

Decoherence from universal tomographic measurements

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The decoherence phenomenon arising from an environmental monitoring of the state of a quantum system, as opposed to monitoring of a preferred observable, is worked out in detail using two equivalent formulations, namely, repeated applications of universal tomographic measurements using positive operator-valued measures, and its continuous time unraveling from the Lindblad equation. The effect of decoherence is analyzed by studying the evolution of Stratonovich-Weyl quasiprobability distributions on the state space of the system. It is shown that decoherence makes an arbitrary-given quasiprobability distribution manifestly positive, thus modeling the emergence of classicality in some sense. The decoherence timescale, the minimum time that quasiprobability distributions of every initial state of the system become non-negative, is shown to decrease in Hilbert-space dimension, and hence larger quantum systems decohere faster.

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The effect of decoherence resulting from a quantum system immersed in an environment is often modeled by an effective monitoring of a preferred observable of the system by the environment [1–5]. This results in the decay of the off-diagonal elements of the density matrix of the system in the basis of the preferred observable. But what if the environment monitors the actual state of the system itself, albeit a fuzzy monitoring, and not merely a single preferred observable? This question can be addressed by use of a tomographic measurement based on a positive operator-valued measure (POVM), where the outcome of a measurement is the point on the state space of the system, and the resulting outcome state is the quantum state corresponding to that point [6].

If a tomographic measurement were to be performed by an external environment of a quantum system, then measurement outcomes will not be recorded, and we are led to a decoherence effect. The purpose of this Letter is to investigate properties of this effect. Our primary focus here concerns the timescale for decoherence. The question of this timescale is investigated by considering an exact unraveling of the POVM measurement using a Lindblad equation. The decoherence timescale can be estimated in a variety of ways, for example, by requiring the off-diagonal elements of the density matrix to fall below a prescribed threshold, but here we define decoherence timescale to be determined by the requirement that a given quasiprobability distribution on the state space becomes non-negative. Of course, there is an entire parametric family of quasiprobability distributions,

including, for example, the Wigner function, so what we will find is that there is a parametric family of decoherence timescales, one for each choice of the quasiprobability distribution. Assuming that the system-environment coupling parameter appearing in the Lindblad equation is independent of the Hilbert space dimension N , we show that the decoherence timescale decays in N like $N^{-1} \ln N$ for large N . This implies that for a macroscopic quantum system, irrespective of the initial state of that system, any quasiprobability distribution will lose its negativity virtually instantaneously under a tomographic decoherence model.

This Letter is organized as follows. We begin by modeling decoherence effects using a universal state-space tomographic measurement [7]. Unlike the conventional projective-measurement-based formulation of decoherence where repeated measurements necessarily yield the same outcome, in the case of a measurement based on POVM the outcome states are in general not orthogonal. As a result, a measurement performed immediately after a previous one will, in general, yield a different outcome. Therefore, one can repeat the measurements, progressively decohering the state of the system. In the case of a tomographic measurement, we first work out the effect on the density matrix of the system, followed by an analysis of its corresponding Stratonovich-Weyl quasiprobability distribution defined on the whole of the state space. We show, in particular, that for a quasiprobability distribution of degree σ , its negativity necessarily disappears when the number of measurements exceeds the smallest integer greater than $(1 + \sigma)/2$.

We then consider a continuous-time version of the model, given by a Lindblad equation, with the property that the solution to the Lindblad equation interpolates the outcome states of the iterated POVM measurements at discrete times. By expressing the solution in terms of the Stratonovich-Weyl quasiprobability distribution, we derive the decoherence timescale arising from a continuous-time tomographic

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monitoring of the system. This timescale is defined by the property that the negativity in the quasiprobability distribution necessarily vanishes for all quantum states. We remark that the identification of a continuous-time unraveling of the universal tomographic measurement in terms of a Lindblad equation, and the investigation of the classicalization timescale in terms of the positivity of the quasiprobability distribution, has recently been pursued in Ref. [8], although the correct timescale was not obtained therein.

Let us begin by remarking that quantum state tomography concerns the reconstruction of the state of a system from measurements on an ensemble of identically prepared copies of the system [9–14]. There are many ways of achieving this, but here we consider a *universal tomographic measurement* [7], by which we mean a maximum POVM measurement where the measurement outcome is the position ψ of the state in the space of rays through the origin of Hilbert space, and the output state is the pure state $|\psi\rangle$ corresponding to that state. Here, by “maximum POVM” we mean a symmetric informationally complete POVM whose resolution involves every quantum state democratically. Such a measurement is called “universal” because it involves only the structure of the state space, that is, the measure on the state space induced by the ambient Fubini-Study metric, without any preferred observable. The fact that such a measurement forms a POVM follows from the resolution of the identity $\int_{\mathcal{PH}} |\psi\rangle\langle\psi| d\mu_\psi = \mathbb{1}$. Here, the integration measure $d\mu_\psi$ is the usual Fubini-Study volume element [15] of the projective Hilbert space $\mathcal{PH}$, scaled by the Hilbert space dimension N . Under such a measurement, if the initial state of the system is given by the density matrix $\hat{\rho}$, then the probability of detecting quantum state ψ in the region A of the state space is

$$\mathbb{P}(\psi \in A) = \int_A Q(\psi) d\mu_\psi, \quad (1)$$

where $Q(\psi) = \langle\psi|\hat{\rho}|\psi\rangle$ is the expectation of the initial state $\hat{\rho}$ in the pure state $|\psi\rangle$, which is non-negative and defines a probability density over the space $\mathcal{PH}$ of pure states [16].

If the state of the system is monitored by the environment, then the measurement outcomes are not recorded. In this case, we average over all possible results to characterize the state of the system. In particular, this “nonselective” measurement results in the update of the state of the system, after a single universal tomographic measurement, given by

$$\hat{\rho}^{(1)} = \int_{\mathcal{PH}} |\psi\rangle\langle\psi| Q(\psi) d\mu_\psi. \quad (2)$$

To analyze the effect of this transformation rule, it is convenient to expand the density matrix in the so-called generalized Bloch representation [17–22]

$$\hat{\rho} = \frac{1}{N} \mathbb{1} + \frac{1}{2} \sum_{a=1}^{N^2-1} r_a \hat{\lambda}_a, \quad (3)$$

where $\{\hat{\lambda}_a\}$ are the generators of the Lie algebra $\mathfrak{su}(N)$. These generators are trace-free Hermitian matrices normalized by $\text{tr}(\hat{\lambda}_a \hat{\lambda}_b) = 2\delta_{ab}$, and obey the commutation and anticommutation relations $[\hat{\lambda}_a, \hat{\lambda}_b] = 2if_{ab}^c \hat{\lambda}_c$ and $\{\hat{\lambda}_a, \hat{\lambda}_b\} = \frac{4}{N}\delta_{ab} \mathbb{1} + 2d_{ab}^c \hat{\lambda}_c$, with f_{ab}^c and d_{ab}^c the antisymmetric and symmetric structure constants, respectively. The real coefficients

$r_a = \text{tr}(\hat{\rho} \hat{\lambda}_a)$ therefore determine the state $\hat{\rho}$ of the system, which can be viewed as forming the generalized Bloch vector $\vec{r}$. In this representation, the action of the universal tomographic POVM channel on the state becomes transparent: The identity component remains invariant, while the Bloch vector contracts by a fixed damping factor set by the Hilbert-space dimension and the number of iterations.

Specifically, substituting the Bloch expansion (3) in Eq. (2) and making use of the identity

$$\int \langle\psi|\hat{A}|\psi\rangle |\psi\rangle\langle\psi| d\mu_\psi = \frac{1}{N+1} (\hat{A} + \text{tr}(\hat{A}) \mathbb{1}) \quad (4)$$

that holds for any observable $\hat{A}$ [7,23], and using the fact that $\hat{\lambda}_a$ are trace free, we deduce at once that

$$\hat{\rho}^{(1)} = \frac{1}{N} \mathbb{1} + \frac{1}{2(N+1)} \sum_{a=1}^{N^2-1} r_a \hat{\lambda}_a. \quad (5)$$

In other words, we have the transformation rule

$$\hat{\rho}^{(1)} = \frac{1}{N+1} (\mathbb{1} + \hat{\rho}) \quad (6)$$

that models the effect of decoherence after a single universal tomographic measurement [7,23]. If the system is repeatedly monitored tomographically by the environment, then by iterating this map k times we find that the state of the system transforms to

$$\begin{aligned} \hat{\rho}^{(k)} &= \frac{1}{N} \left(1 - \frac{1}{(N+1)^k} \right) + \frac{1}{(N+1)^k} \hat{\rho} \\ &= \frac{1}{N} \mathbb{1} + \frac{1}{2} \sum_{a=1}^{N^2-1} \frac{r_a}{(N+1)^k} \hat{\lambda}_a. \end{aligned} \quad (7)$$

Thus, the effect of decoherence is to exponentially damp the trace-free part of the density matrix. In particular, in the limit $k \rightarrow \infty$, we have $\hat{\rho}^{(k)} \rightarrow N^{-1} \mathbb{1}$, the state of complete ignorance, which is the only fixed point of the transformation.

While the density matrix representation makes the contraction of the Bloch vector transparent, the onset of classicality is most naturally characterized in terms of the so-called quasiprobability distributions. Such a distribution is conventionally defined over the “phase space” of the system that parametrize the relevant coherent states [24–26]. However, they can equally be defined over the whole of the state space $\mathcal{PH}$, on account of its symplectic structure [15,23]. With this in mind, here we analyze the action of the tomographic channel on the Stratonovich-Weyl family of quasidistributions. Following standard constructions [27,28], we first introduce the σ -parametrized kernel according to

$$\hat{w}^{(\sigma)}(\psi) = \frac{1}{N} + \frac{1}{2} (N+1)^{\frac{1+\sigma}{2}} \sum_{a=1}^{N^2-1} \langle\psi|\hat{\lambda}_a|\psi\rangle \hat{\lambda}_a, \quad (8)$$

which has unit trace $\text{tr}[\hat{w}^{(\sigma)}(\psi)] = 1$ and satisfies the resolution of the identity with respect to the scaled Fubini-Study measure: $\int_{\mathcal{PH}} \hat{w}^{(\sigma)}(\psi) d\mu_\psi = \mathbb{1}$. The associated quasiprobability distribution defined by

$$W_0^{(\sigma)}(\psi) = \text{tr}(\hat{\rho} \hat{w}^{(\sigma)}(\psi)), \quad (9)$$

although need not be positive, satisfies the normalization condition $\int_{\mathcal{PH}} W_0^{(\sigma)}(\psi) d\mu_\psi = 1$, where the subscript zero refers to the quasiprobability distribution associated with the initial state. Examples of quasiprobability distribution include the Husimi Q function ($\sigma = -1$) [29], the Wigner-type distribution ($\sigma = 0$) [30], and the Sudarshan P representation ($\sigma = +1$) [31], albeit here they are defined on the whole of the state space and not on the classical phase space.

In what follows, we write $W_k^{(\sigma)}$ for the quasiprobability distribution associated with $\hat{\rho}^{(k)}$. Then, substituting the one-step update (6) for $\hat{\rho}^{(1)}$ in Eq. (9) and making use of Eq. (8), we find

$$W_1^{(\sigma)}(\psi) = \frac{1}{N} + \frac{1}{2} (N+1)^{\frac{\sigma-1}{2}} \sum_{a=1}^{N^2-1} \langle \psi | \hat{\lambda}_a | \psi \rangle \text{tr}(\hat{\rho} \hat{\lambda}_a), \quad (10)$$

where we have made use of the fact that the Bloch components in $\hat{\rho}^{(1)}$ are rescaled by $1/(N+1)$. In addition, the relation

$$\text{tr}(\hat{\lambda}_a \hat{w}^{(\sigma)}(\psi)) = (N+1)^{\frac{\sigma+1}{2}} \langle \psi | \hat{\lambda}_a | \psi \rangle \quad (11)$$

allows us to express $W_1^{(\sigma)}(\psi)$ explicitly in terms of $W_0^{(\sigma)}(\psi)$ according to

$$W_1^{(\sigma)}(\psi) = \frac{1}{N+1} (W_0^{(\sigma)}(\psi) + 1). \quad (12)$$

On the other hand, if we make the replacement $\sigma \mapsto \sigma - 2$ in Eq. (8) and substitute the result in Eq. (9), we obtain the right-hand side of Eq. (10), from which we deduce that $W_1^{(\sigma)}(\psi) = W_0^{(\sigma-2)}(\psi)$. This shows that each nonselective universal tomographic measurement shifts the ordering parameter by 2. It thus follows that after k iterations of the tomographic measurement, the quasiprobability density transforms according to

$$W_k^{(\sigma)}(\psi) = W_0^{(\sigma-2k)}(\psi), \quad k \geq 0. \quad (13)$$

In other words, quasiprobability density of order σ after k iterative measurements is the quasiprobability density of order $\sigma - 2k$ of the initial state. We note that an analogous transformation rule was obtained under POVM measurements induced by $\text{SU}(2)$ coherent states [6].

Now exactly what condition constitutes classicality is a matter of debate, but if one adopts the operational viewpoint that positivity of the quasiprobability distribution marks the onset of classical behavior, for instance, the positivity of the Wigner function over the state space, then the foregoing analysis shows that decoherence arising from universal tomographic measurements necessarily drives the system toward classicality. This follows from the fact that for any quantum state $\hat{\rho}$ we have $W^{(-1)}(\psi) = \langle \psi | \hat{\rho} | \psi \rangle \geq 0$. Hence, provided that $\sigma - 2k \leq -1$, the quasiprobability distribution becomes non-negative. It follows, in particular, that the minimal number of iterations $k^*(\sigma)$ required to ensure the positivity of the quasiprobability distribution of order σ is

$$k^*(\sigma) = \max \left\{ 0, \left\lceil \frac{\sigma+1}{2} \right\rceil \right\}, \quad (14)$$

a threshold that is independent of the Hilbert-space dimension N . That is, although the magnitude of the damping of the off-diagonal elements of the density matrix, as seen in Eq. (7),

increases in the Hilbert-space dimension, the number of iterations required to convert a given quasiprobability distribution is independent of the dimension.

Our next objective is to investigate properties of a continuous-time model for the state transformation that interpolates the discrete iterations generated by the universal tomographic measurements. To this end, on account of isotropy, the natural choice of such a continuous-time model is a Lindblad equation in which all $\text{SU}(N)$ generators act as independent Lindblad operators:

$$\frac{d\hat{\rho}}{dt} = \gamma \sum_{i=1}^{N^2-1} \left(\hat{\lambda}_i \hat{\rho} \hat{\lambda}_i - \frac{1}{2} \{ \hat{\lambda}_i^2, \hat{\rho} \} \right), \quad (15)$$

where $\gamma > 0$ sets the system-environment coupling strength. Using the $\mathfrak{su}(N)$ completeness relation

$$\sum_{i=1}^{N^2-1} \hat{\lambda}_i \hat{X} \hat{\lambda}_i = 2 \left(\text{tr}(\hat{X}) \mathbb{1} - \frac{1}{N} \hat{X} \right) \quad (16)$$

that holds for any $\hat{X}$, the Lindblad equation (15) reduces to the simple form

$$\frac{d\hat{\rho}}{dt} = 2\gamma (\mathbb{1} - N\hat{\rho}). \quad (17)$$

The solution can easily be obtained as follows:

$$\hat{\rho}(t) = \frac{\mathbb{1}}{N} + e^{-2\gamma N t} \left(\hat{\rho}(0) - \frac{\mathbb{1}}{N} \right). \quad (18)$$

Thus, for a given Hilbert space dimension N , all trace-free Bloch components of the state decay exponentially at the uniform rate $2\gamma N$, precisely mirroring the uniform contraction $(N+1)^{-k}$ generated by the discrete tomographic map, driving the system toward the maximally mixed state $\mathbb{1}/N$. That is, for a fixed N , the solution to the Lindblad equation (15) at times $t = t_k$ matches exactly the outcome state of the k successive universal tomographic measurements, where

$$t_k = \frac{\ln(N+1)}{2\gamma N} k, \quad (19)$$

in agreement with Ref. [8].

Next, we examine the quasiprobability representation of the Lindblad equation and its solution, which determines the decoherence timescale of a system reaching classicality as measured by the positivity of the quasiprobability distribution. Specifically, writing $W_t^{(\sigma)}(\psi)$ for the quasiprobability distribution of order σ associated with the time-dependent density matrix $\hat{\rho}(t)$, our intention is first to derive the Lindblad equation satisfied by $W_t^{(\sigma)}(\psi)$. Expressing Eq. (15) in the form $d\hat{\rho}/dt = \mathcal{L}(\hat{\rho})$, we have $\dot{W}_t^{(\sigma)} = \text{tr}(\mathcal{L}(\hat{\rho}) \hat{w}^{(\sigma)}) = \text{tr}(\hat{\rho} \mathcal{L}(\hat{w}^{(\sigma)}))$, on account of the self-adjointness of the Lindblad generator $\mathcal{L}$. We then observe that $\mathcal{L}(\hat{w}^{(\sigma)}) = 2\gamma (\mathbb{1} - N\hat{w}^{(\sigma)})$, from which it follows that $\dot{W}_t^{(\sigma)}(\psi) = 2\gamma (1 - NW_t^{(\sigma)}(\psi))$, whose solution is given by

$$W_t^{(\sigma)}(\psi) = \frac{1}{N} + \left(W_0^{(\sigma)}(\psi) - \frac{1}{N} \right) e^{-2\gamma N t}, \quad (20)$$

in line with Eq. (18) and in agreement with Ref. [8]. The effect of decoherence therefore is that the dependence on the state-space coordinate ψ decays exponentially, and in

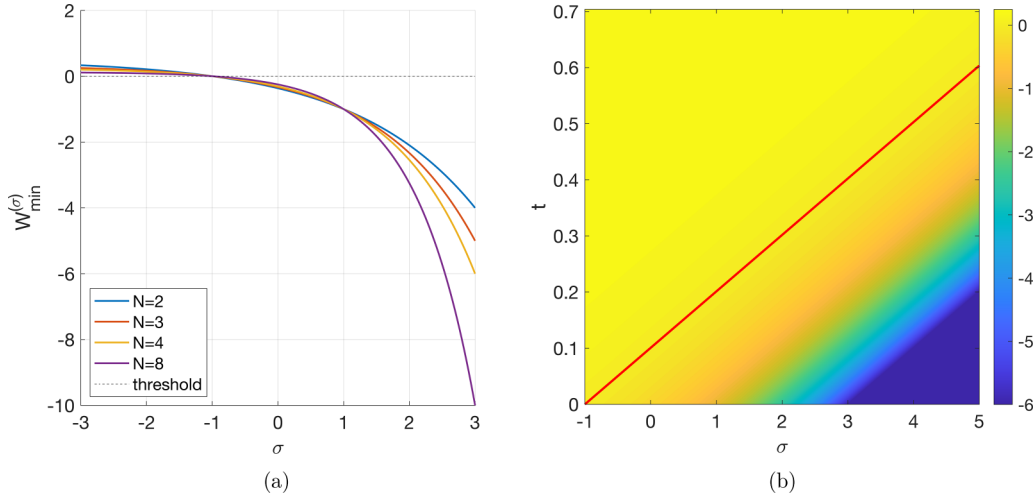


FIG. 1. (a) Minimum value of the quasiprobability distribution $W_{\min}^{(\sigma)}$ as a function of the order parameter σ , shown for Hilbert-space dimensions $N = 2, 3, 4, 5$. For each N , the curves cross the positivity, or classicality, threshold (dashed line) at $\sigma = -1$. (b) Phase diagram of $\min W_t^{(\sigma)}$ in the (σ, t) plane for $N = 4$ and $\gamma = 1$, with the red line marking the boundary $\sigma - \frac{4\gamma N t}{\log(N+1)} = -1$; beyond which all quasiprobability distributions become non-negative.

the long-time limit $t \rightarrow \infty$, the distribution approaches the uniform distribution $W_t^{(\sigma)}(\psi) \rightarrow 1/N$, signaling the complete loss of quantum coherence and convergence to the maximally mixed state.

If we take the condition for classicality, as before, to mean the positivity of $W_t^{(\sigma)}(\psi)$ for a fixed order $\sigma \geq -1$ (recall that $W_t^{(\sigma)} \geq 0$ for all $t \geq 0$ when $\sigma < -1$), then we can define the decoherence timescale as the physical *time-to-classicality*:

$$t^*(\sigma) = \frac{(\sigma + 1) \ln(N + 1)}{4N\gamma}. \quad (21)$$

That is, $t^*(\sigma)$ gives the minimal evolution time after which the quasiprobability distribution of order σ is guaranteed to become everywhere non-negative. To derive Eq. (21), we consider the minimal of the quasiprobability distribution and demand its positivity. To find the extremal negativity attainable by a σ -parametrized distribution for a given dimension N , we substitute Eq. (3) in Eq. (9) to obtain

$$W_0^{(\sigma)}(\psi) = \frac{1}{N} + \frac{(N + 1)^{\frac{\sigma+1}{2}}}{2} \sum_a r_a \langle \psi | \hat{\lambda}_a | \psi \rangle. \quad (22)$$

On the other hand, the expectation of $\hat{\rho}$ in Eq. (3) with respect to the state $|\psi\rangle$ defines the state-space analog of the Husimi function $Q(\psi) = W^{(-1)}(\psi)$:

$$Q(\psi) = \langle \psi | \hat{\rho} | \psi \rangle = \frac{1}{N} + \frac{1}{2} \sum_a r_a \langle \psi | \hat{\lambda}_a | \psi \rangle. \quad (23)$$

Combining Eqs. (22) and (23), we obtain the relation

$$W_0^{(\sigma)}(\psi) = \frac{1}{N} + (N + 1)^{\frac{\sigma+1}{2}} \left(Q(\psi) - \frac{1}{N} \right), \quad (24)$$

but because $Q(\psi) \geq 0$, the minimum of $W_0^{(\sigma)}$ occurs at $Q(\psi) = 0$. It follows that for any quantum state $\hat{\rho}$, the associated quasiprobability distribution of order σ has the

minimum

$$W_{\min}^{(\sigma)} = \frac{1 - (N + 1)^{\frac{\sigma+1}{2}}}{N}. \quad (25)$$

The bound $W^{(\sigma)}(\psi) \geq W_{\min}^{(\sigma)}$ given by Eq. (25) is sharp because it is attained when $\hat{\rho}$ is pure. This follows on account of the fact that for pure states, $\min_{\psi} Q(\psi) = 0$.

Now, returning to Eq. (20), evidently $\min_{\psi} W_t^{(\sigma)}(\psi)$ is attained at $\min_{\psi} W_0^{(\sigma)}(\psi)$, but the minimum of $W_0^{(\sigma)}(\psi)$ is given by Eq. (25). Therefore, substituting Eq. (25) in Eq. (20) and solving for t , we deduce Eq. (21). What is interesting about Eq. (21) is that when the system-environment coupling γ is fixed, the time to classicality decays in Hilbert-space dimension like $N^{-1} \ln N$ for large N . In other words, larger systems “classicalizes” more rapidly under universal tomographic monitoring. We remark that an analogous analysis has been pursued recently in Ref. [8], although the correct timescale (21) was not obtained due to a minimization that involved matrices $\hat{\rho}$ with negative eigenvalues. It is worth adding that the independence of the iteration count (14) for the universal tomographic measurements required to ensure the positivity of $W^{(\sigma)}$ on the Hilbert-space dimension N might at first seem to contradict the N dependence of the decoherence timescale under the continuous-time Lindbladian unraveling. However, Eq. (19) shows that the timescale of the Lindbladian dynamics to interpolate the POVM measurements becomes smaller in increasing N . Therefore, while each measurement takes place instantaneously, the effective timescale of a measurement as interpolated by the Lindblad equation becomes smaller in increasing N .

Next, we show that the effect of the Lindblad dynamics (15) on a quasiprobability distribution of order σ is to merely shift the value of the order parameter linearly in time. Specifically, we have $W_t^{(\sigma)} = W_0^{(\sigma_{\text{eff}}(t))}$, where

$$\sigma_{\text{eff}}(t) = \sigma - \frac{4\gamma N t}{\ln(N + 1)} \quad (26)$$

is the effective order parameter. This follows by first postulating the existence of $\sigma_{\text{eff}}(t)$ such that $W_t^{(\sigma)} = W_0^{(\sigma_{\text{eff}}(t))}$ holds. We then use Eqs. (20) and (24) to represent these in terms of $Q(\psi)$. Eliminating $Q(\psi)$ from the resulting relation, we see that $W_t^{(\sigma)} = W_0^{(\sigma_{\text{eff}}(t))}$ holds if Eq. (26) holds. Our universal decoherence model thus admits a simple representation in terms of the family of quasiprobability distributions on the state space.

In Fig. 1(a), we plot $W_{\min}^{(\sigma)}$ as a function of σ for different values of N . As the Hilbert-space dimension increases, the negativity of quasiprobability distributions for larger σ values becomes highly pronounced. In panel (b), we show $\min W_t^{(\sigma)}$ as a function of σ and t for fixed N , illustrating how the negativity is resolved in time.

In summary, we have analyzed the effect of universal decoherence on quasiprobability distributions over the entire state space resulting from both discrete-time and continuous-time models. The continuous-time model, in particular, shows the existence of decoherence timescale (21) with the property that beyond this time, a quasiprobability distribution of order σ for any initial quantum state is necessarily non-negative.

The result shows, in particular, that the decoherence timescale reduces for large Hilbert-space dimension N according to $N^{-1} \ln N$, thus indicating that larger systems will decohere faster. We conclude by briefly remarking experimental verification of the model, at least at the thought level. The difficulty here is the creation of a generic or “universal” environment that does not isolate a particular preferred observable. For instance, for a spin- $\frac{1}{2}$ particle prepared in a given pure state, immersing it in an external magnetic field, say, in the z direction, will in effect select a preferred observable $\hat{\sigma}_z$. If instead the spin is placed inside of a field-free (hence Hamiltonian-free) chamber for a period of time, after which a tomographic measurement is performed, then the result will confirm whether Eq. (18) is a viable model for the state of the system.

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