

Knowledge integration in student learning of the ideal gas law

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The *ideal gas law* is a fundamental concept in physics, yet students often struggle to develop a deep understanding of its principles. Despite its central role in secondary school physics and introductory college thermodynamics, research indicates that, even after instruction, students frequently retain fragmented knowledge structures and fail to achieve deep conceptual understanding. To address these challenges, this study develops a conceptual framework to model features of knowledge integration in students' learning of the ideal gas law. The conceptual framework identifies essential concepts and their interrelationships, which are used to guide the development of a diagnostic tool that targets knowledge integration and deep learning. Administered to cohorts of Chinese high school and college students, the assessment revealed four distinct knowledge integration levels, including novice, lower-intermediate, upper-intermediate, and expertlike, through a combination of quantitative test results and qualitative interview data. Findings indicate that the conceptual framework can effectively capture features of integration in students' knowledge structures, which can provide valuable insights for developing educational interventions that help students to establish critical connections within their knowledge structures and thereby deepen their understanding of the ideal gas law.

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I. INTRODUCTION

In introductory science education, a key goal is to develop students' deep understanding of fundamental scientific principles and concepts. However, traditional teaching methods often fall short of this aim [1–5]. While students usually show proficiency in solving familiar textbook problems, they frequently struggle with novel or unfamiliar situations. This gap has led to extensive research over the past decades, focusing on students' conceptual understanding and reasoning processes during problem solving. Studies consistently reveal that traditional approaches, which emphasize content-based drills and memorization, are inadequate for building complex problem-solving skills or fostering deep conceptual insight [1–8]. In response, new instructional strategies have emerged, such as peer instruction [9], studio-style teaching [10], and inquiry-based learning [9,11–13]. Although these

methods aim to increase interactive engagement and improve conceptual understanding, a significant gap remains in developing instructional approaches that meet discipline-specific learning needs.

To address the challenge of developing a deep understanding of discipline-specific concepts, the knowledge integration approach has gained significant attention in physics education research [14–21]. This framework models students' conceptual understanding and problem-solving behaviors by emphasizing the connections among different components within a concept domain. Expert learners organize their knowledge around core principles, which not only facilitate problem solving but also help connect various elements and bridge familiar and unfamiliar contexts [3,5–7,10]. In contrast, novice students typically have fragmented knowledge structures and rely on rote memorization, resulting in superficial understanding and poorly organized knowledge [22–24]. As a result, novices tend to focus on surface-level aspects of problems, applying memorized patterns rather than engaging in deeper conceptual analysis [3,5,7,8]. This difference in cognitive processing highlights the fundamental gap between novice and expert learners [1–4].

The conceptual framework serves as a practical tool for modeling students' knowledge structures and evaluating how well knowledge is integrated [9,11–13,15–21].

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Essentially, it is based on a core idea that describes the fundamental principles and mechanisms that govern causal relationships among key elements. This core idea helps to understand complex concepts by offering a clear structure. It connects lower-intermediate processes, related concepts, task goals, and specific contextual features. These connections form a hierarchical network that illustrates how knowledge is organized [7,8,25]. Additionally, the framework aims to address the challenges students encounter when managing the complexity of these interconnected elements. It highlights the importance of tailored instructional strategies and assessment methods to encourage deeper learning.

As a strategy for assessment based on a conceptual framework, recent research has introduced unique question types called typical and atypical questions, which are specifically designed to evaluate students' deep understanding of important scientific concepts and their ability to transfer knowledge across different contexts [9,13–21]. Typical questions refer to standard problems students have encountered during instruction and practice. These questions can often be solved easily using memorized procedures and equations. In contrast, atypical questions are crafted with novel contextual scenarios that demand an in-depth understanding of the concept. By comparing performance across these different contexts, levels of knowledge integration can be empirically assessed. This comparative analysis offers valuable insights into students' reasoning processes, showing the depth of their conceptual understanding. Additionally, the results from these comparisons can help educators identify specific areas needing further development, guiding the design of instructional strategies aimed at improving students' problem-solving skills and deep conceptual comprehension.

Building on previous research [15–21], this study explores how students integrate knowledge when learning about *the ideal gas law* (IGL), a key concept in thermodynamics. Earlier studies have examined students' misconceptions and difficulties in thermodynamics [26–29], showing that naive misconceptions often persist [30–35]. These misconceptions are frequently reinforced by incomplete or misleading explanations in traditional teaching [35], which help maintain nonscientific beliefs. Most existing research focuses on identifying misconceptions and assessing instructional strategies to correct them [30–32,35]. This study was not to increase misconceptions, but to create a conceptual framework for IGL and develop an assessment tool to examine how students integrate knowledge of the IGL, which is carried out in two parts:

Part 1: The initial section develops a conceptual framework to analyze students' struggles in understanding the ideal gas law (IGL) from a knowledge-integration perspective.

Part 2: The second part focuses on designing a multiple-choice assessment based on the conceptual framework

to measure students' levels of knowledge integration in learning IGL.

II. DEVELOPMENT OF THE CONCEPTUAL FRAMEWORK FOR THE IDEAL GAS LAW

A. Expert and novice understanding of the ideal gas law

1. Review of students' difficulties

The ideal gas law (IGL) covers two related conceptual areas: a macroscopic model explained with the ideal gas equation and a microscopic model based on the kinetic theory of gases. Throughout this paper, the term “statistical average” refers to an average over particles; T is the absolute temperature (K). For monatomic ideal gases, $E_k = \frac{3}{2}k_B T$ and $p = \frac{1}{3}\rho v^2$, where ρ is the mass density. Previous research in science education has revealed ongoing misconceptions about the IGL, many of which have proven resistant to traditional teaching methods [35,36]. Students frequently depend on intuitive models from everyday macroscopic experiences, leading to stable yet scientifically incorrect ideas about gas behavior [30,31,33,34,36].

A common challenge is oversimplifying relationships in *the ideal gas equation*. Students often reduce the multi-variable equation to separate two-variable relationships [30]. For example, they might incorrectly assume that pressure and volume are always inversely proportional, that temperature and volume are directly proportional, or that pressure and temperature are always directly proportional [31]. Students also find it challenging to understand the role of mechanical equilibrium in gas systems, especially in situations involving pistons or containment devices where internal gas pressure must balance external forces [31]. Overemphasizing *the ideal gas equation* in instruction can unintentionally worsen these issues by leading students to view macroscopic and microscopic models as separate rather than as parts of a unified system. This separation makes it harder for students to connect both perspectives. It hampers their ability to apply *the ideal gas law* effectively in problem solving, particularly when faced with unfamiliar contexts or complex system dynamics.

Additionally, students often struggle to understand the random nature of molecular velocities. This challenge usually stems from oversimplifications in how gas-particle behavior is taught and a limited grasp of the complexity of molecular motion. For example, the velocity distribution of gas molecules follows the Maxwell-Boltzmann distribution. However, some students incorrectly believe that all gas molecules move at the same speed, ignoring the statistical variation described by the distribution [34]. Others mistakenly think that gravitational forces greatly influence the motion of gas particles. This misconception clashes with the ideal gas assumption of random molecular motion, unaffected by external forces such as gravity [35,37]. These misunderstandings show the need for teaching strategies that connect macroscopic and microscopic models.

Practical approaches should help students critically explore the details of molecular dynamics, thereby linking abstract theory with observable macroscopic phenomena.

Many students face significant challenges in understanding the IGL and often develop persistent misconceptions. A common source of confusion is the difference between temperature and heat. Students frequently fail to realize that temperature reflects the average kinetic energy of particles in a system, not the total heat content [30–32,38,39]. This misunderstanding can cause flawed reasoning, such as blaming increases in temperature or internal energy solely on molecular collisions. Others incorrectly connect temperature changes with shifts in particle density [30]. These misconceptions hinder students' ability to develop a clear understanding of thermal phenomena and limit their accurate application of thermodynamic principles.

Misconceptions about gas pressure are similarly widespread. Some students mistakenly believe that pressure only results from collisions between gas particles, ignoring the crucial role of collisions with the container walls [27,30,32]. Others confuse pressure with particle velocity, average kinetic energy, or density, without fully understanding how these factors influence collision rate and momentum transfer during interactions [30]. These misunderstandings emphasize the need for teaching models that link microscopic processes to the overall behavior of gases.

For the physics of IGL, the *microscopic model of the ideal gas* assumes that gas particles are idealized, rigid, pointlike entities that move randomly in all directions and undergo perfectly elastic collisions. The volume of a gas refers to the spatial region that contains these particles. Temperature is related to the average kinetic energy of the particles; as temperature increases, so does their average kinetic energy. Gas pressure is a macroscopic state variable that reflects the collective momentum transfer of gas particles per unit area. In the kinetic model, this can be explained as the combined effect of particle collisions with an imaginary or real boundary; however, pressure itself does not originate from the walls. It exists even in unconfined systems like the atmosphere, where local pressure results from molecular interactions and can be detected by acoustic, optical, or electrical methods. These relationships illustrate how molecular behavior gives rise to macroscopic properties such as pressure and temperature, and they also provide insight into the statistical nature of thermodynamic systems. However, this model intentionally excludes intermolecular forces and ignores particle size, making it a simplified approximation for many theoretical scenarios.

Previous studies have consistently highlighted the importance of both macroscopic and microscopic perspectives in explaining gas behavior [30,31]. However, these perspectives are often treated separately in instruction without being explicitly emphasized as connected. While textbooks usually present *the ideal gas equation* and aspects of molecular theory [39–41], they rarely develop a dedicated microscopic model based on molecular

dynamics. More importantly, few studies explicitly connect the *microscopic model of the ideal gas* to macroscopic thermodynamic equations.

Although pressure and temperature are often discussed, their roles in linking molecular motion to larger-scale outcomes are frequently underestimated. A notable gap exists in the lack of qualitative reasoning connecting the ideal gas equation to particle-level behavior. This reasoning, based on Newtonian mechanics, helps students understand macroscopic changes in terms of individual particle interactions. However, it is mostly absent from typical textbooks and classroom instruction [38,39,41]. In the above, we therefore summarize only the essential background on the ideal gas law, documented student difficulties, and the knowledge-integration perspective, while detailed theoretical and empirical explanations are provided in the following sections.

2. Experts' view of an integrated knowledge system

Students often show a fragmented or isolated understanding when working with the IGL. In contrast, experts demonstrate a very different way of thinking. Their reasoning is driven by a connected knowledge system that seamlessly links broad and detailed models. To explain the difference between novice and expert thinking, the next section examines how experts' integrated knowledge systems are organized and function within the IGL.

A key trait of expert knowledge is the ability to combine multiple conceptual areas into a single, adaptable framework. This combination is essential in expert reasoning about the IGL. Novice learners often see macroscopic laws, microscopic models, and statistical principles as separate or unrelated ideas. However, experts integrate these domains into a coherent system that explains gas behavior across different levels of understanding [42,43]. Such integration helps experts go beyond merely recalling empirical relationships, such as those in *the ideal gas equation*. They connect these relationships with molecular mechanisms and dynamic particle interactions. This synthesis enables a smooth transition between qualitative reasoning based on particle motion and quantitative analysis grounded in formal derivations. As a result, experts show a deep conceptual understanding that beginners usually lack.

The significance of knowledge integration becomes particularly clear when examining the shift from microscopic to macroscopic models. Experts do not use *the ideal gas equation* in isolation; instead, they grasp the kinetic mechanisms and statistical assumptions underlying macroscopic gas properties. Their ability to move seamlessly across different levels of understanding, molecular dynamics, statistical behavior, and observable macroscopic phenomena defines expert performance. Conversely, novices often apply formulas superficially without understanding the physical principles behind them. Therefore, integrating microscopic reasoning, statistical interpretation,

and macroscopic laws is crucial to experts' understanding of the IGL. This integration highlights a key cognitive difference between experts and novices.

Experts' understanding of the IGL includes three interconnected parts: *the ideal gas equation*, the *microscopic model of the ideal gas*, and a set of transition models that connect the two. At the macroscopic level, *the ideal gas equation*, $pV = nRT$, combines empirical findings from Boyle's, Charles's, Gay-Lussac's, and Avogadro's laws. This equation links pressure (p), volume (V), number of moles (n), and temperature (T) within a single mathematical framework, mediated by the universal gas constant (R). Experts see this equation not just as an empirical law but as a macroscopic view of the underlying molecular behaviors. At the microscopic level, experts view ideal gases as collections of particles moving randomly in straight lines, undergoing elastic collisions and experiencing very weak intermolecular interactions. These assumptions form the basis of the kinetic molecular theory, offering a mechanistic explanation for the macroscopic properties of gases.

Connecting the microscopic and macroscopic realms, a variety of transition models work through both qualitative and quantitative approaches. The qualitative transition models rely on Newton's laws of motion to conceptually explain macroscopic changes and to predict system-level properties from the dynamics of individual particles. In contrast, the quantitative transition models formalize these conceptual pathways with mathematical formulas. For example, pressure caused by molecular collisions can be expressed as $p = \frac{1}{3} \rho v^2$ and temperature is represented as $E_K = \frac{3}{2} k_B T$. Internal energy is similarly described using statistical formulas such as $U = \frac{3}{2} nRT$ or $U = N \times \frac{1}{2} m v^2$. These formulations connect the statistical behaviors of large groups of particles to observable thermodynamic quantities, reinforcing the integration of microscopic processes with macroscopic results.

Overall, the experts' integrated knowledge system shows not only mastery of individual models but also a deep understanding of the dynamic transitions and structural links that connect microscopic mechanisms to macroscopic gas behaviors. This integrated understanding enables experts to adapt their reasoning flexibly depending on the context. Whether engaging in qualitative inference, performing quantitative calculations, or building explanatory models, they function effectively across multiple conceptual domains.

B. Development of the conceptual framework for the ideal gas law

The components and structure of the conceptual framework were identified and developed through expert analysis of core physics content and documented student misconceptions. This framework functions as a hypothetical model that illustrates the knowledge structures of both experts and students. The validity of the proposed pathways and the

overall framework is supported by quantitative assessments and interview data, with detailed analyses of these empirical findings presented in later sections. This framework is a qualitative reasoning map, not a derivation of $pV = nRT$, demonstrating how experts relate microscopic mechanisms, statistical links, and macroscopic observables.

Building on prior work that employs conceptual frameworks in science education [15–21], developing a framework for a specific concept begins with identifying a central idea that organizes related concepts and reasoning patterns. In this study, a team of three faculty members and one doctoral researcher in physics education reviewed the literature on student difficulties [27,30–32] and representative textbooks [38,40,41,44] to determine the central explanatory mechanism for the IGL.

After extensive discussion within the expert team, the microscopic model of the ideal gas was identified as the central idea. This model explains how particle interactions lead to observable macroscopic phenomena. In contrast, the ideal gas equation mainly functions as a calculation tool to determine macroscopic gas properties. This difference emphasizes a fundamental purpose: the equation supports calculation, while the model enables causal understanding of gas behaviors. For this study, we use a simplified version of the microscopic model of the ideal gas that excludes potential energy considerations, in line with the scope of Chinese high school and introductory college physics curricula. To avoid ambiguity while maintaining a layered presentation, we list the classical, interaction-free assumptions once in Sec. II A and cross reference them in Fig. 1 rather than repeating them in the central idea paragraph. Statistical averages are included only in the transitional layer. Formal relations serve as links rather than derivations here, e.g., $p = \frac{1}{3} \rho v^2$, $E_k = \frac{3}{2} k_B T$. The figure shows pressure as an emergent property arising from microscopic particle motion, rather than a consequence of physical boundaries. The wall-collision illustration is only a conceptual tool to help visualize momentum transfer.

Building on this central idea, the conceptual framework for the IGL is presented in Fig. 1, integrating key physics content with well-documented student difficulties. The framework is arranged into four hierarchical layers—microscopic $\rightarrow$ transitional $\rightarrow$ macroscopic $\rightarrow$ contexts—with assumptions (i)–(vi) linked by in-figure tags at the microscopic level and referenced in the transitional layer to show where statistical averaging connects particle motion to p and T . This structure supports qualitative, bidirectional reasoning and clearly indicates where statistical averaging is applied, without repeating the assumptions from the central idea paragraph.

The bottom layer of the framework covers the contextual features and variables often found in IGL problems. These features represent the most tangible aspects of physical scenarios commonly included in such issues. This layer involves various conditions and measures, such

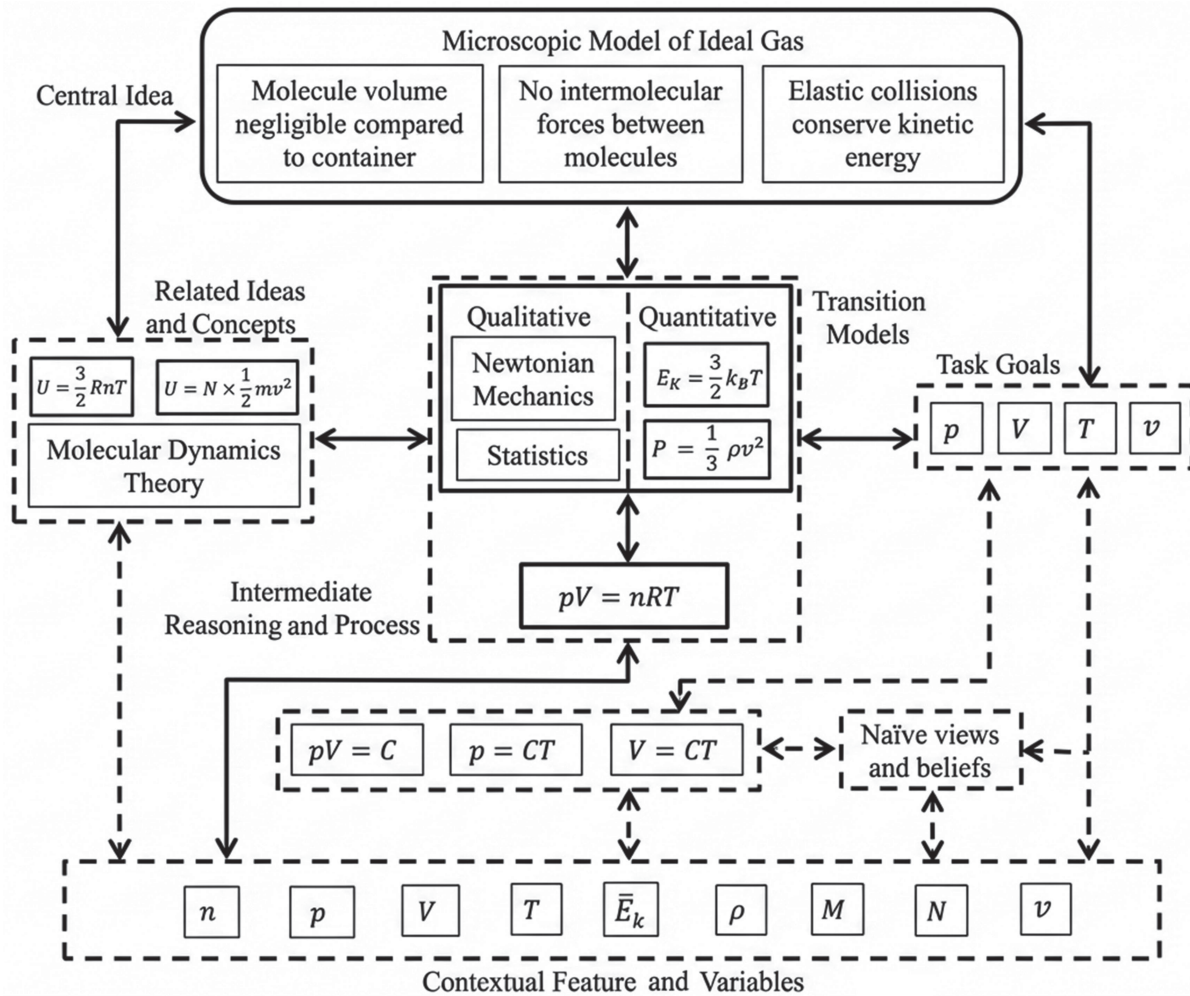


FIG. 1. *Ideal gas law* conceptual framework. The solid arrows represent the expert's conceptual path, while the dashed lines represent the direct connections that novices often make between contextual features and task outcomes. It is intended as a qualitative reasoning map, rather than a derivation of $pV = nRT$.

as the amount of substance, volume, pressure, temperature, and other relevant factors. These elements serve as key variables for understanding and solving IGL problems. They provide the essential context for applying the law in both theoretical and practical situations.

The second layer from the bottom addresses students' naïve or alternative conceptions, as identified in the literature [27,31,32]. These alternative conceptions oversimplify the multivariable relation, focusing on a limited subset of variables and overlooking the specific conditions required for the law's validity. Consequently, the oversimplification will hinder deeper conceptual understanding and impede the development of a more integrated and accurate comprehension of the law's broader applicability.

The third layer from the bottom is the intermediate reasoning and process layer, which represents the operational reasoning and processes that connect the central idea with the contextual features and variables. As defined by experts, this layer comprises two parts: the macroscopic model and its associated operations, and the transitional

models and their corresponding operations. The macroscopic model involves understanding and applying the equation $pV = nRT$. The transformation models encompass both quantitative and qualitative approaches. The quantitative model states that the average kinetic energy of gas molecules is proportional to the system's temperature. In this framework, pressure is seen as the result of molecular collisions with the container walls. These collisions transfer momentum, creating a force that produces pressure. This model effectively links macroscopic and microscopic physical quantities. Conversely, the qualitative transformation model relies on basic principles of *the microscopic model of the ideal gas*. It uses Newtonian mechanics and statistical concepts to explain how particle behaviors relate to macroscopic thermodynamic variables.

The *microscopic model of the ideal gas* is closely linked to key concepts in thermodynamics and statistical physics, especially molecular dynamics. This connection is vital for a thorough understanding of the IGL. These reasoning pathways establish the core mechanisms behind the IGL,

providing students with a solid conceptual foundation for applying the law in practical situations. In typical problem-solving tasks, students are asked to calculate variables like pressure, volume, temperature, and average molecular speed. They may also need to analyze how these quantities relate to one another, as shown on the right side of the framework under “task goals.”

The framework visually shows these relationships with double-headed arrows, which represent the two-way nature of reasoning pathways. These arrows show that conceptual connections can be formed in either direction, highlighting the dynamic and adaptable nature of reasoning. However, for some students, these pathways might only work in one direction. Such limitations often stem from difficulties in integrating all components or in transitioning between different layers of the framework.

The structure of these pathways is based on previous research on expert-novice differences, expert reasoning patterns, curriculum design, and student learning behaviors. Solid arrows represent expert reasoning pathways, characterized by a well-organized integration of knowledge components and clear connections to the main idea. In contrast, dashed arrows indicate novice reasoning pathways, which typically focus on surface features and rely on memorized procedures rather than an integrated understanding. Within the diagram’s knowledge framework, the layer of intermediate reasoning and processes turns theoretical models into practical applications involving contextual variables. The overall integrity of the structure relies on the consistency and connections among these layers.

These reasoning paths typically proceed directly from contextual features and variables to the task goal, often by selecting and applying equations with two variables or following existing misconceptions without engaging in deeper conceptual reasoning. As a result, this approach can reflect fragmented knowledge structures, which are common in novice learners [27–33]. Such structures often lead to an incomplete understanding of the underlying principles and an emphasis on surface-level features. These naive approaches frequently depend on memorized formulas or intuitive but scientifically incorrect beliefs, creating only local and loosely connected links between surface features and macroscopic models.

In contrast, analyses of disciplinary experts’ knowledge structures reveal coherent and conceptually grounded reasoning pathways related to the IGL. One pathway begins with the central idea, specifically the microscopic model of an ideal gas, directly leading to the task goal. This involves microscopic reasoning about microscopic phenomena. A second pathway also starts with the same central idea. Still, it passes through intermediate reasoning and processes before reaching the task goal, reflecting expert reasoning from microscopic models to macroscopic outcomes. A third pathway begins with contextual features and variables, proceeds through the ideal gas equation, and reaches the task goal by solving for unknown macroscopic

quantities. A fourth pathway transforms known macroscopic variables into estimates of microscopic quantities by engaging suitable qualitative or quantitative transitional models. Across all these pathways, models are actively used to manipulate contextual variables with meaningful conceptual understanding. These reasoning patterns showcase the coherent structure of experts’ knowledge, connecting central theoretical principles to applied problem-solving tasks in a conceptually integrated way.

C. Modeling student understanding using the conceptual framework

The conceptual framework offers a structured approach to systematically analyze student misconceptions and reasoning pathways, as documented in prior research [26–36]. It emphasizes the dynamic nature of learning and highlights the integration of knowledge across multiple cognitive levels. The analysis shows that differences in knowledge integration correspond to distinct student understandings and reasoning processes. Building on these insights and existing literature on knowledge integration [14–19], students’ conceptual understanding and problem-solving behaviors can be categorized into four distinct levels of knowledge integration. These levels will be discussed in detail in the upcoming sections.

1. Novice level

Novice learners often face significant challenges in understanding the IGL, resulting in a fragmented knowledge structure. These students tend to rely on their alternative conceptions or memorized examples when problem solving and frequently struggle to make meaningful connections between key concepts. These issues hinder their ability to fully grasp the law’s underlying mechanisms.

- Absence of the microscopic model of the ideal gas
A key issue for novices is their inability to understand the *microscopic model of the ideal gas*, a fundamental concept that explains how gases behave. Instead of this particle-based view, they often take a continuous, macroscopic approach [37,45–48], which hides the discrete nature of gas particles and makes it harder to understand essential features, including particle volume, intermolecular forces, and elastic collisions. As a result, students struggle with problems that require applying these microscopic principles.
- Oversimplified proportional reasoning in the *ideal gas equation*

When working with macroscopic variables such as pressure, volume, or temperature, novices often rely on intuitive naive ideas or memorized equations involving only two variables [30,31]. These methods usually come from everyday observations, like the thermal expansion of gases, which are easy to remember but do not fully capture the complexity of gas behavior [30,34,35]. This dependence on simplified reasoning

often causes difficulties when students encounter unfamiliar problems. Many assume that a linear relationship always exists between two variables, but this breaks down in multivariable systems or when relationships are more complex, with additional conditions.

- The conceptual gaps of the transition model

Novice learners often demonstrate a fragmented understanding of the IGL, tending to see it as a collection of isolated parts. Specifically, they may confuse the *ideal gas equation* or its simplified versions with the full IGL, while ignoring the underlying microscopic model of the ideal gas. This conceptual gap may arise from a lack of understanding of the particulate nature of gases, a key aspect that sets the IGL apart from its macroscopic applications [30,34]. While learners might remember individual formulas, they often fail to grasp the interrelations between variables or why these relationships exist. Such misunderstandings limit their ability to apply the theory effectively in new situations, slowing the development of a complete and integrated scientific understanding.

Without an integrated knowledge structure, students tend to rely on surface cues and memorized examples. When these cues are missing or misused, mistakes often happen. When faced with unfamiliar situations, students frequently resort to guesswork or overly simplistic reasoning. Their problem-solving process typically follows this path: problem context → two – variable gas equations or naive conceptions → solution.

2. Lower-intermediate level

At the lower-intermediate level, learners demonstrate a more organized, methodical understanding of gas behavior than novices. They show improved logical thinking and problem-solving abilities, especially in distinguishing and recognizing physical principles. Although these students are beginning to use conceptual strategies, they often still rely on rote memorization and plug-in formulas. Their problem-solving skills are generally limited to macroscopic-level scenarios, and they encounter significant challenges with tasks involving basic microscopic concepts and transitional reasoning, as summarized below.

- Deficiencies in understanding the microscopic model of the ideal gas

Lower-intermediate learners often have a limited understanding of the basic concepts underlying the IGL. They then adopt a continuous model rather than the particle model, which limits their ability to grasp the microscopic mechanisms that control gas behavior. This misconception blocks their understanding of key parts of the *microscopic model of the ideal gas*, including the size of gas molecules, the influence of intermolecular forces, and the nature of elastic collisions [45–49]. As a result, they usually focus only on macroscopic traits without considering the

microscopic principles that underlie them. This limited view reduces their ability to fully appreciate the explanatory power of the IGL.

- Achieving moderate performance in the macroscopic model

Despite these challenges, lower-intermediate learners generally perform moderately well in solving macroscopic problems using the IGL. They can accurately calculate thermodynamic properties such as pressure, volume, temperature, and the number of moles under standard conditions. This demonstrates a solid understanding of formula-based problem contexts. However, they often misunderstand the concept of mechanical equilibrium in gas systems. In particular, many find it difficult to grasp how a gas's internal pressure force must balance external forces in situations involving piston-containment structures [31]. This gap hinders their ability to connect molecular motion with the resulting macroscopic pressure.

- Challenges in understanding the transition model

A common challenge for lower-intermediate learners is linking the random motion of gas molecules to macroscopic quantities such as pressure and temperature. Learners find it difficult to grasp the quantitative aspect of transition models, which explain the relationship between molecular motion and observable thermodynamic properties [38,39]. This conceptual gap hinders their ability to understand and apply the core principles of thermodynamic phenomena, especially in complex or unusual situations.

Lower-intermediate learners often see microscopic and macroscopic models as separate entities. This lack of integration hampers their ability to connect different levels of representation and hinders a unified understanding of gas behavior. As a result, their explanations usually oversimplify thermodynamic processes, leading to fragmented and incomplete conceptual frameworks. This issue is evident in problem solving, where learners tend to rely on macroscopic reasoning alone without considering the microscopic processes behind those models. Their typical thought process tends to be: problem context → ideal gas equation → problem solving.

3. Upper-intermediate level

Upper-intermediate learners have a fairly well-developed knowledge framework that allows them to solve problems in various contexts. They can effectively handle broad problems and complete quantitative transition tasks. However, their limited understanding of the central idea often hinders their ability to solve unusual problems. While these students may try to apply the central idea in their reasoning, their use of the knowledge is frequently inconsistent. As a result, they can generally solve most typical questions in familiar contexts but usually struggle when faced with unusual questions in new scenarios.

These students show partial proficiency in the transition model and can apply *the ideal gas equation* to predict changes in gas properties under different macroscopic conditions. However, while they have a basic understanding of gas behavior at the microscopic level, they still struggle with the *microscopic model of the ideal gas*. They can relate problems to key ideas such as elastic collisions of gas molecules and the assumptions of negligible molecular volume and intermolecular forces. Yet, when tackling problems involving molecular trajectories, collisions, and related phenomena, they still hold several misconceptions. For example, some students assume gas molecules move at constant speeds, ignoring the Maxwell-Boltzmann distribution, a fundamental aspect of molecular motion [34]. Others believe gas pressure arises solely from molecular collisions, overlooking the role of collisions with container walls. They also often neglect the roles of velocity, average kinetic energy, and particle density in determining pressure [27]. Additionally, some mistakenly think gravitational forces significantly influence gas particles, which conflicts with the idea of random motion in ideal gases [49]. Despite their familiarity with the IGL, they often find it challenging to connect molecular dynamics with macroscopic thermodynamic equations. This shows a missing link between microscopic interactions and observable properties [29]. Moreover, students frequently fail to understand that the volume of an ideal gas equals the container's volume, not the negligible volume taken up by individual molecules [50].

A persistent challenge is connecting macroscopic observations with the molecular interactions that produce these effects. While these learners excel at quantitative transitions, such as relating temperature to average kinetic energy or pressure to gas density and average molecular speed, they struggle with qualitative transition models. This skill indicates an emerging integration of macro and microscopic perspectives. However, they find it challenging to apply Newtonian mechanics or statistical principles in situations that require qualitative reasoning. Learners often link temperature increases directly to molecular collisions or attribute gas pressure solely to particle impacts. This demonstrates a limited understanding of the statistical nature of these processes. Although they use conceptual strategies, their understanding remains shallow and prone to misconceptions. Research on student ideas about molecular gas dynamics supports this [27,34], showing that many learners fail to grasp the statistical distribution of molecular velocities or the role of microscopic interactions in shaping macro gas behavior. Their reasoning often follows the pattern: problem context $\rightarrow$ ideal gas equation $\leftrightarrow$ transition model $\rightarrow$ problem solving.

4. Expertlike level

Expert-level students demonstrate flexible and adaptive reasoning. They can adapt their problem-solving strategies to suit the specific demands of each situation. Their extensive,

well-organized knowledge base enables them to identify meaningful links among contextual factors, the *microscopic model of the ideal gas*, and intermediate-level concepts and operations. This interconnected network helps them succeed with familiar problems and also enables them to tackle new or unfamiliar challenges confidently.

Using this analytical model, the reasoning pathways of learners at various developmental stages can be systematically identified and understood within the framework of knowledge integration [15–21]. In microscopic modeling of particle motion, expert learners excel in applying the *microscopic model of the ideal gas*. They also effectively use transition models to connect microscopic and macroscopic variables. In macroscopic scenarios, expert learners show strong skills in identifying the state of an ideal gas across different contexts. They use the IGL not only for precise calculations but also for nuanced interpretations of how macroscopic properties, such as pressure and volume, arise from molecular interactions. Expert learners can rapidly identify and accurately interpret core ideas in microscopic contexts, including negligible particle volume relative to the container, the absence of intermolecular forces, and elastic particle collisions.

These students demonstrate proficiency with both quantitative and qualitative transition models that connect macroscopic and microscopic domains. They incorporate Newtonian mechanics and statistical methods into their reasoning. Their knowledge structures are well developed and interconnected, enabling them to link contextual variables to fundamental principles. This integration goes beyond the IGL, drawing on elements from statistical mechanics to create a unified, conceptually rich framework. Their deep understanding allows them to explain gas behavior by linking changes in macroscopic quantities back to microscopic mechanisms. They see gas state variables not as isolated observables but as emergent properties rooted in particle-level interactions. This synthesis across domains supports precise, coherent reasoning in both theoretical analysis and practical application. Their reasoning process can be summarized as follows: problem context $\rightarrow$ ideal gas equation $\leftrightarrow$ transition model $\leftrightarrow$ microscopic model of the ideal gas $\rightarrow$ problem solving.

In summary, the conceptual framework developed in this study shows that different cognitive models can effectively distinguish students at various levels of conceptual understanding. Novice students display reasoning pathways that engage with macroscopic contexts only occasionally. In contrast, low-intermediate students consistently use macroscopic models, while upper-intermediate students utilize both macroscopic models and selected transition models. Expertlike students demonstrate complete mastery of *the ideal gas law* (IGL), including macroscopic, transition, and microscopic models. These patterns indicate that mastering the microscopic model sets apart expertlike students from those at the upper-intermediate level,

understanding transition models differentiates upper-intermediate from low-intermediate students, and familiarity with macroscopic models separates low-intermediate students from novices. Notably, the increases in score between adjacent KI levels are not uniform across the five categories. This pattern reflects thresholdlike changes in representational resources: novices and lower-intermediate students both lack a stable particulate view, whereas upper-intermediate and expertlike students increasingly activate microscopic and transitional models, which simultaneously improve performance on atypical microscopic and transitional items.

Building on this framework, the next phase of the research focuses on creating an assessment tool to evaluate students' integration of knowledge. This instrument aims to identify areas of conceptual difficulty and to monitor how students' understanding develops across different contexts. Its use will enable detailed analysis of student performance across levels of knowledge, providing valuable evidence to inform and guide teaching strategies.

III. ASSESSMENT OF THE KNOWLEDGE INTEGRATION IN STUDENT LEARNING OF THE IDEAL GAS LAW

A. Designing the conceptual framework-based assessment

As previously discussed, different contexts and content can be used to distinguish between various levels of students' knowledge integration. This section evaluates students' ability to integrate knowledge in line with the conceptual framework outlined above. The assessment tool includes 23 multiple-choice questions, 10 typical and 13 atypical. The test questions were partly adapted from validated sources, such as the Thermodynamic Concept Survey, the Kinetic-Molecular Theory of Gases Concept Inventory, and the Chinese college entrance exams. Additionally, custom items were created to match the instructional content of introductory physics courses.

Typical questions reflect standard problem formats often seen in textbooks, classroom instruction, and homework tasks. These tasks emphasize procedural fluency and focus on students' ability to apply the *ideal gas equation* in familiar situations. In contrast, atypical questions are designed to engage with the core concept. These questions, either adapted from existing literature or created by the research team, present unfamiliar or new contexts. In this research, atypical questions aim to involve students in particle-level reasoning. They require students to connect perspectives across microscopic, transitional, and macroscopic domains. This approach helps assess students' ability to integrate knowledge across multiple conceptual levels.

The test structure includes four main conceptual categories: Macroscopic items (7 questions) focus on students' ability to interpret and apply the IGL equation in observable,

real-world contexts. These tasks reflect procedural and operational knowledge. Microscopic items (5 questions) evaluate students' reasoning about gas behavior at the particle level, requiring explicit application of the microscopic model. Quantitative transitional items (7 questions) measure students' ability to connect microscopic and macroscopic representations mathematically. These include tasks that link molecular average kinetic energy with temperature or relate pressure to particle velocity. Qualitative transitional items (4 questions) assess students' ability to reason qualitatively across different levels of representation. These items often require Newtonian mechanics and statistical principles to explain how microscopic interactions give rise to observable macroscopic phenomena.

Detailed descriptions of each item category, the correct answers, and the intended reasoning pathways for each item, along with the complete 23-item instrument, are included in Supplemental Material [51]. This organized categorization allows for a thorough assessment of students' knowledge integration across different representation levels. It also closely aligns with instructional goals focused on promoting a deeper conceptual understanding of thermodynamics.

Although eight item types were theoretically possible based on the combinations of contextual saliency and levels of conceptual connectedness, only five were included in the final instrument. These questions are systematically designed to assess students' reasoning across various representational domains. These domains are categorized as follows: macroscopic-typical, quantitative transitional-typical, qualitative transitional-atypical, quantitative transitional-atypical, and microscopic-atypical (see Table I). They were selected for their alignment with instructional content and their diagnostic usefulness. Notably, items set in atypical contexts that require integrated understanding, specifically microscopic and qualitative transitional items, demonstrated the strongest discriminatory power. These items are particularly effective at distinguishing different levels of knowledge integration among students.

B. Data collection

A total of 709 students participated. After excluding 42 incomplete responses, 667 valid responses were retained, resulting in a response rate of 94.1%. Participants were recruited from 11th and 12th grades across five high schools and from third-year college students at a university in northwestern China. The sample included 245 11th-grade students, 366 12th-grade students, and 54 college students. All high school participants were enrolled in required physics courses, and all college participants were majors in physics. This selection ensured a representative demographic distribution across the central and western regions. All participants had previously completed instruction on fundamental thermodynamics concepts as part of the standard high school physics curriculum.

TABLE I. Summary of students' performance on the IGL design across four levels of knowledge integration.

Knowledge integration	Macro typical (Q9, Q18, Q19, Q20, Q21, Q22, Q23)	Quantitative typical (Q10, Q13, Q14)	Quantitative atypical (Q4, Q15, Q16, Q17)	Qualitative atypical (Q7, Q8, Q11, Q12)	Micro atypical (Q1, Q2, Q3, Q5, Q6)
Novice	Medium	Low	Low	Low	Low
Intermediate-lower	Medium	Medium	Low	Low	Low
Intermediate-upper	High	High	Medium	Medium	Medium
Expertlike	High	High	High	High	High

High school students took the assessment two weeks after completing three weeks of instruction on thermodynamics in their physics classes. The instructional content covered essential topics, including isothermal, isovolumetric, and isobaric processes, along with an introduction to *the microscopic model of the ideal gas*. The teaching involved 3 h of lectures and 2 h of exercises each week. The assessment was administered during a 40-min session held during supervised self-study periods. The college students had studied thermodynamics and statistical physics shortly before the evaluation. Their understanding of thermodynamics, including the IGL, was gained approximately one year prior to the test.

In addition to the written assessment, qualitative data were collected to explore students' reasoning processes. A stratified sampling method was used to select 30 students for in-depth "thinking-aloud" interviews, conducted one week after the assessment. Each interview lasted about 40 min. During the interviews, students were first asked to recall their responses to the assessment items and then to give a detailed explanation of their reasoning. With participants' consent, all sessions were audio-recorded and transcribed for later analysis. The sample included five expert-level, eight upper-intermediate, nine lower-intermediate, and eight novice students, ensuring representation across the full range of proficiency levels. As students verbalized their thought processes, they reviewed the assessment questions and explained their reasoning behind the answers. Additionally, they described their understanding of *the microscopic model of the ideal gas*, the composition of air, and the role of each variable in *the ideal gas equation*.

C. Analysis method

Both quantitative and qualitative methods were used to examine students' performance on *the ideal gas law* test and to explore their reasoning processes. Quantitatively, test reliability was evaluated using Cronbach's alpha, and psychometric properties—including item difficulty, discrimination indices, and score distribution features—were analyzed within the framework of classical test theory. Validity was confirmed through expert review (content validity), factor analysis measures (construct validity), and student interviews (face validity), with interrater reliability determined between independent coders. Descriptive statistics

summarized performance patterns, while inferential analyses included one-way ANOVA, pairwise *t* tests with effect sizes, and cluster analysis to identify levels of knowledge integration. We used stratified purposive sampling to select 30 students from a pool of 667 to represent four KI levels. Two researchers independently coded transcripts for reasoning-path categories aligned with the micro-translational-macro framework and four KI levels, blind to students' test scores. Intercode agreement was high (Cohen's $\kappa = 0.89$, $p < 0.001$). Using a constant-comparison approach, saturation was reached by the 22nd interview. Transcripts were independently coded and collaboratively reviewed to identify reasoning patterns, which were mapped to the knowledge integration framework to complement the quantitative results.

IV. RESULTS

A. Quantitative results: Statistical analysis of test item performance

Pretest data were used to evaluate the psychometric properties of the IGL test. Using classical test theory, the overall difficulty index was 0.432, and the discrimination index was 0.454. Both values fall within established benchmarks for acceptable test performance [52]. An assessment of the distributional characteristics indicated approximate normality. The skewness was 0.175, and the kurtosis was -0.259 . These values are within the widely accepted range of ± 1 . Reliability analysis using Cronbach's alpha yielded a coefficient of 0.781, surpassing the conventional threshold of 0.65, indicating a satisfactory level of internal consistency [53].

The validity of the IGL test was evaluated through a comprehensive approach that included content, construct, and face validity. Content validity was confirmed by expert review, carried out by a panel of three professors and one graduate student specializing in physics and physics education. This group went through multiple iterative cycles of evaluation and refinement to ensure it aligned with curricular objectives and maintained conceptual coherence. The Kaiser-Meyer-Olkin (KMO) measure produced a value of 0.769, indicating an acceptable level of sampling adequacy for factor analysis [54]. Additionally, Bartlett's test of sphericity was statistically significant ($\chi^2 = 2532.625$, $df = 253$, $p < 0.001$).

TABLE II. Students' scores on three concept types and two context types.

Concept type	Context type	Score of concepts		Score of contexts	
		Mean	SE	Mean	SE
Micro	Atypical	23.9	0.91		
Qualitative transition	Atypical	36.5	1.26	31.2	0.76
Quantitative transition	Atypical	35.0	1.03		
Macro	Typical	57.7	1.33	58.9	0.89
	Typical	59.4	0.93		

To evaluate face validity, 30 university students participated in interviews during the pilot and revision stages of test development. The primary goal of these interviews was to determine whether students' interpretations of the test items matched the constructs the instrument aimed to measure. Two independent researchers analyzed the participants' responses. The interrater reliability was high, with a standard consistency score of 0.86 and a Cohen's kappa coefficient of 0.92. These values indicate strong agreement between students' interpretations and the intended design of the measurement instrument [55].

The descriptive results of students' performances across different question types are summarized in Table II. These results show that students performed better on questions presented in typical contexts and targeting macroscopic models compared to those in atypical contexts and focusing

on microscopic models. A one-way ANOVA was conducted to assess the significance of these differences, revealing a significant effect of question type on student performance [$F(4, 3330) = 195.638$, $p < 0.001$]. This statistical evidence indicates that variation in student scores is mainly attributable to differences in question type. Pairwise t tests further support these findings.

The results indicate that students' performance across the five question types can be grouped into three distinct clusters: macroscopic and typical quantitative transition questions, atypical quantitative and qualitative transition questions, and microscopic questions. This grouping is supported by several pairwise comparisons among category means. First, students scored much higher on macroscopic than microscopic items, $t(666) = 33.9$, $p < 0.001$, $d = 1.31$, with macroscopic items producing the highest and microscopic items the lowest scores. Second, qualitative- and quantitative-atypical questions showed virtually identical performance, $t(666) = 1.07$, $p = 0.284$, $d = 0.06$, but both exceeded microscopic questions (qualitative-atypical vs microscopic: $t(666) = 9.26$, $p < 0.001$, $d = 0.76$; quantitative-atypical vs microscopic: $t(666) = 9.90$, $p < 0.001$, $d = 0.01$). Third, macroscopic and quantitative-typical questions yielded similarly high scores, $t(666) = 1.37$, $p = 0.172$, $d = 0.08$. Finally, typical questions were answered far more successfully than atypical ones, $t(666) = 34.5$, $p < 0.001$, $d = 1.34$, indicating that contextual variation in item format is a sensitive indicator of students' IGL understanding. Figure 2 and Table III show how students perform at different proficiency levels of the

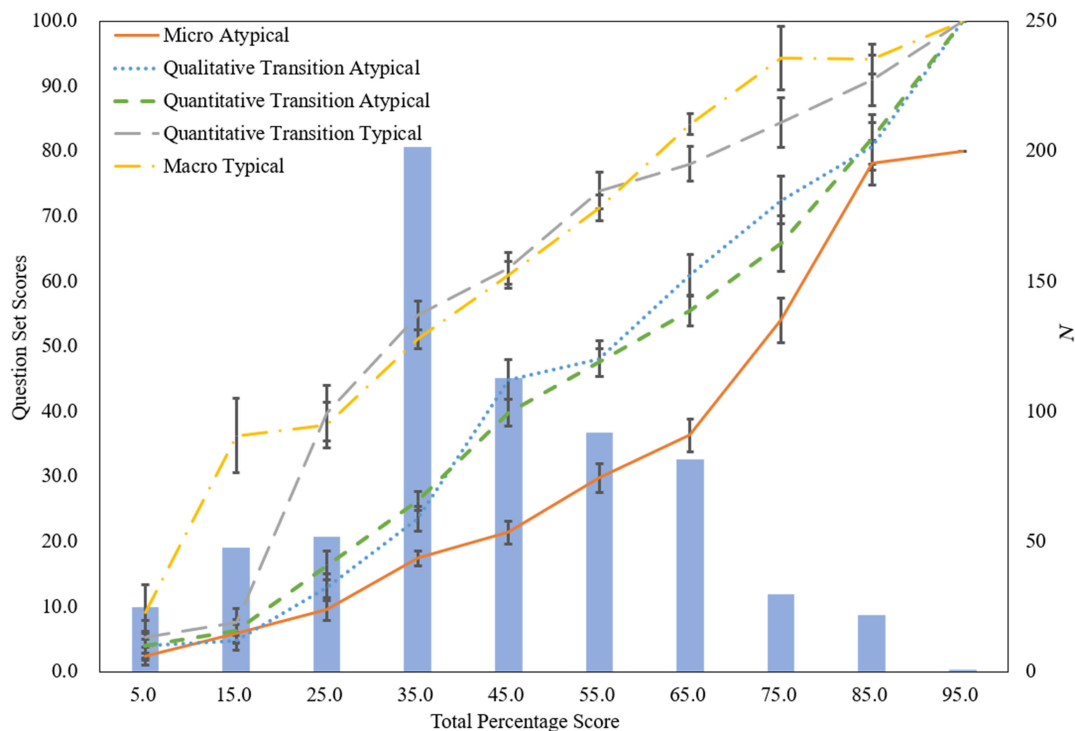

 FIG. 2. Analysis of learner differences at different levels of *ideal gas law* knowledge integration.

TABLE III. Mean attribute scores for each level of knowledge integration. Standard errors are given in brackets. Results are based on assessment data from all students.

Level	Score range	N	Total	Concept and context type				
				Micro atypical	Qualitative transition atypical	Quantitative transition atypical	Quantitative transition typical	Macro typical
Novice	0.0–25.0	96	14.3 (0.61)	5.8 (1.02)	7.3 (1.17)	7.6 (1.44)	11.1 (2.08)	29.5 (1.82)
Lower intermediate	25.1–50.0	344	37.7 (0.37)	18.1 (0.95)	29.6 (1.60)	28.9 (1.14)	56.9 (1.58)	53.3 (0.88)
Upper intermediate	50.1–75.0	194	60.4 (0.51)	35.1 (1.65)	55.4 (1.98)	52.3 (1.53)	77.1 (1.82)	78.9 (1.11)
Expertlike	75.1–100.0	33	83.3 (0.69)	70.9 (3.38)	81.8 (2.93)	75.8 (3.69)	86.9 (3.54)	95.7 (1.16)
<i>p</i> value			<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

IGL. Based on students' typical learning behaviors associated with various levels of knowledge integration, we can define four percentage score ranges (0–25, 25–50, 50–75, 75–100) to categorize students into four levels of understanding.

Novice-level students (scores below 25) consistently underperformed on both microscopic and transitional items. Their scores on macroscopic tasks were slightly higher but still limited, indicating reliance on rote memorization rather than conceptual reasoning. They struggled especially with unfamiliar or conceptually demanding tasks, reflecting a lack of cohesive understanding of the IGL.

Students at the lower-intermediate level (scores between 25 and 50) performed reasonably well on macroscopic and typical quantitative transition tasks, which mainly involved the procedural application of *the ideal gas equation*. However, they continued to struggle with qualitative and atypical quantitative transition items, as well as microscopic tasks. These difficulties highlight ongoing challenges in connecting particle-level explanations to observable gas behavior. Their reasoning was often procedural and superficial, reflecting fragmented knowledge and limited integration between microscopic models and macroscopic implications. Interview data further supported these findings, showing that many students could not explain the relationship between *the microscopic model of the ideal gas* and *the ideal gas equation*.

Upper-intermediate students (scores between 50 and 75) showed significant improvement, especially on qualitative and atypical quantitative transitional items. Their performance indicated partial knowledge integration and increased flexibility in representation, although their understanding of microscopic tasks remained weak. This suggests that a complete, coherent particle-level conceptual framework had yet to be developed. Therefore, these students were classified as upper-intermediate in terms of knowledge integration.

In contrast, expertlike students (scores between 75 and 100) consistently achieved high results across all concept types, including those involving integrative reasoning, such as qualitative transitions and microscopic representations.

Their performance showed a well-developed, interconnected understanding of the IGL. These students demonstrated the ability to apply particle-level reasoning flexibly and coherently across a range of complex and unfamiliar contexts, indicating expert-level knowledge integration.

Figure 2 and Table III summarize the four empirically distinct KI levels and their performance patterns, which, together with mapped interview reasoning pathways, describe trajectories from fragmented to coherent integration rather than just replicating known difficulties. The quantitative findings further reinforce these distinctions, revealing significant performance differences across concept types and contextual settings (typical vs atypical). These results suggest that well-designed assessment items can effectively differentiate among various levels of student knowledge integration of the IGL. To confirm the levels identified through the quantitative analysis, follow-up interviews were conducted to explore the reasoning pathways used by students at different performance levels. The interview data provided supporting evidence for the categorized levels of knowledge integration, which are discussed in the following section.

B. Qualitative findings: Students' reasoning pathways were indicated from interviews

A total of 30 students were selected for interviews from the pool of tested participants, including five expert-level, eight upper-intermediate, nine lower-intermediate, and eight novice students. This stratified sampling ensured representation across all proficiency levels, as determined by students' total scores on the TIGL described earlier. The interviews took place in the week immediately after the test, with each session lasting about 40 min. All interviews were audio-recorded and later transcribed for analysis. During the interviews, students were asked to recall their test answers and explain the reasoning behind their responses. The transcripts were first independently analyzed by two graduate researchers, then reviewed collaboratively with faculty members to reach consensus on the classification of reasoning types. The final analysis involved categorizing

and aligning students' responses with the proposed knowledge integration model. Interview quotes are presented as evidence → framework dimension → level feature and are cross-referenced to Fig. 2 and Table III.

1. Novice-level students

Eight students (total score ≤ 25) were interviewed as part of the novice-level sample. Their understanding of the IGL was mostly fragmented and limited to a macrolevel model. Their reasoning usually involved isolated two-variable relationships and relied heavily on rote memorization and superficial pattern recognition. For example, in response to Q22, student A relied on memorized algebraic relationships, such as the proportional relationship between volume and temperature, as evidenced in the interview excerpt.

Student A: (Answered C to Q22) At first, the gas is $\frac{3}{4}V_0$. After heating, it becomes V_0 . I think the temperature goes up because the volume gets bigger. Honestly, I don't remember all the details of the ideal gas equation. When gas takes up more space, it feels like the temperature should increase. I believe that heating increases volume and raises temperature. Therefore, I chose option C.

Novice students had not yet formed a microscopic model of the ideal gas. They often thought of gases as uniform substances, like water. For example, in question 15, when asked to compare two gas samples, each initially at the same temperature but with different numbers of particles, after both received the same amount of heat, these students usually believed that equal heat input would lead to equal final temperatures, regardless of the particle count differences. This belief is shown in the following excerpt from the interview.

Student B: (Answered B to Q15) I think the temperature will stay the same in both containers once they settle. Since they both receive the same amount of heat and start at the same temperature, the heat should spread out the same way in both cases. So, after they warm up, they'll both end up at the same temperature, reaching balance or equilibrium.

Furthermore, novice students had not yet constructed a transition framework to connect microscopic dynamics with observable macroscopic phenomena. For instance, when asked to explain the temperature change during the free expansion of a gas, their responses reflected a lack of such a framework. This is clear in the following excerpt:

Student C: (Answered C to Q7) At first, all the gas is in compartment A. When the partition is opened,

the gas flows into compartment B, spreading out like a liquid. The total volume increases, so I think the temperature should drop.

Overall, novice students showed fragmented knowledge structures and relied heavily on surface features or memorized procedures. They struggled to adopt a particulate view of gases and to coordinate multiple variables. These misconceptions hinder the development of reasoning and block the integration of micro- and macro-level understanding. Most students (7/8) at the novice level displayed the reasoning patterns described above. However, one student (1/8) was able to memorize *the ideal gas equation* but performed poorly due to calculation errors.

2. Lower-intermediate-level students

The nine lower-intermediate-level students ($25 < \text{total score} \leq 50$) showed improved recall of key properties and could generally apply *the ideal gas equation* accurately when solving typical problems, as seen in the interview excerpt.

Student D: (Answered B to Q22) The original volume of the gas cylinder is V_0 . Before it was heated, the volume inside the cylinder was $\frac{3}{4}V_0$, and the temperature was 300 K. As the cylinder is heated slowly, the volume expands to fill the entire cylinder. Since this is an isobaric change, we can use the relationship $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$. As the temperature increases, we get $\frac{V_0}{T_1} = \frac{\frac{3}{4}V_0}{4 \times 300}$, which gives $T_2 = 400$ K.

As revealed by the interviews, lower-intermediate-level students had not developed a robust understanding of the microscopic model of an ideal gas. Their responses were primarily procedural, relying heavily on mathematical manipulation with little reference to particle-level mechanisms that explain the observed behavior. For example, when asked about the IGL, student E showed a significant gap in understanding *the microscopic model of the ideal gas*, which is evident from the student's explanation: "*the ideal gas law is the ideal gas equation, which is the formula $pV = nRT$.*"

The interviews also revealed students' difficulties with tasks that require transition reasoning. Although these students may understand the relationship between temperature and the average kinetic energy of gas particles, they often struggle to fully grasp the broader implications of this connection.

Student F: (Answered A to Q12) It's true that the faster the particles move, the higher the temperature is. As for pressure, I think we can estimate it from the particles' speed. The faster they move, the higher the pressure should be, too.

In summary, students at the lower-intermediate level can apply macroscopic relationships with moderate competence and sometimes include quantitative transitions. However, their reasoning often lacks accuracy and coherence. This is especially clear when they need to combine multiple representational levels. As a result, although their performance improves compared to novice learners, it remains limited in less typical contexts. All lower-intermediate students interviewed showed the reasoning patterns described above.

3. Upper-intermediate-level students

All eight upper-intermediate-level students ($50 < \text{total score} \leq 75$) demonstrated proficiency in applying *the ideal gas equation* to macroscopic problems. In microscopic contexts, they recognize that gases are made up of molecules, yet they often struggle with key aspects of the *microscopic model of the ideal gas*. Specifically, they find it challenging to understand concepts such as elastic collisions, the exclusion of intermolecular forces, and the assumption that molecular volume is negligible, as shown in the interview excerpt below.

Student G: (Answered D to Q2) I think the particles should move in a random, chaotic way, following curvy paths. However, sketches A, B, and C all have sharp turns. This makes it look like the particles are colliding, which doesn't match the idea of random motion. So, there shouldn't be any sudden twists or turns in the path. It should be a chaotic, curved path instead.

While upper-intermediate students make initial attempts to bridge the gap between microscopic and macroscopic principles, challenges still exist. Overall, they can accurately solve problems related to the relationship between temperature and particle velocity, demonstrating their ability to apply key principles effectively and connect microlevel concepts with macroscopic phenomena, as shown in the following response:

Student H: (Answered B to Q16) Particles with smaller masses move faster because the question states that $T_A = T_B$ and $M_A > M_B$. Since the temperature is the same for both, the larger mass of A means its particles move more slowly, while B's smaller particles move faster at the same temperature.

However, these students still face challenges when answering transition questions that require a deeper understanding of the microscopic model. For example, their conclusion about the temperature increase shows a misunderstanding of the principle that a particle's velocity stays constant during free expansion.

Student I: (Answered A to Q12) I think the temperature of the gas will go up because part A has hydrogen and part B has nitrogen. When the baffle is removed, the hydrogen and nitrogen molecules mix and start colliding in double the space they had before. This causes both gases to expand, and the overall volume of the system increases. Once everything settles down and the system reaches equilibrium, the pressure stays the same as it was at the start, but the volume is bigger. So, based on the ideal gas equation, I think the temperature of the gas will also rise.

Overall, upper-intermediate students show proficiency in applying *the ideal gas equation* to macroscopic situations but struggle with the microscopic model, particularly in areas such as elastic collisions and molecular volume. While they can generally solve typical transition model problems, they often misunderstand key principles when asked questions that require a deeper understanding of transition models. All upper-intermediate students interviewed displayed reasoning patterns consistent with those described above.

4. Expert-level students

The five expert-level students ($75 < \text{total score} \leq 100$) demonstrated a deep and scientifically precise understanding of the microscopic model, showing the ability to synthesize the central idea, intermediate reasoning steps, and contextual factors. This integrated knowledge provides a strong basis for problem solving in various situations. This skill boosts their cognitive flexibility. As a result, they can tackle not only relatively simple macroscopic problems but also more complex questions involving atypical problems that include microscopic or transitional elements.

At the microscopic level, expert-level students possess a deep understanding of the *microscopic model of the ideal gas*. This allows them to describe particle behavior accurately. They demonstrated a firm grasp of the *microscopic model of the ideal gas* by recognizing the random, linear motion of gas particles between collisions, the core principle of kinetic theory. This understanding enabled the student to reason accurately about particle trajectories and interactions.

Student J: (Answered C to Q2) The motion of gas particles is random. The difference between options A and C is that if there were only one gas particle in the container, its trajectory would follow a straight line and reflect off the container walls. However, option C suggests there are multiple gas particles in the container, and they will collide with each other. As a result, the trajectory of the particles cannot be a single straight line but becomes more

complex due to these collisions. Regarding option B, since the gas particles are confined to a minimal range, their trajectories would be highly restricted, which doesn't match reality. As for option D, the motion of gas particles is unrealistic because if their momentum were to change instantaneously countless times during their motion, the trajectory would become more complex, not a smooth straight line.

These students also demonstrate an integrated understanding of microscopic particle behavior and its effects on macroscopic gas properties, such as temperature. They accurately apply principles of kinetic theory to analyze thermodynamic outcomes in idealized systems.

Student K: (Answered B to Q7) Each gas molecule in an ideal gas can be thought of as an elastic ball. In this model, gas molecules can collide with each other or with the walls of the container. Since elastic collisions do not result in energy loss, the average kinetic energy of each gas molecule remains constant, even if the volume of the container increases. Therefore, the average kinetic energy of individual gas molecules stays the same, and as a result, the temperature remains unchanged. This explanation is reasonable, as it leads to the conclusion that the temperature does not change, based on the principle of elastic collisions.

In summary, expert-level students demonstrate a thorough understanding of microscopic, macroscopic, and transition models, effectively integrating the central idea, reasoning processes, and contextual factors. They excel at solving both typical and atypical problems, showing a deep knowledge of particle behavior and thermodynamic principles. All interviewed expert-level students exhibited reasoning patterns aligned with those described above.

V. CONCLUSION AND DISCUSSION

A. Summary of findings

This study developed a conceptual framework for the IGL to guide the design of a test to assess knowledge integration (TIGL). The test included two main measurement strategies: (a) contextual saliency, comparing typical vs atypical contexts, and (b) conceptual model categories, such as microscopic, transitional, and macroscopic models. Analyses of the assessment data, supported by interview findings, confirmed that these design features effectively distinguished students of four levels of knowledge integration, including novice, lower-intermediate, upper-intermediate, and expertlike.

Novice-level students demonstrated fragmented knowledge and relied primarily on contextual clues to recall memorized equations, limiting their ability to solve only simple, two-variable problems. They lacked understanding of both microscopic and transitional models, which are crucial for deeper reasoning. Lower-intermediate students developed a partial understanding, with some sense of the ideal gas equation, but they remained within typical problem contexts and focused on macroscopic or quantitative transitional models. Upper-intermediate students expanded their knowledge to include transitional models but still showed weak links to the microscopic model of the ideal gas. In contrast, expertlike students displayed a highly integrated knowledge system that enabled them to apply microscopic, transitional, and macroscopic models flexibly in both typical and atypical contexts.

B. Implications for teaching practice

The four KI levels identified in this study can serve as actionable markers for differentiating instruction on the ideal gas law. Rather than assuming homogeneous understanding, instructors can align learning activities with students' current integration level and deliberately scaffold transitions toward more expertlike micro-transitional-macro reasoning, which can be interpreted and used as follows:

At the novice level (0–25), instruction should prioritize robust macroscopic reasoning about the relationships among p , V , and T . Guided-inquiry activities that foreground how two variables co-vary while the third is held fixed (e.g., $p - V$ or $V - T$ explorations) can be paired with short, focused exercises or clicker questions to consolidate basic macroscopic patterns before introducing microscopic explanations.

At the lower-intermediate level (25–50), students can use some macroscopic relations but still rely on “continuous-fluid” intuitions. Teaching at this stage should begin to surface and challenge their implicit particle models. Simple microscopic visualizations, such as molecule-collision simulations, particle animations, or sketching tasks, can be used to contrast discrete-particle behavior with naive continuous-medium images and to link pressure and temperature to particle motion.

At the upper-intermediate level (50–75), students often articulate both macro relations and some particle ideas, but their links between levels are fragile. Instruction should therefore emphasize transitional models that explicitly connect microscopic assumptions to macroscopic equations. Qualitative and quantitative bridging tasks (e.g., predicting how changes in collision rate or average kinetic energy affect p , V , and T) can be used to practice systematic micro $\rightarrow$ macro inference.

At the expertlike level (75–100), students are approaching expertlike profiles, and instruction can shift toward transfer and refinement. Assigning less familiar scenarios,

such as mixing gases at different temperatures, free expansion into a vacuum, or multistep thermodynamic processes, can help reveal residual gaps in microscopic models and promote flexible application of micro-translational-macro reasoning beyond routine textbook problems.

C. Methodological contributions

This research methodologically advances by introducing an assessment framework that combines macroscopic, translational, and microscopic models. By differentiating macroscopic operational rules, translational explanatory models, and microscopic mechanistic models, the framework offers a new way to measure knowledge integration. The inclusion of the translational model fills a vital gap in earlier frameworks, enabling links between otherwise separate microscopic and macroscopic representations [56]. Additionally, designing both qualitative and quantitative translational connections aligns with previous calls in physics education to focus not only on quantitative calculations but also on students' qualitative reasoning [35,57].

Further analyses indicate that total test scores can reliably reflect knowledge integration levels, providing a practical tool for diagnosing learning difficulties and customizing instruction. The framework can be refined and expanded to other areas involving microscopic mechanisms, such as electromagnetism and thermodynamics, as well as interdisciplinary fields. Targeted instructional interventions are critical, as most students in this study reached only lower-intermediate levels of knowledge integration under traditional lecture-based methods. Increasing the sample size to include a larger, more diverse group of learners, including exceptionally high-performing students, can yield deeper insights into the development of knowledge integration in the IGL.

D. Limitations and future research

Several limitations must be acknowledged. The use of multiple-choice questions, despite careful design and pilot testing, still carries the risk of random guessing, which may slightly impact the accuracy of the results. The limited representation of high-performing students restricted the analysis of expert groups. The cultural and educational context, an examination-oriented East Asian system, also limits how directly the findings can be applied to similar environments that emphasize high-stakes testing, so caution is needed when extending the results to inquiry-driven classrooms. We promote cross-cultural comparisons to examine how educational traditions influence knowledge integration. Finally, the scoring scheme for categorizing knowledge integration levels was based on this study's sample and might not be universally applicable.

Future research can proceed in four directions guided by the KI framework and the micro-translational-macro perspective. First, future studies can expand the content coverage by applying the same lens and KI diagnostics to other thermodynamics topics to test whether similar KI levels and reasoning paths appear beyond the ideal gas law, such as entropy, the first law, and heat engines. Second, methodological work is needed that combines KI instruments with process data such as eye tracking, response times, or digital trace data to examine when and how students switch between microscopic, translational, and macroscopic models. Third, comparative studies across East Asian and Western settings can explore how curriculum, exam culture, and representational practices shape KI trajectories. Fourth, quasiexperimental or design-based studies can evaluate targeted, level-specific instructional sequences to determine whether they produce measurable gains in KI levels, reasoning quality, and transfer.

In summary, this study demonstrates that applying the ideal gas law conceptual framework yields an assessment that effectively examines and categorizes students' understanding and integration of the concepts. The results confirm that variations in context (typical vs atypical) and model-based reasoning (microscopic, translational, macroscopic) are key design features for identifying different levels of integration. However, the findings also suggest that traditional teaching methods are inadequate for fostering more profound understanding, underscoring the need for future instructional innovations to strengthen students' conceptual understanding of *the ideal gas law*.

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DATA AVAILABILITY

Data that support the findings of this article are contained within the article and its Supplemental Material [51].

- [1] M. Chiu, C. Guo, and D. Treagust, Assessing students' conceptual understanding in science: An introduction about a national project in Taiwan, *Int. J. Sci. Educ.* **29**, 379 (2007).
- [2] E. Kim and S.-J. Pak, Students do not overcome conceptual difficulties after solving 1000 traditional problems, *Am. J. Phys.* **70**, 759 (2002).
- [3] W. J. McKeachie, *Teaching and Learning in the College Classroom: A Review of the Research Literature* (National Center for Research to Improve Postsecondary Teaching and Learning, Ann Arbor, 1986).
- [4] M. H. Chiu, Algorithmic problem solving and conceptual understanding of chemistry by students at a local high school in Taiwan, *Proc. Natl. Sci. Coun. Repub. China* **11**, 20 (2001).
- [5] C. Hoellwarth, M. J. Moelter, and R. D. Knight, A direct comparison of conceptual learning and problem solving ability in traditional and studio style classrooms, *Am. J. Phys.* **73**, 459 (2005).
- [6] M. B. Nakhleh, Are our students conceptual thinkers or algorithmic problem solvers? Identifying conceptual students in general chemistry, *J. Chem. Educ.* **70**, 52 (1993).
- [7] M. B. Nakhleh and R. C. Mitchell, Concept learning versus problem solving: There is a difference, *J. Chem. Educ.* **70**, 190 (1993).
- [8] D. Stamovlasis, G. Tsaparlis, C. Kamilatos, D. Papaoikonomou, and E. Zarotiadou, Conceptual understanding versus algorithmic problem solving: Further evidence from a national chemistry examination, *Chem. Educ. Res. Pract.* **6**, 104 (2005).
- [9] L. C. McDermott, *Physics by Inquiry: An Introduction to the Physical Sciences* (John Wiley & Sons, New York, NY, 1996).
- [10] R. J. Beichner, J. M. Saul, D. S. Abbott, J. J. Morse, D. Deardorff, R. J. Allain, and J. S. Risley, The student-centered activities for large enrollment undergraduate programs (SCALE-UP) project, in *Research-Based Reform of University Physics* (PERC Publishing, Rochester, NY, 2007), Vol. 1, p. 2.
- [11] E. Etkina and A. Van Heuvelen, Investigative science learning environment: Using the processes of science and cognitive strategies to learn physics, in *Proceedings of the 2001 Physics Education Research Conference* (John Wiley and Sons, Inc., Rochester, 2001).
- [12] P. W. Laws, Workshop physics activity guide, module 4: Electricity and magnetism, in *Workshop Physics Activity Guide* (Wiley-VCH, New York, 2004).
- [13] R. K. Thornton and D. R. Sokoloff, Assessing student learning of Newton's laws: The force and motion conceptual evaluation and the evaluation of active learning laboratory and lecture curricula, *Am. J. Phys.* **66**, 338 (1998).
- [14] Q. Chen, G. Zhu, Q. Liu, J. Han, Z. Fu, and L. Bao, Development of a multiple-choice problem-solving categorization test for assessment of student knowledge structure, *Phys. Rev. Phys. Educ. Res.* **16**, 020120 (2020).
- [15] L. Bao and J. Fritchman, Knowledge integration in student learning of Newton's third law: Addressing the action-reaction language and the implied causality, *Phys. Rev. Phys. Educ. Res.* **17**, 020116 (2021).
- [16] Y. Nie, Y. Xiao, J. Fritchman, Q. Liu, J. Han, J. Xiong, and L. Bao, Teaching towards knowledge integration in learning force and motion, *Int. J. Sci. Educ.* **41**, 2271 (2019).
- [17] L. Xie, Y. Li, Q. Fu, Y. Xiao, L. Yang, and L. Bao, Assessing and enhancing university students' knowledge integration in learning electromagnetic induction, *Int. J. Sci. Educ.* **47**, 1 (2024).
- [18] W. Xu, Q. Liu, K. Koenig, J. Fritchman, J. Han, S. Pan, and L. Bao, Assessment of knowledge integration in student learning of momentum, *Phys. Rev. Phys. Educ. Res.* **16**, 010130 (2020).
- [19] D. Tong, J. Liu, Y. Sun, Q. Liu, X. Zhang, S. Pan, and L. Bao, Assessment of student knowledge integration in learning work and mechanical energy, *Phys. Rev. Phys. Educ. Res.* **19**, 010127 (2023).
- [20] L. Xie, Q. Liu, H. Lu, Q. Wang, J. Han, X. Feng, and L. Bao, Student knowledge integration in learning mechanical wave propagation, *Phys. Rev. Phys. Educ. Res.* **17**, 020122 (2021).
- [21] R. Dai, J. Fritchman, Q. Liu, Y. Xiao, H. Yu, and L. Bao, Assessment of student understanding on light interference, *Phys. Rev. Phys. Educ. Res.* **15**, 020134 (2019).
- [22] A. H. Schoenfeld and D. J. Herrmann, Problem perception and knowledge structure in expert and novice mathematical problem solvers, *J. Exp. Psychol. Learn. Mem. Cognit.* **8**, 484 (1982).
- [23] M. T. Chi, M. Bassok, M. W. Lewis, P. Reimann, and R. Glaser, Self-explanations: How students study and use examples in learning to solve problems, *Cognit. Sci.* **13**, 145 (1989).
- [24] P. T. Hardiman, R. Dufresne, and J. P. Mestre, The relation between problem categorization and problem solving among experts and novices., *Mem. Cognit.* **17**, 627 (1989).
- [25] C. Crouch and E. Mazur, Peer instruction: Ten years of experience and results, *Am. J. Phys.* **69**, 970 (2001).
- [26] P. Wattanakasiwich, P. Taleab, M. D. Sharma, and I. D. Johnston, Construction and implementation of a conceptual survey in thermodynamics, *Int. J. Innov. Sci. Math. Educ.* **21**, 29 (2013).
- [27] D. E. Meltzer, Investigation of students' reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course, *Am. J. Phys.* **72**, 1432 (2004).
- [28] M. Baser, Fostering conceptual change by cognitive conflict based instruction on students' understanding of heat and temperature concepts, *Eurasia J. Math. Sci. Technol. Educ.* **2**, 96 (2006).
- [29] A. G. Harrison, D. J. Grayson, and D. F. Treagust, Investigating a grade 11 student's evolving conceptions of heat and temperature, *J. Res. Sci. Teach.* **36**, 55 (1999).
- [30] C. H. Kautz, P. R. Heron, P. S. Shaffer, and L. C. McDermott, Student understanding of the ideal gas law, part II: A microscopic perspective, *Am. J. Phys.* **73**, 1064 (2005).
- [31] C. Kautz, P. Heron, M. Loverude, and L. McDermott, Student understanding of the ideal gas law, part I: A macroscopic perspective, *Am. J. Phys.* **73**, 1055 (2005).

- [32] M. E. Loverude, C. H. Kautz, and P. R. Heron, Student understanding of the first law of thermodynamics: Relating work to the adiabatic compression of an ideal gas, *Am. J. Phys.* **70**, 137 (2002).
- [33] D. L. Gabel and D. M. Bunce, Research on problem solving: Chemistry, in *Handbook of Research on Science Teaching and Learning*, edited by D. L. Gabel (Macmillan, New York, 1994), pp. 301–326.
- [34] N. Erceg, I. Aviani, V. Mešić, M. Glunčić, and G. Žauhar, Development of the kinetic molecular theory of gases concept inventory: Preliminary results on university students' misconceptions, *Phys. Rev. Phys. Educ. Res.* **12**, 020139 (2016).
- [35] J. P. Jong, M. H. Chiu, and S. L. Chung, The use of modeling-based text to improve students' modeling competencies, *Sci. Educ.* **99**, 986 (2015).
- [36] Stefanie Schwedler and Michael Lyczek, Punktmassen oder Mini-Billiardkugeln? Moleküldynamiksimulationen stärken Lernervorstellungen zu idealen Gasen im Chemie-studium/Point mass or little billiard ball? molecular dynamics simulations enhance students' mental models on the ideal gas law in tertiary chemistry studies, *CHEM-KON* **26**, 179 (2019).
- [37] D. L. Benson, M. C. Wittrock, and M. E. Baur, Students' preconceptions of the nature of gases, *J. Res. Sci. Teach.* **30**, 587 (1993).
- [38] K. J. Laidler and J. H. Meiser, *Physical Chemistry* (Benjamin Cummings, Belmont, CA, 2003).
- [39] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 12th ed. (Pearson, San Francisco, CA, 2008).
- [40] Chris Hamper and Stephen Pople, *Physics for the IB*, 1st ed. (Oxford University Press, Oxford, UK, 2014).
- [41] Douglas C. Giancoli, *Physics: Principles with Applications*, 7th ed. (Pearson Education, Boston, MA, 2014).
- [42] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics: Volume 1* (Addison-Wesley, Reading, MA, 1963).
- [43] Wolfgang Pauli, *Pauli Lectures on Physics* (MIT Press, Cambridge, MA, 1973), Vol. 3.
- [44] Walker Jearl, Halliday David, and Robert Resnick, *Fundamentals of Physics*, 10th ed. (John Wiley, Hoboken, NJ, 2014).
- [45] P. Johnson and G. Papageorgiou, Rethinking the introduction of particle theory: A substance-based framework, *J. Res. Sci. Teach.* **47**, 130 (2010).
- [46] E. J. Gómez, A. Benarroch, and N. Marín, Evaluation of the degree of coherence found in students' conceptions concerning the particulate nature of matter, *J. Res. Sci. Teach.* **43**, 577 (2006).
- [47] B. J. Gustafson, M. C. Shanahan, and S. Gentilini, Elementary children's shifting views of models and the nature of matter, *Can. J. Sci. Math. Technol. Educ.* **10**, 103 (2010).
- [48] D. Stamovlasis, G. Tsitsipis, and G. Papageorgiou, The effect of logical thinking and two cognitive styles on understanding the structure of matter: An analysis with the random walk method, *Chem. Educ. Res. Pract.* **11**, 173 (2010).
- [49] D. L. Benson, M. C. Wittrock, and M. E. Baur, Students' preconceptions of the nature of gases, *J. Res. Sci. Teach.* **30**, 587 (1993).
- [50] G. Gutiérrez and J. M. Yáñez, Can an ideal gas feel the shape of its container?, *Am. J. Phys.* **65**, 739 (1997).
- [51] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/vv1g-7yl8> for the full assessment items used in the study and the item parameters from the item response theory analysis.
- [52] N. Bedduside, A. Hadis, A. A. Jalal, Syamsiah, and D. D. P. U. S. Patongai, Innovation of biology learning through the development of authentic assessment based on scientific literacy for students of senior high school, *J. Phys. Conf. Ser.* **1899**, 012142 (2021).
- [53] R. F. DeVellis, *Scale Development: Theory and Application* (Sage Publications, Thousand Oaks, CA, 2012), Vol. 26.
- [54] Andy Field, *Discovering Statistics Using IBM SPSS Statistics*, 4th ed. (SAGE Publications, London, 2013).
- [55] M. L. McHugh, Interrater reliability: The kappa statistic, *Biochem. Med.* **22**, 276 (2012).
- [56] Zengze Liu, Sudong Pan, and Lei Bao, Effectiveness of conceptual-framework-based instruction on promoting knowledge integration in learning simple electric circuit, *Phys. Rev. Phys. Educ. Res.* **20**, 020113 (2024).
- [57] Brianna L. Martinez, Ryan D. Sweeder, Jessica R. VandenPlas, and Deborah G. Herrington, Improving conceptual understanding of gas behavior through the use of screencasts and simulations, *Int. J. STEM Educ.* **8**, 5 (2021).