

Reshaping quantum device noise via repetition code circuits

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Noise of a quantum processor can be an important resource for simulating open quantum dynamics. However, this requires characterizing the device noise and then transforming it into a target structure. Here, we take the first step toward this goal: We analytically and numerically study reshaping the noise associated with native trapped-ion two-qubit entangling gates via quantum circuits based on repetition codes, and demonstrate our findings on the IonQ Aria-1 quantum hardware. We investigate all the building blocks, including the quantum channels describing noisy two-qubit entangling gates, the compilation of the encoding circuits into native gates, and the propagation of two-qubit errors across ideal single-qubit gates.

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I. INTRODUCTION

All quantum computers [1] suffer from noises (uncontrollable interactions), some of which lead to undesired evolutions (errors) [2,3] (we use “noise” and “error” interchangeably in this manuscript). Two categories of methods have been developed to effectively remove noise: quantum error correction (QEC) [4–16] and quantum error mitigation (QEM) [17]. However, a question much less explored but more relevant to the current noisy intermediate-scale quantum (NISQ) era is, how the noise can be exploited, one important application being assisting simulations of open quantum dynamics. This application has been demonstrated in the context of QEM [18,19], and in some theoretically designed algorithms [20,21]. But these works mostly assume single-qubit noise models, which do not directly correspond to the noisiest element in realistic quantum devices. The complicated structures of native noise have not been involved in any of these studies.

Here, we address this challenge of directly exploiting native device noise. We focus on the trapped-ion quantum devices, where the mediator for qubit-qubit interactions, the common vibrational mode, is also a quantum degree of freedom [22–24]. The two-qubit entangling gates, which are a source of seriously noisy quantum operations, have been experimentally characterized [25–27], but the highlights are not the native noise structure. In this paper, we first derive this noise structure. Then, we demonstrate reshaping it into quantum channels that have different noise structures, analytically, numerically, and on the IonQ Aria-1 device, via quantum circuits based on the three-qubit bit-flip QEC code. Our choice

of QEC circuits does not mean that we aim at correcting errors. Instead, the three-qubit QEC circuit represents one of the smallest meaningful quantum computing units, in contrast to random circuits that are much more widely used but are arguably of limited practical usage [28].

II. NOISY MØLMER-SØRENSEN GATES

Mølmer-Sørensen (MS) gates [22,23] implement the two-qubit entangling gates in trapped-ion quantum devices. Qubits (ions’ electronic states) effectively interact with each other via their common vibrational mode. The dominant noise for MS gates has been pointed out to be from the motional dephasing [29], whose Lindblad form is $C = \sqrt{\Gamma_P} a^\dagger a$, with Γ_P the dephasing rate and $\hat{a}$ the annihilation operator for the vibrational mode with frequency ν . The dynamics are analytically solvable under certain *approximations* [22]: In the interaction picture with respect to all the unitary dynamics, the state of the two qubits is always separable from the state of the vibrational mode, and the latter is kept in a thermal state with mean phonon number n_{th} . For the fully entangling gate that is equivalent to the cnot gate up to single-qubit gates [22,30,31], we find that the canonical Kraus operators of the quantum channel that maps the target state to the state produced by the noisy gate have the following structure:

$$\begin{aligned} K_1 &= a_1 I + a_3 J_y^2, \\ K_2 &= k_2 J_y, \\ K_3 &= a_2 I + a_4 J_y^2. \end{aligned} \quad (1)$$

Here, $J_y = (Y_1 \otimes I_2 + I_1 \otimes Y_2)/2$ is the total angular momentum and Y_i are Pauli operators. The coefficients have closed-form solutions and, in particular, the value of k_2 uniquely determines the values of a_1, a_2, a_3 , and a_4 (the derivations and results are provided in Appendix A). Our results of canonical Kraus operators go beyond Ref. [22], where only the fidelity is derived.

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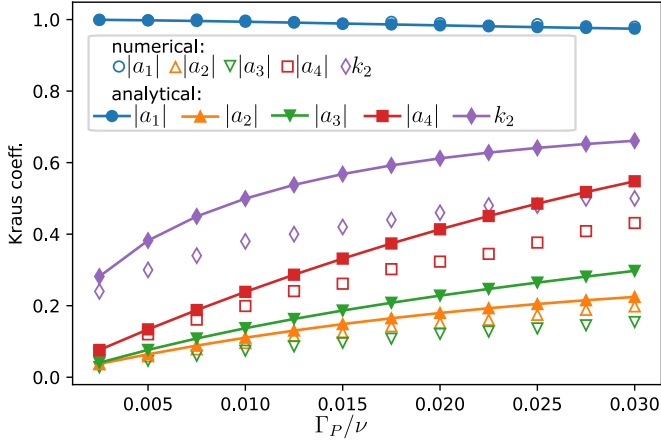


FIG. 1. Coefficients of the canonical Kraus operators in Eq. (1), as the value of Γ_P changes, comparing the analytical approximate solutions (solid line with solid markers) with the numerical exact solutions (open markers).

An example of how these coefficients change with the dephasing rate is shown in Fig. 1. We have chosen the parameters [22] according to the weak-coupling regime that most experimental realizations of MS gates follow [32–35]: The Lamb-Dicke parameter is $\eta = 0.1$, the Rabi frequency is $\Omega = 0.1\nu$, $n_{\text{th}} = 0.05$, and $K = 25$ is an integer describing the number of loops traversed in the vibrational phase space [24]. For comparison, in Fig. 1, we also show numerical simulations obtained without the approximations above, where we find that the canonical Kraus operators are still in the form of Eq. (1), and the coefficients have only reasonably small deviations from the analytical ones. Importantly, K_2 is the dominant error term. Here, the Pauli Y operator originates from the conventional choice of basis in theoretical derivations [22], where the entangling gate is also defined in the y basis. For trapped-ion quantum devices such as IonQ Aria-1, commonly the fully entangling gate is in the x basis [31]; therefore, the dominant error in the noisy MS gate is $X_1 \otimes I_2 + I_1 \otimes X_2$, i.e., the superposition of the bit-flip error for each qubit.

Having derived the noise structure of the MS gate, we want to explore how to reshape it. As the first step, we use one of the smallest meaningful quantum circuit units—the circuit for the three-qubit bit-flip repetition code. We increase the tunability by adding redundant MS gate pairs and an additional single-qubit gate. One benefit of a QEC circuit is that we map multiple physical qubits into one logical qubit, so that the output has a manageable Hilbert space dimension. We are not actually trying to recover the original state.

III. NOISY ENCODING THROUGH BIT-FLIP REPETITION CODE

QEC codes work by encoding quantum information into a larger Hilbert space such that detection of errors will not completely collapse the state, thus allowing recovery operations after syndrome measurements. For the three-qubit bit-flip repetition code, a state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is mapped into a three-qubit state, $|\bar{\psi}\rangle = \alpha|000\rangle + \beta|111\rangle$. Each of the three qubits is called a physical qubit, while $|\bar{\psi}\rangle$ defines a

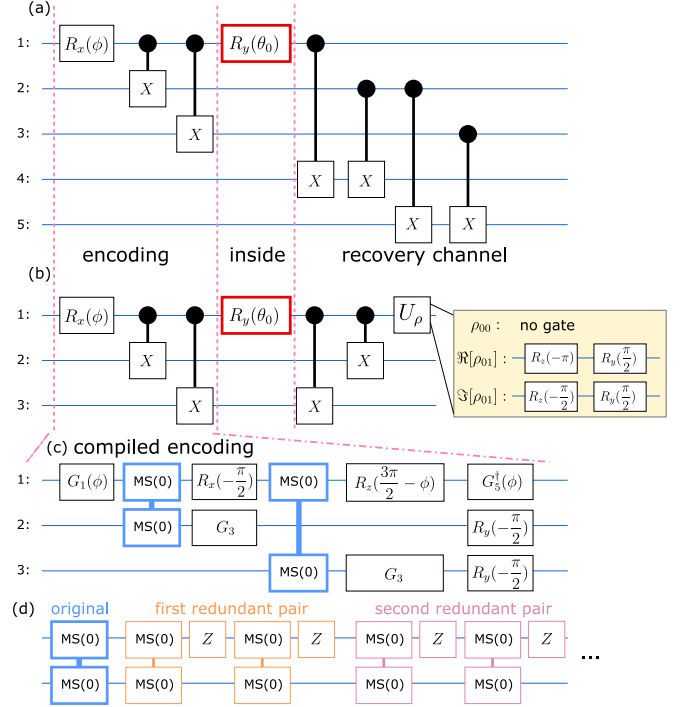


FIG. 2. Quantum circuits to implement the modified repetition code for noise reshaping. In panels (a) and (b), we show the circuits before any compilation into native gates. They are partitioned into three sections as marked. All qubits are measured at the end in the computational basis. The recovery unitaries are implemented via classical postprocessing. (a) Two ancillae are included for measuring ρ_{00} with reduced measurement errors. (b) Choosing three different U_ρ corresponds to measuring ρ_{00} , $\Re[\rho_{01}]$ or $\Im[\rho_{01}]$. (c) The encoding section is first compiled by converting cnot gates to the MS gates. (d) Redundant pairs of MS gates can be inserted after each encoding MS gate to effectively tune the noise level.

logical qubit: The logical space is spanned by $|\bar{0}\rangle = |000\rangle$ and $|\bar{1}\rangle = |111\rangle$. The most straightforward way of implementing this encoding process is via the first section of the quantum circuits in Figs. 2(a) and 2(b). We use $R_x(\phi)$ to initialize $|\psi\rangle$ with $\alpha = \cos(\phi/2)$ and $\beta = -i\sin(\phi/2)$. The subsequent two CNOT gates encode this into the logical state $|\bar{\psi}\rangle$. For standard QEC theoretical study, this encoding process is considered as noiseless, while single-qubit error happens after encoding. However, it is not what we consider.

In order to be implemented on a real trapped-ion quantum device, this encoding circuit section needs to be compiled into an equivalent version consisting of native gates. Of particular interest are the two-qubit entangling gates, as they are noisier than single-qubit gates. Moreover, we are interested in reshaping the native noise of two-qubit gates into quantum channels that have different types of noise. The native MS gate is related to the cnot gate by applying additional single-qubit gates (see Appendix B, where all the conventions are also clarified). Compiling the two encoding cnot gates into two MS gates results in the compiled encoding circuit section as shown in Fig. 2(c). There, the single-qubit gates are defined as $G_1(\phi) = -\frac{1}{\sqrt{2}}\cos\frac{\phi}{2}I + \frac{i}{\sqrt{2}}\sin\frac{\phi}{2}X + \frac{i}{\sqrt{2}}\cos\frac{\phi}{2}Y - \frac{i}{\sqrt{2}}\sin\frac{\phi}{2}Z$, $G_3 = \frac{1}{2}I + \frac{i}{2}X - \frac{i}{2}Y - \frac{i}{2}Z$, and

$G_5^\dagger(\phi) = \frac{1}{\sqrt{2}} \cos \frac{\phi}{2} I + \frac{i}{\sqrt{2}} \cos \frac{\phi}{2} X - \frac{i}{\sqrt{2}} \sin \frac{\phi}{2} Y - \frac{i}{\sqrt{2}} \sin \frac{\phi}{2} Z$. Although this is not the full compilation that should include converting all the single-qubit gates into trapped-ion native gates and is shown in the Supplemental Material [36] for the case of IonQ Aria-1 quantum device, this intermediate circuit is illustrative for keeping track of the propagation of two-qubit noise while assuming ideal single-qubit gates, which we will focus on later.

We have derived that the noisy implementation of each MS gate is represented by applying the quantum channel with Kraus operators in Eq. (1) (replacing the y axis with x axis) after the noiseless MS gate. For a given quantum device, the amount of error is fixed and roughly corresponds to the left side of Fig. 1. As our main interest is about how to reshape this native noise, we would like to have tunability over the strength of MS gate noise. We achieve this by adding redundant pairs of MS gates [25,26] after each of the two encoding MS gates, as shown in Fig. 2(d). Each pair consists of the following sequence repeated two times: one MS gate followed by one Z gate on the first qubit. In the noiseless case, each redundant pair is equal to identity. In practice, adding redundant pairs changes the noise strength and structure associated with the circuit, which will be discussed later. Note that the noise in implementing the MS gates already brings the state of the three physical qubits out of the logical space.

IV. ADDITIONAL TUNABILITY INSIDE THE CODE

We define “inside the code” as operations after the encoding circuit section and before the recovery channel (defined later). Traditional QEC studies assume that errors happen here, i.e., bit-flip repetition code is to correct single-qubit stochastic bit-flip error X . However, as we do not want to inject noise that is not native, we instead add tunability by inserting a parameterized single-qubit gate. One of the simplest options is to add a unitary gate associated with rotation around the y axis [37],

$$R_y(\theta_0) = \begin{pmatrix} \cos(\theta_0/2) & -\sin(\theta_0/2) \\ \sin(\theta_0/2) & \cos(\theta_0/2) \end{pmatrix}, \quad 0 \leq \theta_0 \leq \pi, \quad (2)$$

on physical qubit 1 (see the Supplemental Material [36] for more generalizations). This is illustrated as the red box in Figs. 2(a) and 2(b). Adding this R_y gate may bring the three-qubit state further away from the logical space. This may thus be viewed as a coherent error from the standard QEC perspective. It is not how we interpret it here. We are interested in how the code reshapes the noise profile of quantum computers, rather than the classical number of the QEC success rate: The output logical state from the repetition code can be viewed as the initial logical state going through a quantum channel that is different from the one on the physical qubit level. We will next elaborate it by explicitly describing how the recovery channel bring the state back to the logical space.

V. RECOVERY CHANNEL TO THE LOGICAL SPACE

As per the standard definition of the three-qubit bit-flip repetition code, we perform syndrome measurements on the stabilizers ($Z_1 Z_2, Z_2 Z_3$), which are expressed as projectors, and apply unitary operators conditioning on the measurement

results. This process is formally defined as the recovery channel $\mathcal{R}(\tilde{\rho}) = \sum_{i,j=0}^1 R_{1+2i+j} P^{(ij)} \tilde{\rho} P^{(ij)} R_{1+2i+j}$, where the projectors are $P^{(ij)} = (I + (-1)^i Z_1 Z_2 + (-1)^j Z_2 Z_3 + (-1)^{i+j} Z_1 Z_3)/4$, with index 0 (1) meaning the syndrome measurement result is +1 (−1) and the applied unitaries are $(R_1, R_2, R_3, R_4) = (I, X_3, X_1, X_2)$. The recovery channel maps any three-qubit state $\tilde{\rho}$ into the output logical state $\tilde{\rho}_f$ in the logical code space. Here, $\tilde{\rho}$ is the outcome of applying the qubit 1 $R_y(\theta_0)$ gate onto the state resulting from the noisy encoding circuit. The latter regarding error propagation will be explained later.

In Figs. 2(a) and 2(b), we show the practical implementations of the recovery channel. Note that we assume perfect single-qubit gates and two-qubit gates in this section of the circuits as we do not introduce any redundant gates in contrast to what we do in the encoding section. Considering the limitations of quantum devices, we do not explicitly apply midcircuit measurements or multiqubit operators. Instead, we measure all qubits in the computational basis at the end of the circuits and perform classical postprocessing. The circuit in Fig. 2(a) explicitly uses two ancilla qubits (labeled 4 and 5) to compensate for measurement errors: Only results that are compatible with the code space basis $\{|000\rangle_{123}, |111\rangle_{123}\}$ are counted in the statistics. The circuit in Fig. 2(b) does not include ancilla, but is able to estimate both $\langle 000 | \tilde{\rho}_f | 000 \rangle \equiv \rho_{00}$ and the complex-valued $\langle 000 | \tilde{\rho}_f | 111 \rangle \equiv \rho_{01}$ once the corresponding single-qubit gate U_ρ is selected at the end. More details are provided in the Supplemental Material [36].

VI. ERRORS ASSOCIATED WITH NOISY NATIVE GATES

The noisy implementation of each MS gate can be approximated keeping the first order as mapping any state ρ into

$$MS_{ij} \rho MS_{ij}^\dagger + \sqrt{\frac{r_1}{2}} (X_i + X_j) MS_{ij} \rho MS_{ij}^\dagger (X_i + X_j) \sqrt{\frac{r_1}{2}}, \quad (3)$$

where we approximate $a_1 \approx 1$, $k_2 \approx \sqrt{2r_1}$, and $a_2 \approx a_3 \approx a_4 \approx 0$, and have replaced y with x based on Eq. (1), and i and j are labels indicating which two qubits the MS gate acts on. Note the invariance of exchanging i and j .

Appending redundant MS gate pairs to one original MS gate as illustrated in Fig. 2(d) changes the structure of errors. By keeping track of how $X_{i,j}$ terms propagate across Z gates, we find that the unit in Fig. 2(d) with n pairs of redundant MS gates corresponds to mapping any state ρ into

$$\begin{aligned} & MS_{ij} \rho MS_{ij}^\dagger \\ & + \sqrt{\frac{r_1 n}{2}} (-X_i + X_j) MS_{ij} \rho MS_{ij}^\dagger (-X_i + X_j) \sqrt{\frac{r_1 n}{2}} \\ & + \sqrt{\frac{r_1 (n+1)}{2}} (X_i + X_j) MS_{ij} \rho MS_{ij}^\dagger (X_i + X_j) \sqrt{\frac{r_1 (n+1)}{2}}. \end{aligned} \quad (4)$$

The strength of noise is indeed enhanced by a factor of n . However, the difference of bit-flip error now contributes together with the existing sum of bit-flip error: Both indicate coherent superposition of single-qubit errors.

Further error propagation toward the end of the encoding circuit section will further transform the error structures. We

find that, if we insert n redundant pairs of MS gates after each of the two encoding MS gates, the physical state of the three qubits at the end of the encoding circuit section is approximated by

$$\begin{aligned} |\bar{\psi}\rangle\langle\bar{\psi}| &+ \frac{r_1(n+1)}{2}(Z_1 + X_2)|\bar{\psi}\rangle\langle\bar{\psi}|(Z_1 + X_2) \\ &+ \frac{r_1(n+1)}{2}(Z_1 + X_3)|\bar{\psi}\rangle\langle\bar{\psi}|(Z_1 + X_3) \\ &+ \frac{r_1 n}{2}(-Z_1 + X_2)|\bar{\psi}\rangle\langle\bar{\psi}|(-Z_1 + X_2) \\ &+ \frac{r_1 n}{2}(-Z_1 + X_3)|\bar{\psi}\rangle\langle\bar{\psi}|(-Z_1 + X_3), \end{aligned} \quad (5)$$

where $|\bar{\psi}\rangle$ is the ideal encoded initial state. The existence of single-qubit gates has converted some bit-flip errors into phase-flip errors.

VII. NOISE RESHAPING INTO THE LOGICAL CHANNEL

The state as approximately expressed in Eq. (5) is then acted upon by the tunable R_y gate inside the code as marked in Figs. 2(a) and 2(b). Then the recovery channel is implemented. This channel maps any three-qubit physical state into the logical space (also defined as the code space). The error structure contained in this output logical state is the result of noise reshaping that we are interested in. To be specific, we have shown that $\bar{\rho}_f = \rho_{00}|\bar{0}\rangle\langle\bar{0}| + \rho_{01}|\bar{0}\rangle\langle\bar{1}| + \rho_{01}^*|\bar{1}\rangle\langle\bar{0}| + (1 - \rho_{00})|\bar{1}\rangle\langle\bar{1}|$. This state contains the same quantum information as the single-qubit state $\rho_f = \rho_{00}|0\rangle\langle 0| + \rho_{01}|0\rangle\langle 1| + \rho_{01}^*|1\rangle\langle 0| + (1 - \rho_{00})|1\rangle\langle 1|$. The quantum channel that maps the initial state $|\psi\rangle$ into ρ_f is the result of reshaping the native device noise: The noise associated with MS gates, as shown in Eq. (1), has a fundamentally different structure.

We can derive the analytical form of $\bar{\rho}_f$ following the linear approximation of two-qubit entangling gate errors as shown in Eqs. (3) and (4). However, for the effective noise to be large enough to be resolvable numerically and on quantum hardware, the number of pairs of redundant gates needs to be large enough, i.e., $n \gg 1$. This in fact breaks down the linear approximation. Attempting to generalize Eqs. (3) and (4) beyond the linear approximation will require keeping track of terms polynomial or even exponential in n —either way intractable for $n \gg 5$. We did not choose to proceed in this way because this exponential dependence is an artifact resulting from our naive way of tuning the error rate by introducing redundancies. It is an open question as to how to efficiently modify the effective error rate of the native gates. We thus numerically calculate $\bar{\rho}_f$ based on the full Kraus operator description of noisy MS gates. Specially, we use the numerically simulated result of the coefficients in Eq. (1) that corresponds to the leftmost values in Fig. 1 (for exact numbers, see the Supplemental Material [36]), which is compatible with the current IonQ Aria-1 fidelity of two-qubit entangling gates. The results are shown as lines in Fig. 3, which will be compared with the data we get from the IonQ Aria-1 quantum device.

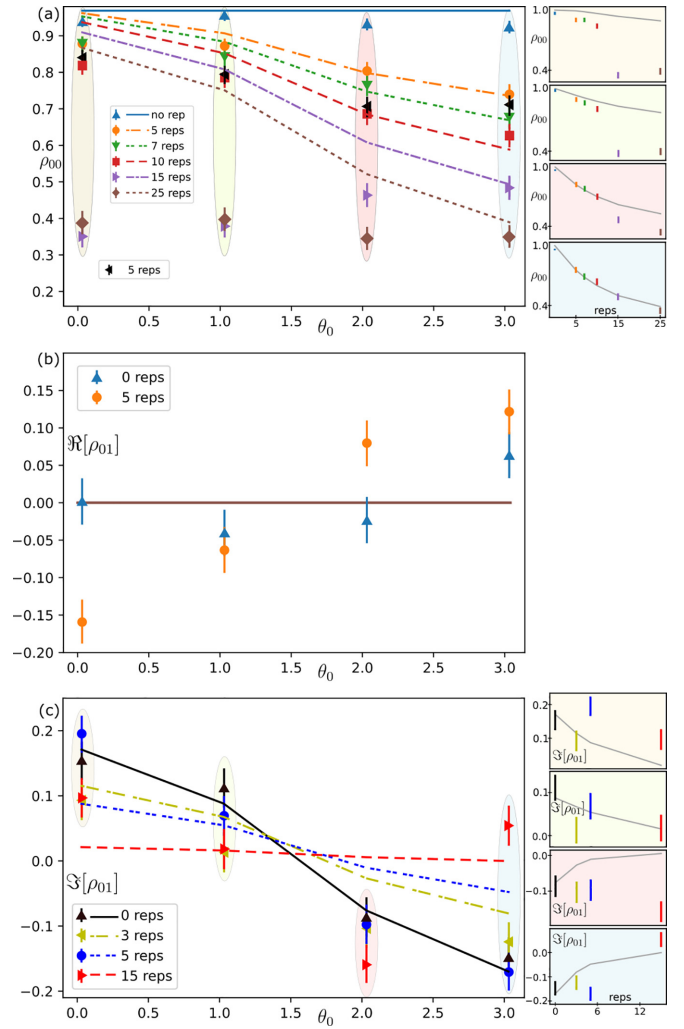


FIG. 3. Data obtained from the IonQ Aria-1 quantum device (points), in comparison with our model (lines). Each data point is averaged over 1000 shots. Error bars are 95% confidence intervals computed from the Wilson score [38]. The number of repetitions refers to the number of the inserted pairs of redundant MS gates after each encoding gate. (a) ρ_{00} . The data come from the compiled version of Fig. 2(a), except the black left-pointing triangles that are from the compiled circuit of Fig. 2(b) for reference (see the main text for details). (b), (c) The real and imaginary parts of ρ_{01} , respectively. The data come from the compiled circuit of Fig. 2(b).

VIII. IONQ ARIA-1 RESULTS

It is possible to further compile the circuit [Fig. 2(c)] into circuits entirely made of IonQ Aria-1 native gates without swapping the positions of the encoding MS gates with any other gate (see the Supplemental Material [36]). The results obtained from the quantum device, for using the compiled version of circuit [Fig. 2(a)] to estimate ρ_{00} , are shown as the data points in Fig. 3(a). Without any redundant pair of MS gates, the dependence of ρ_{00} on θ_0 is weak. Once we add five, seven, or ten redundant pairs, it becomes clear that ρ_{00} decreases as θ_0 increases from 0 to π . We can also see the decrease of ρ_{00} as the number of redundant pairs increases (see the insets). If we add 15 or 25 redundant pairs, the dependence of ρ_{00} on θ_0 becomes less clear. Altogether, we can see that our model

(plotted as lines) matches reasonably well with the data, given that we do not run any direct device characterization. The noticeable discrepancies between the theory and the Ion-Q Aria-1 data for large numbers of redundant gate pairs (15 and 25) may result from various additional sources of error. These quantum circuits involve hundreds of gates [36]; therefore, effects from even tiny imperfection per gate may accumulate to a significant level. However, we do observe that as θ_0 increases from 0 to π , the data for how ρ_{00} changes with the number of redundant gate pairs match better with the theory, as shown in the colored insets of Fig. 3(a). This may be associated with the effect of the $R_y(\theta_0)$ gate. As discussed at the end of Sec. IV, $R_y(\theta_0)$ acts as a coherent error that rotates the bit-flip errors of the MS gates to a different basis that do not match the original bit-flip repetition code structure, resulting in a wide range of ρ_{00} values that are significantly different from that of the initial state.

The results from the compiled version of the circuits in Fig. 2(b), which reconstruct ρ_{00} , $\Re[\rho_{01}]$, or $\Im[\rho_{01}]$ without using ancilla qubits, are included in Figs. 3(a)–3(c). In Fig. 3(a), the black data points (left-pointing triangles) are based on the compiled version of Fig. 2(b) for five repetitions. The smaller ρ_{00} compared with the orange circles for each θ_0 results from the lack of postselection for compensating measurement errors as there is no ancilla qubit. In Fig. 3(b), we compare the data for $\Re[\rho_{01}]$ with our model that predicts $\Re[\rho_{01}] = 0$ for all θ_0 and n . The data indeed show small values of $\Re[\rho_{01}]$ around 0. The deviations from 0 can be associated with noise sources other than the MS gate noise. Reading out $\Re[\rho_{01}]$ does not reveal any features from the latter; thus, various effects from the former are unveiled. In Fig. 3(c), we compare the data for $\Im[\rho_{01}]$ with our model. For zero, three, or five repetitive MS gate pairs, increasing θ_0 reduces $\Im[\rho_{01}]$, as predicted by our model (lines). However, for each fixed θ_0 , how $\Im[\rho_{01}]$ changes with the number of repetitions cannot be fully resolved. This is because $\Im[\rho_{01}]$ – θ_0 curves for different n are close to each other and cross each other, as demonstrated in the lines in Fig. 3(c). In fact, this crossing is visible in the hardware data. For the third value of θ_0 , $\Im[\rho_{01}]$ for zero, three, and five repetitions are the closest to each other compared with other θ_0 . For 15 repetitive MS gate pairs, the deviation from our model is more evident. We have also included the insets to illustrate the dependence of $\Im[\rho_{01}]$ on the number of redundant pairs n while fixing θ_0 . These deviations suggest that unlike ρ_{00} , $\Im[\rho_{01}]$, together with $\Re[\rho_{01}]$, is not ideal for resolving the dependence of the transformed logical channel on n and θ_0 . Finding suitable observables in addition to ρ_{00} remains an open question, which will be beneficial not only for reliably reshaping the device noise but also for efficiently characterizing the MS gates.

IX. CONCLUSION AND DISCUSSION

We demonstrate exploiting the native structure of quantum device noise and reshaping it. Focusing on trapped-ion setups, we analytically characterize noisy entangling gates and investigate how the native noises propagate across and get transformed by modified repetition code circuits. And we demonstrate our results on the IonQ Aria-1 quantum device. Our work opens up a new research direction fundamentally different from the current mainstream effort on making qubits

and gates with higher fidelity. We suggest to focus on exploiting the benefits brought forward by the physics behind the noise, a promising application of which is simulating open quantum dynamics on quantum devices, which is very difficult under existing approaches.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: DETAILS OF NOISY MØLMER-SØRENSEN GATES

The implementation of Mølmer-Sørensen gates [22,23] in trapped-ion systems involves two ions whose intrinsic energy levels are considered as qubits, and their common vibrational mode that is modeled as a mechanical oscillator whose virtual excitation by the laser field is used to entangle the intrinsic states of the two ions. The Hamiltonian is generally referred to as [22]

$$H = f(t)J_{yx} + g(t)J_y p, \quad (A1)$$

where $J_y = (\sigma_{y1} + \sigma_{y2})/2$, σ_{yi} are Pauli operators, $f(t) = -\sqrt{2}\eta\Omega \cos(\nu - \delta)t$, $g(t) = -\sqrt{2}\eta\Omega \sin(\nu - \delta)t$, where η is the Lamb-Dicke parameter, Ω is the Rabi frequency, ν is the mechanical frequency, δ is the laser detuning, and x and p are the dimensionless position and momentum operators, respectively. We want to derive the quantum channel that connects the final state subject to noise with the target state. We therefore work in the interaction picture with respect to the unitary evolution governed by Eq. (A1) [22]. In this way, the dynamics in the interaction picture directly correspond to our desired quantum channel.

The dominant source of noise for trapped ion systems is the motional dephasing [29]. In the Lindblad form, this corresponds to the dissipator $C = \sqrt{\Gamma_P} a^\dagger a$. After moving into the interaction picture, the dissipator becomes

$$\tilde{C} = \sqrt{\Gamma_P} \left(a^\dagger + J_y \frac{G(t) + iF(t)}{\sqrt{2}} \right) \left(a + J_y \frac{G(t) - iF(t)}{\sqrt{2}} \right), \quad (A2)$$

where $G(t) = \int_0^t dt' g(t')$ and $F(t) = \int_0^t dt' f(t')$, and the dynamics become

$$\frac{d}{dt} \tilde{\rho}(t) = -\frac{1}{2} \tilde{C}^\dagger \tilde{C} \tilde{\rho} - \frac{1}{2} \tilde{\rho} \tilde{C}^\dagger \tilde{C} + \tilde{C} \tilde{\rho} \tilde{C}^\dagger. \quad (A3)$$

We approximate that throughout the evolution, in this interaction picture, the state of the two qubits remains separable from the state of the oscillator, which stays in the initial thermal state. Specifically, we assume $\tilde{\rho}(t) \approx \tilde{\rho}_q(t) \otimes \gamma_\nu$, where $\tilde{\rho}_q(t)$ is the joint state of the two qubits and γ_ν is the thermal state

of the oscillator with mean phonon number n_{th} . Under this approximation and taking the partial trace over the oscillator mode, Eq. (A3) becomes the dynamical equation for only the state of the qubits,

$$\begin{aligned} \frac{d}{dt} \tilde{\rho}_q(t) = & (G^2(t) + F^2(t)) \left(n_{\text{th}} + \frac{1}{2} \right) \Gamma_P \left(-\frac{1}{2} J_y^2 \tilde{\rho}_q - \frac{1}{2} \tilde{\rho}_q J_y^2 \right. \\ & \left. + J_y \tilde{\rho}_q J_y \right) + \frac{1}{4} (G^2(t) + F^2(t))^2 \Gamma_P \\ & \times \left(-\frac{1}{2} J_y^4 \tilde{\rho}_q - \frac{1}{2} \tilde{\rho}_q J_y^4 + J_y^2 \tilde{\rho}_q J_y^2 \right). \end{aligned} \quad (\text{A4})$$

The equation can be straightforwardly solved in an element-wise manner by expanding all operators in the eigenbasis of J_y [22]. We are interested in interaction times when the state of the qubits disentangles with the state of the oscillator in the original picture, and when the maximal entanglement between the two qubits is generated (up to local unitary operators that rotate Y into X , this will be referred to as XX gates later). These correspond to the constraints between parameters [22], $\tau = 2\pi K/(\nu - \delta)$ and $\eta\Omega/(\nu - \delta) = 1/2\sqrt{K}$, where K is an integer referring to how many phase-space loops of the oscillator have been traversed [24]. For simplicity, we define two parameters

$$r_1 = \frac{1}{4} (2n_{\text{th}} + 1) \Gamma_P \frac{\tau}{K}, \quad (\text{A5})$$

$$r_2 = \frac{1}{8} \Gamma_P \frac{3\tau}{2K^2}. \quad (\text{A6})$$

The quantum channel that maps the initial state $\tilde{\rho}_q(0)$ to the final state $\tilde{\rho}_q(\tau)$ is found to have the following canonical Kraus operators ($\text{Tr}(K_i^\dagger K_j) = 0$ for $i \neq j$):

$$\begin{aligned} K_1 &= a_1 I + a_3 J_y^2, \\ K_2 &= \sqrt{\frac{1}{2}(1 - e^{-4r_1})} J_y, \\ K_3 &= a_2 I + a_4 J_y^2, \end{aligned} \quad (\text{A7})$$

where a_1, a_2, a_3 , and a_4 are solutions of the coupled equations

$$\begin{aligned} a_1^2 + a_2^2 &= 1, \\ a_1 a_3 + a_2 a_4 &= e^{-r_1 - r_2} - 1, \\ 2a_1 a_3 + a_3^2 + 2a_2 a_4 + a_4^2 &= \frac{1}{2}(e^{-4r_1} - 1), \\ 2a_1 a_2 + a_3 a_4 + a_1 a_4 + a_2 a_3 &= 0. \end{aligned} \quad (\text{A8})$$

Note that the canonical Kraus representation of a quantum channel is unique. The nonuniquenesses in the solution of Eq. (A8) are that we can simultaneously swap a_1 with a_2 and a_3 with a_4 , and that there are global phases for each Kraus operator in Eq. (A7). Considering that practical Mølmer-Sørensen gates operate in the weak-coupling regime [32–35], i.e., small phase-space loops are traversed many times, one canonical Kraus operator has a dominant identity component. We fix this to be K_1 . We find that $a_1 \approx 1$ has closed-form solution, after removing the global phases by requiring $a_1 > 0$ and $a_2 > 0$,

$$a_1 = \sqrt{\frac{e^{-r_2}(4e^{2r_1} - e^{2r_2} + e^{4r_1+2r_2} + e^{r_2}\sqrt{16e^{6r_1} + e^{2r_2}(-1 + e^{4r_1})^2})}{2\sqrt{16e^{6r_1} + e^{2r_2}(-1 + e^{4r_1})^2}}}. \quad (\text{A9})$$

The solution for a_2, a_3 , and a_4 will be found once Eq. (A9) is inserted into Eq. (A8). It should be pointed out that for the weak-coupling regime, $r_2 \ll r_1$ and $r_2 \approx 0$. This implies the coefficient of K_2 , which is defined as k_2 in the main text, will uniquely determine what a_1, a_2, a_3 , and a_4 are through r_1 .

As a practical example, we consider the weak-coupling parameters given in Ref. [22], $\eta = 0.1$, $\Omega = 0.1\nu$, and $K = 25$, which come from taking $\delta = 0.9\nu$. For $\Gamma_P = 0.02\nu$ and $n_{\text{th}} = 0.05$, we find $r_1 = 0.3456$ and $r_2 = 0.0094$. These lead to the solution $a_1 = 0.9837$, $a_2 = 0.1797$, $a_3 = -0.2282$, and $a_4 = -0.4136$. In particular, $K_2 = 0.612J_y$ is the leading noise source, while K_1 is close to the identity operator. Importantly, K_2 represents the sum of bit-flip errors for each of the two qubits. Note that in theoretical derivations [22], the target gate is in the YY form, while for trapped ion systems such as IonQ, the target gate is in the XX form. This implies that K_2 in the XX gate setup will be proportional to $\sigma_{x1} + \sigma_{x2}$, which is indeed the sum of bit-flip errors.

We also numerically simulated the exact dynamics following the Hamiltonian in Eq. (A1) and the dephasing dissipator C . Using the diagonalization of the Choi matrix, we find that the structure of the canonical Kraus operators in Eq. (A7) remains the same, while each parameter is slightly different. The results are shown in Fig. 1. The numerical

results match reasonably well with the analytical ones. In particular, K_2 represents the dominant error term. Here, we also list the relevant numbers. For the numerical results shown in Fig. 3, the coefficients of the channel Kraus operators are numerically calculated based on assuming $\Gamma_P = 0.0025\nu$, i.e., the leftmost set of points in Fig. 1. The result is $a_1 = 0.999 + 0.00024i$, $a_2 = 0.03647 + 0.00063i$, $a_3 = -0.028 - 0.00048i$, $a_4 = -0.074 - 0.00126i$, and $k_2 = 0.24$.

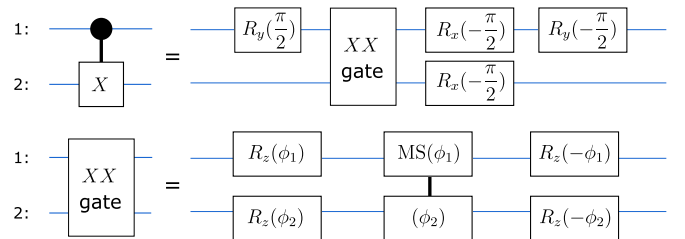


FIG. 4. The relation between the cnot gate, the XX gate, and the Mølmer-Sørensen gates. (Top) The cnot gate can be decomposed as a combination of single-qubit rotations and the XX gate. (Bottom) The XX gate can be decomposed as the Mølmer-Sørensen gate $\text{MS}(\phi_1, \phi_2)$ sandwiched by single-qubit R_z rotations.

For comparison, we also derive the quantum channel for the heating of the oscillator. The approximate analytical solution of the master equation is already derived in Ref. [22]; however, there the fidelity was calculated rather than the quantum channel. We find that the canonical Kraus form of the quantum channel is also represented by Eq. (A7). The coefficients correspond to directly replacing $r_1 \rightarrow r$, $r_2 \rightarrow 0$, $\Gamma_p \rightarrow \Gamma$, and n_{th} becoming the mean bath phonon number for the relaxation operators [22]. Exact numerical simulation also leads to the same structure as Eq. (A7). We can therefore deduce that based on the state of the two qubits alone, we cannot distinguish whether the oscillator mode is subject to dephasing noise or heating noise.

APPENDIX B: CONVENTIONS AND GATE DECOMPOSITIONS

In this section, we first clarify the conventions we have followed for using the IonQ Aria-1 quantum device via Amazon Braket. We then illustrate the gate decompositions, namely, how cnot gate, XX gate, and Mølmer-Sørensen gates with different phases are related.

For expressing quantum operators as matrices, we follow the conventions of Amazon Braket [37]. Specifically, the qubit computational basis is given by

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (\text{B1})$$

The cnot gate, where qubit 1 is the control and qubit 2 is the target, is

$$\begin{aligned} \text{CNOT} &= |0\rangle_1\langle 0| \otimes I_2 + |1\rangle_1\langle 1| \otimes X_2 \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \end{aligned} \quad (\text{B2})$$

The single-qubit rotations are

$$R_x(\phi) = \begin{pmatrix} \cos(\phi/2) & -i \sin(\phi/2) \\ -i \sin(\phi/2) & \cos(\phi/2) \end{pmatrix} \quad (\text{B3})$$

and

$$R_z(\phi) = \begin{pmatrix} \exp(-i\phi/2) & 0 \\ 0 & \exp(i\phi/2) \end{pmatrix}, \quad (\text{B4})$$

while R_y is already introduced in Eq. (2). Mølmer-Sørensen gates, which are the native two-qubit entangling gates implemented on IonQ Aria-1, include two phase parameters ϕ_1 and ϕ_2 ,

$$\text{MS}(\phi_1, \phi_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & -ie^{-i(\phi_1+\phi_2)} \\ 0 & 1 & -ie^{-i(\phi_1-\phi_2)} & 0 \\ 0 & -ie^{i(\phi_1-\phi_2)} & 1 & 0 \\ -ie^{i(\phi_1+\phi_2)} & 0 & 0 & 1 \end{pmatrix}. \quad (\text{B5})$$

Their relation to the CNOT gate via the XX gate is illustrated in Fig. 4.

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