales. As sketched in Fig. 9, near-surface convergence flow associated with the Hadley cells amplifies the KE of smaller scales through an anisotropic pressure-strain interaction. Although planetary-scale horizontal convergence is balanced by vertical divergence, the vertical contribution to Π_ℓ is much weaker because the sub-planetary-scale stress is highly anisotropic. Specifically, the horizontal stress associated with synoptic and mesoscale KE is much larger than the vertical stress component. This reflects the constraints of rotation and stratification on atmospheric motion: Synoptic-scale horizontal velocities are typically about 3 orders of magnitude larger than vertical velocities, so the corresponding KE or stress components can be about 6 orders of magnitude larger [50]. As a result, the horizontal contribution dominates, leading to a net positive transfer of KE from large to small scales. This anisotropic pressure-strain mechanism is analogous to a piston acting against the pressure of a gas, except that the effective pressure here is tensorial and highly anisotropic: It resists compression or rarefaction primarily along horizontal directions where synoptic and mesoscale motions are strongest.

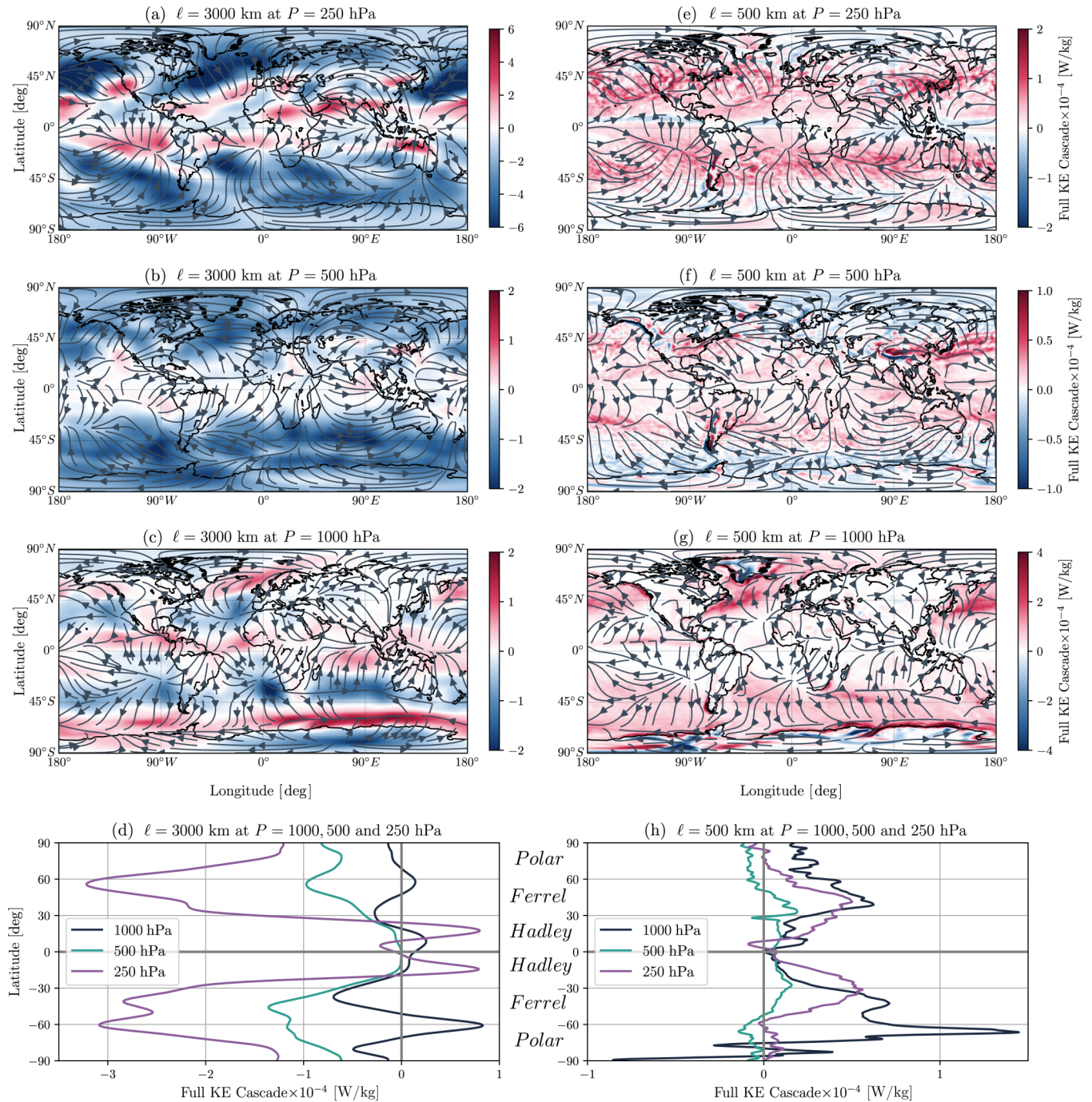


FIG. 1. Maps of KE cascades and streamlines of large-scale laterally divergent circulation. Spatial maps of two-year averaged Π [W/kg] for (a)–(c) $\ell = 3000$ km and (e)–(g) $\ell = 500$ km at (a), (e) 250 hPa; (b), (f) 500 hPa; and (c), (g) 1000 hPa. In each of panels (a), (b), (c), (e), (f), and (g), streamlines show the laterally divergent flow at scales larger than ℓ . Bottom two panels [panels (d) and (h)] show the zonal means of panels (a)–(c) and (e)–(g), respectively.

A schematic of the anisotropic pressure-strain mechanism is provided in Fig. 9. This schematic shows that, even when the 3D flow is divergence-free, persistent time-mean convergence or divergence in horizontal directions can produce significant interscale KE transfer. This mechanism is distinct from a classical turbulence cascade: Its key ingredients are inhomogeneity, here imposed by the surface boundary along the vertical direction, and anisotropy of the turbulent fluctuations. Similar to the surface convergence of the Hadley

cells, the midlatitude divergence of the Hadley and Ferrel cells as seen from the streamlines in Fig. 1(c) around 30°N and 30°S attenuates the KE of sub-planetary-scale motions. The upscale KE transfer from sub-planetary-scale motions to the planetary-scale circulation is seen as two upscale transfer (blue) zonal bands around 30°N and 30°S flanking the red equatorial band in Fig. 1(c) [see also Fig. 1(d)]. Within latitudes 45° – 70° in both hemispheres, convergence between the Ferrel and Polar cells near the surface is associated with

downscale KE transfer, which energizes motions smaller than 3000 km. Poleward of 70° , especially near the thermal poles in East Antarctica (Antarctic Plateau) and Greenland, we can see upscale KE transfer associated with surface divergence due to air subsidence from aloft within the polar cells.

Further support is provided in Figs. 10–12 showing flow divergence (annually and seasonally averaged) with overlaid streamlines. At 250 hPa, which is just below the tropopause at midlatitudes and is a convenient proxy for upper-tropospheric dynamics, the convergence and divergence patterns are reversed [Fig. 1(a)], leading to a switch in sign of the zonal bands. The reversed band structure is also clear from the zonally averaged Π in Fig. 1(d) (violet plot): The upscale transfer at the equator flanked by downscale transfer in the subtropics (around 15°) and upscale transfer around 60° are anticorrelated with the zonally averaged Π at 1000 hPa [black plot in Fig. 1(d)]. An exception is near the poles, where we would expect downscale transfer ($\Pi > 0$) associated with convergence of the polar cells in the upper troposphere, whereas Figs. 1(a) and 1(d) show upscale transfer. We shall show later [Fig. 2(f)] that a downscale transfer due to polar cells convergence is indeed present but is masked by strong upscale transfer ($\Pi < 0$) from the quasigeostrophic (2D-like) dynamics, which we can disentangle. Because precipitation is also associated with upper-level divergence, Π and precipitation show strong geographical covariation (Figs. 8 and 13). In the midtroposphere at 500 hPa, which is often considered as the level of nondivergence [50,51] (see also Fig. 10), the Π map in Fig. 1(b) shows that scale transfer is predominantly upscale outside the tropics. This is consistent with a dominance of quasigeostrophic dynamics in the extratropical midtroposphere, whereby synoptic-scale motions energize the planetary-scale circulation.

The cascade exhibits hemispheric asymmetry that is clearest in Figs. 1(d) and 1(h) and is consistent with a weaker surface drag over the Southern Ocean. Less drag allows for a stronger circulation, especially at the planetary scales (Fig. 14), contributing to a stronger KE exchange. The cascade's hemispheric asymmetry decreases with elevation as the influence of surface drag diminishes [Figs. 1(d) and 1(h)]. The KE scale-transfer latitudinal bands exhibit marked seasonal trends (Figs. 6 and 7), including winter intensification and meridional shifts that are consistent with the planetary-scale circulation.

(b) *Mesoscales.* Figures 1(e)–1(g) show maps of KE transfer across mesoscale length of 500 km. We can see that the transfer is predominantly downscale (positive Π) through most of the troposphere. This downscale transfer is consistent with the mesoscales being less constrained by geostrophic balance and exhibiting a 3D-like turbulence behavior with a downscale cascade [12]. From Fig. 1(g), the strongest KE transfer at the surface ($P = 1000$ hPa) seems to be correlated with regions of strong temperature gradients, such as near coastlines and western boundary ocean currents, primarily the Gulf Stream and Kuroshio [52]. In the midtroposphere [$P = 500$ hPa, Fig. 1(f)], the KE transfer map shows footprints of the Himalayas and Andes mountain ranges. This is consistent with orography exciting gravity waves and barrier jets, which can produce organized precipitation bands and mesoscale turbulence at scales resolved in the dataset. Despite

a predominantly downscale KE cascade at the mesoscales, we can see in the upper troposphere [250 hPa, Fig. 1(e)] a narrow equatorial blue band of upscale transfer ($\Pi < 0$). This is due to upper-level divergence of the Hadley cells in the ITCZ, which removes KE from mesoscales smaller than 500 km via the anisotropic pressure-strain interaction described earlier. This equatorial blue band migrates northward/southward during the northern summer/winter (Figs. 6 and 7), consistent with the ITCZ seasonality [49]. Further support of this assertion will be provided by analysis below. The anisotropic pressure-strain interaction also yields upscale transfer (blue) over the Greenland and Antarctic ice sheets in the lower troposphere [1000 hPa, Fig. 1(g)] associated with descending, divergent planetary-scale flow. In the midtroposphere [500 hPa, Figs. 1(f) and 1(h)], where planetary-scale divergence is weak, we find upscale transfer (blue) poleward of approximately 60° , consistent with ~ 500 km motions remaining rotation constrained (small Rossby number) at high latitudes.

B. Transfer across broad scale range

We can generate Π maps as in Fig. 1 for the entire range of scales resolved in the dataset. Figure 2 is one representation of such maps that are zonally integrated $\rho\Pi$ as a function of length scales from 40×10^3 to 100 km while retaining meridional information along the y axis. In Fig. 2(a) for the lower troposphere, alternating upscale/downscale bands seen at large scales ($\gtrsim 2000$ km) seem to vanish at smaller scales, where a downscale ($\Pi > 0$) cascade prevails. In the upper troposphere [Fig. 2(b)], the extratropics show a dominant upscale ($\Pi < 0$) cascade for $\ell \gtrsim 2000$ km, consistent with quasigeostrophic dynamics, whereas for $\ell \lesssim 1000$ km the cascade is downscale, consistent with the mesoscales being less rotation constrained and more 3D-like (here, “3D-like” means larger contributions from laterally divergent and ageostrophic motions, not isotropic 3D turbulence).

(a) *Exchange between planetary-scale flow and weather.* We now show that the anisotropic pressure-strain interaction drives direct KE exchange between the planetary circulation and weather-scale motions, down to the smallest resolved mesoscales. Following Ref. [43], we perform an exact decomposition of the cascade (Π_ℓ) into contributions from laterally divergent and nondivergent motions (see Sec. IV). A laterally divergent flow is associated with vertical motion. Figures 2(c) and 2(d) repeat Figs. 2(a) and 2(b) for the lower and upper troposphere, respectively, after removing the divergent flow contributions at subplanetary scales ($\ell < 3000$ km). The resulting cascade, denoted by Π^Δ , excludes interactions with the 3D-like divergent subplanetary flow that produce the downscale transfer at $\ell \lesssim 2000$ km in Figs. 2(a) and 2(b). With the 3D-like weather contributions removed, Fig. 2(d) shows a clearer upscale cascade in the upper troposphere, consistent with quasigeostrophic dynamics. In the lower troposphere, Fig. 2(c) reveals that the alternating zonal bands due to interactions with the planetary-scale circulation penetrate to the smallest mesoscales (≈ 100 km) resolved in the dataset. Figures 2(e) and 2(f) provide support that the direct KE transfer between the planetary scales and subplanetary scales is indeed driven by the anisotropic

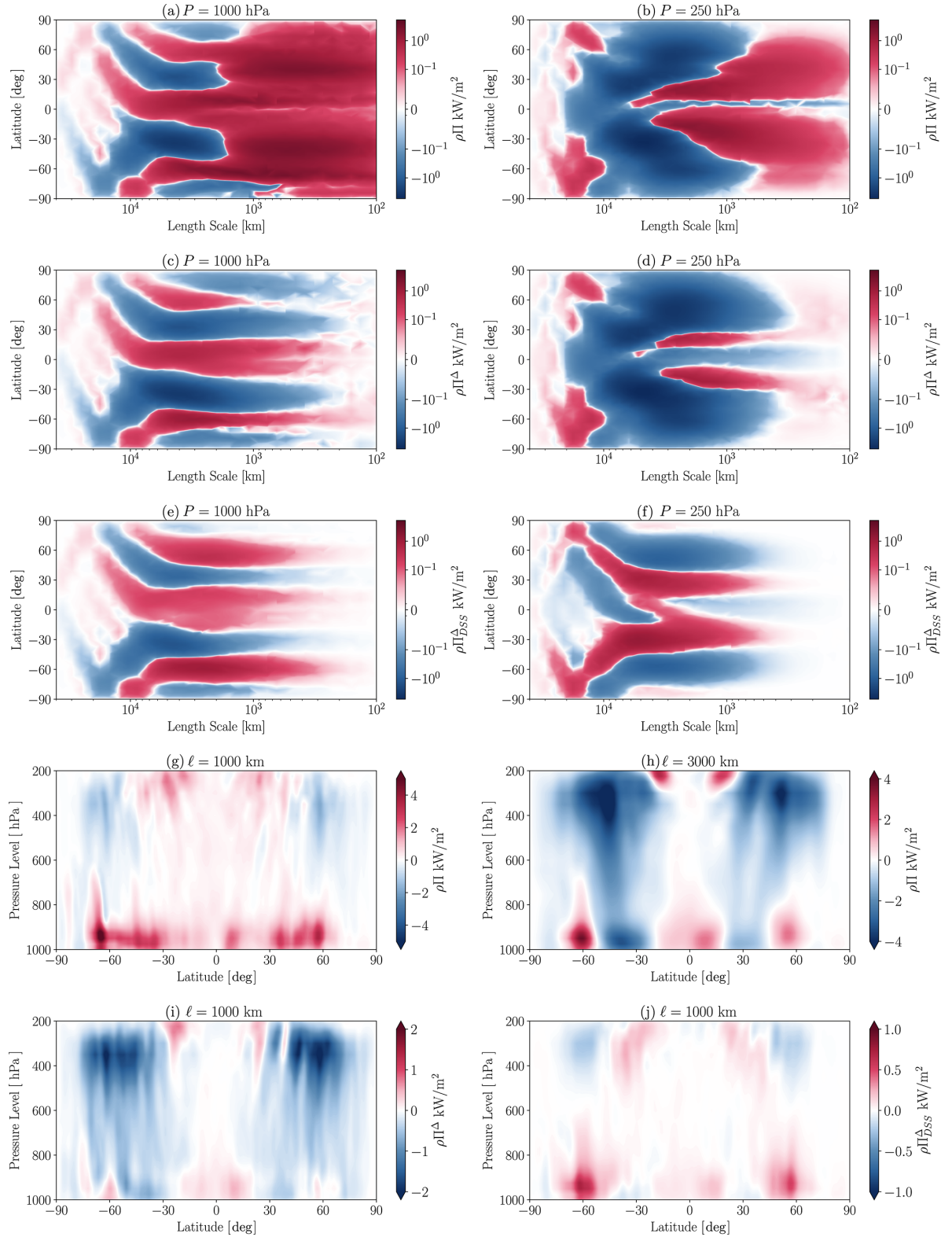


FIG. 2. Length scale and depth dependence of KE cascades and the impact of divergent flow. (a), (b) Time-averaged and zonally integrated $\rho\Pi$ shows the scale dependence of the KE cascade. Panels (c) and (d) remove the contribution from small-scale divergent flow ($\rho\Pi^\Delta$), while panels (e) and (f) consider only the action of large-scale divergent strain on small-scale nondivergent stress ($\rho\Pi_{DSS}^\Delta$). Panels (a), (c), and (e) correspond to 1000 hPa and panels (b), (d), and (f) to 250 hPa. The bottom four panels show latitude-depth profiles of time-averaged and zonally integrated (g) $\rho\Pi$ at $\ell = 1000$ km, (h) $\rho\Pi$ at $\ell = 3000$ km, (i) $\rho\Pi^\Delta$ at $\ell = 1000$ km, and (j) $\rho\Pi_{DSS}^\Delta$ at $\ell = 1000$ km ($\Delta = 3000$ km in all panels).

pressure-strain interaction. Figures 2(e) and 2(f) are similar to previous panels in Fig. 2, but show only the scale transfer ($\Pi_{\text{DSS}}^{\Delta}$) between the planetary-scale, laterally divergent flow (D for “divergent”; streamlines in Fig. 1) and the laterally nondivergent or solenoidal flow (S for “solenoidal”). $\Pi_{\text{DSS}}^{\Delta}$ is one of eight terms contributing to scale transfer and follows from a Helmholtz decomposition of the flow (see Sec. IV; cf. Ref. [43]). $\Pi_{\text{DSS}}^{\Delta}$ quantifies KE scale transfer across ℓ due to the planetary-scale divergent flow acting against an effective stress due to the nondivergent flow at scales smaller than ℓ . The alternating zonal bands are clear in both the lower and upper troposphere [Figs. 2(e) and 2(f)], providing evidence for the anisotropic pressure-strain interaction proposed above. The bands span nearly all scales smaller than 10×10^3 km. This deep penetration of the bands reveals a direct KE exchange between the planetary-scale circulation and the synoptic and mesoscale motions that underpin most weather systems. It should be noted that the aforementioned KE exchange due to anisotropic pressure-strain interaction is distinct from KE exchange between the nondivergent planetary scales and nondivergent mesoscales. Quasigeostrophic and two-dimensional turbulence theories predict that nondivergent flows transfer KE upscale [13,14]. However, local deviations from this idealized picture can occur [43], especially where the underlying assumptions are weakened, such as near orography, coastlines, or strong surface heterogeneity. In such regions, processes such as filamentation or vortex shedding can create smaller-scale vortical structures and produce localized downscale KE transfer.

(b) *Vertical structure of KE transfers.* Figures 2(g)–2(j) provide a view throughout the troposphere of the scale transfer presented in earlier panels of Fig. 2. The sign reversal of Π between the lower and upper troposphere, unlike flow divergence itself, yields nontrivial vertical profiles of the scale transfer [Figs. 2(g)–2(j)] and the magnitude remains significant when vertically integrated (Fig. 15). This is because Π is a nonlinear (tensor) product of planetary-scale velocity gradients with the effective stress due to sub-planetary-scale motions [Eq. (6)]. The latter’s vertical inhomogeneity can yield considerable transfer of $O(100)$ TW (see Table I). Figures 2(g) and 2(h) present latitude-depth profiles of zonally averaged Π across $\ell = 1000$ km and $\ell = 3000$ km, respectively. Both Figs. 2(g) and 2(h) show upscale transfer in the upper troposphere around 40° – 70° latitude, which is associated with the polar jet streams and is especially pronounced across $\ell = 3000$ km [Fig. 2(h)]. This correlation is consistent with the dominance of laterally nondivergent, quasigeostrophic dynamics in the extratropical upper troposphere. Even across $\ell = 1000$ km, the upscale transfer due to the jet streams becomes pronounced after removing the sub-planetary-scale divergent flow (Π^{Δ}) in Fig. 2(i). From Fig. 2(g), we can also see ubiquitous downscale transfer within the atmospheric boundary layer (ABL; 900–1000 hPa), which is absent from the transfer across $\ell = 3000$ km [Fig. 2(h)]. The downscale transfer across $\ell = 1000$ km within the ABL relies on interactions with 3D-like flow at subplanetary scales and mostly vanishes after removing those contributions (Π^{Δ}) in Fig. 2(i). From Π across $\ell = 3000$ km [Fig. 2(h)] and $\Pi_{\text{DSS}}^{\Delta}$ across $\ell = 1000$ km [Fig. 2(j)], we can see the clear imprint of planetary-scale circulation cells on the

KE exchange across scales throughout the troposphere [see also Fig. 16(b)].

C. Hemispheric and global averages

A well-known shortcoming of using spherical harmonics to analyze hemispheric energy budgets across scales, such as their seasonality, is the need to impose artificial symmetry conditions that distort the energetically active tropics [27,53]. Such data distortion is not necessary using the coarse-graining approach since it allows us to diagnose length scales locally in space. We will also present globally averaged results (Fig. 3), which reproduce prior work using spherical harmonics—thereby demonstrating that coarse graining is a more general framework that can capture spatial inhomogeneity. In fact, coarse graining reduces to a purely spectral approach (e.g., Fourier or spherical harmonics) when using a filtering kernel with an infinite number of vanishing moments (Dirichlet-like kernel), which is justified only for homogeneous fields given the highly nonlocal nature of such kernels in physical space [54]. Note that Fig. 3 plots Π (in W/kg) to facilitate the comparison between different altitudes (plots of $\rho\Pi$ in W/m^3 are provided in Fig. 17).

(a) *Transition from downscale to upscale.* Figure 3(a) shows that in the lower troposphere ($P = 1000$ hPa and $P = 900$ hPa), scales larger than ≈ 2500 km transfer KE upscale, while smaller scales transfer KE downscale. This dual cascade behavior has been reported in past studies using data from idealized and realistic models, and reanalyses [31,32,55,56], albeit not near the surface as we report here. The transition scale between upscale and downscale transfer is smallest at ≈ 700 km in the midtroposphere (500 hPa), increasing again with elevation to ≈ 1500 km in the upper troposphere (250 hPa) and reaching ≈ 4000 km in the stratosphere (≤ 100 hPa). Vertical variation of the transition scale is physically expected because of the altitudinal change in stratification [57] and dominant energy injection mechanisms [58,59]. Baroclinic energy input, in particular, occurs over a range of synoptic scales rather than at a single characteristic scale [59]. Variation of the transition scale with altitude has also been observed in previous studies [31,32,55,60] and is attributed to different dynamical balances, e.g., dominance of surface friction in the ABL [61], eddy-driven dynamics in the troposphere [62], and planetary- and wave-driven dynamics in the stratosphere [63,64]. Strong baroclinic forcing in the troposphere, along with convective/shear production [65], can contribute to a stronger tropospheric KE cascade (both upscale and downscale) seen in Fig. 3(a). The weaker stratospheric cascade is likely due to stable stratification [66], which suppresses vertical mixing and leads to the dominance of inertia-gravity waves [67,68].

(b) *Importance of divergent motions.* Figures 3(a) and 3(b) show that laterally divergent motions associated with ascending and descending air cause a significant departure from the upscale KE transfer associated with 2D-like flows. The downscale transfer is in a sense *catalyzed* by the laterally divergent 3D-like motions, which are very weak and constitute order of 1% of the full circulation’s KE (Fig. 18). Figure 3(b) shows that the downscale transfer at the mesoscales seen in Fig. 3(a) is completely suppressed after removing interactions with the

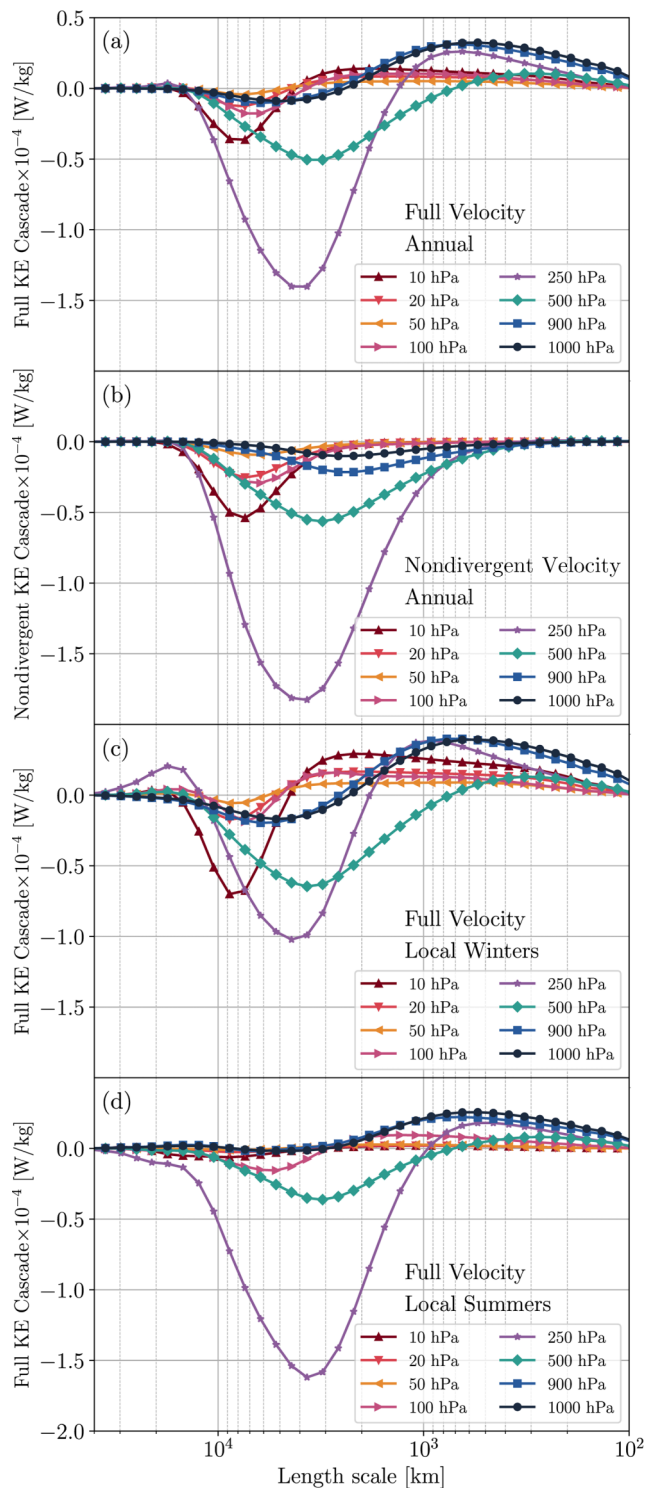


FIG. 3. Area-mean KE transfers, their seasonal dependence, and the impact of divergent flow. Area-mean KE scale transfer [Π (W/kg)] for eight pressure levels spanning the troposphere and stratosphere (see legend) for length scales ranging from 100 km to the circumference of the Earth. (a), (b) Averages of the entire globe, while panel (c) uses hemispheric averages over local winters and panel (d) over local summers. Local winters are defined as January-February-March (JFM) in Northern Hemisphere (NH), and local summers as July-August-September (JAS) in Southern Hemisphere (SH). Panels (a), (c), and (d) use the full velocity field, while panel (b) restricts to only the laterally nondivergent flow.

laterally divergent motions. Moreover, the upscale transfer at scales larger than 1000 km is enhanced in Fig. 3(b) compared to Fig. 3(a) at all pressure levels. The downscale transfer effect of divergent motions is notable at these large scales since they are expected to be mostly in geostrophic balance. This is consistent with past analysis of divergent motions using spherical harmonics [36]. The globally averaged Π restricted to the laterally divergent flow is presented in Fig. 19. It shows that the self-interactions of laterally divergent component of the flow yield downscale transfer at all pressure levels, with a weak up-scale exception in the upper troposphere near the tropopause. This exception should not be interpreted as evidence that the divergent component dominates the upscale cascade near the tropopause. Rather, it reflects the dynamical coupling between the divergent and balanced rotational components of the flow in the upper troposphere. This interpretation is reinforced by recent spectral budget analyses [36] showing strong coupling between divergent and rotational KE budgets in the upper troposphere, including KE conversion from divergent to rotational motions at synoptic scales. It is worth noting that the weak upscale Π_{DDD} near the tropopause is not identical to the divergent KE spectral flux in spectral budgets such as in Ref. [36]. Li *et al.* [36] measure the transfer of divergent KE by the full flow, and much of that transfer corresponds to mixed divergent/nondivergent cross-interaction terms in our decomposition. In Fig. 20, we show that the residual term $\Pi - \Pi_{SSS} - \Pi_{DDD}$ is large and downscale near 250 hPa. Note also that the transfer in Fig. 19 is much weaker than that due to the full flow in Fig. 3, which includes contributions from self-interactions among nondivergent motions, self-interactions among divergent motions, and cross-interactions between divergent and nondivergent motions.

(c) *Jet stream energization by subplanetary scales.* As shown in Figs. 3(a) and 3(b), the most prominent upscale cascade occurs in the upper troposphere (250 hPa, even after density weighting Π in Fig. 17). The prominent upscale transfer at this altitude is due to the energization of the polar jet streams by sub-planetary-scale motions, evidence for which is presented in Fig. 2. This is consistent with established knowledge that polar jet streams are eddy driven, whereby eddy momentum flux convergence accelerates and sharpens the jets [12].

(d) *Seasonality of the cascade.* The ability to analyze scale transfer locally in space allows us to naturally probe its hemispheric seasonality. Comparing local winters to summers in Figs. 3(c) and 3(d), respectively, we see that KE transfer intensifies in winter at most altitudes. Strengthening of the upscale cascade in winter is most pronounced in the midstratosphere at planetary scales (at $\approx 8 \times 10^3$ km), consistent with enhanced planetary-wave activity and vortex-edge dynamics in that season (e.g., Refs. [69–71]). A seeming exception is in the upper troposphere (250 hPa), where the upscale transfer at scales larger than 1000 km is stronger in summer; this may reflect the higher summer tropopause and associated changes in static stability at 250 hPa (e.g., Ref. [12]). We note that the smallest scale shown in Fig. 3 (≈ 100 km, which is roughly 3–4 grid spacings at the equator) is close to the effective resolution of ERA5 and may be affected by numerical diffusion, physical parametrizations, and data assimilation. The main conclusions, however, are based on scales of several

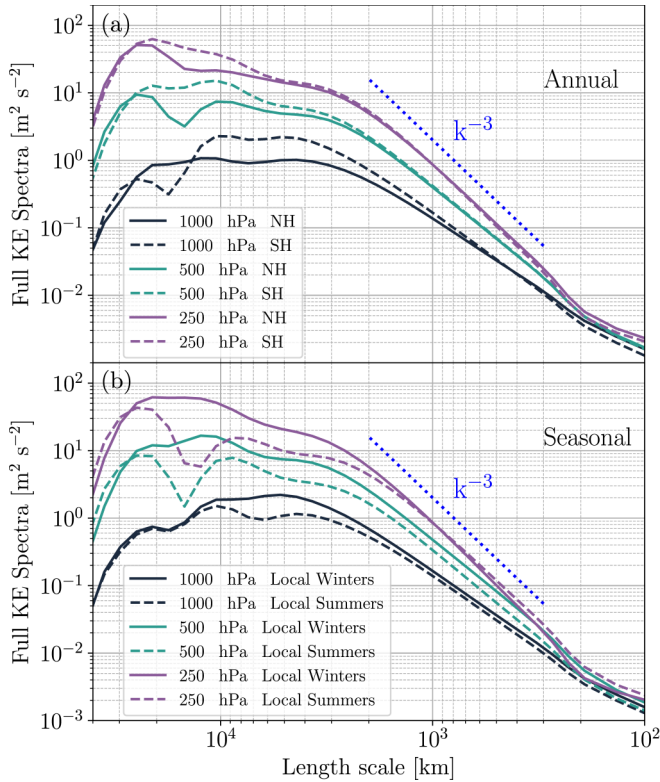


FIG. 4. Atmospheric KE spectra. Full kinetic energy spectra partitioned into (a) northern (solid) and southern (dashed) hemispheres and (b) winter (solid) and summer (dashed) for 1000, 500, and 250 hPa. The inset dotted line shows a k^{-3} power law. The horizontal axis is length scale, ranging from the circumference of the Earth to 100 km.

hundred to several thousand kilometers and are expected to be insensitive to unresolved-scale effects.

D. Kinetic energy spectra

For completeness and to connect with previous studies, we now discuss KE spectra derived from coarse graining. Geographic maps of atmospheric spectra from coarse graining were presented recently in Ref. [44]. Global KE spectra in the troposphere and stratosphere have been extensively studied using spherical harmonics [25,26,28–31,33,34,72]. Figure 21 shows that globally averaged KE spectra obtained through coarse graining and those from spherical harmonics agree and are consistent with previous studies across pressure levels spanning the troposphere and stratosphere. Similar to Π , KE spectra of each hemisphere can be obtained naturally using coarse graining, thereby giving us insights into the seasonality of different scales.

(a) *Hemispheric asymmetry.* Figure 4(a) shows KE spectra averaged over each of the Northern Hemisphere (NH) and Southern Hemisphere (SH), separately. The NH and SH spectra are almost identical over the mesoscales ($\lesssim 1000$ km). However, at scales larger than ~ 1000 km, the SH has greater KE. The disparity is most pronounced near the surface (1000 hPa) likely due to greater friction over land in the NH, but the disparity persists throughout the troposphere (at 500 and

250 hPa) and stratosphere [100, 50, and 10 hPa; see Fig. 22(a)]. This may be indicative of other causes for the asymmetry, such as known disparities in meridional temperature gradients between the two hemispheres due to radiative processes and ocean heat transport [73].

(b) *Seasonality.* Figure 4(b) shows spectra during the local winter and local summer of both hemispheres. We define NH winter and summer as January–March (JFM) and July–September (JAS), respectively; SH seasons are reversed. As expected [26,27], the circulation during winter is more energetic compared to summer at most scales in the troposphere and stratosphere [Fig. 22(b)]. The seasonal KE contrast is especially pronounced at planetary scales ($\gtrsim 3000$ km).

(c) *Power-law spectral slopes.* Figure 4 shows that KE spectra follow putative power laws at scales smaller than $\approx 2 \times 10^3$ km. At these scales, we can expect that meridional differential heating and baroclinic instabilities are less prominent [60]. Yet, this scale should not be interpreted as a sharp transition scale below which baroclinic instabilities become unimportant. The k^{-3} scaling in the upper troposphere is consistent with quasi-2D phenomenology [58]. In line with this interpretation, a downscale enstrophy flux has been observed in the upper troposphere using atmospheric reanalysis data [31], and a related enstrophy-controlled regime has been reported in atmosphere-like rotating laboratory experiments [74]. It should be noted that a spectral slope by itself does not prove the presence of a downscale enstrophy cascade [75,76], nor does it identify the scale at which divergent inertia-gravity-wave motions become dominant [60]. Our decomposition of the KE cascade shows that, across this range of scales, the self-interaction of the nondivergent flow transfers KE upscale, while the downscale transfer arises mainly from the mixed interactions between the divergent and nondivergent components. Since the large-scale nondivergent flow is close to geostrophic balance and the divergent motions are primarily ageostrophic, this is consistent with an enstrophy-controlled regime in which the KE transferred to small scales is primarily the KE of unbalanced motions.

Figure 4 shows that power-law scaling is similar in both hemispheres and also in both seasons. The scaling is close to k^{-3} in the upper troposphere [250 hPa, violet plots in Fig. 4(a)], consistent with spectral scaling of quasi-two-dimensional flows at those scales [58]. Approaching the surface, the spectra become shallower and near $k^{-5/3}$ at 1000 hPa [black curves in Fig. 4(a); see also Fig. 23]. However, such putative power laws, here and in prior atmospheric studies, are not by themselves evidence for an upscale or downscale cascade as in idealized turbulence theory. Because the atmospheric circulation is subject to complex forcing/damping processes at various scales that can alter spectra (e.g., radiative forcing, atmosphere-ocean coupling, baroclinic forcing, wave drag, topography), establishing the cascade direction requires direct diagnostics of the energy scale transfer (e.g., Π), not spectra alone.

III. DISCUSSION

These results show that coarse graining is a viable framework for interrogating multiscale coupling within Earth's climate system. A key advantage is that it preserves a deter-

ministic description of the dynamics without the need to resort to statistical averaging.

Rather than being distinct from conventional spectral analysis such as spherical harmonics, coarse graining can be viewed as a generalization of conventional spectral analysis to spatially inhomogeneous fields: It yields a scale decomposition and an energy-preserving (Plancherel-type) identity without relying on the global orthogonality of Fourier modes [54]. For homogeneous fields, coarse-grained diagnostics converge (in a global-average sense) to their Fourier counterparts in the formal limit of filters with infinitely many vanishing moments [48,54,77–79]. However, this limit corresponds to highly oscillatory, nonlocal kernels in physical space and is therefore inappropriate in the presence of spatial inhomogeneity; attempts to form scale-aware geographic maps by spectral truncation (or filtering) can exhibit substantial ringing artifacts [80].

Another salient issue is that classical Fourier decompositions are energy preserving (Plancherel) only in a globally averaged sense: Defining low- and high-pass components by spectral truncation, $\mathbf{u}^{<K}(\mathbf{x})$ and $\mathbf{u}^{>K}(\mathbf{x})$, generally yields $|\mathbf{u}^{<K}|^2 + |\mathbf{u}^{>K}|^2 \neq |\mathbf{u}|^2$ pointwise. Coarse graining instead provides a spatially local, energy-preserving decomposition (while allowing spectral filters as one possible kernel choice) [44]. The coarse-graining approach [43,54,81,82] departs from traditional Fourier frameworks not necessarily in the filtering kernel choice, but early in the formulation of local energy at different scales.

These advantages of coarse graining come at a price: One must forgo exact localization in both physical and Fourier space as prohibited by the uncertainty principle. Retaining geographic information therefore implies a less sharp notion of length scale (a weighted average over a band of Fourier modes [Eq. (16) in Ref. [48]] and a controlled “fuzziness” in spatial localization ($\approx \ell/2$) when probing scale ℓ [40]. The inevitability of the uncertainty principle is evident in spatial derivatives, which themselves encode scale information: Evaluating a derivative at a point requires information from a neighborhood whose extent reflects the scale of the variations being resolved.

The analysis here did not attempt to analyze the coupling between individual weather events and the planetary-scale circulation; instead, it examined their collective interactions. Future work should test how well this framework diagnoses event-specific coupling.

IV. MATERIALS AND METHODS

A. Datasets

This study employs velocity fields from the ERA5 [47] dataset. ERA5 is the fifth-generation European Centre for Medium-Range Weather Forecasts (ECMWF) reanalysis for the global climate and weather for the past 8 decades, and can be obtained in Ref. [83]. The ERA5 dataset provides global coverage at sufficiently high spatial and temporal resolutions, which together are needed for a global, scale-resolved analysis of the atmospheric KE transfer. For a detailed description of the ERA5 dataset, see Ref. [84]. The ERA5 data are available hourly for 37 pressure levels covering 1000–1 hPa on a

$0.25^\circ \times 0.25^\circ$ latitude-longitude grid. We analyze two years of the reanalysis data (2020 and 2021), using 4 individual hourly fields per day (0, 6, 12, and 18 h UTC) at 8 pressure levels: 1000, 900, 500, 250, 100, 50, 20, and 10 hPa. A different time and vertical resolution is used to only conduct the analysis of vertical structure of KE transfers. For results requiring higher vertical resolution [Figs. 2(g)–2(j)], we analyzed the first day of each month of the two years at 0 and 12 h UTC, and for 23 pressure levels ranging from $P = 1000$ hPa to $P = 200$ hPa. We also validated our methodology using a different ECMWF dataset from the Stratospheric Processes and Their Role in Climate (SPARC) International Polar Year (IPY) project (data can be obtained from Ref. [85]). From this dataset, velocity fields are available at 0, 6, 12, and 18 h UTC on 91 pressure levels on a $0.25^\circ \times 0.25^\circ$ (horizontal) latitude-longitude grid. To compare with a previous study [31], we considered January 2008 when validating our methodology. Figure 24 shows a comparison of KE spectra and cascade between ERA5 and ECMWF-IPY for January 2008. Data used to validate spectra from coarse graining against those from spherical harmonics (Fig. 21) are also obtained from the SPARC-IPY project for January 2008.

B. Coarse graining

Commutative coarse graining on the sphere [41] has been previously applied in oceanographic contexts [42,43,80], where many of the implementation details are provided. Here, we outline the methodology and highlight novelties in the approach. Commutative coarse graining is essentially a spatially weighted local average, but performed in a way [41,86] that guarantees that spatial filtering commutes with differential operators. For example, $\overline{\nabla \cdot \mathbf{u}} = \nabla \cdot \overline{\mathbf{u}}$. This is necessary to preserve scale-dependent flow properties and also to derive explicitly the governing equations at different scales [42,43]. Ensuring that convolutions and spatial derivatives commute is mathematically nontrivial on the sphere and relies on generalizing the convolution operation [41]. The difficulties with commutation are due to curvature, irrespective of discretization on numerical grids. The source of the problem is deeper than merely performing a spherical convolution and including the appropriate metric terms [41]. For a chosen length scale ℓ and scalar function $F(\mathbf{x})$ at location $\mathbf{x}$ on the spherical surface, a filtered scalar field is

$$\overline{F}_\ell(\mathbf{x}) = \int_{\Omega} F(\mathbf{y}) G_\ell(\gamma(\mathbf{x}, \mathbf{y})) dS(\mathbf{y}), \quad (1)$$

where G_ℓ is the filtering kernel, $dS(\mathbf{y})$ is the area measure on the sphere, Ω is the entire spherical shell, and $\gamma(\mathbf{x}, \mathbf{y})$ is the geodesic distance between points $\mathbf{x}$ and $\mathbf{y}$. Lack of commutation using common averaging on the sphere, even after accounting for the appropriate metric terms due to curvature, can lead to cascade diagnostics that are incorrect both qualitatively and quantitatively, with relative errors of $O(100)$ across almost the entire globe. Following a corollary in Ref. [41], we implement commutative coarse graining of a vector field $\mathbf{u}$ by first performing a Helmholtz decomposition,

$$\mathbf{u} = \begin{bmatrix} u_\lambda \\ u_\phi \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial \phi} & \sec(\phi) \frac{\partial}{\partial \lambda} \\ \sec(\phi) \frac{\partial}{\partial \lambda} & \frac{\partial}{\partial \phi} \end{bmatrix} \begin{bmatrix} \Psi \\ \Phi \end{bmatrix}. \quad (2)$$

The Helmholtz scalars Φ and Ψ are then filtered as scalars following Eq. (1), and the coarse-grained velocity, $\bar{\mathbf{u}}_\ell$, computed from Eq. (2) after replacing Φ and Ψ with Φ_ℓ and Ψ_ℓ , respectively.

1. Filtering kernel

We use the third-order kernel [48]

$$G_\ell(\gamma) = \frac{1}{2} \left[1 - \tanh \left(\frac{D-1}{0.1} \right) \right] - \alpha \exp \left[- \left(\frac{D-1}{0.5} \right)^2 \right], \quad (3)$$

where $D = \gamma/(\ell/2)$ is normalized distance and $\alpha = 0.215\,340\,290\,414\,741\,62$ ensures that the second-order moment vanishes (the third-order moment vanishes due to the kernel's symmetry [48]). Following Ref. [48], this kernel allows us to measure spectral slopes as steep as -5 by sequential low-pass filtering. Note that commutation of the convolution operation with differential operators does not depend on the kernel choice as long as the kernel is spatially homogeneous [41].

2. Diagnostic quantities

The KE (per unit mass) at scales larger than ℓ is defined by

$$\text{KE}^{>\ell} = \frac{1}{2} |\bar{\mathbf{u}}_\ell|^2, \quad (4)$$

and, following Ref. [48], the filtering spectrum is obtained via

$$\bar{E}(k_\ell) = \frac{d}{dk_\ell} \text{KE}^{>\ell}, \quad (5)$$

where $k_\ell = \ell^{-1}$ is the filtering wave number. The rate of KE transfer (per unit mass) from scales larger than ℓ into scales smaller than ℓ arising from the non-linear dynamics is given by [78,79,82,87,88]

$$\Pi_\ell := -\frac{1}{2}(\bar{u}_{i;j} + \bar{u}_{j;i})\tau^{ij}, \quad (6)$$

where repeated indices denote summation, indices after semicolon (;) denote (covariant) differentiation, and τ^{ij} are (contravariant) components of the subscale stress (per unit mass) tensor $\bar{\mathbf{u}}\bar{\mathbf{u}}_\ell - \bar{\mathbf{u}}_\ell\bar{\mathbf{u}}$ [40]. Using this definition of the KE cascade, $-\Pi_\ell$ appears on the right-hand side of the large-scale KE budget and $+\Pi_\ell$ appears on the right-hand side of the small-scale KE budget [81,89,90]. Thus, positive values of Π_ℓ are sink of large-scale KE and a source of small-scale KE, corresponding to downscale KE transfer. In contrast, negative values of Π_ℓ have the opposite physical meaning and correspond to upscale KE transfer. As a technical note, the decompositions in Figs. 1, 6, 7, 8, 10, 11, 12, 13, and 14 are evaluated at length scales of 3100 and 545 km. For readability, these values are rounded to 3000 and 500 km in the main text, figure labels, and captions. All other plots, maps, and tables use the exact length scales indicated.

Two related clarifications are useful here. First, the 3D flow is divergence-free in pressure coordinates due to the use of the primitive hydrostatic equations in the ERA5 reanalysis to compute the vertical velocity. Therefore, density appears only as a prefactor in the KE cascade [91,92], which is calculated using horizontal velocities. Contributions from vertical velocity are negligible, which is consistent with the

strong anisotropy of atmospheric motion at the scales studied here. Indeed, vertical velocities are typically about 2 orders of magnitude smaller than horizontal velocities over the scales considered here. Second, Π_ℓ computed in the present work using coarse graining is conceptually distinct from frameworks based on Reynolds-type decompositions, such as the *E*-vector formalism of Hoskins *et al.* [93]. *E*-vectors use a zonal-mean/eddy decomposition (typically followed by time averaging) to diagnose eddy forcing of the mean flow, whereas the present coarse-graining framework quantifies KE transfer across an entire range of spatial scales while simultaneously retaining geographic localization.

C. Comparison to spherical harmonics

Spherical harmonics methods (or Fourier, in general) are standard and valuable tools for studying scale interactions in wave number space. In particular, triadic and shell-to-shell analyses directly quantify interactions between separated scale ranges. Past work [78] reconciled Π_ℓ within the coarse-graining framework with the traditional view of the cascade based on triadic Fourier interactions. However, spectral approaches are typically unable to provide spatial maps. It was demonstrated in Ref. [80] that such attempts produce heavy spectral ringing and truncation artifacts. The coarse-graining approach can be regarded as a generalization of the Fourier approach to spatially inhomogeneous fields, including a generalization of Parseval's energy-preserving relation without the need for the orthogonality structure provided by Fourier modes [48,54].

D. Reference density

Power spectra $\bar{E}(k_\ell)$, Eq. (5)] are reported in units of $\text{m}^3/\text{s}^2 = \text{J m/kg}$, so do not require a density value. KE scale transfers [Eq. (6)] are reported both in units of both W/kg (Π_ℓ) and W/m^3 ($\rho\Pi_\ell$). For the latter, reference densities used are given in Table II.

E. FLOWSIEVE

The software package used to perform the coarse-graining calculations is FLOWSIEVE [86], which was developed by the authors. Source code is available in Ref. [94], with documentation in Ref. [95].

F. Decomposing KE scale transfer

Following Ref. [43], we perform a Helmholtz decomposition of the velocity $\mathbf{u}$ into a laterally nondivergent (S, for solenoidal) and laterally divergent (D, for divergent) components. Substituting this decomposition into Eq. (6) yields an exact sum of eight terms: $\Pi_{\text{SSS}}, \Pi_{\text{SSD}}, \Pi_{\text{SDS}} \equiv \Pi_{\text{SSD}}, \Pi_{\text{SDD}}, \Pi_{\text{DSS}}, \Pi_{\text{DSD}}, \Pi_{\text{DDS}} \equiv \Pi_{\text{DSD}}$, and Π_{DDD} , where the stated equivalences are due to symmetry of the strain tensor τ^{ij} and reduce these to six unique terms that can represent different KE transfer mechanisms. For example, the nondivergent flow (S) is predominantly geostrophic and the Π_{SSS} term expresses transfer due to self-interactions among 2D-like flow structures, which generally yield an upscale cascade. Π_{DSS} denotes interactions between large-scale divergent

motions with small-scale stress from nondivergent motions. When the filter scale is chosen to be 3×10^3 km, Π_{DSS} quantifies the KE gained by nondivergent sub-planetary-scale motions ($< 3 \times 10^3$ km) from planetary-scale divergent motions, which include the Hadley, Ferrel, and polar cells. Note that the KE cascades due to the divergent components alone, Π_{DDD} , and due to the nondivergent components alone, Π_{SSS} , do not add up to the full KE cascade without including cross terms.

G. Removing interactions with small-scale divergent flow

To remove interactions with small-scale divergent flow, we fix a length scale Δ and partition the flow into four components: sub- Δ nondivergent, sub- Δ divergent, super- Δ nondivergent, and super- Δ divergent. We then remove the sub- Δ divergent component before performing the full coarse-graining analysis on the resulting flow field. Fields computed using this approach are denoted by a superscript Δ (e.g., Π^Δ , $\mathbf{u}^\Delta$). In this work, $\Delta = 3000$ km.

H. Down-sampling

To reduce the high computational cost associated with filtering at large scales, we utilized a down-sampling approach when coarse graining large scales. This is analogous to the decimation step that speeds up the fast Fourier transform algorithm. Only for scales larger than 10×10^3 km, we down-sampled the fields from a $0.25^\circ \times 0.25^\circ$ grid to a $1^\circ \times 1^\circ$ computational grid after performing a local average. To validate the accuracy of results using down-sampling for scales larger than 10^4 km, we performed a series of tests comparing KE spectra and cascades at different pressure levels between the original fields and the down-sampled fields. These results are provided in Fig. 25. For this validation, we employed only the northern winter (JFM) and northern summer (JAS) of 2020 from the ERA5 dataset.

ACKNOWLEDGMENTS

We thank the anonymous reviewers for their careful reading and insightful comments. We also thank Morgan O'Neill for valuable comments. This research was funded by a University Research Award from the University of Rochester. H.A. was also supported by U.S. NSF Grants No. OCE-2123496,

No. OCE-2446475, No. TI-2449340, No. PHY-2020249, and No. PHY-2206380, US DOE Grant No. DE-SC0020229, and by U.S. NNSA Grants No. DE-NA0004144 No. and DE-NA0004134. Computing time was provided by the Texas Advanced Computing Center (TACC) at The University of Texas at Austin, under an ACCESS allocation Grant No. EES220052.

DATA AVAILABILITY

The data that support the findings of this article are openly available (see Refs. [47,86]).

APPENDIX A: SUPPLEMENTARY TABLES

Table I reports the time averaged KE transfer, volumetrically integrated between lower and upper troposphere, over all longitudes, and meridionally between 50°S and 50°N . These values show that vertical inhomogeneity can yield a substantial transfer of order 100 TW. Table II lists the reference density at each pressure level based on the U.S. Standard Atmosphere.

TABLE I. KE transfer in terawatts, time averaged and volumetrically integrated: vertically between 1000–200 hPa, zonally, and meridionally between 50°S and 50°N .

Length scale ℓ	Π	Π_{DSS}
3000 km	−117.32 TW	+18.26 TW
1000 km	+45.05 TW	+10.07 TW

TABLE II. Area-mean density values used at each pressure level based on the U.S. Standard Atmosphere model.

Pressure level (hPa)	ρ (kg/m ³)
1000	1.225
900	1.11
500	0.69
250	0.39
100	0.17
50	0.098
20	0.051
10	0.034

APPENDIX B: SUPPLEMENTARY FIGURES

Figure 5 presents individual hourly maps of the full KE transfer across $\ell = 3000$ km during northern winter and summer at different pressure levels throughout the troposphere. Figures 6 and 7 present seasonal average maps of the KE cascade, complementing the two-year averaged maps in the main text (Fig. 1). Figure 8 presents the correlation between spatial maps of the KE cascade/flow divergence and precipitation. It shows that the near surface equatorial convergence of the Hadley cells is associated with downscale transfer and is strongly correlated with the two-year averaged precipitation belt of the ITCZ. Figure 13 presents the joint probability density functions of the full KE cascade/flow divergence at $\ell = 3000$ km and averaged hourly precipitation at different pressure levels throughout the troposphere. Figure 9 provides a schematic of the anisotropic pressure-strain mechanism. Figures 10–12 present annually and seasonally averaged flow divergence, with overlaid streamlines, at different pressure levels throughout the troposphere. Figure 14 presents zonally averaged vertical velocity and horizontal velocity divergence in the lower and upper troposphere. Figure 15 presents zonally and vertically integrated KE cascades, showing that the magnitude of the KE transfer remains significant after vertical integration. Figure 16 presents the

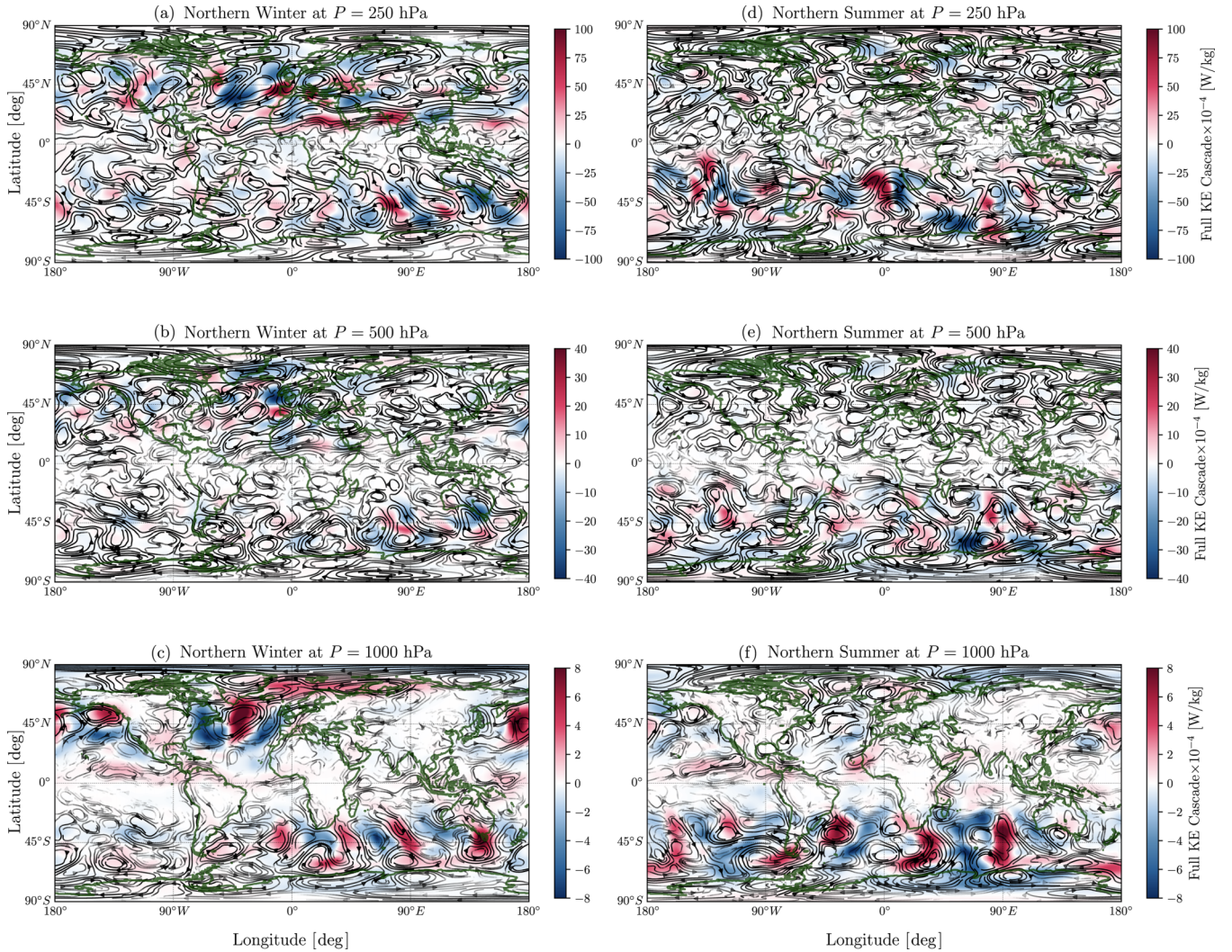


FIG. 5. Individual hourly maps of the full KE transfer across $\ell = 3000$ km during northern winter and northern summer at different pressure levels. The spatial distributions of the KE cascade at 3000 km are illustrated for two specific time instances: January 10th, 2020, 00:00 UTC [panels (a)–(c)] and July 10th, 2020, 00:00 UTC [panels (d)–(f)]. Panels (a) and (d) correspond to the upper troposphere ($P = 250$ hPa), whereas panels (b) and (e) represent the midtroposphere ($P = 500$ hPa), and panels (c) and (f) show the lower troposphere ($P = 1000$ hPa). The streamlines plotted on top of the KE cascade maps are derived from the fine (small scale) laterally nondivergent component of the horizontal velocities filtered (high pass) at 3000 km. For all panels, the streamlines are visualized using a logarithmic opacity filter, ranging from fully transparent (black) at low KE values ($1 \text{ m}^2/\text{s}^2$) to fully opaque (black) at high KE values ($100 \text{ m}^2/\text{s}^2$).

time-averaged latitude-pressure level distributions of the KE cascade due to the interaction of divergent large scale flow with nondivergent small scale motions. Figure 17 presents the area-mean, density-weighted KE transfers, complementing Fig. 3 in the main text. Figure 18 presents the globally averaged KE spectra as a function of length scale at different pressure levels throughout the troposphere, tropopause, and stratosphere. Figure 19 presents the globally averaged KE cascade restricted to contributions from the laterally divergent flow. Figure 20 presents the contributions of the Helmholtz-decomposed velocity components to the globally averaged KE cascade at different pressure levels. Figure 22 presents the annual and seasonal full KE spectra at three stratospheric pressure levels, complementing Fig. 4 in the main text. Figure 23 presents the globally averaged KE spectral slope as a function of length scale at different pressure levels throughout the troposphere, tropopause, and stratosphere. Finally, Figs. 21, 24, and 25 provide validation through, respectively, a comparison of global KE spectra computed using coarse-graining and spherical harmonics, a comparison of the computed KE spectra and cascade between two different datasets, and a comparison of the computed KE spectra and cascade between the original and down-sampled grids.

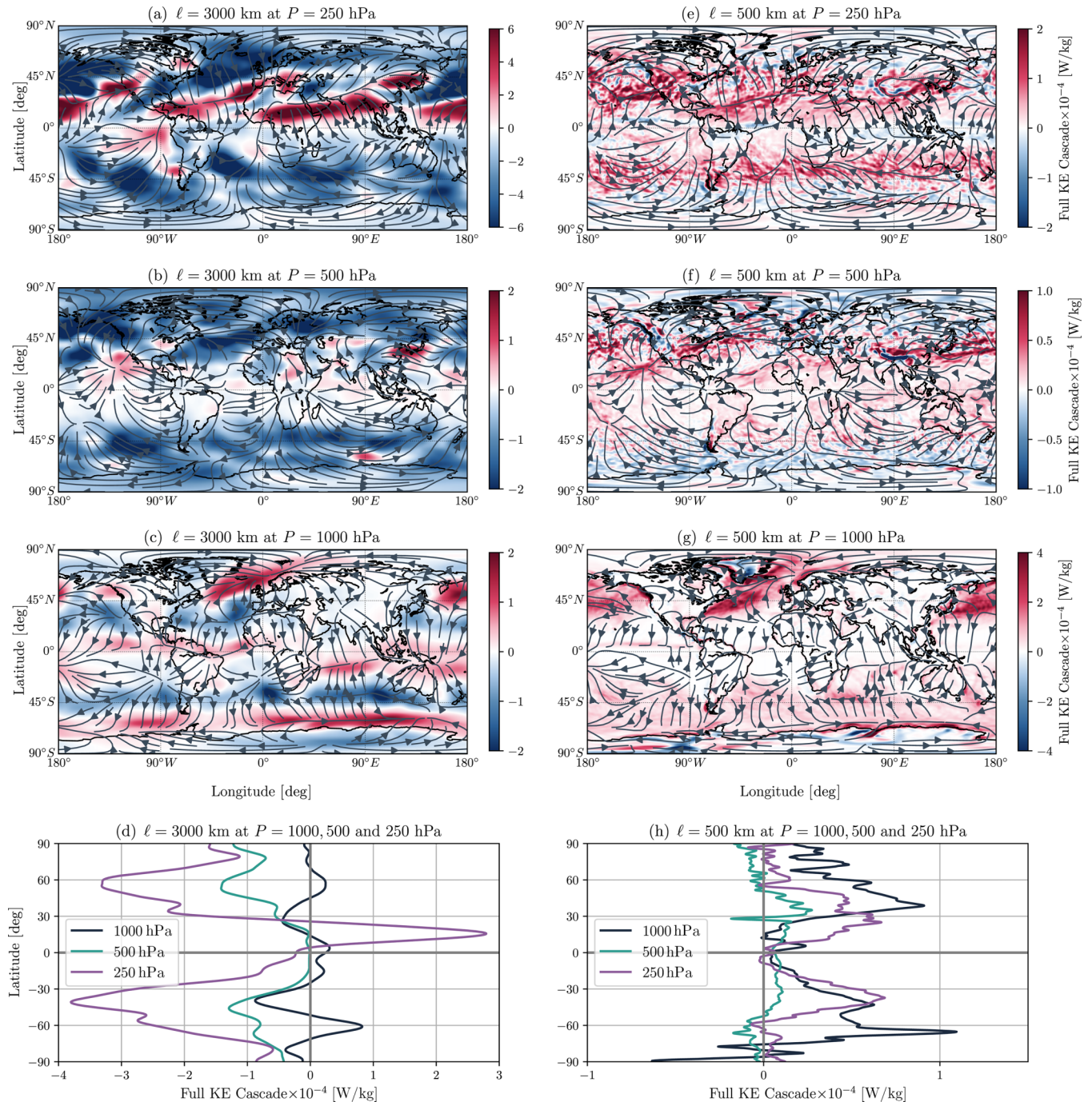


FIG. 6. Cascade maps similar to Fig. 1 but averaged over northern winter months (JFM).

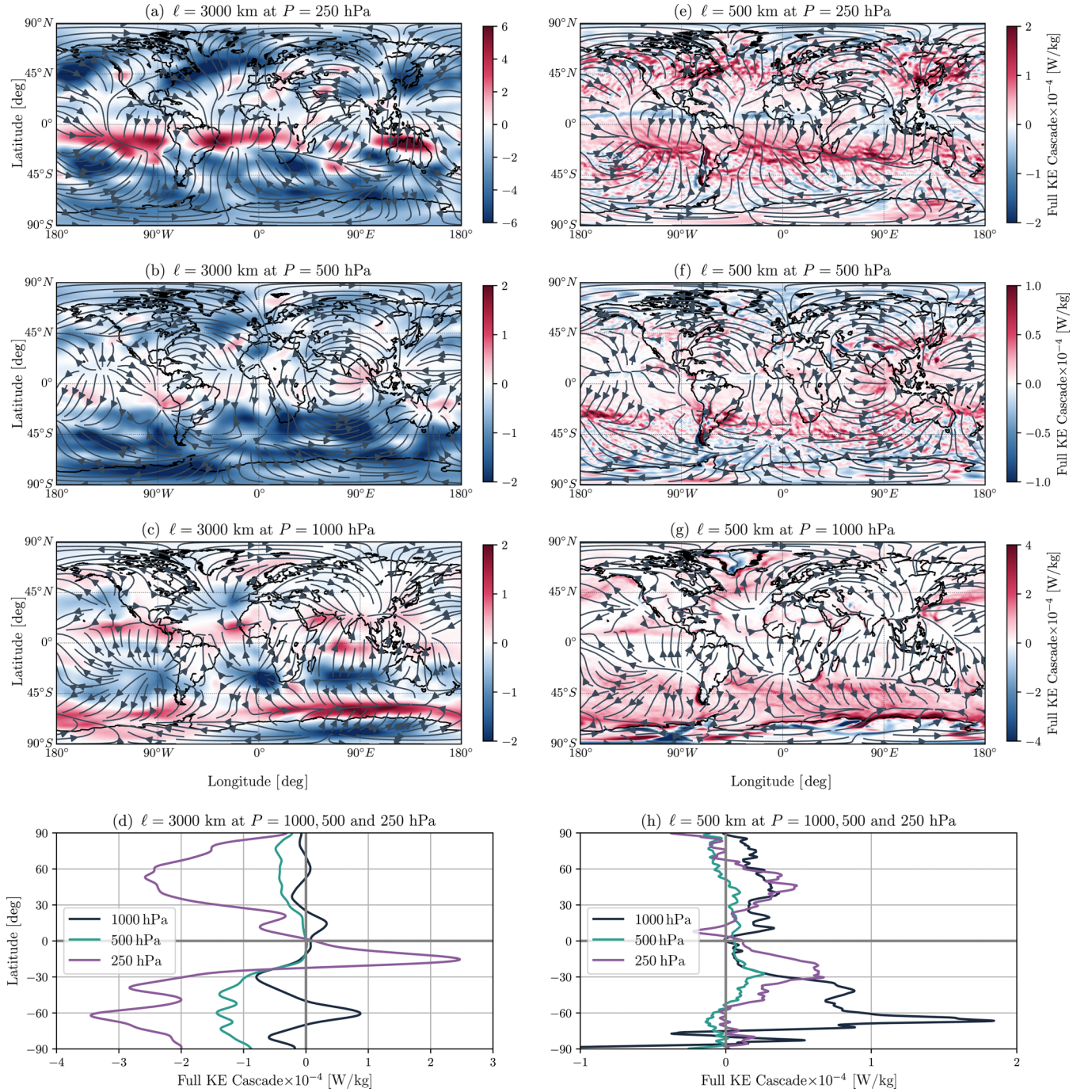


FIG. 7. Cascade maps similar to Fig. 1 but averaged over northern summer months (JAS).

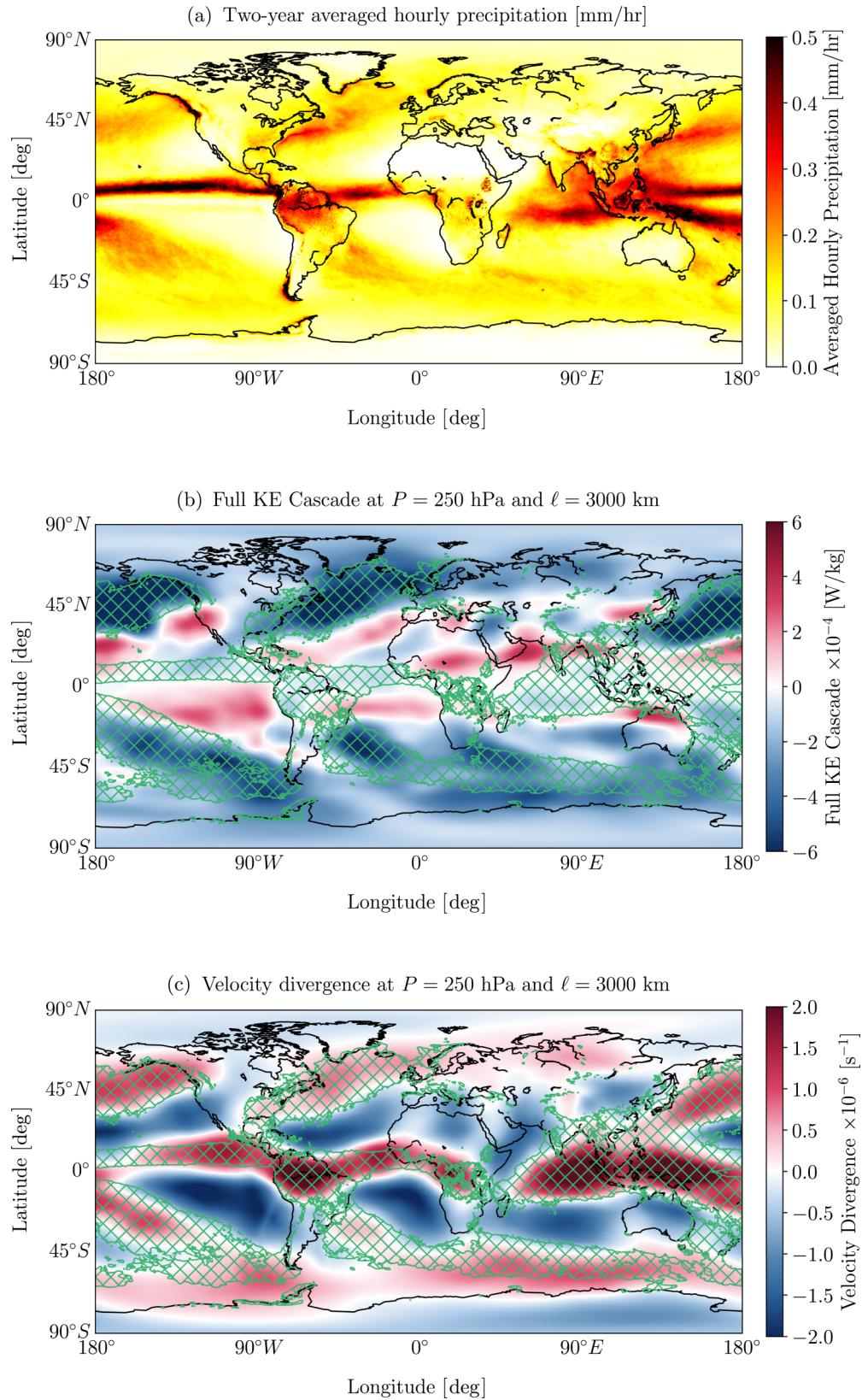


FIG. 8. Correlation between spatial maps of the KE cascade/flow divergence and precipitation. (a) Spatial distribution of the two-year averaged hourly precipitation. (b) Spatial distribution of the two-year averaged full KE cascade across $\ell = 3000$ km at $P = 250$ hPa. (c) Spatial distribution of the two-year averaged flow divergence at $\ell = 3000$ km and $P = 250$ hPa. In both panels (b) and (c), the green hatched regions are the high precipitation areas (average hourly precipitation above 0.1 mm/h).

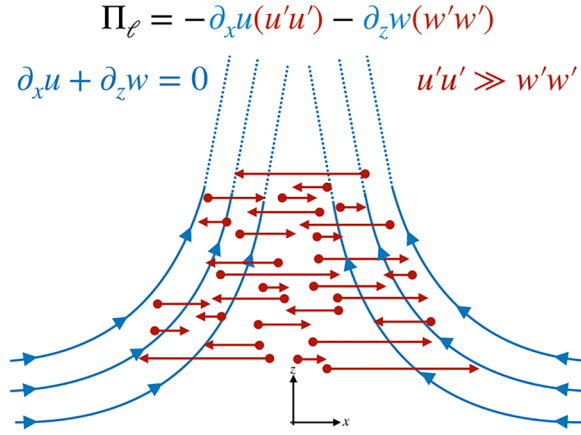


FIG. 9. Schematic of the anisotropic pressure-strain interaction. Blue streamlines represent a planetary-scale horizontally convergent flow in the x direction, balanced by upward motion in the vertical z direction, consistent with mass continuity ($\partial_x u + \partial_z w = 0$) to leading order. Red arrows represent sub-planetary-scale (synoptic and mesoscale) motions with an associated anisotropic stress tensor, where the horizontal component $\tau_{xx} \sim u'u'$ is much larger than the vertical component $\tau_{zz} \sim w'w'$. In this simplified two-dimensional schematic, the KE transfer is $\Pi_\ell = -\partial_x u \tau_{xx} - \partial_z w \tau_{zz}$. Although planetary-scale horizontal convergence is balanced by vertical divergence, $\partial_x u = -\partial_z w$, the vertical contribution to Π_ℓ is relatively much weaker because the vertical stress component is very small, $w'w' \ll u'u'$. Therefore, the horizontal contribution dominates, leading to a net positive transfer of KE from large to small scales. This anisotropic pressure-strain mechanism is analogous to a piston acting against the pressure of a gas, except that the effective pressure here is tensorial and highly anisotropic: It resists compression or rarefaction primarily along horizontal directions where synoptic and mesoscale motions are strongest. The mechanism remains significant after vertical integration because the sub-planetary-scale motions, and hence their effective stress, are vertically inhomogeneous. This schematic shows that, even when the 3D flow is divergence-free, persistent time-mean convergence or divergence in horizontal directions can produce significant interscale KE transfer. This mechanism is distinct from a classical turbulence cascade: Its key ingredients are inhomogeneity, here imposed by the surface boundary along the vertical direction, and anisotropy of the turbulent fluctuations.

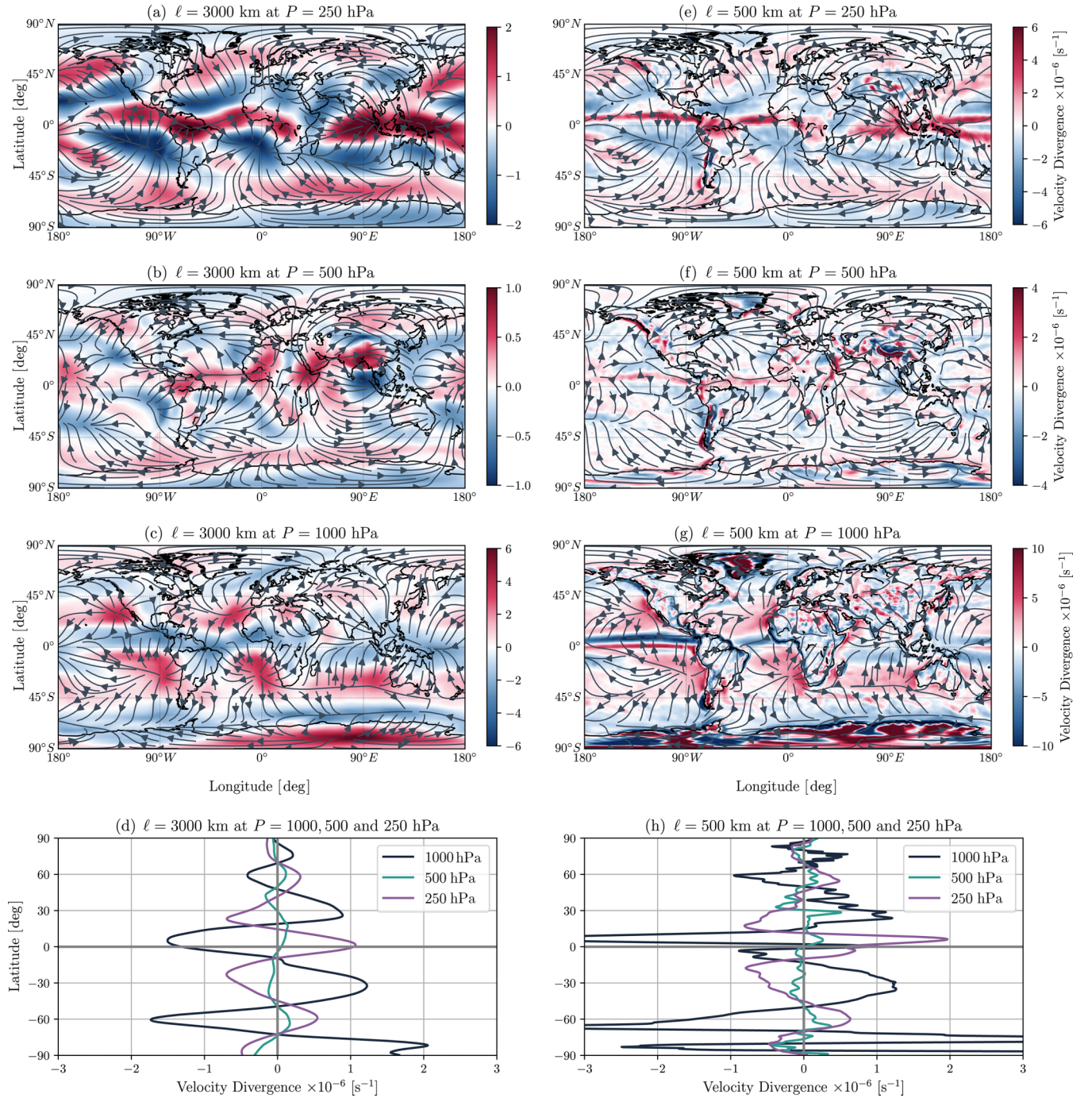


FIG. 10. Large-scale flow divergence maps to compare to cascade maps of Fig. 1. Spatial maps of two-year averaged velocity divergence at scales larger than (a)–(c) $\ell = 3000$ km and (e)–(g) $\ell = 500$ km at (a), (e) 250 hPa; (b), (f) 500 hPa; and (c), (g) 1000 hPa. In panels (a), (b), (c), (e), (f), and (g), streamlines are identical to those in Fig. 1. Bottom two panels [panels (d) and (h)] show the zonal means of panels (a)–(c) and panels (e)–(g), respectively.

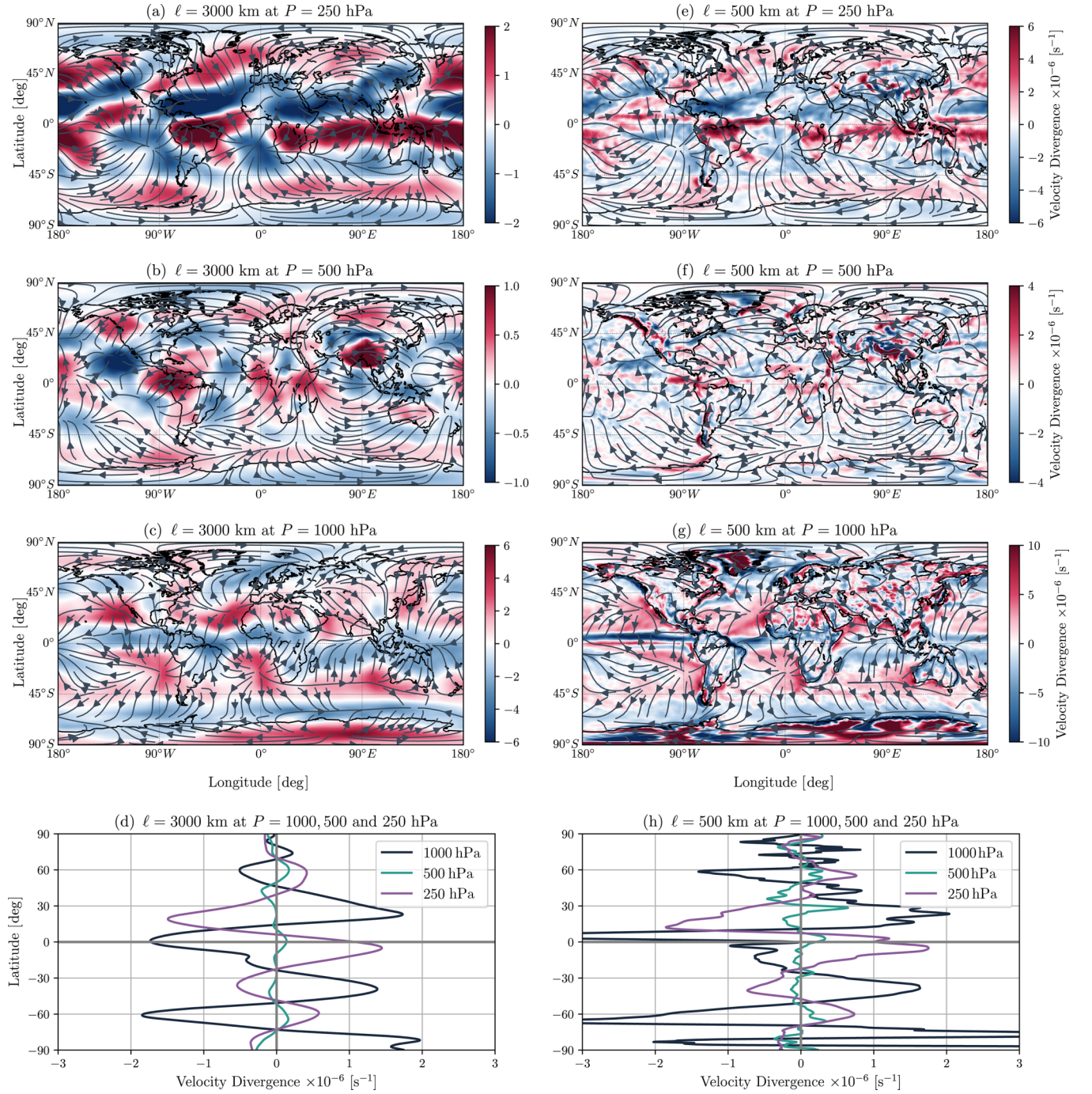


FIG. 11. Divergence maps similar to Fig. 10 but averaged over northern winter (JFM).

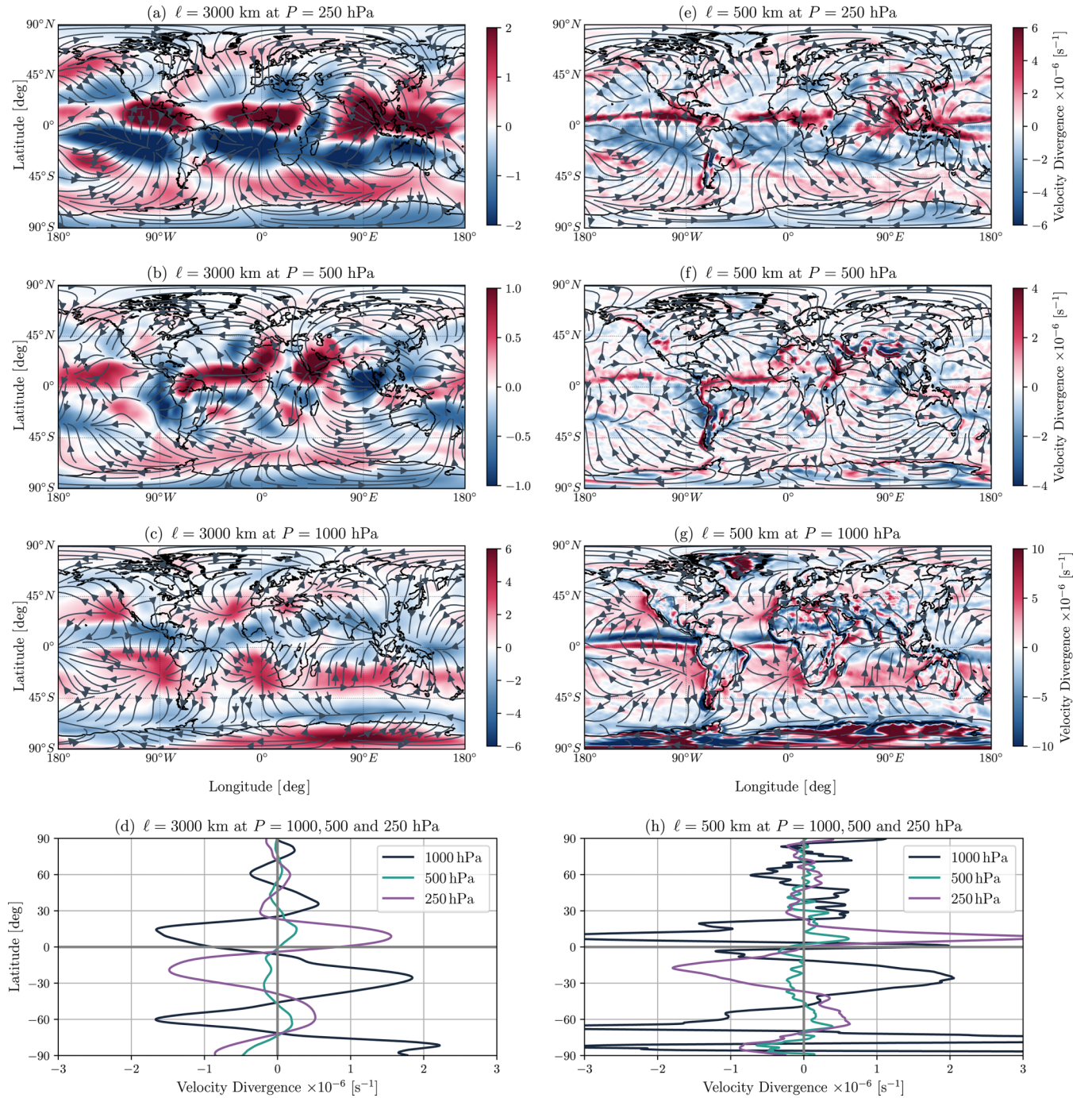


FIG. 12. Divergence maps similar to Fig. 10 but averaged over northern summer (JAS).

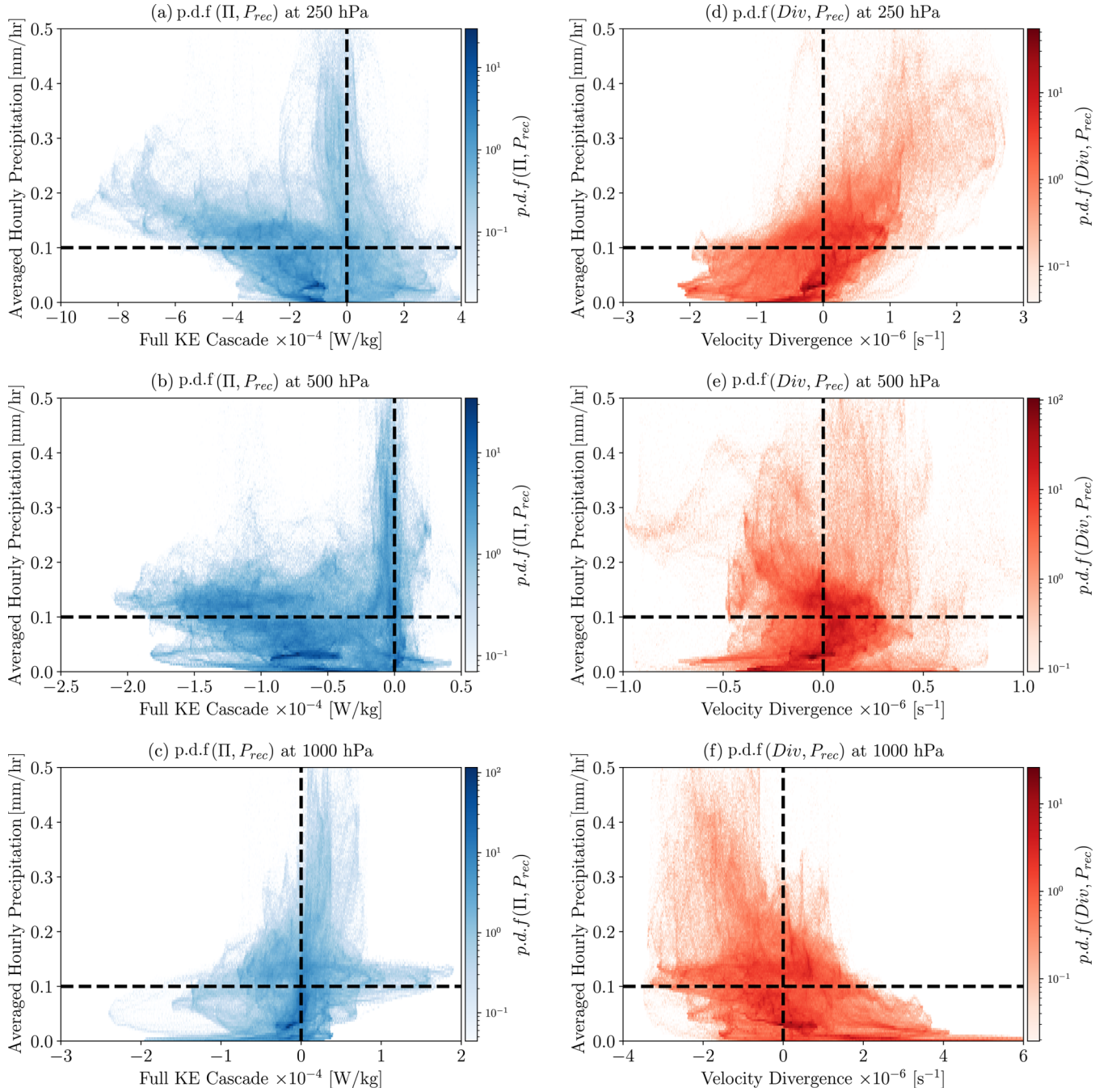


FIG. 13. Joint probability density function (p.d.f.) of full KE cascade (Π)/flow divergence (Div) at $\ell = 3000$ km and averaged hourly precipitation (P_{rec}). Joint probability density function of Π at $\ell = 3000$ km and averaged hourly precipitation are shown in panels (a)–(c). The p.d.f. of velocity divergence at $\ell = 3000$ km and P_{rec} are illustrated in panels (d)–(f). Joint probability density functions are evaluated at (a), (d) 250 hPa; (b), (e) 500 hPa; and (c), (f) 1000 hPa.

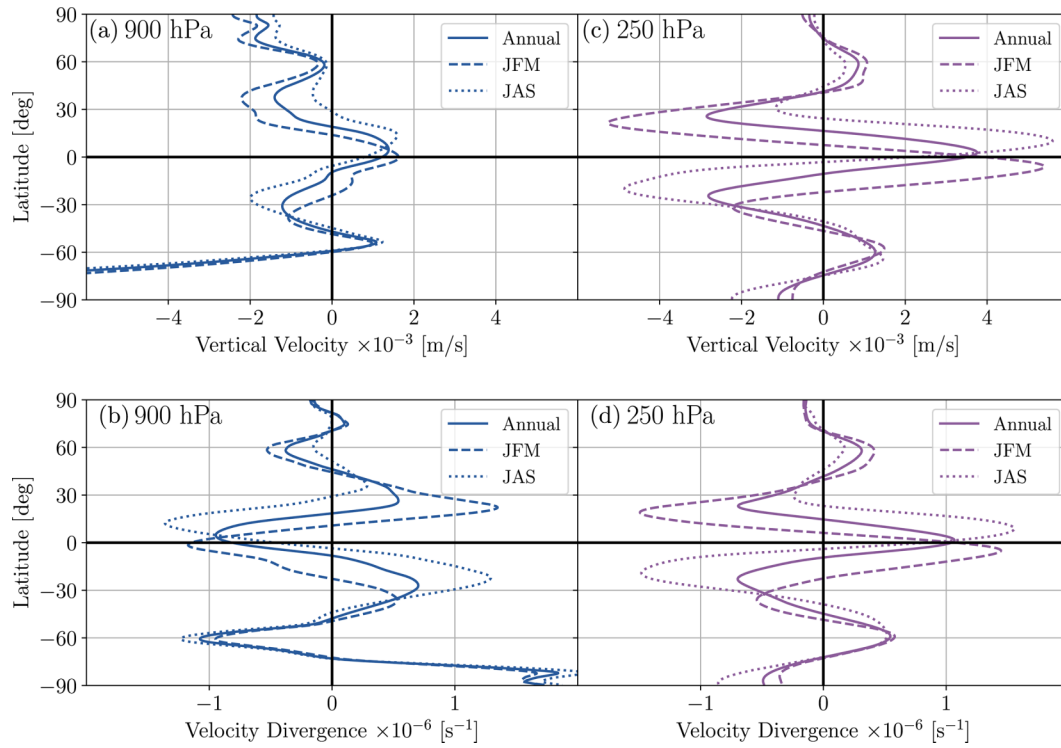


FIG. 14. Zonally averaged vertical velocity and horizontal velocity divergence. Zonally averaged vertical velocity filtered at $\ell = 3000$ km is shown in panels (a) and (c). Velocity divergence filtered at $\ell = 3000$ km is illustrated in panels (b) and (d). Panels (a) and (b) are representative of the lower troposphere ($P = 900$ hPa), and panels (c) and (d) are representative of upper troposphere ($P = 250$ hPa). Solid lines show the annually averaged (two years), while dashed lines show the time average over the northern winter (JFM), and the dotted lines indicate the time average over the northern summer (JAS).

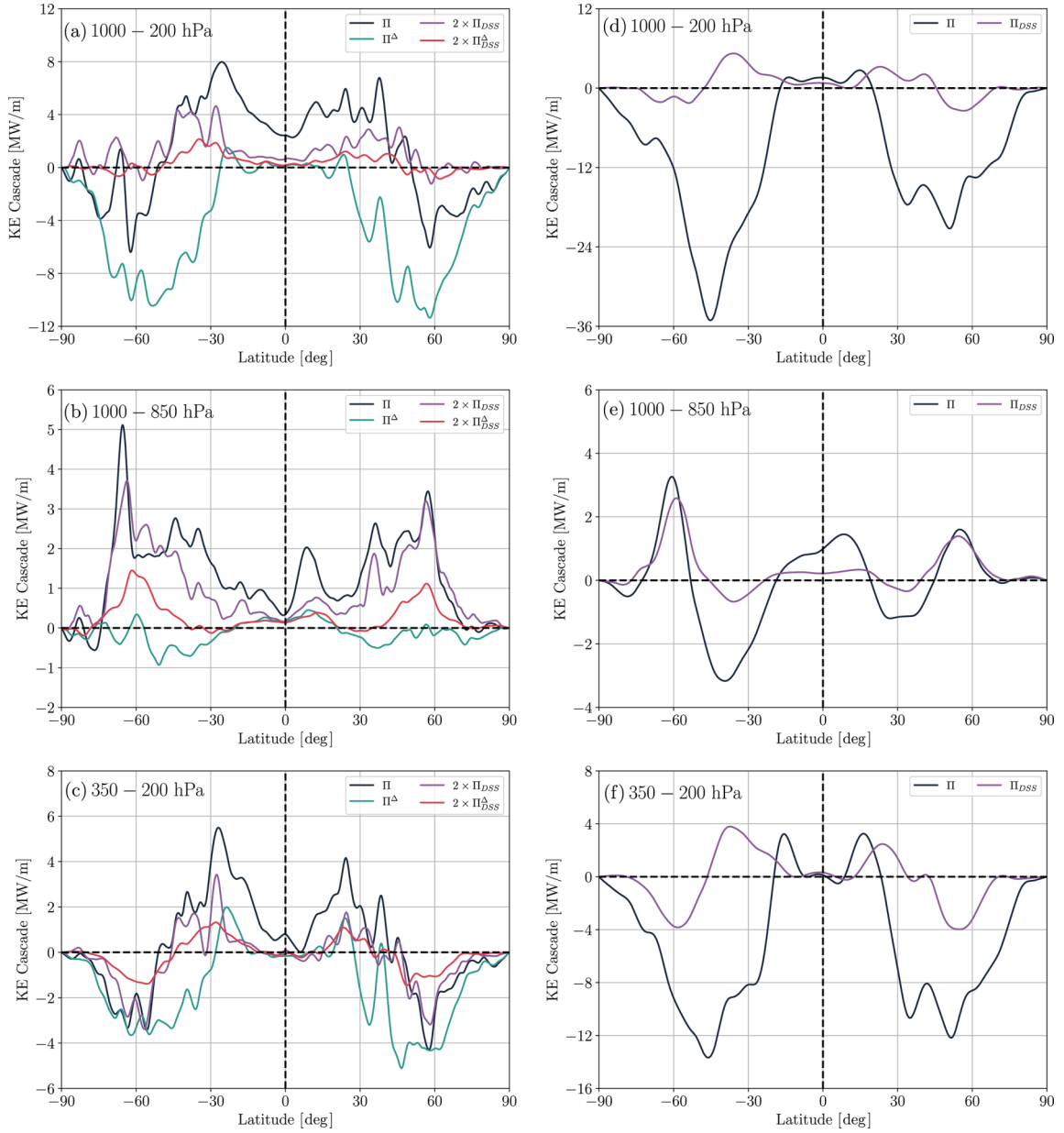


FIG. 15. Zonally and vertically (in altitude) integrated KE cascades. Panels (a)–(c) show different contributing terms (Π , Π^Δ , Π_{DSS} , and Π_{DSS}^Δ) to the KE scale transfer across $\ell = 1000$ km. Panels (d)–(f) illustrate Π and Π_{DSS} across $\ell = 3000$ km. The vertical integration (in altitude) is performed for the entire troposphere (1000–200 hPa) [panels (a) and (d)], lower troposphere (1000–850 hPa) [panels (b) and (e)], and upper troposphere (350–200 hPa) [panels (c) and (f)].

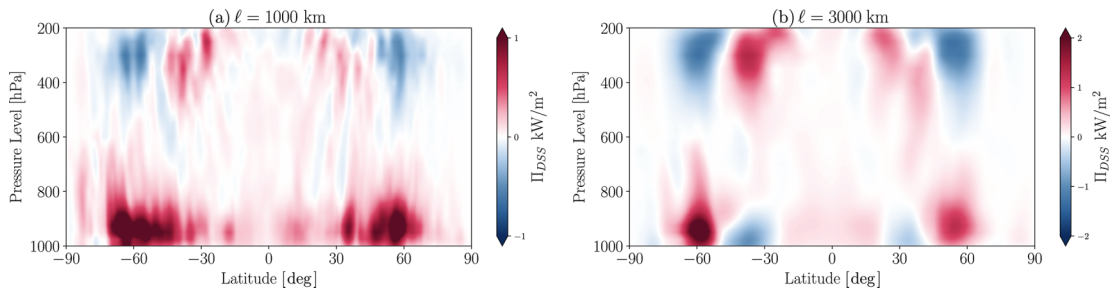


FIG. 16. Time-averaged latitude-pressure level distributions of Π_{DSS} . Latitude-depth profiles of time averaged and zonally integrated Π_{DSS} across $\ell = 1000$ km [panel (a)] and $\ell = 3000$ km [panel (b)].

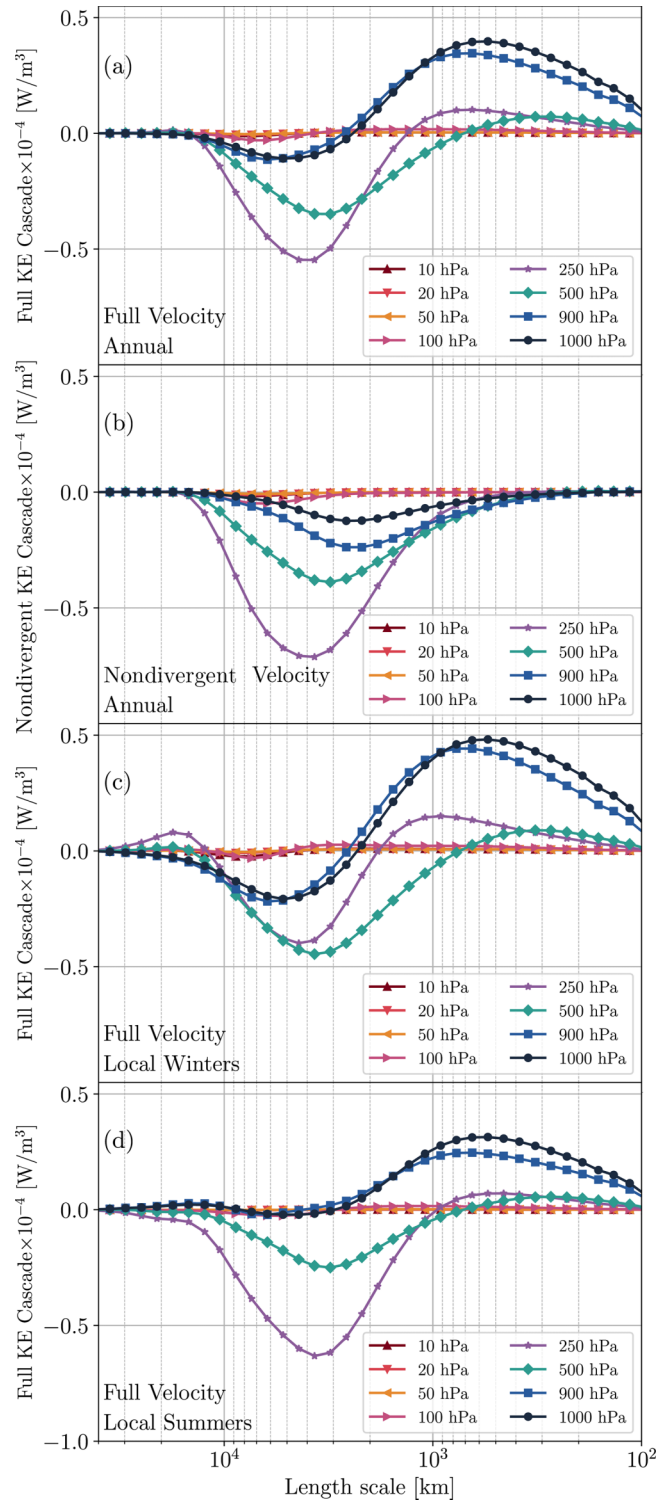


FIG. 17. Area-mean KE transfers, their seasonal dependence, and the impact of divergent flow. Similar to Fig. 3, but here Π is density weighted (in W/m^3) using the U.S. Standard Atmosphere model for calculating the averaged density at each pressure level (Table II). Kinetic energy cascade per unit volume as a function of length scale for different pressure levels in troposphere and stratosphere (1000, 900, 500, 250, 100, 50, 20, and 10 hPa). (a) Globally averaged full KE cascade as a function of length scale for different pressure levels. (b) Globally averaged nondivergent KE cascade as a function of length scale for different pressure levels. (c) Hemispherically averaged full KE cascade over local winters (NH JFM and SH JAS) as a function of length scale for different pressure levels. (d) Hemispherically averaged full KE cascade over local summers (NH JAS and SH JFM) as a function of length scale for different pressure levels.

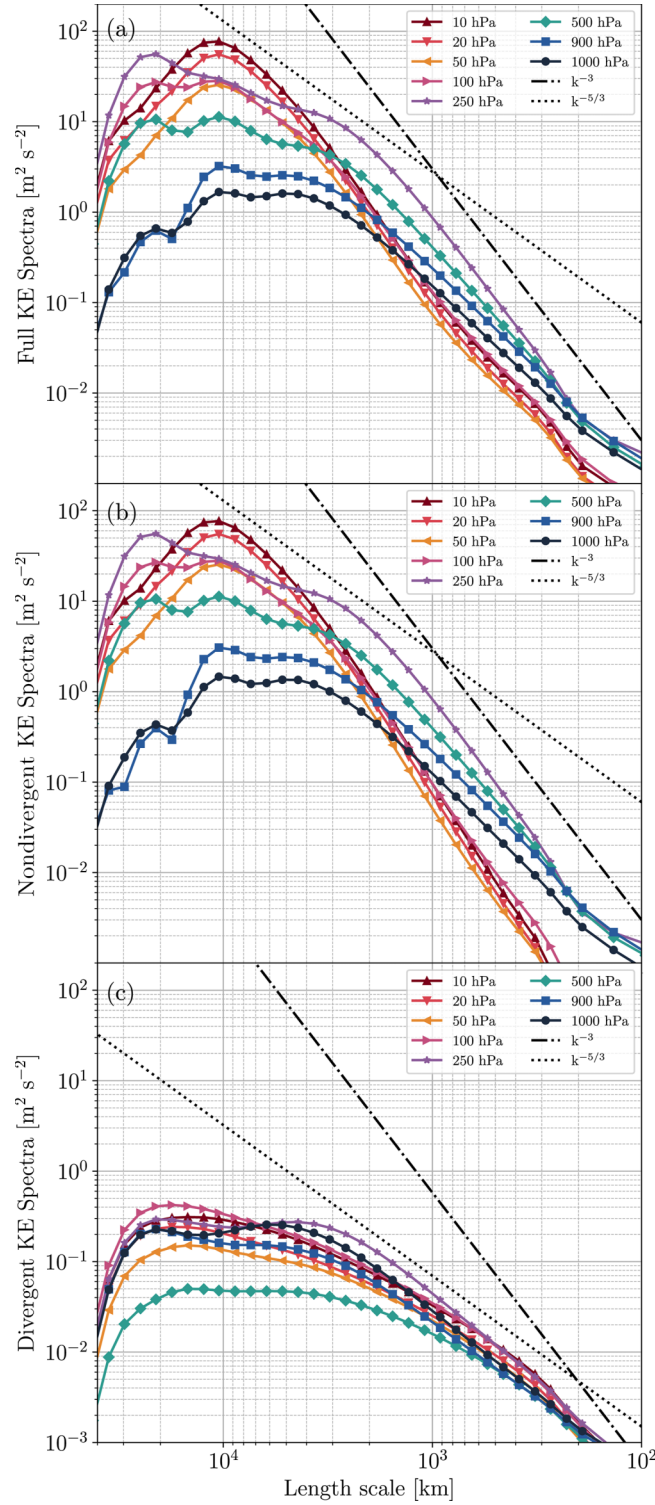


FIG. 18. Globally averaged KE spectra as a function of length scale at different pressure levels throughout troposphere, tropopause, and stratosphere. KE spectra for full [panel (a)], laterally nondivergent [panel (b)], and laterally divergent [panel (c)] flow contributions. Dashed and dotted black lines indicate the k^{-3} and $k^{-5/3}$ scaling for classical 2D and 3D turbulence, respectively.

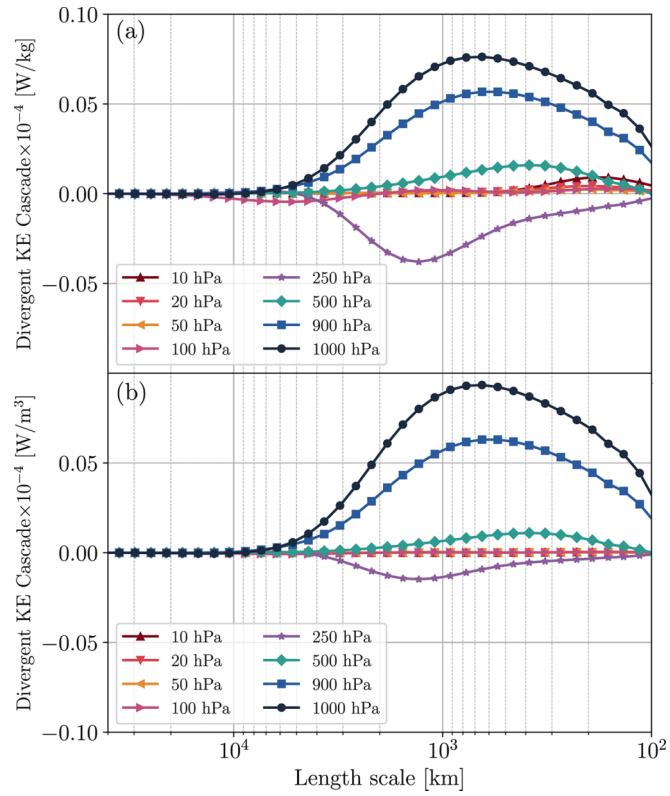


FIG. 19. Globally averaged KE cascade restricted to contributions from the laterally divergent flow. KE cascade per unit mass [panel (a)] and per unit volume [panel (b)] as a function of length scale at different pressure levels (1000, 900, 500, 250, 100, 50, 20, and 10 hPa). Plots show that the self-interactions of laterally divergent component of the flow yield downscale transfer at all pressure levels, with a weak upscale exception in the upper troposphere near the tropopause. This exception should not be interpreted as evidence that the divergent component dominates the upscale cascade near the tropopause (see discussion in main text).

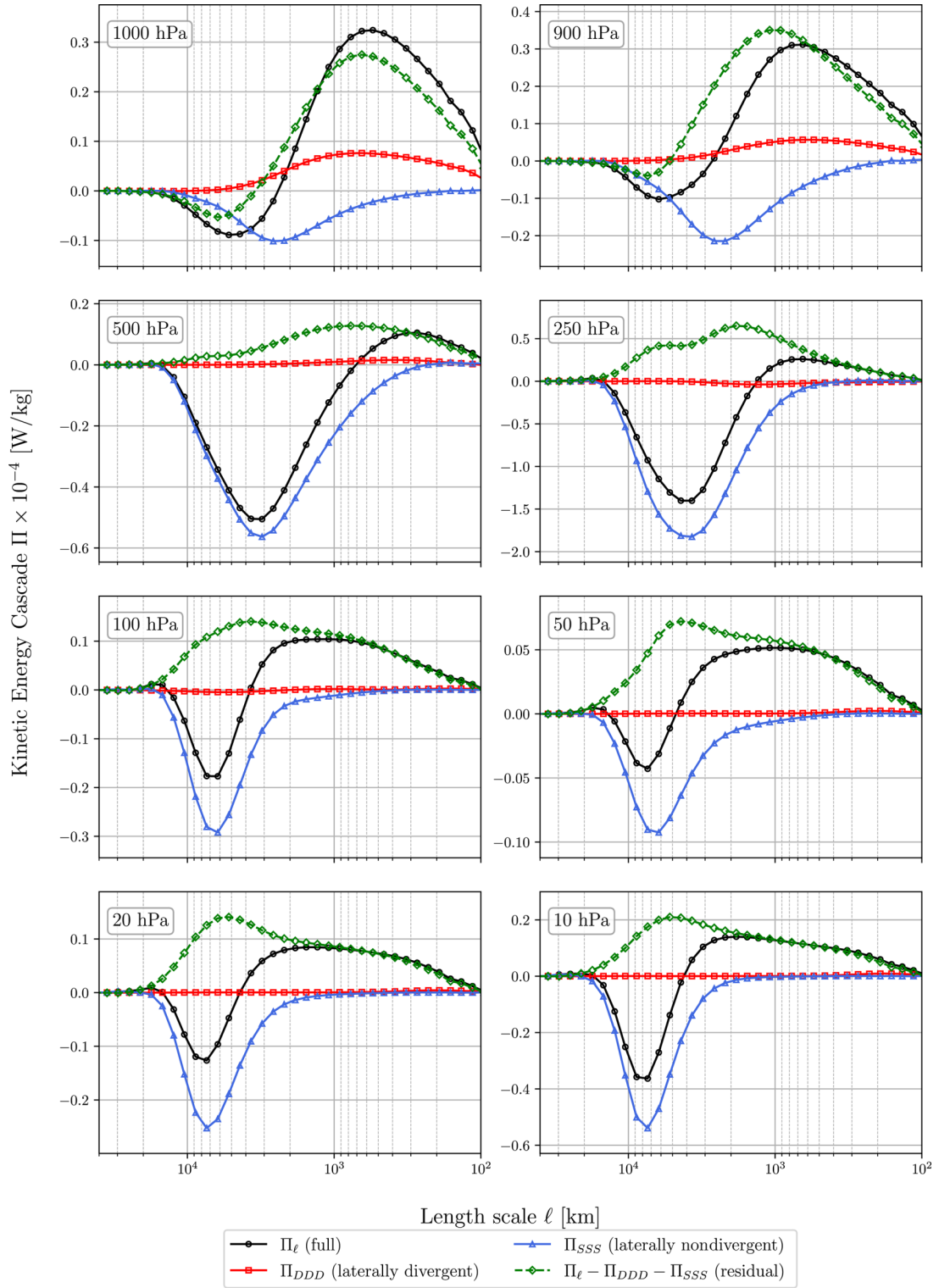


FIG. 20. Contributions of the Helmholtz decomposed velocity components to the globally averaged KE cascade at different pressure levels spanning the troposphere and stratosphere. In each panel, the full transfer Π_ℓ (black curves) is decomposed into the laterally nondivergent self-interaction Π_{SSS} (blue curves), the laterally divergent self-interaction Π_{DDD} (red curves), and the residual $\Pi_\ell - \Pi_{SSS} - \Pi_{DDD}$ (green curves), which is the sum of the mixed divergent/nondivergent cross-interaction terms, as a function of length scale ℓ . Positive and negative values denote downscale and upscale transfer, respectively. Π_{SSS} and Π_{DDD} are only the two pure self-interactions and do not sum to Π_ℓ . At every pressure level, Π_{DDD} stays much smaller than the residual across all scales: It is downscale in the lower troposphere and becomes small or weakly upscale from the midtroposphere upward, including at 250 hPa. Most of the transfer that cancels the upscale Π_{SSS} comes from the residual, so the divergent flow shapes the cascade mainly through its interaction with the nondivergent flow rather than through its own self-interaction.

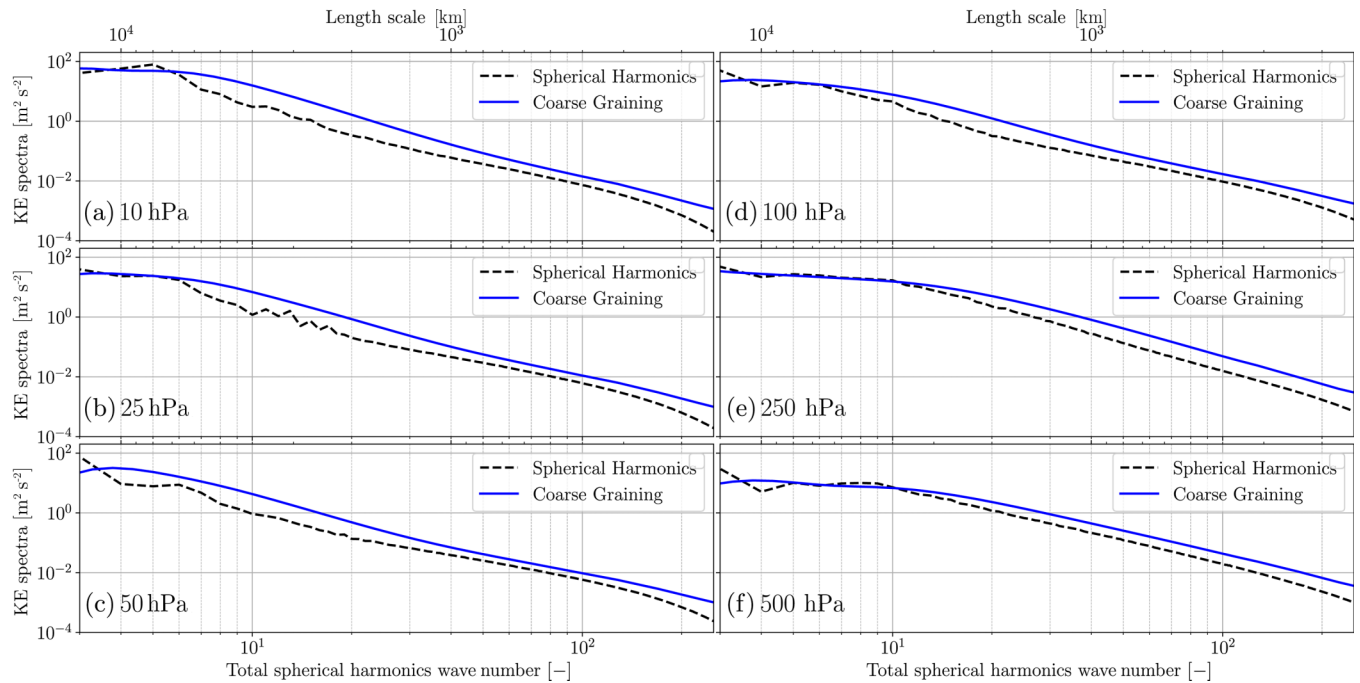


FIG. 21. Global KE spectra computed using coarse graining (solid blue lines) compared to spherical harmonics (dashed black lines) for different pressure levels: (a) 10 hPa, (b) 25 hPa, (c) 50 hPa, (d) 100 hPa, (e) 250 hPa, and (f) 500 hPa. The upper x axis in all panels corresponds to length scale, whereas the lower x axis is representing the full spherical harmonics wave number (full spherical harmonics wave number = $2\pi R/\ell$, where R is the Earth's radius and ℓ is the length scale).

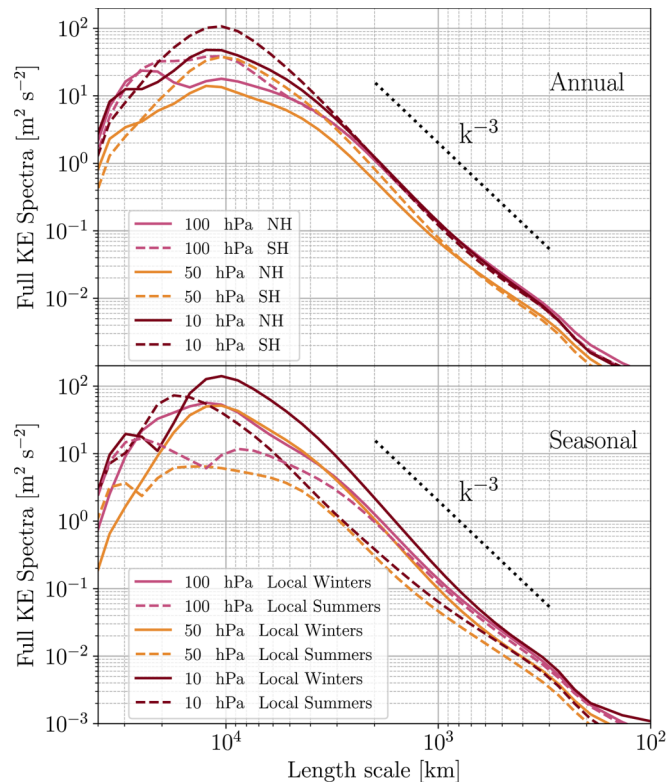


FIG. 22. Full kinetic energy spectra as a function of length scale for three different stratospheric pressure levels. (a) Averaged full KE spectra over the Northern Hemisphere (solid lines) and Southern Hemisphere (dashed lines). (b) Averaged full KE spectra over the local winters (solid lines) and local summers (dashed lines).

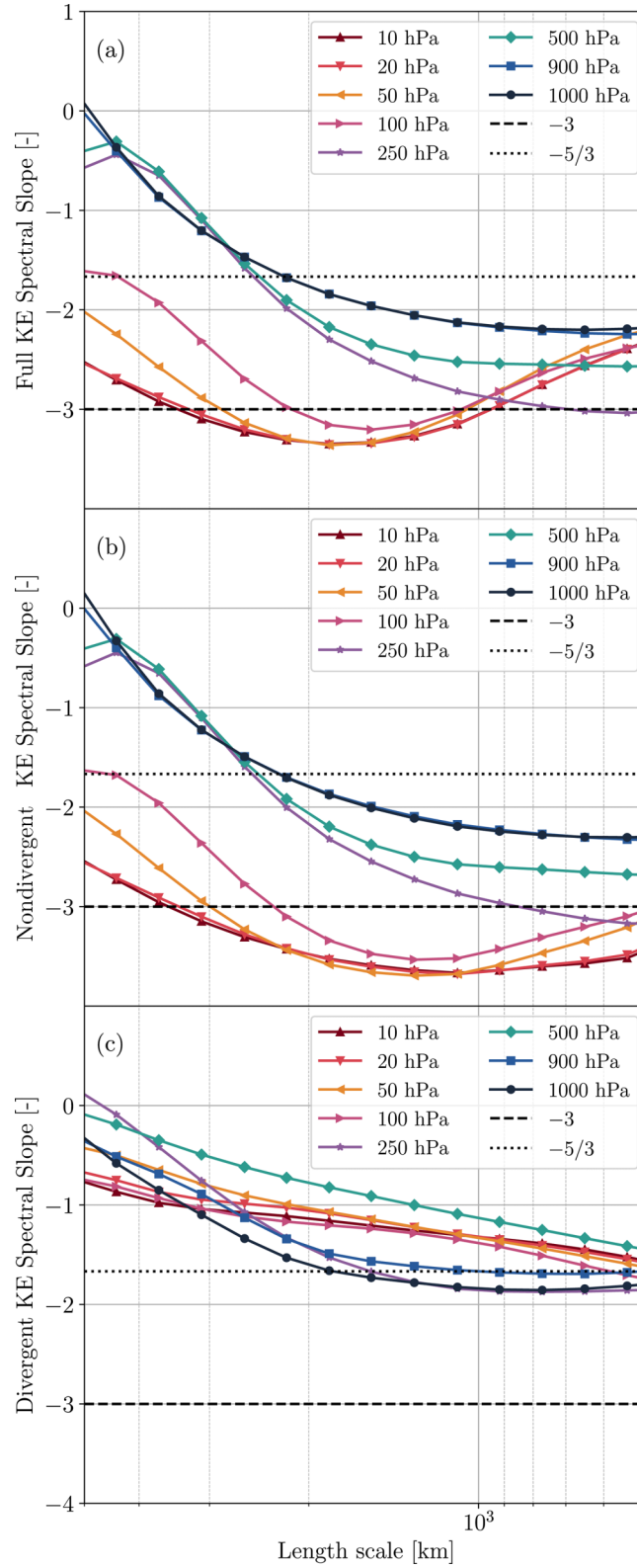


FIG. 23. Globally averaged KE spectral slope as a function of length scale at different pressure levels throughout troposphere, tropopause, and stratosphere. Spectral slopes are shown for full [panel (a)], laterally nondivergent [panel (b)], and laterally divergent [panel (c)] flow contributions. Dashed and dotted black lines indicate the -3 and $-5/3$ spectral slopes for classical 2D and 3D turbulence, respectively.

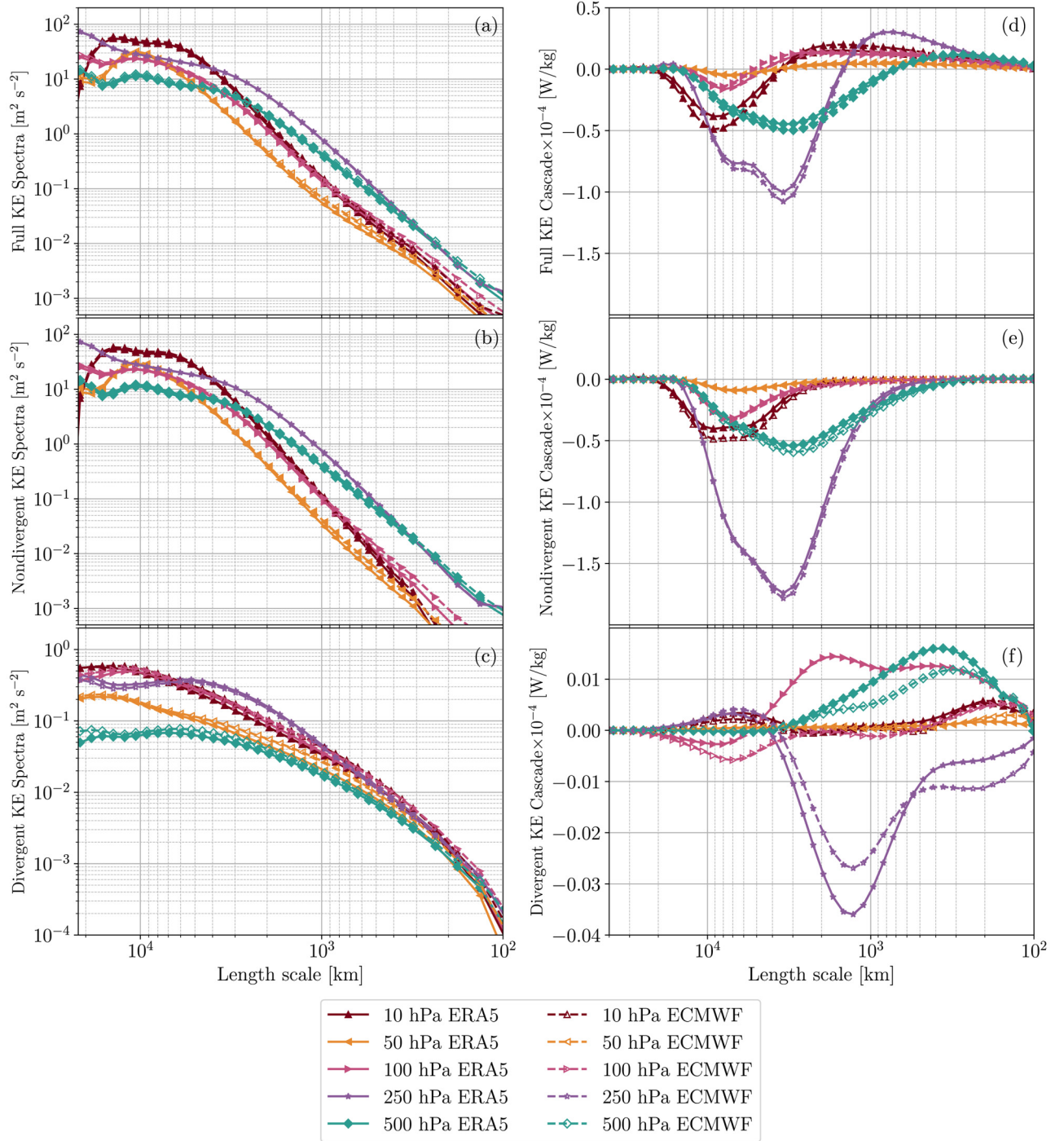


FIG. 24. Comparison of the computed KE spectra and cascade between ERA5 and ECMWF-IPY datasets, using coarse graining. Panels (a)–(c) illustrate the full, nondivergent, and divergent KE spectra at different pressure levels, respectively. Panels (d)–(f) show the full, nondivergent, and divergent KE cascade at different pressure levels, respectively. The solid lines with filled symbols represent data derived from ERA5, while the dashed lines with open symbols indicate data from ECMWF-IPY.

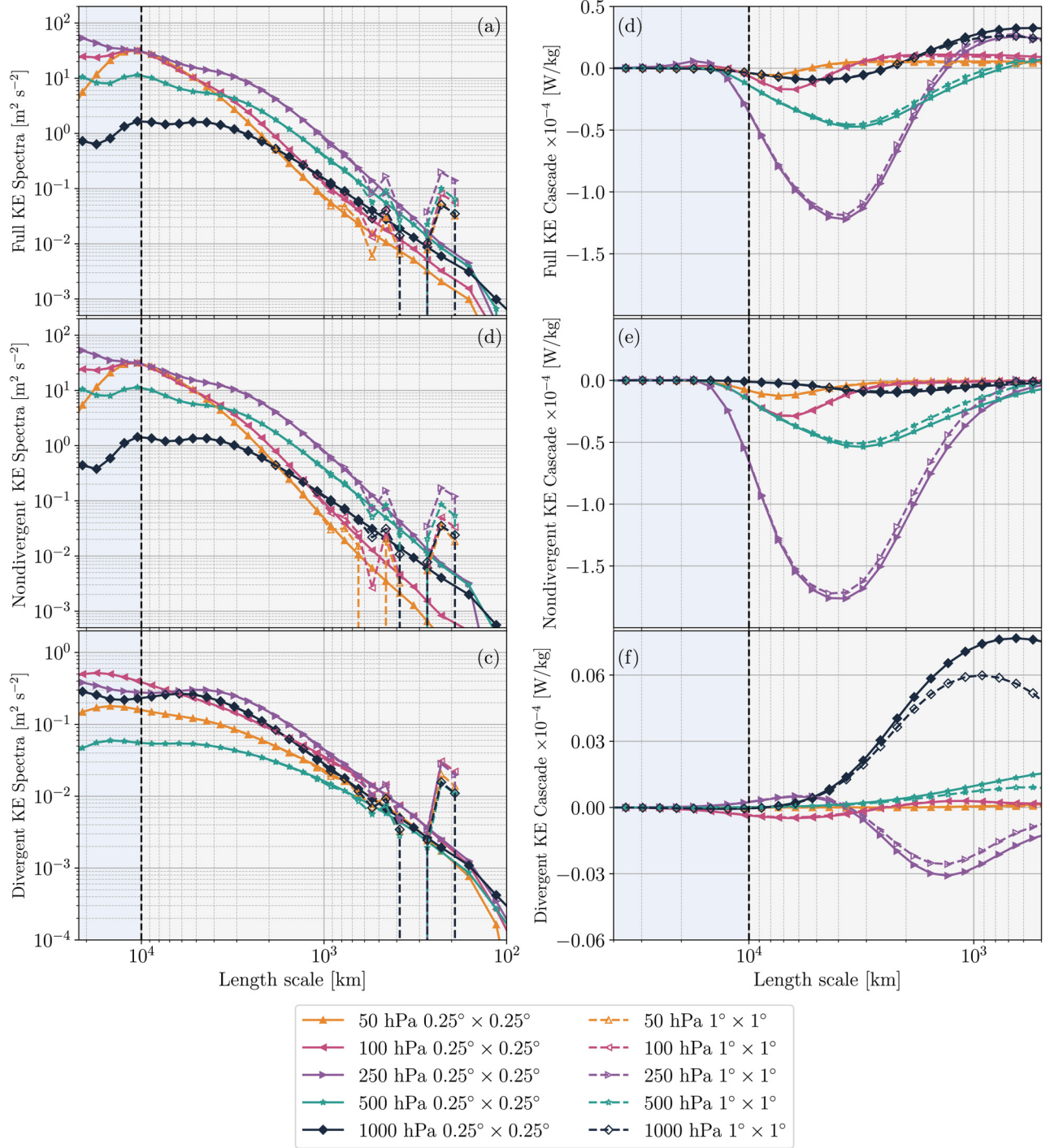


FIG. 25. Comparison of the computed KE spectra and cascade between the original grid and the down-sampled grid. Panels (a)–(c) illustrate the full, nondivergent, and divergent KE spectra at different pressure levels, respectively. Panels (d)–(f) show the full, nondivergent, and divergent KE cascade at different pressure levels, respectively. The solid lines with filled symbols represent data derived from the original $0.25^\circ \times 0.25^\circ$ grid, while the dashed lines with open symbols indicate data from the down-sampled $1^\circ \times 1^\circ$ grid. The blue shading regions indicate scales larger than 10×10^3 km, where the down-sampled $1^\circ \times 1^\circ$ grid is used throughout the article. The gray shading regions show scales smaller than 10×10^3 km, where the original $0.25^\circ \times 0.25^\circ$ ERA5 grid is retained. The dashed vertical line marks the 10×10^3 km threshold separating the two ranges.

- [1] F. Judt, Insights into atmospheric predictability through global convection-permitting model simulations, *J. Atmos. Sci.* **75**, 1477 (2018).
- [2] F. Zhang, Y. Q. Sun, L. Magnusson, R. Buizza, S.-J. Lin, J.-H. Chen, and K. Emanuel, What is the predictability limit of midlatitude weather? *J. Atmos. Sci.* **76**, 1077 (2019).
- [3] B.-W. Shen, R. A. Pielke, X. Zeng, and X. Zeng, Exploring the origin of the two-week predictability limit: A revisit of Lorenz's predictability studies in the 1960s, *Atmosphere* **15**, 837 (2024).
- [4] E. N. Lorenz, The predictability of a flow which possesses many scales of motion, *Tellus* **21**, 289 (1969).
- [5] R. Fjørtoft, On the changes in the spectral distribution of kinetic energy for twodimensional, nondivergent flow, *Tellus* **5**, 225 (1953).
- [6] R. H. Kraichnan, Inertial ranges in two-dimensional turbulence, *Phys. Fluids* **10**, 1417 (1967).
- [7] J. G. Charney, Geostrophic turbulence, *J. Atmos. Sci.* **28**, 1087 (1971).
- [8] L. F. Richardson and G. T. Walker, Atmospheric diffusion shown on a distance-neighbour graph, *Proc. R. Soc. A* **110**, 709 (1926).
- [9] A. N. Kolmogorov, The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers, *Proc. R. Soc. A* **434**, 9 (1991); *Dokl. Akad. Nauk SSSR* **30**, 301 (1941).
- [10] L. Onsager, Statistical hydrodynamics, *Nuovo Cim.* **6**, 279 (1949).
- [11] G. L. Eyink and K. R. Sreenivasan, Onsager and the theory of hydrodynamic turbulence, *Rev. Mod. Phys.* **78**, 87 (2006).
- [12] G. K. Vallis, *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-Scale Circulation*, 2nd ed. (Cambridge University Press, Cambridge, 2017).
- [13] J. C. McWilliams, The emergence of isolated coherent vortices in turbulent flow, *J. Fluid Mech.* **146**, 21 (1984).
- [14] G. Boffetta and R. E. Ecke, Two-dimensional turbulence, *Annu. Rev. Fluid Mech.* **44**, 427 (2012).
- [15] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, 1995).
- [16] A. Alexakis and L. Biferale, Cascades and transitions in turbulent flows, *Phys. Rep.* **767–769**, 1 (2018).
- [17] K. S. Gage, Evidence for a $k^{-5/3}$ law inertial range in mesoscale two-dimensional turbulence, *J. Atmos. Sci.* **36**, 1950 (1979).
- [18] D. K. Lilly, Stratified turbulence and the mesoscale variability of the atmosphere, *J. Atmos. Sci.* **40**, 749 (1983).
- [19] G. D. Nastrom and K. S. Gage, A climatology of atmospheric wavenumber spectra of wind and temperature observed by commercial aircraft, *J. Atmos. Sci.* **42**, 950 (1985).
- [20] K. S. Gage and G. D. Nastrom, Theoretical interpretation of atmospheric wavenumber spectra of wind and temperature observed by commercial aircraft during GASP, *J. Atmos. Sci.* **43**, 729 (1986).
- [21] J. T. Bacmeister, S. D. Eckermann, P. A. Newman, L. Lait, K. R. Chan, M. Loewenstein, M. H. Proffitt, and B. L. Gary, Stratospheric horizontal wavenumber spectra of winds, potential temperature, and atmospheric tracers observed by high-altitude aircraft, *J. Geophys. Res.: Atmos.* **101**, 9441 (1996).
- [22] E. Lindborg and J. Y. N. Cho, Horizontal velocity structure functions in the upper troposphere and lower stratosphere: 2. Theoretical considerations, *J. Geophys. Res.: Atmos.* **106**, 10233 (2001).
- [23] Q. Jiang, J. D. Doyle, V. Grubišić, and R. B. Smith, Turbulence characteristics in an elevated shear layer over Owens Valley, *J. Atmos. Sci.* **67**, 2355 (2010).
- [24] D. Byrne and J. A. Zhang, Height-dependent transition from 3-D to 2-D turbulence in the hurricane boundary layer, *Geophys. Res. Lett.* **40**, 1439 (2013).
- [25] W. R. Burrows, A diagnostic study of atmospheric spectral kinetic energetics, *J. Atmos. Sci.* **33**, 2308 (1976).
- [26] G. J. Boer and T. G. Shepherd, Large-scale two-dimensional turbulence in the atmosphere, *J. Atmos. Sci.* **40**, 164 (1983).
- [27] S. J. Lambert, A global available potential energy-kinetic energy budget in terms of the two-dimensional wavenumber for the FGGE year, *Atmos. Ocean* **22**, 265 (1984).
- [28] T. G. Shepherd, A spectral view of nonlinear fluxes and stationary-transient interaction in the atmosphere, *J. Atmos. Sci.* **44**, 1166 (1987).
- [29] G. J. Boer, Mean and transient spectral energy and enstrophy budgets, *J. Atmos. Sci.* **51**, 1765 (1994).
- [30] D. M. Straus and P. Ditlevsen, Two-dimensional turbulence properties of the ECMWF reanalyses, *Tellus A: Dyn. Meteorol. Oceanogr.* **51**, 749 (1999).
- [31] B. H. Burgess, A. R. Erler, and T. G. Shepherd, The troposphere-to-stratosphere transition in kinetic energy spectra and nonlinear spectral fluxes as seen in ECMWF analyses, *J. Atmos. Sci.* **70**, 669 (2013).
- [32] P. Augier and E. Lindborg, A new formulation of the spectral energy budget of the atmosphere, with application to two high-resolution general circulation models, *J. Atmos. Sci.* **70**, 2293 (2013).
- [33] J. Peng, L. Zhang, and J. Guan, Applications of a moist nonhydrostatic formulation of the spectral energy budget to baroclinic waves. Part I: The lower-stratospheric energy spectra, *J. Atmos. Sci.* **72**, 2090 (2015).
- [34] Y. Q. Sun, R. Rotunno, and F. Zhang, Contributions of moist convection and internal gravity waves to building the atmospheric $-5/3$ kinetic energy spectra, *J. Atmos. Sci.* **74**, 185 (2017).
- [35] P. L. Read, F. Tabataba-Vakili, Y. Wang, P. Augier, E. Lindborg, A. Valeanu, and R. M. B. Young, Comparative terrestrial atmospheric circulation regimes in simplified global circulation models. Part II: Energy budgets and spectral transfers, *Q. J. R. Meteorol. Soc.* **144**, 2558 (2018).
- [36] Z. Li, J. Peng, and L. Zhang, Spectral budget of rotational and divergent kinetic energy in global analyses, *J. Atmos. Sci.* **80**, 813 (2023).
- [37] Z. Li, J. Peng, and L. Zhang, Impact of convective parameterizations on atmospheric mesoscale kinetic energy spectra in global high-resolution simulations, *Geophys. Res. Lett.* **50**, e2023GL105513 (2023).
- [38] G. J. Boer, Homogeneous and isotropic turbulence on the sphere, *J. Atmos. Sci.* **40**, 154 (1983).
- [39] D. Faranda, V. Lembo, M. Iyer, D. Kuzay, S. Chibbaro, F. Daviaud, and B. Dubrulle, Computation and characterization of local subfilter-scale energy transfers in atmospheric flows, *J. Atmos. Sci.* **75**, 2175 (2018).
- [40] H. Aluie, M. Hecht, and G. K. Vallis, Mapping the energy cascade in the North Atlantic Ocean: The coarse-graining approach, *J. Phys. Oceanogr.* **48**, 225 (2018).
- [41] H. Aluie, Convolutions on the sphere: Commutation with differential operators, *GEM Int. J. Geomath.* **10**, 9 (2019).

- [42] B. A. Storer, M. Buzzicotti, H. Khatri, S. M. Griffies, and H. Aluie, Global energy spectrum of the general oceanic circulation, *Nat. Commun.* **13**, 5314 (2022).
- [43] B. A. Storer, M. Buzzicotti, H. Khatri, S. M. Griffies, and H. Aluie, Global cascade of kinetic energy in the ocean and the atmospheric imprint, *Sci. Adv.* **9**, eadi7420 (2023).
- [44] S. Kouhen, B. A. Storer, H. Aluie, D. P. Marshall, and H. M. Christensen, Convective and orographic origins of the mesoscale kinetic energy spectrum, *Geophys. Res. Lett.* **51**, e2024GL110804 (2024).
- [45] O. Reynolds, On the dynamical theory of incompressible viscous fluids and the determination of the criterion, *Phil. Trans. R. Soc. A* **186**, 123 (1895).
- [46] D. G. Andrews and M. E. McIntyre, Planetary waves in horizontal and vertical shear: The generalized Eliassen–Palm relation and the mean zonal acceleration, *J. Atmos. Sci.* **33**, 2031 (1976).
- [47] H. Hersbach, B. Bell, P. Berrisford, G. Biavati, A. Horányi, J. Muñoz Sabater, J. Nicolas, C. Peubey, R. Radu, I. Rozum, D. Schepers, A. Simmons, C. Soci, D. Dee, and J.-N. Thépaut, ERA5 hourly data on pressure levels from 1940 to present, Copernicus Climate Change Service (C3S) Climate Data Store (CDS), 2023, <https://doi.org/10.24381/cds.bd0915c6>.
- [48] M. Sadek and H. Aluie, Extracting the spectrum of a flow by spatial filtering, *Phys. Rev. Fluids* **3**, 124610 (2018).
- [49] T. Schneider, T. Bischoff, and G. H. Haug, Migrations and dynamics of the intertropical convergence zone, *Nature (London)* **513**, 45 (2014).
- [50] J. R. Holton and G. J. Hakim, *An Introduction to Dynamic Meteorology*, 5th ed., International Geophysics Series, Vol. 88. (Academic Press, Waltham, Massachusetts, USA, 2012).
- [51] K. E. Trenberth, D. P. Stepaniak, and J. M. Caron, The global monsoon as seen through the divergent atmospheric circulation, *J. Clim.* **13**, 3969 (2000).
- [52] H. Seo, L. W. O'Neill, M. A. Bourassa, A. Czaja, K. Drushka, J. B. Edson, B. Fox-Kemper, I. Frenger, S. T. Gille, B. P. Kirtman, *et al.*, Ocean mesoscale and frontal-scale ocean–atmosphere interactions and influence on large-scale climate: A review, *J. Clim.* **36**, 1981 (2023).
- [53] X. Luan, W. Wang, A. Burns, S. Solomon, Y. Zhang, and L. J. Paxton, Seasonal and hemispheric variations of the total auroral precipitation energy flux from TIMED/GUVI, *J. Geophys. Res.: Space Phys.* **115**, 2009JA015063 (2010).
- [54] D. Zhao and H. Aluie, Calculating spectra by sequential filtering, *J. Renewable Sustainable Energy* **17**, 013303 (2025).
- [55] J. N. Koshyk and K. Hamilton, The horizontal kinetic energy spectrum and spectral budget simulated by a high-resolution troposphere–stratosphere–mesosphere GCM, *J. Atmos. Sci.* **58**, 329 (2001).
- [56] A. Alexakis, R. Marino, P. D. Mininni, A. van Kan, R. Foldes, and F. Feraco, Large-scale self-organization in dry turbulent atmospheres, *Science* **383**, 1005 (2024).
- [57] K. M. Grise, D. W. J. Thompson, and T. Birner, A global survey of static stability in the stratosphere and upper troposphere, *J. Clim.* **23**, 2275 (2010).
- [58] R. Salmon, Baroclinic instability and geostrophic turbulence, *Geophys. Astrophys. Fluid Dyn.* **15**, 167 (1980).
- [59] V. D. Larichev and I. M. Held, Eddy amplitudes and fluxes in a homogeneous model of fully developed baroclinic instability, *J. Phys. Oceanogr.* **25**, 2285 (1995).
- [60] J. Callies, R. Ferrari, and O. Bühler, Transition from geostrophic turbulence to inertia–gravity waves in the atmospheric energy spectrum, *Proc. Natl. Acad. Sci. USA* **111**, 17033 (2014).
- [61] D. S. Adamson, S. E. Belcher, B. J. Hoskins, and R. S. Plant, Boundary-layer friction in midlatitude cyclones, *Q. J. R. Meteorol. Soc.* **132**, 101 (2006).
- [62] F. Ait-Chaalal and T. Schneider, Why eddy momentum fluxes are concentrated in the upper troposphere, *J. Atmos. Sci.* **72**, 1585 (2015).
- [63] M. Shiotani, Planetary wave activity in the troposphere and stratosphere during the Northern Hemisphere winter, *J. Atmos. Sci.* **43**, 3200 (1986).
- [64] W. J. Randel, The seasonal evolution of planetary waves in the Southern Hemisphere stratosphere and troposphere, *Q. J. R. Meteorol. Soc.* **114**, 1385 (1988).
- [65] L. M. Polvani and P. J. Kushner, Tropospheric response to stratospheric perturbations in a relatively simple general circulation model, *Geophys. Res. Lett.* **29**, 18 (2002).
- [66] T. Birner, Fine-scale structure of the extratropical tropopause region, *J. Geophys. Res.: Atmos.* **111**, 2005JD006301 (2006).
- [67] E. Lindborg, The energy cascade in a strongly stratified fluid, *J. Fluid Mech.* **550**, 207 (2006).
- [68] F. Zonta, P. Hadi Sichani, and A. Soldati, Interaction between thermal stratification and turbulence in channel flow, *J. Fluid Mech.* **945**, A3 (2022).
- [69] J. G. Charney and P. G. Drazin, Propagation of planetary-scale disturbances from the lower into the upper atmosphere, *J. Geophys. Res.* **66**, 83 (1961).
- [70] D. G. Andrews, J. R. Holton, and C. B. Leovy, *Middle Atmosphere Dynamics* (Academic Press Inc., Orlando, FL, 1986).
- [71] R. A. Plumb, Stratospheric transport, *J. Meteorol. Soc. Jpn. Ser. II* **80**, 793 (2002).
- [72] K. E. Trenberth and A. Solomon, Implications of global atmospheric spatial spectra for processing and displaying data, *J. Clim.* **6**, 531 (1993).
- [73] T. A. Shaw, O. Miyawaki, and A. Donohoe, Stormier Southern Hemisphere induced by topography and ocean circulation, *Proc. Natl. Acad. Sci. USA* **119**, e2123512119 (2022).
- [74] S.-S. Ding, H. Bobas, H. Scolan, R. M. B. Young, and P. L. Read, Stratification-dependent enstrophy-controlled regime in geostrophic turbulence, *Phys. Rev. Lett.* **136**, 114101 (2026).
- [75] H. Aluie and S. Kurien, Joint downscale fluxes of energy and potential enstrophy in rotating stratified Boussinesq flows, *Europhys. Lett.* **96**, 44006 (2011).
- [76] M. Rivera, H. Aluie, and R. E. Ecke, The direct enstrophy cascade of two-dimensional soap film flows, *Phys. Fluids* **26**, 055105 (2014).
- [77] B. Vreman, B. Geurts, and H. Kuerten, Realizability conditions for the turbulent stress tensor in large-eddy simulation, *J. Fluid Mech.* **278**, 351 (1994).
- [78] H. Aluie and G. L. Eyink, Localness of energy cascade in hydrodynamic turbulence. II. Sharp spectral filter, *Phys. Fluids* **21**, 115108 (2009).
- [79] M. Buzzicotti, M. Linkmann, H. Aluie, L. Biferale, J. Brasseur, and C. Meneveau, Effect of filter type on the statistics of energy transfer between resolved and subfilter scales from a-priori

- analysis of direct numerical simulations of isotropic turbulence, *J. Turbul.* **19**, 167 (2018).
- [80] M. Buzzicotti, B. Storer, H. Khatri, S. Griffies, and H. Aluie, Spatio-temporal coarse-graining decomposition of the global ocean geostrophic kinetic energy, *J. Adv. Model. Earth Syst.* **15**, e2023MS003693 (2023).
- [81] G. L. Eyink and H. Aluie, Localness of energy cascade in hydrodynamic turbulence. I. Smooth coarse graining, *Phys. Fluids* **21**, 115107 (2009).
- [82] M. Germano, Turbulence: The filtering approach, *J. Fluid Mech.* **238**, 325 (1992).
- [83] cds.climate.copernicus.eu.
- [84] H. Hersbach, B. Bell, P. Berrisford, S. Hirahara, A. Horányi, J. Muñoz-Sabater, J. Nicolas, C. Peubey, R. Radu, D. Schepers, *et al.*, The ERA5 global reanalysis, *Q. J. R. Meteorol. Soc.* **146**, 1999 (2020).
- [85] aparc-climate.org.
- [86] B. A. Storer and H. Aluie, FlowSieve: A coarse-graining utility for geophysical flows on the sphere, *J. Open Source Software* **8**, 4277 (2023).
- [87] C. Meneveau, Statistics of turbulence subgrid-scale stresses: Necessary conditions and experimental tests, *Phys. Fluids* **6**, 815 (1994).
- [88] G. L. Eyink, Local energy flux and the refined similarity hypothesis, *J. Stat. Phys.* **78**, 335 (1995).
- [89] H. Aluie, Hydrodynamic and magnetohydrodynamic turbulence: Invariants, cascades, and locality, Ph.D. thesis, The Johns Hopkins University, 2009.
- [90] H. Aluie, Coarse-grained incompressible magnetohydrodynamics: Analyzing the turbulent cascades, *New J. Phys.* **19**, 025008 (2017).
- [91] H. Aluie, Compressible turbulence: The cascade and its locality, *Phys. Rev. Lett.* **106**, 174502 (2011).
- [92] H. Aluie, Scale decomposition in compressible turbulence, *Physica D* **247**, 54 (2013).
- [93] B. J. Hoskins, I. N. James, and G. H. White, The shape, propagation and mean-flow interaction of large-scale weather systems, *J. Atmos. Sci.* **40**, 1595 (1983).
- [94] github.com/husseinaluie/FlowSieve.
- [95] flowsieve.readthedocs.io.