# Exploring preservice teachers' use of ontologies and stances toward the wave function through a classical-quantum analogy lesson

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Physics students, teachers, and preservice teachers (PSTs) struggle with the abstract nature of quantum mechanics (QM), which has led to calls for a more conceptual approach to QM teaching and learning. In response, this study uses a novel analogy between Kundt's tube and the infinite quantum well to elicit discussion of ontological issues. We investigated PSTs' use of ontologies, including their interpretation of the wave function. Nine PSTs participated in the analogy lesson; dialogues were recorded, transcribed, and thematically analyzed. We identified five different ontologies: one classical and four quantum ontologies. The importance of probabilities in PSTs' quantum ontologies stood out. Three ontologies (probabilistic particle, particle-wave, and probabilistic wave) expand the particle-wave duality, while the fourth (wave collapse) added a dynamic element to this axis. PSTs often used a single ontology in response to one aspect of the analogy but switched flexibly between ontologies across aspects of the analogy that they discussed. Concerning the wave function, we found both realist and formalist stances, revealing a diversity of interpretations that contrasts with the uniformity of the formalist QM courses PSTs had attended. We conclude that explicit attention to ontological issues in QM, supported by classical-quantum analogies, provides insight into PSTs' ontological reasoning and offers a conceptual approach to teaching and learning QM. We present recommendations for PST training and implications for QM teaching reform.

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## I. INTRODUCTION

Quantum mechanics (QM) has increasingly become part of secondary school physics curricula worldwide [1]. Yet students, in-service teachers, and preservice teachers (PSTs) alike continue to struggle with its abstract nature and find it conceptually difficult [2,3]. This has led researchers to call for the development of and research into a more conceptual approach to teaching and learning quantum physics [4,5]. Suggested strategies include the use of analogies [6], making the differences and similarities between classical and QM explicit [7,8], engaging students in ontological discussion [9], and the use of computer simulations to facilitate visualization of the abstract formalism and to foster students' intuitive understanding of quantum concepts [10].

In this study, we build on these suggestions by investigating PSTs' use of classical and quantum ontologies through a lesson that introduces a novel analogy: Kundt's tube vs the infinite quantum well. The lesson design seeks to prompt participants to compare the visible classical system with the abstract quantum system thereby paying explicit attention to ontological issues.

PSTs form a particularly relevant group to study. Although their university training in QM is primarily formalist, their future role is to teach QM conceptually in secondary schools. Research shows that many PSTs struggle with this transition [2,11–14]. One contributing factor is the lack of explicit attention to conceptual and ontological issues in university quantum physics courses that PSTs attend as physics students. These courses tend to leave students' conceptual and ontological reasoning largely unattended [7,15]. Our lesson was designed to explicitly elicit and support this form of reasoning.

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The research objective of this study is to gain insight into which classical and quantum ontologies the nine PSTs used and how flexibly they applied them when discussing the analogy. In addition, we examined their interpretive stance toward the wave function, in particular whether they regarded the wave function as a physical entity existing in space and time or as a mathematical representation used to describe and predict measurement outcomes. Together, this study provides insight into PSTs' ontological reasoning and offers a conceptual approach to teaching and learning QM. These findings also give suggestions for PST training and implications for QM teaching reform.

## II. THEORETICAL BACKGROUND

This theoretical background first elaborates on the QM problems with ontologies in Sec. II A and on the interplay between pedagogy, ontology, and interpretive stance in QM in Sec. II B. For the design of our lesson, we consulted the literature on the use of analogies in science education in Sec. II C. In Sec. II D, we present our research questions.

### A. The problems with ontologies in QM teaching and learning

In this study, we distinguish three levels of ontology: general, subject, and student. In philosophy, ontology refers to the study of being in the broadest sense of "everything that is," i.e., *general ontology*. A *subject ontology* is instrumental in deciding if or what entities are present in a given context or phenomenon [8]. In physics, for example, the discovery of the electron in 1897 by Thomson and the neutron in 1932 by Chadwick changed physics' subject ontology. Subject ontologies are also critically reflected upon in general ontology, which assesses the implications of new scientific developments for general ontology [16]. A *student's ontology* then, finally, determines what the fundamental nature of an identified thing might be for that student [8].

For this study, two subject ontologies in physics are especially relevant: *classical* and *quantum ontologies*. Arguments to split physics' subject ontology into these two separate domains can be found in Dreyfus *et al.* [17], who argue that students' epistemological views are domain-specific and that "domain" can be finer-grained than often assumed. They show that physics itself divides into subdomains, with students holding distinct beliefs about classical and quantum entities.

The ontology of classical mechanics (CM), i.e., "classical ontology," assumes massive particles with definite position and momentum whose future states follow Newton's laws. Waves are treated as emergent from collective particle motion, hence reducible to particles. This classical ontology became an issue when, at the end of the 19th century, it was extended to the atomic domain in the thermodynamics of gases. The reality of those atoms was contested in the Mach–Boltzmann controversy. Mach dismissed atomism as producing valueless images, while Boltzmann defended atoms as "inner images" of reality [16,18]. Boltzmann's constructive realism enabled him to use classical mechanical models, as analogies, and visualizations productively in thermodynamics [19]. Through his successful application of atomic models and aided by contemporary experiments, particle realism was extended to the molecular and atomic domain. In the same decades around 1900, particle realism also entered the domain of electromagnetism, first as "ion-hypothesis" and then as "electron-theory," constituting microphysical explanations of macroscopic electromagnetic fields ([20] pp. 250–258). Thus, by the early 20th century, particle realism became the main underlying ontology in many areas of classical physics.

In quantum mechanics too, a "quantum ontology" determines the interpretation of quantum concepts and influences how educators present quantum mechanics in their courses [21]. But unlike in classical mechanics, there is no consensus on a coherent quantum ontology or on the interpretation of quantum concepts [4,9,22]. At the heart of the interpretation issue in QM is the question of the ontological status of de Broglie's matter waves: are they real or merely instrumental? The realist-formalist debate thus shifted from thermodynamics to QM, but this time, the debate never settled, and in this climate, the instrumentalist stance became dominant. The most prominent advocates of the formalist interpretation of matter waves were the neopositivist physicists of the Copenhagen School, i.e., Heisenberg, Pauli, Born, and Dirac. For them, their theories were not about a possible reality behind the observations. According to them, those theories were just tools for mathematically organizing laboratory observations, preferably in such a way that some future predictions regarding observables were possible [23]. This formalist stance was based on a philosophical choice, i.e., neopositivist, regarding ontology in general, with direct implications for QM's subject ontology and indirect implications for students' ontologies in QM learning.

At the tertiary level, teaching practice reflects the vision of the Copenhagen School: the dominant view is that a formal-mathematical approach is sufficient [24,25] and that a quantum ontology and visualization are unnecessary [26]. The pedagogical elaboration of this instrumentalist vision, or formalist stance, has been succinctly expressed as "shut up and calculate!" [27]. Thus, the absence of consensus on QM's subject ontology, and the subsequent decision to avoid the topic where possible, has consequences for teaching at all levels. In secondary education, the formalist quantum pedagogy is not feasible because pupils lack the required calculus tools [28]. In tertiary education, most university teachers pay minimal attention to the interpretation of quantum mechanics in their

formal-mathematical approach. But students still try to visualize the abstract course material [9] and try to connect it conceptually and ontologically with classical mechanics [29]. Without explicit attention, these learning processes remain invisible to instructors and go "underground" [7], and so do students' ontologies.

When researchers investigate students' ontologies, they find that students often adopt realist interpretations—such as matter waves visualized as electrons with definite trajectories—especially when instructors avoid ontology and interpretation [29]. This results in misconceptions that remain unaddressed [7,15]. Dreyfus *et al.* described the ensuing pedagogical situation as a tension between constructivist theories on the one hand, telling us that students (should) try to connect the new to the already known, and QM content presented as a breach from classical physics, a discontinuity [8]. We thus face the dilemma of how to teach and learn QM in a more constructivist manner in an environment where the lead scientists (Heisenberg, Born, Pauli, and Dirac) opted for a "do-not-construct" (models of reality) formalist stance. How do we deal with the development of QM students' ontologies when the dominant neopositivist approach is to avoid its subject ontology?

### B. Pedagogical approaches to ontologies in QM

Research literature describes several pedagogies regarding the use of subject ontologies in the context of reforming (or not reforming) the teaching and learning of QM. First, we have Rohrlich's (1999) formalist approach of ignoring subject ontologies, leading to the unknown formation of students' ontologies [26]. Second, several science education researchers recommend selecting a realist quantum subject ontology, for example, Bunge [30] and Greca and Freire [24]. Third, Baily and Finkelstein promote the formalist stance, not by ignoring ontologies but through discussion of the limits of subject ontologies [29]. Finally, Dreyfus *et al.* [8] and Hoehn [31] suggest creating room for flexible use of subject ontologies. The three reformist approaches can be placed along a realist to formalist axis. On the realist side of the axis, Greca and Freire, who follow Bunge, are positioned, and Baily and Finkelstein represent the formalist stance. Dreyfus and Hoehn can be placed in the middle, as they advocate appreciation of students' flexibility in QM ontologies. We describe these approaches in more detail in the next paragraphs. We note that this axis represents idealized positions. In practice, both instructors and students may adopt intermediate stances, for example, treating the wave function as a meaningful mathematical entity with its own structure and dynamics without necessarily interpreting it as a physical entity in space and time [32]. Such positions are also reflected in instructional approaches that emphasize visualization and the development of intuition about the wave function [10] and are consistent with the flexible use of ontologies described above.

The most prominent advocate of a realist stance in QM reform is Bunge, who aligned himself with the quantum realism of Bohm and de Broglie [22]. Bunge introduced the *quanton* as a quantum entity without classical analogy, displaying integrated particle and wave properties [30]. At the secondary level, Weissman *et al.* [33] used Bunge's *quanton* in their course design, as is done in some French schools [34]. At the tertiary level, physics education researchers such as Lévy-Leblond [35] and Greca and Freire [24] selected variations of Bunge's *quanton* to reform their QM courses.

Taking such a realist stance, Greca and Freire first investigated students' use of ontologies in standard QM courses. They distinguished the ontologies of particle, wavy, synthetic (particle-wave), blurred electron (probability cloud), and formal [24]. They categorized the first four ontologies as classical constructs and considered them undesirable outcomes for learning about QM. To reduce reliance on such constructs, they designed a course to help students visualize the quantum world and construct more adequate mental models. Following Bunge and Bohm, they adopted a realist interpretation in which the quantum state represented physical reality, independent of measurement. Greca and Freire's quantum state described real properties concerning real objects. As such, their subject ontology opposed the positivist branch of the Copenhagen School, which regards the wave function as a mathematical representation used for predicting measurement outcomes, without an associated physical ontology. Greca and Freire described their approach as "orthodox realist." Importantly, their lesson material placed greater emphasis on quantum concepts, which reduced the mix-up of classical and quantum concepts in subject ontologies in students' post-tests.

Baily and Finkelstein valued the formalist stance as the preferred outcome of QM education and similarly found a reduction of classical-quantum mix-ups for students [4]. Like Greca and Freire, they first mapped students' ontologies and additionally surveyed instructors. They found that students typically entered QM courses with a classical ontology of localized particles and often retained semiclassical or trajectory-based views during and after those QM courses. After instruction, students' interpretations fell into five categories, each representing a combination of three interpretive choices, namely: 1. Are there hidden variables? 2. Is the wave function just a mathematical representation or also a physical entity? 3. Is the wave function collapse a physical process or solely a change in knowledge? The five resulting interpretations were realist/satistical, Copenhagen matter-wave formalist, matter-wave realist, pilot-wave realist, and agnostic. Students were more likely to adopt realist interpretations when instructors gave less attention to ontologies. Based on

these findings, Baily and Finkelstein redesigned their course. Rather than endorsing one interpretation, they used an inventory of existing ones, with formalism as the preferred outcome. They integrated ontological dilemmas into course activities, designing questions that prompted students to argue "against realist interpretations of quantum phenomena" [36]. They later reported that this approach improved students' sophistication regarding interpretative issues [15]. Their work highlighted an apparent paradox: strict formal-mathematical teaching often led to students adopting realist interpretations aligned with classical physics, whereas integrating ontological dilemmas into instruction supported a shift toward formalist, less realist interpretations.

A mixed approach away from pure formalism or realism can be found in the work of Dreyfus *et al.* [8] and Hoehn [31]. Dreyfus and colleagues developed a tutorial on the quantum well and studied students' collaborative reasoning. They found that conceptual understanding in QM involved activating new ontologies, with the "quantum particle" as an important example. Like Bunge's *quanton*, this entity combined particle- and wavelike features with additional properties lacking classical analogues. Conceptual expertise in QM, according to Dreyfus *et al.*, requires understanding how such entities behave and when classical ontologies can or cannot be applied. They showed that students were "capable of switching between multiple ontologies when reasoning about a given physical entity" [8]. Unlike Greca and Freire, who preferred realist outcomes, or Baily and Finkelstein, who preferred formalist ones, Dreyfus *et al.* valued this flexibility as a positive outcome. This *flexible use* of subject ontologies by students while working on quantum mechanics problems was also found by Hoehn and Finkelstein [37]. In follow-up work, Hoehn observed that students used different combinations of six ontologies on the particle-wave axis as "flexible structures that can be cued in certain contexts and that a reasoner can move between" ([31] p. 15). Hoehn identified three modes of use of the six ontologies: unitary (i.e., the use of a single ontology), parallel (switching between two unitarily applied ontologies), and blended (combining ontologies into a new entity, such as Dreyfus' quantum particle) [38]. Ultimately, Hoehn defined flexible ontological reasoning as the ability to draw on multiple ontologies in diverse ways across contexts [31].

In this paper, we adopt Hoehn's definition of unitary use ("an application of a single stable ontology in a given local reasoning episode"; see Ref. [38] p. 010124-4) and contrast it with flexible use (including both parallel and blended use as its variations).[1] We take the student ontologies found by Greca and Freire [24], Baily and Finkelstein [4], and Hoehn [31] as a starting point for our coding of student ontologies. We use the stances toward the matter wave of QM, i.e., realist, formalist, or flexible, as an analytic tool for categorizing students' stances.

### C. Use of analogies in science and QM teaching

In science education research, use of analogies has been studied extensively. Taber defined an analogy as "a recognition of some similarity between two different systems" [39]. Teachers and curriculum materials use analogies as a way to reduce the gap between the unknown and the familiar [40]. Analogies thus embody a characteristic element of constructivist learning processes [41]. The familiar system is called the "base" and the unknown system the "target" of the analogy [41]. Through the addition of the category of the "model" of the analogy to the base and the target, Duit arrived at his base-target-model analogy scheme; see Fig. 1 [41]. In science education, analogies are often used when the target situation is relatively unknown, and students need a comparison to a familiar situation, i.e., the base of the analogy [42]. Bunge, in his discussion of analogy in quantum physics, distinguishes between ontological and formal analogies [40]. Sound waves in water and in air are examples of the first, as both involve longitudinal oscillations of a material medium. Formal analogies, in contrast, arise when systems obey similar mathematical laws despite differing in their physical nature. For example, pressure waves are formally analogous to light waves and matter waves, as they satisfy similar wave equations while differing in their underlying physical nature.

The use of analogies has also been criticized. Many things can go partly wrong while teaching with analogies, such as taking the analogy too literally, which may even create misconceptions [43–45]. This is also the case for the use of classical physics analogies for teaching QM, due to the revolutionary character of QM. For instance, QM requires a probabilistic way of thinking, whereas CM relies on deterministic thinking [46]. Vilarta Rodriguez *et al.* [46] suggest that the use of analogies with classical physics might be detrimental to the learning process, as some students took the analogies too literally. As a mediating strategy, students should additionally be taught where the analogy breaks down, which does increase the teaching and learning load. Analogies are not just about what is similar between two systems. Any two items that are analogous in some way will be disanalogous in other ways [47], a distinction formulated by Hesse as positive versus negative analogy [48]. However, negative analogies can also be leveraged as learning opportunities because learners can learn as much about the target by analyzing the differences as by looking at the similarities [39]. Thus, using analogies in science teaching preferably involves a

[1]In formal QM, "unitary" is a technical term with a well-established mathematical meaning, i.e., as a property of operators. This should not be confounded with the ontological use of the term in Ref. [38], which we adopt in this paper.

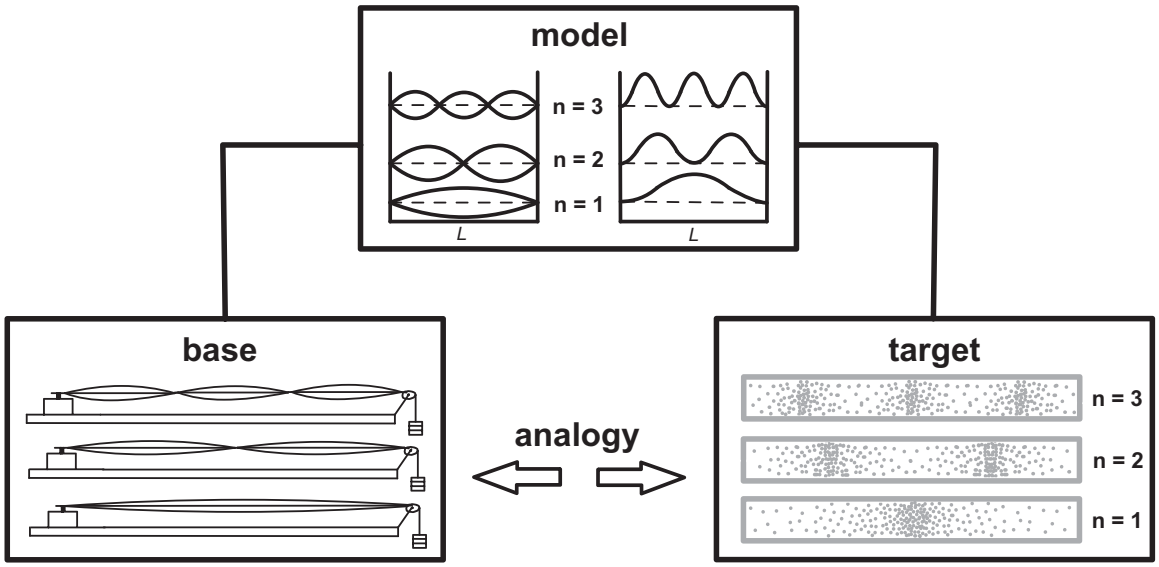



FIG. 1. Base-target-model analogy scheme (see Ref. [41]) applied to Melde's experiment as the base and the quantum well as the target of the comparative analysis. The picture for the target is based on pictures from a physics textbook in use in Dutch upper-grade secondary classes.

full comparative analysis between the analogy base and target.

One of the well-known models for the use of analogies in science, constructed in such a way that it prevents the most common risks, is Glynn's teaching-with-analogies model (TWA) [49]. A recently adapted version of Glynn's TWA model has six elements: 1. introduce the target; 2. examine the student's prior knowledge about the base (and reminding and explaining it); 3. identify the relevant features of the base and target; 4. map similarities between the base and target; 5. identify dissimilar features between the base and the target, where analogy breaks down; and 6. arrive at a conclusion [12]. Still, substantial teacher guidance is seen as crucial when working with analogies because group discussions between students can develop in unexpected directions and different groups of students often arrive at different levels of understanding [43].

### D. Research questions

The four pedagogical approaches we found in the literature toward the role of subject ontologies in QM teaching were: ignoring subject ontologies, selecting a realist quantum subject ontology, promoting formalism through discussion of the limits of subject ontologies, and stimulating flexible use of subject ontologies. The first approach is the dominant teaching pedagogy at the tertiary level and combines with a formalist stance toward the matter wave of QM. The other three are strategies to reform this dominant pedagogy. Discussion of the limits of subject ontologies as a reform strategy is still most in line with the formalist stance of the dominant pedagogy. The reform strategy aimed at promoting a realist quantum subject ontology is in direct conflict with the dominant teaching pedagogy. Facilitating flexible use of subject ontologies as a reform strategy offers a way to bridge the realist-formalist schism.

Our research aimed at prompting PSTs to use ontologies in their dialogues when discussing a classical-quantum analogy, so we could map PSTs' use of quantum ontologies, their possible flexibility in use, and their stance toward the realist versus formalist wave function schism. The ensuing research questions were exploratory and qualitative in nature:

RQ. 1 What classical and quantum ontologies do PSTs use when discussing analogies between a classical base and a quantum target?

RQ. 2 To what extent do PSTs use ontologies in a unitary or flexible manner?

RQ. 3 What stance toward the wave function do PSTs show?

To investigate our research questions, we developed a new lesson design built around an analogy between Kundt's tube and the infinite quantum well. This design was intended to elicit PSTs' use of ontologies as they discussed the lesson's comparative analogy questions. We describe the lesson and the rest of the research design in the methods section.

## III. METHOD

We adopted an exploratory, qualitative design-based research approach to investigate how PSTs used classical and quantum ontologies during the comparative analogy lesson. We aimed to explore which ontologies PSTs used, whether they used them in a unitary or flexible manner,

and what stance, realist, or formalist, they adopted toward the wave function. PSTs followed the lesson plan in the role of students, and we audio- and video-recorded their discussions, which we then analyzed for their use of ontologies.

### A. Design of our TWA-modeled lesson plan

As announced, this study used a new analogy for a quantum particle in an infinite square potential well (the quantum well). Almost all standard physics textbooks that treat the quantum well use the analogy with Melde's experiment; see Fig. 1. Melde's experiment visualizes the amplitude of a standing transverse wave in a closed system. With Melde's experiment, the standing wave in the rope is the formal analog for the matter wave. An analog for the particle aspect is missing [50].

Our study introduces a new analogy: Kundt's tube compared to a simulation of an electron in a quantum well; see Fig. 2. An extensive description of Kundt's tube can be found in Ref. [51]. The two analogies complement each other relative to the model in Figs. 1 and 2: Melde's experiment visualizes the amplitude distribution of the standing waves, the graph on the left in the model; Kundt's tube visualizes the energy distribution and the square of the amplitude of the standing waves, the graph on the right in the model. Compared to Melde's experiment analogy, the Kundt's tube analogy has more scaffolded layers, i.e., vacuum—air—Styrofoam. Each layer plays a distinct role in the comparative analysis following the TWA model. There exists a positive analogy between sound waves and matter waves, which are both standing waves in this case. There exists a negative analogy relative to the medium of the waves: sound waves need air, and matter waves exist in a vacuum. There is again a positive analogy between Styrofoam granules and electrons. And then there is the analogy of the interaction between sound waves and Styrofoam granules in the base and the duality of matter waves and electrons in the target. This fourth analogy makes the context especially useful for the visualization of and explicit reasoning about the particle-wave duality, including the breakdown of visualizations in the quantum target.

In our analogy, the mathematical model representing the laws for standing waves is given in the graph in Fig. 2. Within the base-target-model structured analogy, the model provides the mathematical structure that connects base and target while remaining ontologically flexible. The model itself can be understood as a well-defined abstract mathematical object, prior to additional ontological commitments about how it corresponds to physical facts ([32] p. 33). The left-side graph of the model gives the amplitude of the sound waves in the air. The visual height distribution of the Styrofoam granules is represented in the right-side graph, representing the square of the left-side graph. In the case of Kundt's tube, the physical link between the two graphs is the invisible energy of the sound waves in the air that gets converted into kinetic and gravitational energy of the Styrofoam granules. For the particle in the quantum box, the left-side graph represents the amplitude of the matter waves (the de Broglie waves or the quantum state), and the right-side graph represents the probability distribution for electron localization in the box. Vertically, the three pictures represent standing waves at increasing sound frequencies for the classical base and higher electron energies for the quantum target.

We organized the comparative analysis lesson activity as a guided group discussion; see Table I for the lesson plan. The core of the lesson plan was a table with 14 comparative analogy questions; see Appendix. The

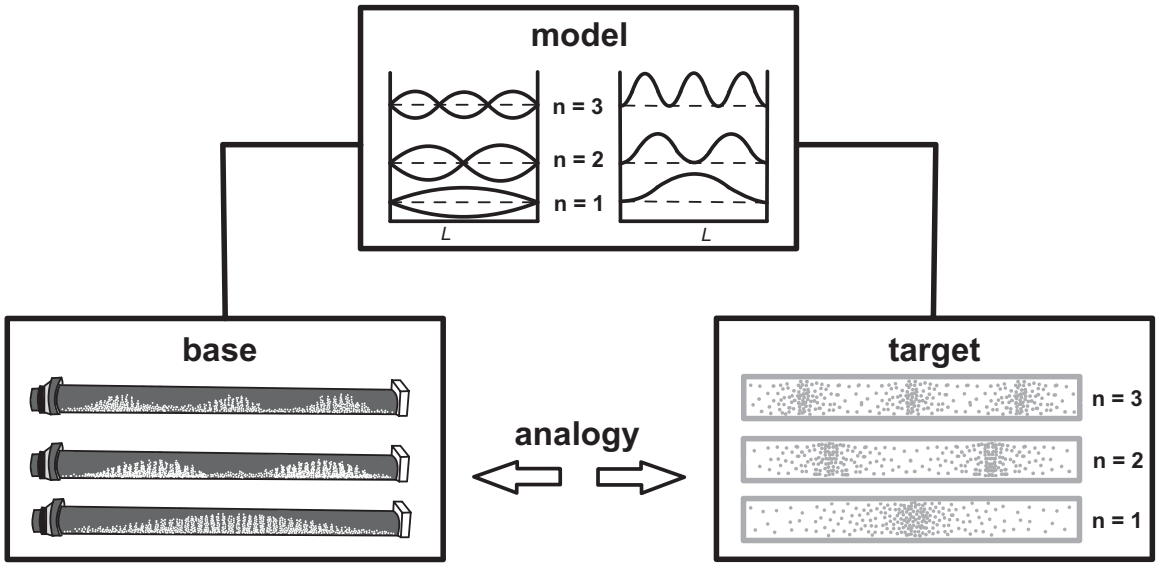



FIG. 2. Base-target-model analogy scheme (Duit, 1991) applied to the Kundt's tube vs quantum well comparative analysis.

TABLE I. Lesson plan.

| Lesson phases (timing) | Learning activities |
|---|---|
| Class introduction (10 min) | Teacher presents the theme quantum well; teacher presents the video on Kundt's tube. |
| Instruction to group assignment (5 min) | Teacher and students go through the worksheet; teacher explains the group assignment. |
| Carrying out group assignment (25 min) | Comparative analysis of the quantum well and Kundt's tube; students discuss the questions and note results in the worksheet; teacher walks around, motivates, and gives feedback. |
| Class discussion (10 min) | Sharing and discussion of groups' comparative analysis results; teacher leads whole-class discussion. |

questions were developed to guide the participants through the comparative analysis of the one-dimensional quantum potential well relative to the standing waves in the classical Kundt's tube. After an introductory question 1, we organized questions 2–14 in three categories. The first category of questions, 2–6, treated the wave theory aspects of target and base. The second category of questions, 7–10, related to the particle-wave interaction and duality. The third category of questions, 11–14, related to the model, i.e., the graphs. In the formulation of questions 11–14, we were motivated by reported misconceptions regarding the double meaning of the vertical axis in the case of the quantum well [2].

The 14 prestructured questions were intended to prompt discussions in groups regarding the conceptual transfer or breakdown of transfer of the classical concept of standing waves in a tube to the said quantum well. For several questions, we chose to give the answers for the base in the worksheet, in order to activate participants' prior conceptual knowledge of the classical base and to prevent a false start of the comparison. The setup still made it possible for students to dynamically switch between base and target while discussing the issues raised by the questions. The comparative analysis questions focused on the ontological aspects of the standing waves analogy, with a special emphasis on the dissimilar aspects, so on where the analogy of the classical base with the quantum target breaks down. The lesson was designed to have the potential to be implemented on all levels of the vertical chain: secondary school students, bachelor physics students, master physics students, PSTs, and in-service teachers.

### B. Participants

Nine PSTs participated in this study, attending the comparative analysis lesson as learners. The PSTs were in the first or second year of their preservice physics teacher master's program at universities in the Netherlands and could participate in the study on a voluntary basis. The participants had previously completed quantum mechanics courses as part of their Bachelor of Physics program. During the analogy lesson, they were organized into: group 1 with PSTs 4 and 5; group 2 with PSTs 1, 2, and 3; group 3 with PSTs 6 and 7; and group 4 with PSTs 8 and 9. We worked with human participant data and obtained informed consent from all participants following institutional guidelines (ethical approval case number: REC2314).

### C. Data collection

The lesson was taught twice to accommodate students' schedules: groups 1 and 2 participated in the first session and groups 3 and 4 in the second. The first author, also a high school physics teacher with 29 years of teaching experience, taught both lessons. To collect data on PSTs' use of ontologies, we placed an audio and a video recorder on the tabletop of each PST group and recorded the group discussions during activity 5 of the lesson plan. The average audio recording time was 27 min. At the end of the lesson, the groups' worksheets were collected. We additionally asked each PST to respond to a single individual feedback prompt: "Describe the most important thing you learned from this comparative-analyses lesson." The written responses to this prompt were collected.

### D. Data analysis

After transcription of the audio recordings, we used Atlas.ti software to code the transcripts for PSTs' use of ontologies (RQ1). We divided the coding process into two phases. In phase 1 of our coding activity, we wanted to produce our own custom codebook. We used previous work for an initial codebook [15,38] and adapted codes to better capture the specific use of ontologies by PSTs as observed in their dialogues. For instance, we decided to fuse initial codes "Quantum Particle as associated to a wave" with "Quantum Particle as wave carrier" [31] because they were very much alike in our data. After several iterations of adapting the codebook and again coding the transcribed dialogues, five codes emerged; see Table II. In phase 2, we counted the frequency of codes per group dialogue and per comparative analysis question; see Table IV. During both phases, the first author was the main coder; the second author critically assessed the first author's coding process and results until they reached agreement, after which the third and fourth authors in turn assessed the coding process and results until these authors could also give their consent. In this way, we reached intercoder consistency within the research team [52].

TABLE II. Codebook for PSTs' use of ontologies.

| Code | Description | Examples |
|---|---|---|
| 1. Classical particle (CP) | PST refers to the particle nature of electrons (e.g., with a mass, charge, velocity, impulse, and/or trajectory on which forces can act, following Newton–Lorentz, or electrons which exist in stationary state orbits and can quantum jump to excited states, following Bohr). | Q3, G1—PST4: "*so due to the repulsion there is zero chance that it [the electron] is at the end. Yes, because of the same charge [of electron and wall.*" Q12, G1—PST4: "*Isn't it just like in an electron, if you increase the n, in an atom, it moves into a higher shell, it gets more energy until it ionizes?*" |
| 2. Q-particle as probabilistic (QPp) | PST refers to a quantum particle as showing probabilistic behavior (e.g., when observed) without mention of wave characteristics present in this stochastic behavior. | Q6, G3—PST7: "*But in quantum mechanics there is no medium that is set into vibration. It is a probability distribution of where you can find a particle.*" |
| 3. Q-particle associated to wave (QPw) | PST mentions a connection between particle and wave, without priority of one over the other, or they refer to the particle as a wave carrier without mention of a stochastic aspect. | Q11, G3—PST6: "*because the electron behaves like a wave.*" |
| 4. Q-wave as probability wave (QWp) | PST refers to the quantum wave as a probability wave without mentioning the connected particle that is the subject of the probability. PST refers to the wave as just a calculus tool to produce probabilities. | Q2, G3—PST6: "*Yes, a probability wave distribution actually, which interferes with each other [i.e. with itself].*" |
| 5. Q-wave 'collapse' (QWc) | PST uses a description of quantum measurement as leading to the sudden disappearance of the wave aspect and the (stochastic) localization of the particle aspect. | Q9, G1—PST5: "*so you look at, then the wave function in the graph stops and then you no longer have the waves and in the same place the electron*" |

TABLE III. Particle-wave axis, with the QPp on the left, the QPw in the middle, the QWp on the right, and the dynamic QWc at the bottom, connecting QWp and QPp.

| Particle-wave axis | | |
|---|---|---|
| QPp | QPw | QWp |
| ⇑ | QWc | ⇓ |

Second, for examining the extent to which PSTs used ontologies in a flexible manner (RQ2), we used the code frequency (Table III) that resulted from our coding. We examined whether PSTs used the same or different ontologies within each question and when moving from one question to the next. Building on the descriptions of Hoehn [31] and Dreyfus *et al.* [8], we defined the use of ontologies as unitary when PSTs used a single ontology while discussing a given question or when they used the same single ontology in a direct follow-up question. We defined PSTs' use of ontologies as flexible when they used two or more ontologies while discussing a given question or when they switched to or added a different ontology in a direct follow-up question.

Finally, for analyzing the stance PSTs show toward the wave function (RQ3), we selected PSTs' discussions involving the wave function. We examined to what extent, if at all, PSTs connected ontologies to the wave. We characterized PSTs as showing a realist stance when they attached an ontology to the wave function or talked about the wave as something that exists in the physical world. We characterized PSTs as showing a formalist stance when they talked about the wave function as a mathematical representation used to describe and predict measurement outcomes. Observations of patterns in regard to research questions 2 and 3 were discussed within the research team until agreement was reached.

## IV. FINDINGS

### A. Which ontologies did the PST groups use? (RQ1)

Following RQ1, we found that the PSTs used five ontologies: one classical ontology and four quantum ontologies (see the codebook in Table II, the outcome of phase 1 of our coding). Of the four quantum ontologies identified in PSTs' dialogues, three—probabilistic quantum particle (QPp), quantum particle-wave (QPw), and probabilistic quantum wave (QWp)—could be positioned along an axis that nuances the particle-wave duality; see Table III. On the left of this axis lies the QPp, where PSTs mentioned the electron's probability distribution without invoking wave aspects. In the middle lies the QPw, in which PSTs loosely connected particle and wave without reference to probabilities. On the right lies the QWp, where a wave was described as a probability distribution without reference to particles. The last quantum ontology, quantum wave collapse (QWc), added a dynamic to this

TABLE IV. Coding results for the use of ontologies in PSTs' dialogues where they discussed the target of the analogy.

| | Group 1 | | | | | Group 2 | | | | | Group 3 | | | | | Group 4 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Code | CP | QPp | QPw | QWp | QWc | CP | QPp | QPw | QWp | QWc | CP | QPp | QPw | QWp | QWc | CP | QPp | QPw | QWp | QWc |
| **Q 2** | | | | QWp | | | | | | | | | | QWp | | | | | QWp | |
| **Q 3** | CP | | | | | | QPp | | | | CP | | | | | CP | | | | |
| **Q 4** | | | | | | CP | | | | | | | | QWp | | CP | | | | |
| **Q 5** | | | | QWp | | | | QPw | | | | | | QWp | | | | | QWp | |
| **Q 6** | CP | | QPw | QWp | | | | | | | | QPp | | | QWc | | QPp | | | |
| **Q 7** | CP | | | | | | | QPw | | | | | | | QWc | | QPp | | | |
| **Q 8** | | | QPw | | | | | QPw | | | | | QPw | | | | QPp | | | |
| **Q 9** | | | QPw | | QWc | | | | | QWc | | QPp | QPw | | QWc | | QPp | | | |
| **Q 10** | | | | | QWc | | QPp | | | | | | | | QWc | | | | | |
| **Q 11** | | | | QWp | | | QPp | | | | | | QPw | QWp | | | | | QWp | |
| **Q 12** | CP | | | | | CP | | | | | CP | | | | | CP | | | | |
| **Q 13** | CP | | | | | CP | | | | | CP | | | | | CP | | | | |
| **Q 14** | | | | QWp | | | QPp | | | | | QPp | | | | | QPp | | | |
| **N** | **5** | **0** | **3** | **5** | **2** | **3** | **4** | **3** | **0** | **1** | **3** | **3** | **3** | **4** | **4** | **4** | **5** | **0** | **3** | **0** |

axis, as PSTs described observation as making the wave disappear and the particle appear, thus producing a shift from the right to the left end of the particle-wave axis.

Phase 2 of our coding process resulted in a frequency table (Table IV). We observed that group 3 used all five ontologies, group 1 and 2 each used a different subset of four ontologies, and group 4 used a subset of three ontologies. All groups used the CP ontology. Group 1 did not use the QPp, group 2 did not use the QWp, and group 4 did not use the QPw nor the QW). In four instances, PST groups did not use any of the five ontologies while answering a question (e.g., they gave a short answer without an apparent ontology). The groups used the ontologies defined in the codebook in different frequencies, as can be seen in Table IV. We can frame the group differences using the concept of the particle-wave axis; see Table III. Group 1 used the center-right part of the axis (QPw-QWp), group 2 used the center-left part of the axis (QPp-QPw), group 3 used the whole axis, and group 4 used only the left and right extremes. Group 4 never used the QWc dynamic ontology, and group 3 used this dynamic ontology the most, as shown in Table IV.

### B. To what extent did PSTs use the ontologies in a flexible manner? (RQ2)

#### *1. Unitary and flexible use of ontologies within comparative analogy questions*

In the first description of our observations regarding PSTs' flexible use of ontologies, we focused on the separate questions as compartmentalized contexts or cases. We observed that PSTs answered most questions using only one ontology in their dialogues. As explained in Sec. II B, we defined this as unitary use of ontologies. An example is how group 4 answers question 9 (If you remove the waves, can the particles from question 1 still be present in the tube and well?) by use of QPp only:

> Q9, G4—PST 8: *Yes oh yes we don't have to say, indeed, what happens with those movements, if you have the waves removed? I don't get that... if you remove the waves? The electron can't be in there without it having such a probability distribution. [QPp]*

In five cases, we observed PSTs using more than one ontology in their dialogues while working toward answering the question at hand. For group 1, this happened with questions 6 and 9, and for group 3, this happened with questions 6, 9, and 11. As explained in Sec. II C, we defined this as flexible use of ontologies. An illustrative example is the dialogue of group 1 regarding question 6 (waves can be transverse or longitudinal. Which applies here?). The PSTs of group 1 struggled with this question. PST 4 started by taking the picture of a wave in a cord, as in the graphs of the Model in Fig. 1 as a real representation of the propagation of probability waves [QWp]:

> Q6, G1—PST 4: *I think transverse because probability waves [QWp] go like this. [On the video the PST made the gesture of transverse wave moving through a cord.]*

In reply to PST 4, PST 5 applied the concept of transverse and longitudinal directly to a classical electron that is moving and vibrating, and arrived at a different, formal conclusion, in which PST 5 pointed to the dissimilarity between the classical base and quantum target or the breakup of the analogy:

TABLE V. Frequency of groups' unitary and flexible use of ontologies within questions.

| | Unitary use within questions | Flexible use within questions |
|---|---|---|
| Group 1 | 10 | 2 |
| Group 2 | 11 | 0 |
| Group 3 | 10 | 3 |
| Group 4 | 12 | 0 |

> Q6, G1—PST 5: *Then you have to think of, electron, has the same or the opposite direction as the vibration [CP]. Yes, transverse or longitudinal. Then I would say that it doesn't even have anything to do with quantum mechanics.*

PST 4 objected and came up with an additional analogy between light particles and electrons, and a subsequent identification of light particles with light waves, to support the answer "transverse':

> Q6, G1—PST 4: *It does, because according to quantum mechanics an electron can behave like a light particle, like a wave. Just like light waves and they are transverse so electrons can also behave like waves, they can do that too. [QPw]*

In this short dialogue, the PSTs 4 and 5 of group 1 applied the ontologies of QWp, CP, and QPw in order to arrive at a common conclusion based on QPw, the one brought forward by PST 4.

Table V describes the number of times we observed unitary and flexible use within questions per group.

### *2. Unitary and flexible use of ontologies from question to question*

We observed both unitary and flexible use of ontologies within the dialogues of all four groups when PSTs went from question to question (see also Table IV). We had 13 questions, so the PST switched 12 times from question to question. When they continued to use the same ontology in the next question as in the previous question, we categorized that as unitary use. If they used a different or an additional ontology in the subsequent question, we categorized that as flexible use.

Unitary use and flexible use were especially remarkable because they were almost the same for all groups, regarding the model questions 11 to 14 about the different graphs of Fig. 2. Questions 11 and 14 were about the stationary state in a single energy level and prompted use of the new QM of de Broglie–Schrödinger–Born. Questions 12 and 13 were about quantum jumps between energy levels and prompted use of the Old QM of Bohr. All groups showed flexible use while switching from Q11 to Q12, unitary use while switching from Q12 to Q13, and flexible use while switching from Q13 to Q14. They all used quantum probability ontologies, (QWp) and/or (QPp), in the context of the stationary state question 11, for example:

> Q11, G1—PST 4: *I know at least that the right one is the probability distribution. [QWp] Yes. Maybe here it is the amplitude of standing de Broglie waves?*

They all switched to the classical particle ontology (specifically, Bohr's model) within the quantum jumps context of question 12 (in the picture above, as n increases, what increases in the Kundt tube and what increases in the quantum tube?), for example:

> Q12, G1—PST 4: *Here the energy of the electrons increases. Just like in an electron, if you increase the n then, in an atom, then it goes into a higher shell, it gets more energy until it ionizes? [CP]*

In question 13, the PSTs were asked what action was needed to realize this transition. All groups used CP to answer this question, for example:

> Q13, G1—PST5: *Increase energy. And how do they do that? Shooting electrons at it, er, shooting photons at it, I would say. Yes. [CP]*

Then, when they moved on to Q14, where they were asked to explain a detail of the same wave model, namely, what was increasing from the left to the middle in the graphs of level 1, all groups switched back to quantum probability ontologies, QWp and/or QPp, for example:

> Q14, G1—PST4: *Well, at least here it should be the probability, right? Yes, the probability of finding the electron there [QWp].*

The result of all observed unitary and flexible use while switching from question to question is given in Table VI.

TABLE VI. Group use of ontologies while switching from question to question.

| | Unitary use between questions | Flexible use between questions |
|---|---|---|
| Group 1 | 2 | 9 |
| Group 2 | 4 | 6 |
| Group 3 | 4 | 8 |
| Group 4 | 5 | 6 |

### C. To what extent do the groups attach ontological status to the wave function (RQ3)?

As for RQ3, we found that all four groups leaned toward quantum particle realism, as reflected in their frequent use of the classical particle ontology. The main differences lay in how they approached or understood the quantum-mechanical wave function.

Of the four groups, only group 3 had a clear formalist stance toward the wave function, as can be observed in the following two quotes, when they discuss the wave function:

> Q14, G3—PST 7: *[…] in quantum mechanics there is no medium that is set into vibration. It is a probability distribution of where you can find a particle, yes. [QPp] And, to give value to that, we attach numbers in the form of vibrations and waves, but there is no actual vibration or wave present.[…]*

> Q14, G3—PST 7: *No, it was really just, it is uninterpretable or something. It has no interpretation*.

They clearly state that there is no matter-wave ontology, i.e., ontological interpretation, only a matter-wave calculus that is used to predict where to find the particle.

Groups 1, 2, and 4 showed a realist stance toward the wave function, meaning that in their dialogues the quantum wave appeared as an existing entity. Of the three groups, group 1's stance could be characterized more specifically as the quantum particle stance. Group 1 used the particle-wave duality as two aspects belonging to one single entity, the electron as "quanton," to support most of their answers. An example of this stance can be found in group 1's discussion of question 6 (are the waves transverse or longitudinal?), where they first made gestures to simulate how probability waves move transversely through space and then stated that an electron can behave as such a wave, (see quotes in Sec. IV B 1).

To position the realist stances of group 2 and group 4, we need the particle-wave axis as an expansion of the quanton or "quantum-wave duality" realist stance. The ontology of group 2 could be positioned on the right side of the axis. For them, the vibration of the particle was the seat of the wave behavior. For students of this group, the wave did not have independent ontological status because it existed literally as the particle's vibration. For group 2, wave realism was reduced to, or contained in, their particle realism, as is illustrated in the following quote:

> Q5, G2—PST 2: *Well, wave behaviour of particles. What do you call the waves in the underlying medium? A wave behaviour of particles? [QPw]*

The most prominent was this "particle absorbing the wave" stance when they discussed question 10 (does looking at it affect what is in the tube and well?):

> Q10, G2—PST 2: *If you have a larger particle, then the chance that a quantum effect occurs is much smaller. Look, all particles have certain vibrations, so to speak, but if you continue calculating and we have the mass of such a grain or something [Styrofoam], it is so much larger than the mass of an electron that you don't see that vibration at all anymore. Okay, because in principle if you were to see the human body as a particle, then you really just don't have any vibration anymore because the mass is so large. And because those particles on the right are so small. What is it called? […] then it becomes a probability distribution*.

This variation on the realist stance was not anticipated in our theoretical analysis, where we only had the choice between wave realist, wave formalist, or a flexible use of both stances. But for group 2, all wave aspects were literally reduced to particle behavior and were made particle-intrinsic, so to say.

Group 4 had a strong quantum probability focus, with the probability either connected to the particle or to the wave. On the particle-wave axis, group 4 only used the extremes, QPp and QWp, never the middle QPw. On the wave side of the axis, group 4 fused wave and probability distribution into one physical entity, the "probability wave." This entity was given a realist interpretation, in the sense of something that can really exist in empty space, as exemplified in their answering of question 5 (what do you call the waves in that underlying medium?):

> Q5, G4—PST 9: *Yes, you can also have a wave that is in a vacuum. Yes*. Q5, G4-PST 8: *Yes, again a probability distribution. [QWp]*

The realist stance of group 4 toward probability wave held out as long as they could fuse wave and probability, but their realism turned into doubt when those two aspects were clearly set apart. This was illustrated in their response to question 11 (which aspect do you associate with the left (the wave), and which aspect with the right pattern (its square)?), where the aggregation of wave and probability distribution wasn't possible:

> Q11, G4—PST9: *Yeah, That's tricky, isn't it? With us it's just really... the wave function, let's say a vague concept. And the square is clearly a probability distribution, but I really don't know what to do with the normal wave function*.

This last exclamation of group 4 indicated that they were in the dark in regard to the ontological status of the wave function once the wave and probability distribution were clearly separated.

In summary, we found one group to have a formalist stance toward the wave function and three groups to have various realist stances. We could identify the stance of group 1 as a realist stance in line with Bunge and Greca and Freire. The stance of group 4 was strongly tied to the statistical interpretation of quantum mechanics but connected to separate particle and wave realism instead of to formalism. The stance of group 2 was realist, but such that wave ontology was reduced to particle ontology.

## V. DISCUSSIONS AND CONCLUSION

Given the well-documented conceptual difficulties that students, teachers, and PSTs encounter in quantum mechanics, our study sought to contribute to research into more conceptual approaches to QM teaching. We focused on PSTs because they are expected to teach QM conceptually in secondary education but have been trained largely in a formalist manner. This calls, at least, for learning environments that make their reasoning about quantum concepts explicit. Our lesson design, based on the classical Kundt's tube—quantum well analogy with a TWA-informed lesson plan—was developed to meet this need. We formulated the following three research questions: which ontologies did PSTs use in their discussions (RQ1), the extent to which they used them unitarily or flexibly (RQ2), and what stance, realist, or formalist, toward the wave function did they show (RQ3). The findings from our study allow us to answer these questions and to discuss their implications for conceptual QM teaching and teacher education.

Regarding RQ1 (what classical and quantum ontologies do PSTs use?), we identified five ontologies in PSTs' dialogues: one classical particle ontology and four quantum ontologies. The probability aspect featured prominently in three of the four quantum ontologies: QWp, QPp, and the QWc. This finding suggests that probabilities played a more central role in students' reasoning than has been previously reported. In contrast to earlier studies with bachelor students, see Refs. [24,31], where the probabilistic aspect was only mentioned in the descriptions of ontologies or in parentheses, our analysis shows that the PSTs explicitly incorporated probability into their ontological reasoning. PSTs' emphasis on probabilities may stem from their exposure to formalist QM courses, where the wave function and its square—as an observable probability distribution—play a dominant role but are seldom connected to an ontology. It may also be that this lack of explicit subject ontology guidance in formalist courses prompted PSTs to independently link the probability distribution in various ways to the particle-wave duality. Our finding regarding the central role of probability extends previous work by demonstrating that PSTs can construct student ontologies that integrate probabilistic aspects into both particle and wave conceptions. To illustrate this observation, we conceptualized the observed quantum ontologies along a QPp-QPw-QWp axis, representing an expansion of the traditional particle-wave duality (see Table III). This nuanced axis helped reveal differences in how PSTs used quantum ontologies in different ways.

For RQ2 (to what extent do PSTs use ontologies in a flexible manner?), we characterized PSTs' use of ontologies as a mix of unitary and flexible. PSTs predominantly used the one ontology that provided the best solution for a given comparative analogy question (unitary use), while they often switched ontologies when moving to the next question (flexible use). We can characterize this as *pragmatic use* of ontologies: given the multiple ontologies in their toolbox, they *flexibly* select the one that *uniquely* fits the given question. If we refer back to Hoehn's ([31] p. 102) definition of flexible use as "making use of a variety of ontologies in a variety of ways within and across contexts" and compare this to our findings, we can report PSTs' unitary use to be strongly dominant within questions and PSTs' flexible use to be dominant across questions. We therefore propose a more nuanced approach to the characterization of flexible use by explicitly positioning it alongside unitary use rather than not mentioning unitary at all, as in Ref. [8], or just as an opposite. In our view, flexible use and unitary use coexist as two sides of the same coin, i.e., pragmatic use.

For RQ3 (what stance toward the wave function do PSTs show?), we found mainly realist stances and a very pronounced formalist stance. In contrast to Baily and Finkelstein [29], who identified five distinct interpretations based on combinations of three ontological choices (concerning hidden variables, the ontological status of the wave function, and the nature of its collapse), our smaller dataset did not allow such fine-grained differentiation. Nevertheless, both stances, the realist (Greca and Freire) and the formalist (Baily and Finkelstein), were represented within our small cohort of PSTs. Additionally, we observed realist stances that diverged from previously characterized realist interpretations. The observed variability supports the perspective of Dreyfus and Hoehn, who emphasize ontological flexibility as a valuable learning outcome rather than a deficiency. Pedagogically, this suggests that courses framed exclusively within a realist or formalist ontology may conflict with PSTs' diverse interpretive positions. A teaching approach that acknowledges and builds on such diversity, encouraging reflective and flexible engagement with competing ontologies, may therefore be most effective. The diversity of PSTs' ontological interpretations should have implications for

reform strategies toward more conceptual teaching for those courses. The observed diversity corroborates previous research [7,15], showing that the outcome of the process of students' interpretation of quantum mechanics does not reflect the relative uniformity of those quantum course intentions and results in rather diverse interpretational outcomes. However, we think that the practical implications might first of all lie in teacher training, because we expect most bachelor-level QM courses to remain formalist for the foreseeable future. Teacher training institutes could try to make the diversity of ontologies transparent to PSTs and reflect with them on its teaching implications.

Our lesson design and its implementation in teacher training can contribute to this goal of transparency. Our lesson revealed PSTs' ontological reasoning, made their position on the realist-formalist spectrum explicit, and created opportunities for conceptual and ontological discussion. Our experience during the lesson taught to PSTs was that the approach with explicit attention to ontological issues proved conceptually challenging and provided a new experience. For instance, PST 3 said afterwards, "*The comparison [assignment] made me think deeply about the subject matter." PST 2 expressed this as: "Is this for [a secondary education] class? Because this is quite a tough job, you know!*" We conclude that the lesson plan, with its conceptual rather than mathematical focus, is well suited for PST training and responds to calls for more conceptual QM teaching materials.

This study also yields suggestions for future research. The lesson, with its novel analogy, and this study's data analysis approach can be used as an instrument to examine students' use of ontology in the relatively short time span of a single lesson. Second, we used the TWA model, in combination with the base-target-model map, as an instrument in our lesson design. But the lesson design can also serve as a tool for researching aspects of the TWA model with the base-target-model map itself.

For example, future studies might investigate how to support learners in recognizing where analogies break down and how such recognition can facilitate their acceptance of new concepts needed to understand the target domain. One could also study the effect of active switching between base and target. As Dreyfus *et al.* (2019) observed, spontaneous transitions between the two can signal epistemological comparisons between classical and quantum domains. To investigate whether this occurred in our study, future research could analyze transcripts for dialogue segments in which PSTs spontaneously shifted between Kundt's tube and the quantum well.

Other possible future studies can be mentioned. A follow-up study with a larger sample could further validate the lesson as a research instrument, possibly uncover greater diversity in PSTs' ontological reasoning, and test the generalizability of the particle-wave axis scheme introduced in Table III. The limited QM context of our study, namely, the particle in the infinite quantum well, should be viewed as a constraint, and future work should include additional quantum contexts, such as tunneling, superposition, or entanglement. In our subsequent studies, we added a nature of science (NOS) activity to the lesson plan, which will be presented in a separate paper. We also asked PSTs to reflect on the lesson design from a teacher perspective, to investigate their pedagogical design capacity (PDC)—that is, their ability to critique and adapt curriculum materials. This focus follows Davis *et al.*, who examined how teachers modify instructional designs and on what basis such decisions are made [53]. Examining PSTs' pedagogical capacities concerning QM teaching with analogies will yield further insight into how teacher educators may prepare PSTs for conceptual QM teaching. Taken together, our findings demonstrate that explicitly engaging PSTs with ontological issues through classical-quantum analogies not only reveals the diversity of their reasoning but might also offer a productive pathway toward more conceptual approaches to teaching quantum mechanics to all learners.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they contain sensitive personal information. The data are available from the authors upon reasonable request.

## APPENDIX APPENDIX: THE QUESTIONS OF THE COMPARATIVE ANALYSIS ASSIGNMENT

### 1. Introductionary question that introduces the key entities.

Q1 At first glance, which particles are involved in the experiment?

### 2. Questions 2 to 6 treat the wave theory aspects.

Q2 What do you call the pattern that becomes visible in the standing waves?

Q3 The tube and well are closed at both ends so that knots are formed at those ends. Explain why the "walls" of the quantum well "consist" of negative charge.

Q4 What is the underlying medium that brings those particles (from question 1) into their state? In other words, what else is in the tube and well other than those particles from question 1.

Q5 What do you call the waves in that underlying medium?

Q6 Waves can be transverse or longitudinal. Which applies here?

### 3. Questions 7 to 10 relate to the particle-wave interaction and duality.

Q7 How many times do you have to look, and how many observations are needed to make the pattern visible in the picture?

Q8 If you remove the particles from question 1, can the waves still be present in the tube and well?

Q9 If you remove the waves, can the particles from question 1 still be present in the tube and well?

Q10 Does looking at it affect what is in the tube and well?

### 4. Questions 11 to 14 relate to the model, i.e., the graphs.

Q11 Look at the model below. In this model, you see a left pattern and a right pattern. Which aspect do you associate with the left and which aspect with the right pattern?

Q12 In the picture above, as $n$ increases (vertically), what increases in the Kundt tube and what increases in the quantum tube?

Q13 What action or input is needed to achieve that transition from $n = 1$ to $n = 2$ and $n = 3$?

Q14 At $n = 1$ in the above picture, you see half a wavelength. From left to center, $A$ and $A^2$ increase from 0 to $A_{\max}$ and $A^2_{\max}$. What increases in the Kundt tube, and what increases in the quantum tube?


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