Global blind spot in understanding trigonometric derivatives: A multinational analysis

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Trigonometric derivatives are fundamental in both mathematics and physics, yet their correct application, particularly the distinction between radians and degrees, remains a significant challenge for college students worldwide. This study identifies a widespread "blind spot" in understanding trigonometric derivatives and their implications for physical systems, highlighting a critical gap in physics education. A multinational survey of 769 college students, primarily undergraduate and graduate science, technology, engineering, and mathematics (majors, from Israel, the United States, China, and India assessed their ability to differentiate between radians and degrees in mathematical and physical contexts, focusing on harmonic motion. Results reveal that only 26.3% of students correctly identified that the well-known expressions for trigonometric derivatives hold exclusively in radians, while 70.7% incorrectly assumed that both radians and degrees are valid. Notably, students demonstrated improved recognition of radians in physical contexts (59% correct responses) compared to mathematical ones, suggesting that students rely on familiar physical equations as cognitive reference points when applying mathematical concepts. These misunderstandings appear globally, suggesting a universal challenge. The findings highlight the need for curriculum reforms to better connect mathematical formalism with physical application.

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I. INTRODUCTION

> An alien culture might not use 360 at all. On the other hand, I believe 2π would be discovered in any advanced civilization.–Shankar, *Fundamentals of Physics I* [1]

Shankar's remark highlights a fundamental reality in mathematical physics: radians are not an arbitrary choice, but a natural consequence of how we define and manipulate angular quantities. Despite their fundamental role in calculus and physics, students often fail to grasp why radians, rather than degrees, form the correct mathematical framework for differentiation and physical equations. This confusion extends beyond simple unit conversion; it likely reflects a deeper conceptual gap in the way students bridge mathematical formalism with physical intuition. In this study, we investigate how this misunderstanding manifests across diverse educational backgrounds and international contexts and examine its implications for learning and applying fundamental principles in physics.

The successful learning of physics fundamentally depends on students' ability to apply mathematical concepts in physical contexts. This connection between mathematical understanding and physics comprehension becomes particularly critical when dealing with trigonometric functions and their derivatives. Long-standing research in physics and mathematics education has highlighted the challenges students face and developed models for the coordination of mathematical and physical reasoning [2–15].

In recent years, to understand and address the documented divides between mathematical and physical reasoning systems, researchers have explored the roles of symbolic forms, drawing from the work of Sherin [16], and conceptual blending of Fauconnier and Turner [11,17–21]. For example, Schermerhorn and Thompson [19] posit a framework of "symbolic blending" that links the conceptual representations embedded within mathematical forms, with the layering of meaning through blending processes. Such an approach links both the formalisms of mathematics and physics with the contextual interpretation of abstract

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formalisms. In related work, Gifford and Finkelstein [22,23] apply both the formalisms of symbolic representation and conceptual layering in their framing of mathematical sensemaking (MSM). This MSM framework notes how mathematics and physics representations may be used as both tools for and objects of understanding of either mathematics or physics concepts. We draw on such approaches when interpreting the finding of students' reasoning around trigonometric derivatives in a physics context.

Focussing on the particular context of trigonometry, research also indicates that trigonometry poses significant challenges for physics students at all levels, from beginners to advanced learners. Galle and Meredith [24], in their study of life science students, identified a common difficulty in understanding the physical applications of trigonometric functions. They highlighted students' struggles to connect the mathematical representation of trigonometric functions with their physical manifestations, such as harmonic motion or waves. Additionally, students often found it challenging to interpret angular measurements quantitatively, especially when transitioning between graphical representations and algebraic solutions.

Weber [25] further demonstrated how these challenges persist among advanced physics students. His work documented how, despite procedural fluency in basic mathematical analyses, many students fail to grasp the deeper conceptual meaning of trigonometric functions. For instance, students tend to perceive these functions merely as computational tools, without understanding the relationship between their geometric properties and physical applications. This leaves a substantial gap between theoretical comprehension of trigonometry and its use in solving complex physics problems.

Van den Eynde *et al.* [26] explored the challenges undergraduate physics and mathematics students face when formulating boundary conditions for the diffusion equation. Through think-aloud interviews, the study revealed that while students often demonstrated a basic understanding of the physical system, they struggled significantly with connecting mathematical formalism to physical intuition. Notably, difficulties arose in interpreting partial derivatives, specifying correct boundary conditions, and blending mathematical and physical reasoning, highlighting the persistent gap between procedural skills and conceptual understanding in advanced physics contexts.

In this study, we will particularly draw from prior work from Beichner [27] on student interpretation of graphs in physical contexts, Christensen and Thompson's [28] studies of on students' ability to interpret derivatives in mathematical contexts, and the work of Carli *et al.* [10] assessing students' ability to solve isomorphic questions in mathematics and physics. These are discussed in more detail in the relevant Discussion section below.

While previous research has extensively documented students' difficulties with trigonometry, little attention has been given to the specific impact of angular representation—degrees versus radians—on their ability to correctly apply differentiation in physics. This study aims to fill that gap by systematically analyzing how students across diverse educational systems understand and utilize angle units in differentiation and assessing the implications for problem solving in physics. Bridging this gap is essential for ensuring that students can apply trigonometry effectively across a wide range of physical problems.

Angular measurements are ubiquitous in physics, from basic kinematics to quantum mechanics. These angles can be quantitatively represented using either degrees or radians, with each system having distinct historical and practical origins. The division of a circle into 360 degrees, originating from ancient Babylonian mathematics, remains deeply ingrained in everyday understanding. In contrast, the radian measure, representing the ratio of arc length to radius, emerged as a mathematically natural unit, with 2π radians corresponding to a full circle. This duality in representation creates a fascinating interface between mathematical formalism and physical understanding.

The everyday prevalence of degrees (e.g., "he made a 180° turn in his opinion") contrasts sharply with the mathematical preference for radians in advanced studies. This dichotomy is reflected in educational progression: students typically first encounter angles in degrees during early mathematics education, with radians introduced later. At the university level, particularly in calculus and trigonometry courses, radians become the exclusive representation, to the extent that degree-based trigonometric functions are rarely used in textbooks and instruction. This discontinuity in teaching different representations and the transition between them can create a fundamental confusion among students, serving as a primary motivation for the current research.

In introductory physics courses, especially at the high school level or in algebra-based college physics, there exists a hybrid approach to these representations. Early mechanics and kinematics typically employ degrees, allowing students to visualize problems using familiar units (e.g., projectile motion at 30° rather than $\pi/6$ radians). However, when discussing periodic motion and using trigonometric functions to represent position as a function of time, instruction often shifts to radians. This transition is often justified with the simple explanation that radians are "more natural" due to their relationship to arc length, without deeper exploration of the mathematical implications. This approach is commonly found in introductory physics textbooks [1,29–31].

The critical distinction lies not in the dimensionless nature of these units—both degrees and radians are

dimensionless—but in their derivatives. The rate of change in radians differs from that in degrees by a factor of $\pi/180$:

$$\frac{d}{dx}\sin(x) = \cos(x) \text{ [in radians]} \tag{1}$$

$$\frac{d}{dx}\sin(x^\circ) = \frac{\pi}{180}\cos(x^\circ) \text{ [in degrees]} \tag{2}$$

This distinction becomes particularly crucial in the context of harmonic oscillation, where the fundamental equation of motion:

$$\frac{d^2x}{dt^2} = -\omega^2 x \tag{3}$$

relies on the radian representation for its standard solutions. The familiar expression for position as a function of time in harmonic motion,

$$x(t) = A\cos(\omega t + \phi) \tag{4}$$

where $\omega = \sqrt{(k/m)}$ is valid only when using radians, a fact that remains unclear to many students throughout their academic careers (demonstrated below). This distinction becomes particularly crucial not only in harmonic oscillation but also in a wide range of physical systems where accurate mathematical representations are essential for modeling and understanding systems' behaviors.

This study investigates students' understanding of trigonometric derivatives in both mathematical and physical contexts, with particular emphasis on the distinction between degrees and radians. We examine how this understanding varies across different countries, educational systems, and academic backgrounds, while also exploring the relationship between students' confidence levels and their conceptual understanding. Through this comprehensive analysis, we aim to identify patterns in student reasoning that may be used to inform pedagogical interventions to address this fundamental gap in physics and mathematics education.

II. RESEARCH QUESTIONS

The study focuses on the following primary research questions:

1. To what extent do university students correctly identify the derivatives of trigonometric functions in radians versus degrees?
2. To what extent do students correctly identify the difference between degrees and radians in a physical system, such as a harmonic oscillator?
3. Are there significant differences in understanding these questions between students from different countries and educational systems?
4. What is the correlation between students' confidence levels and the correctness of their answers to conceptual questions involving the trigonometric expression for harmonic oscillators?

In addition to these primary questions, the following subquestions will be explored:

1. What patterns emerge from students' explanations regarding the validity of the trigonometric expression for a harmonic oscillator, and how do these patterns correlate with the correctness of their answers and their confidence levels?
2. To what extent do students' academic departments (e.g., physics, mathematics, engineering) influence their understanding of trigonometric derivatives, and how do these departments influence their application of trigonometric functions in physical systems like harmonic oscillators?
3. To what extent do grades in foundational courses, such as mechanics and calculus, predict students' ability to correctly identify the conditions under which trigonometric derivatives are valid, and how do these grades predict their ability to apply this knowledge in physical systems?

III. METHODS

A. The survey

The survey included 6 core questions:

Q1: What is the derivative of sin(t) with respect to t? (Open-ended question)

Q2: What is the derivative of cos(t) with respect to t? (Open-ended question)

Q3: The answer you gave in the previous questions is correct in the case of:

- Angle in radians ("Radians").
- Both are correct ("Both").
- Angle in degrees ("Degrees").

Q4: An object with a mass of m performs simple harmonic motion under a spring connection with an elastic coefficient of k, and its position is described as $x(t) = A\cos(\omega t + \varphi)$, where A and φ are determined by the initial state, and $\omega = \sqrt{(k/m)}$.

The expression $x(t) = A\cos(\omega t + \varphi)$ is correct in the case of:

- Angle in radians (Radians).
- Both are correct (Both).
- Angle in degrees (Degrees).

Q5: Please briefly explain your answer to Q4. (Open-ended question)

Q6: How sure are you that the answer you gave (in question Q4) is correct?

- Very sure.
- Somewhat sure.
- Natural.
- Somewhat unsure.
- Not sure at all/very unsure.

In addition to the core survey questions, participants were asked to complete a demographic questionnaire. These elements enabled analysis by academic and personal background and are presented in full in Appendix C.

The questionnaire was as identical as possible across all countries, with translations provided as necessary. In each case, the survey was conducted in the language of instruction. In the United States and India, the survey was conducted in English. In Israel, it was given in Hebrew, and in China, it was given in Mandarin Chinese. The translations were performed by physics experts fluent in the respective languages to ensure accuracy and consistency across versions.

The instrument was subjected to face validation by the course instructors, ensuring that the language and symbolic forms aligned with the content of the undergraduate physics courses in which the survey was administered. The representational formats of the questions were also consistent with the textbooks commonly used in each country. Before finalizing the survey, a draft version was tested with selected university students from the United States ($N = 12$), Israel ($N = 10$), India ($N = 5$), and China ($N = 5$). Follow-up interviews were conducted to verify that the students interpreted the questions as intended, and adjustments were made accordingly.

B. Participants

The study sample consisted of 769 participants from four primary countries: Israel (35.4%), the United States (21.3%), China (24.2%), and India (19.1%). The average age of the participants was 21.37 years (SD = 3.01). Regarding the gender distribution, the majority identified as male (67.1%), followed by female (31.6%), and a small proportion identified as other (1.3%). Most participants were pursuing a bachelor's degree (84.3%), while smaller proportions were pursuing master's (7.8%), doctorate (6.9%), or postdoctoral/other degrees (1%). Participants were predominantly in their first year of study (50.3%), with fewer in their second (28.3%), third (14.1%), fourth (5.9%), fifth (1.1%), or sixth year (0.4%). The largest group of participants majored in physics (50.3%), followed by engineering (21.9%), chemistry, life sciences, or neuroscience (14.4%), math and computer science (8.1%), and other fields (5.3%) (see Table I). Given the different degree requirements across disciplines and by country, we note that in our specific sample, physics, engineering, and mathematics/computer science students typically have calculus-based backgrounds, while most students from chemistry, life sciences, neurosciences, and other fields have more limited calculus exposure. For brevity in our analysis, we refer to this latter group collectively as life sciences students.

The inclusion criteria for the study were as follows: all students who were present in the classroom during the lecture in which the questionnaire was distributed and who completed it within the time allocated for answering were included in the final sample.

TABLE I. Demographic characteristics of the sample.

Variable	*N*	%
Country		
Israel	272	35.4
United States	164	21.3
China	186	24.2
India	147	19.1
Gender		
Male	516	67.1
Female	243	31.6
Other	10	1.3
Degree		
Bachelor's	647	84.3
Master's	60	7.8
Doctorate	53	6.9
Postdoctoral and other	8	1
Year of study		
Year 1	312	41.1
Year 2	173	22.8
Year 3	105	13.8
Year 4+	54	7.1
Graduate students (all years) and other	116	15.3
Major field		
Physics	386	50.3
Engineering	168	21.9
Math and computer science	62	8.1
Chemistry, life sciences, or neuroscience	111	14.4
Other	41	5.3

C. Data collection description

The study utilized a structured questionnaire to examine students' understanding of trigonometric derivatives and their impact on physics comprehension. To minimize potential biases associated with self-reporting or online research during questionnaire completion, data were collected in class during course lectures.

Data collection was conducted at four leading research universities:

- United States: University of Colorado Boulder.
- Israel: Bar-Ilan University.
- China: Shandong University.
- India: Indian Institute of Technology Hyderabad.

These institutions were selected for their strong academic programs in science and their ability to provide a diverse and representative sample of students from various educational systems and cultural backgrounds.

D. Coding procedure and classification of student reasoning

To avoid pre-prediction effects and to gain insights into the conceptual understanding of the students, Q5 was

formulated as an open question (similar to Q1 and Q2). This allowed participants to freely express their reasoning without being constrained by predefined answer choices.

After collecting the responses, we used a bottom-up (emergent) coding approach: The seven explanation categories were developed directly based on the patterns observed in the responses of the students. This inductive method ensured that the categories reflected the actual ways of thinking of the students rather than fitting them into preset frameworks.

Then each of these seven explanation categories was classified under one of three overarching reasoning lines: meaningful, misguided, or meaningless. This secondary classification captures broader differences in the depth and accuracy of student conceptual reasoning.

The reasoning lines were defined as follows:

1. *Meaningful reasoning*: These responses demonstrate a correct or near-correct understanding of the physical or mathematical principles involved, even if the explanation is not entirely precise. They reflect a deeper engagement with the problem.
2. *Misguided reasoning*: These responses attempt to explain the answer but rely on incorrect assumptions or misunderstandings of the principles. Although inaccurate, they show an effort to think critically about the problem.
3. *Meaningless reasoning*: These responses do not provide meaningful insight into students' understanding, often due to a lack of knowledge, unsupported statements, or irrelevant content.

This classification of reasoning lines is valuable for both qualitative and quantitative analysis. Meaningful reasoning provides insight into genuine understanding, misguided reasoning highlights conceptual difficulties, and meaningless reasoning identifies gaps in knowledge or engagement. By analyzing these categories, we can better understand student reasoning patterns and target specific areas for educational improvement.

The seven categories of explanation, along with their description, significance, example, and associated reasoning are.

1. *Derivative distinction*
   - Description: Correctly explains that derivatives differ between degrees and radians or explicitly mentions the need for a scaling factor when converting between the two (e.g., recognizing that degrees require adjusting the rate of the function).
   - Significance: This represents a correct and sophisticated reasoning, indicating an understanding of solving differential equations and the mathematical differences in derivatives.
   - Reasoning line: *Meaningful reasoning.*
   - Example: "Since the derivative of cos is sin only in radians, then the solution must be in radians to satisfy the wave equation."
2. *Dimensional analysis*
   - Description: Claims that radians must be used for physical or mathematical reasons, often referring to the necessity of specific units. This category includes justifications based on the units of omega and phi.
   - Significance: This is a correct justification, though it does not necessarily indicate a full understanding of derivatives.
   - Reasoning line: *Meaningful reasoning.*
   - Example: "Omega is angular frequency and is measured in radians/seconds, so I would use radians here."
3. *Units equivalence*
   - Description: Claims that both degrees and radians can be used interchangeably, emphasizing conversion between them without adjusting constants like omega or phi. This type includes transitions that do not account for the need to adjust the constants themselves when changing the units.
   - Significance: This is an incorrect justification that reveals a misunderstanding of the distinction between radians and degrees.
   - Reasoning line: *Misguided reasoning.*
   - Example: "Both are correct, as they are essentially a measurement of the same thing."
4. *Experience/authority-based*
   - Description: Relies on personal experience, memory, or authority, such as referencing past instruction or standard practices. Responses often include statements like, "That's how we've always done it," or mention what was taught by a teacher.
   - Significance: This reasoning is often unsupported by a clear understanding of the underlying mathematical principles. These responses may be accompanied by either correct or incorrect answers.
   - Reasoning line: *Meaningless reasoning.*
   - Example: "I remember when I learned about circular motion (which was then applied to simple harmonic motion), I was told that units of omega are radians per second."
5. *Unsupported statement*
   - Description: Provides a statement without further explanation or justification.
   - Significance: While sometimes technically correct, these responses lack depth and clarity. These statements may coincide with correct or incorrect answers.
   - Reasoning line: *Meaningless reasoning.*
   - Example: "Radians are the natural units."
6. *Explicit uncertainty*
   - Description: Admits a lack of knowledge or states that the answer was guessed.
   - Significance: Reflects uncertainty or a lack of engagement with the question.

- Reasoning line: *Meaningless reasoning.*
- Example: "I guessed."

7. *No response*
   - Description: Provides no answer or an irrelevant response, including challenges in understanding, such as "only radians are unitless."
   - Reasoning line: *Meaningless reasoning.*
   - Significance: Indicates a failure to engage with the question or a fundamental misunderstanding.

The coding process was conducted blindly by the first author and two additional independent reviewers. The final coding for each response was determined by the majority agreement among the three coders, except in 11 cases out of 770 (1.4%), where the author opted for a different classification based on contextual analysis. Interrater reliability was assessed using Cohen's kappa coefficient, which yielded moderate (0.56) to high values (0.76) between the coders, indicating robust consistency among the reviewers. The Cohen's kappa values (0.56–0.76) reflect the initial agreement levels among coders before applying majority-rule decisions. This indicates that while agreement was generally high, there was still some variation in initial classifications that was resolved through consensus.

E. Missing data handling

Most students answered almost all questions, and descriptive and bivariate analyses (e.g., distributions, group comparisons, and item-level success rates) were conducted using all available responses per item. As a result, sample sizes for these analyses were typically very close to the full sample of 769 participants. However, regression models required complete responses on all demographic and academic background variables. Therefore, all regression analyses were conducted on the subset of 610 students who provided full data on these variables, using listwise deletion.

F. Analysis plan

We used *SPSS* version 28.0 for descriptive statistics, chi-square tests (to assess associations between categorical variables), and one-way analysis of variance (to compare confidence levels across response categories).

To model predictors of correctness and confidence while accounting for the nested structure of students within countries, we implemented multivariate mixed-effects regression models in *R* using the LME4 package. The country was included as a random intercept in all models to account for between-country variance.

Statistical significance was defined as $p < 0.05$.

IV. RESULTS

In our analyses, we distinguish between descriptive and multivariate analyses. The primary interpretations are based on statistically controlled models—hierarchical (mixed-effects) logistic and linear regressions—that isolate the independent contributions of demographic and academic variables to students' responses while accounting for the nested structure of students within countries. Descriptive and unadjusted analyses are reported to illustrate broad patterns and support interpretive context.

TABLE II. Descriptive statistics of the primary dependent variables. The bold values indicate the correct answers.

	Correct case code for trigonometric derivatives	Correct case code for harmonic oscillator expression
Radians	**26.3% (202)**	**59% (454)**
Both	70.7% (544)	34.7% (267)
Degrees	3% (23)	6.3% (48)

A. Descriptive statistics of student success

Table II presents the descriptive statistics of the primary dependent variables, highlighting participants' accuracy in identifying the correct case codes for trigonometric derivatives and harmonic oscillator expressions.

For trigonometric derivatives (Q3), 26.3% of participants (202) correctly identified that the angle must be in radians for the derivative to be valid. However, 70.7% (544) incorrectly selected the option indicating that both radians and degrees were correct, reflecting a significant misinterpretation of the conditions. A smaller fraction, 3% (23), incorrectly assumed that the angle was in degrees.

For the harmonic oscillator expression (Q4), 59% of participants (454) correctly identified the angle as being in radians. However, 34.7% (267) again misinterpreted the conditions, selecting the option that both radians and degrees were correct. Additionally, 6.3% (48) incorrectly assumed that the angle was in degrees.

These results underscore widespread challenges in understanding among participants, particularly regarding the distinction between radians and degrees in both mathematical and physical contexts. The high percentage of incorrect responses in Q3 and Q4, even among students familiar with trigonometric derivatives from Q1 and Q2, highlights the "blind spot" this study aims to investigate.

For completeness, we note that over 95% of respondents correctly identified the derivatives of sine and cosine in Q1 and Q2. These open-ended questions allowed students to freely write their answers, revealing that all respondents used the standard radian-based expressions, with no attempts to express them in degrees. The most common errors involved misplacing the minus sign, a procedural mistake that is not the focus of this study.

B. Student success by country

Error rates for trigonometric derivatives (Q3) were consistently high across all countries, ranging from 68.7% to 81.7%. Israel exhibited the highest rate of correct

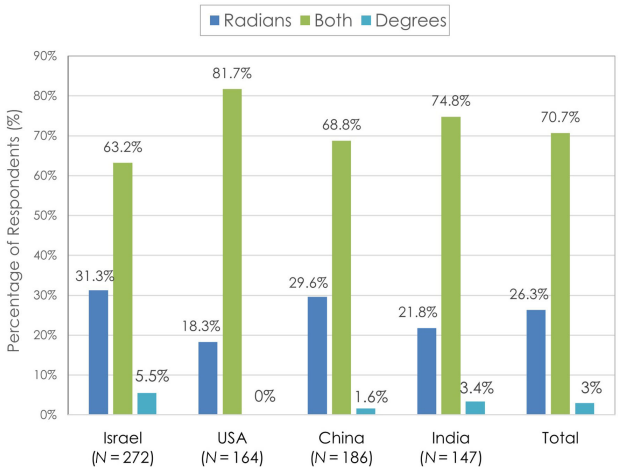


FIG. 1. Distribution of responses for Q3 (trigonometric derivatives) across countries in percentages, highlighting the correct and incorrect responses. A detailed breakdown of percentages and absolute counts is provided in Table IX in the Appendix A.

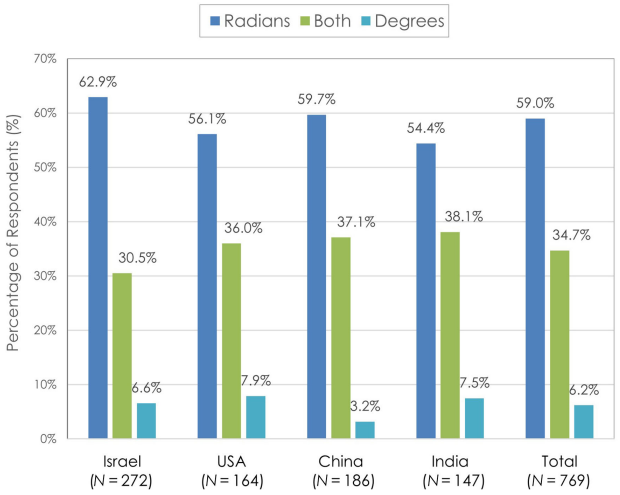


FIG. 2. Distribution of responses for Q4 (harmonic oscillator expression) across countries in percentages, showing the observed trends in correct and incorrect responses. A detailed breakdown of percentages and absolute counts is provided in Table X in the Appendix A.

responses, Radians at 31.3%, followed by China (29.6%). Lower rates were observed in the United States (18.3%) and India (21.8%). The consistently high rates of incorrect responses across countries underscore the widespread nature of the "blind spot."

Figure 1 visualizes the distribution of responses for Q3 across all countries, emphasizing the observed trends in correct and incorrect responses.

In examining the association between responses and country, a significant correlation was observed [$\chi^2(6) = 25.80$, $p < 0.001$]. It is important to note that these results may partially reflect differences in student populations across countries, including variations in academic year and major distribution, which are not identical across the sample. The multivariate mixed-effects regression analysis presented later accounts for these confounding variables and provides a more precise assessment of cross-national differences. Nonetheless, the consistently high rates of incorrect responses suggest that this conceptual blind spot is a global phenomenon.

Error rates for the harmonic oscillator expression (Q4) were similarly elevated across countries, ranging from 37.1% to 45.6%. Israel exhibited the highest rate of correct responses, Radians at 62.9%, followed by China (59.7%). The United States and India had slightly lower rates at 56.1% and 54.4%, respectively. The consistently high error rates across all educational systems underscore the universal nature of this angular unit confusion.

Figure 2 illustrates the distribution of responses for Q4 in all countries, showing the consistent patterns in correct and incorrect answers.

In analyzing the association between responses and country, no significant correlation was found [$\chi^2(6) = 7.58$, $p = 0.27$]. This lack of cross-national variation is consistent with the mixed-effects regression analysis presented later, which detected essentially no between-country variance for this concept. These results reinforce the notion that the confusion regarding angular units represents a truly global cognitive blind spot, regardless of educational system differences.

C. Analysis of respondent types

We classified the responses by the following criteria:

1. Consistent correct: Respondents who answered both Q3 and Q4 correctly.
2. Q3 correct only: Respondents who answered Q3 correctly but Q4 incorrectly.
3. Q4 correct only: Respondents who answered Q4 correctly but Q3 incorrectly.
4. Consistent incorrect: Respondents who answered both Q3 and Q4 incorrectly.

In examining the respondent types across countries, we observe a consistent global pattern. Overall, the highest proportion of respondent types was in the category "Q4 correct only" (38.6%) and the category "consistent incorrect" (35.1%). About 20% of the total sample demonstrated consistent correct responses, while only 6% fell into the "Q3 correct only" category. This distribution pattern is remarkably similar across all four countries. These trends are further illustrated in Fig. 3, which visually represents the distribution of the four respondent types across all countries.

While there are some variations between countries—with Israel and China showing relatively higher consistent correct responses (26.1% and 21.0% respectively) compared to the United States (15.2%) and India (15%)—the overall pattern remains consistent globally. A statistical association was found between country and respondent

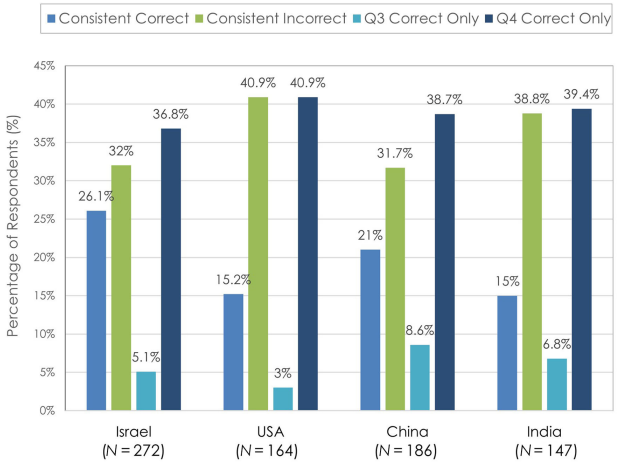


FIG. 3. Distribution of respondent types across countries. The figure categorizes participants based on their accuracy in answering Q3 (trigonometric derivatives) and Q4 (harmonic oscillator expression). The "consistent correct" group is largest in Israel and China, whereas the "Consistent Incorrect" and "Q4 Correct Only" groups dominate in all countries. A detailed breakdown of percentages and absolute counts is provided in Table XI in the Appendix A.

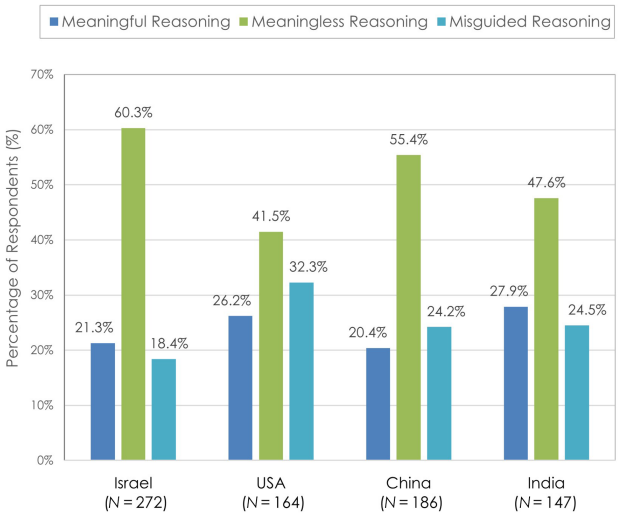


FIG. 4. Distribution of reasoning lines across countries. The figure categorizes participants based on their reasoning classification: Meaningful, misguided, and meaningless reasoning. A detailed breakdown of percentages and absolute counts is provided in Table XII in the Appendix A.

types $[\chi^2(9) = 17.63, p = 0.04]$, though the practical differences are modest given the universal nature of the underlying cognitive challenges.

D. Common challenges in understanding radians in harmonic motion

Table III presents the distribution of common challenges in understanding radians in harmonic motion—Q5. The most common explanation, given by 23.9% of participants (184), was the units equivalence category, which claims that both methods can be used and emphasizes the possibility of converting between them. Dimensional analysis, which asserts that radians must be used for physical or mathematical reasons and refers to the nature and necessity of units, was cited by 17% of participants (131). Unsupported statements, consisting of explanations without any justification, accounted for 13.5% (104).

TABLE III. Distribution of the seven explanation categories of student reasoning. Note: Categories marked with an asterisk (*) indicate correct reasoning.

Explanation	*N*	%
Derivative distinction*	49	6.4
Dimensional analysis*	131	17
Units equivalence	184	23.9
Experience/authority based	80	10.4
Unsupported statement	104	13.5
Explicit uncertainty	79	10.3
No response	142	18.5

Experience/authority-based reasoning, where participants relied on personal experience, teacher authority, or tradition, was given by 10.4% (80). Additionally, 10.3% (79) of participants expressed explicit uncertainty, admitting a lack of knowledge or guessing. Finally, the least common theme was derivative distinction, mentioned by 6.4% (49), which reflected an understanding that derivatives differ between degrees and radians.

To further examine the distribution of reasoning lines across different educational systems, we analyzed the classification of reasoning by country. Across all countries, the most common response pattern was meaningless reasoning (approximately 50% of students), followed by meaningful and misguided reasoning (each around 25%). This general distribution pattern is observed globally, though with some variations between countries.

As shown in Fig. 4, students in the United States (26.2%) and India (27.9%) exhibited the highest rates of meaningful reasoning, whereas lower rates were observed in Israel (21.3%) and China (20.4%). Conversely, the proportion of students classified under meaningless reasoning was highest in Israel (60.3%) and China (55.4%), while the lowest rates were observed in the United States (41.5%) and India (47.6%). Misguided reasoning responses were relatively balanced across countries, ranging from 18.4% in Israel to 32.3% in the United States.

While a statistical association was found between country and reasoning types $[\chi^2(6) = 19.25, p = 0.004]$, the underlying pattern of student reasoning challenges appears consistent across educational systems, with meaningless reasoning being the predominant response globally.

TABLE IV. Confidence level by response.

Correct case code for harmonic oscillator expression	*M*	SD
Radians	3.78	1.09
Both	3.76	1.08
Degrees	2.96	1.34

E. Confidence level

We examined confidence levels among students based on their responses to the harmonic oscillator expression (Q4). A striking pattern emerged: students who answered correctly (Radians) and those who incorrectly believed both units were valid ("Both") exhibited virtually identical confidence levels ($M = 3.78$ vs $M = 3.76$). Only students who selected Degreesshowed meaningfully lower confidence ($M = 2.96$), though this group comprised only 6.3% of the sample.

This finding reveals a critical insight: students who hold the misconception that both radians and degrees are equally valid maintain the same level of confidence as those who answer correctly. This pattern reinforces the hypothesis of a global "blind spot" in trigonometric differentiation—students are confidently incorrect rather than uncertain.

Table IV presents the mean confidence levels for each response type on a five-point Likert scale, with 5 being the most confident. *Post hoc* analysis confirmed no difference between Radians and Both groups ($p = 0.95$), while both differed significantly from the Degrees group ($p < 0.001$).

Students demonstrated remarkably similar confidence levels regardless of their performance patterns. Those who answered both questions correctly ($M = 3.89$) were nearly as confident as those who answered both incorrectly ($M = 3.63$), with students who succeeded on only one question falling in between ($M = 3.69 - 3.73$). This striking consistency reveals that confidence does not track accuracy—students maintain similar self-assurance whether they demonstrate consistent understanding, partial understanding, or persistent misconceptions.

Table V presents the descriptive statistics across all respondent types, confirming minimal variation in confidence levels $[F(2, 765) = 1.80, p = 0.14, \eta^2 = 0.007]$.

An analysis of confidence levels across reasoning categories revealed a striking pattern: students who provided meaningful reasoning ($M = 4.04$) and those who provided misguided reasoning ($M = 3.94$) showed virtually identical confidence levels. Only students who provided meaningless reasoning showed meaningfully lower confidence ($M = 3.48$). This finding suggests that even incorrect but structured reasoning can sustain high confidence levels, further reinforcing the "blind spot" phenomenon where students maintain confidence despite fundamental misconceptions.

TABLE V. Confidence level by respondent types.

	M	SD
Consistent correct	3.89	1.04
Q3 correct only	3.69	1.24
Q4 correct only	3.73	1.11
Consistent incorrect	3.63	1.15

TABLE VI. Confidence level by reasoning.

	M	SD
Meaningful reasoning	4.04	0.93
Misguided reasoning	3.94	0.96
Meaningless reasoning	3.48	1.19

Post hoc analysis confirmed no difference between meaningful and misguided reasoning groups ($p = 0.95$), while both differed from the meaningless reasoning group ($p < 0.001$). These findings are summarized in Table VI, which presents the descriptive statistics for confidence levels across reasoning categories.

F. Hierarchical logistic regression predicting correct responses

To investigate how students' background characteristics relate to their conceptual understanding, we employed a hierarchical modeling framework using generalized linear mixed-effects models (GLMMs). These models are appropriate for data with nested structure, such as students clustered within national education systems.

Specifically, we modeled country as a random intercept, which allowed us to account for baseline differences between countries (e.g., grading norms or curricular emphasis) while estimating the independent contributions of student-level variables such as academic discipline and performance.

This approach avoids oversimplified between-group comparisons and reduces the risk of false positives due to uncontrolled population differences.

Below, we present two models: one predicting correct responses on the trigonometric derivatives question (Q3) and one predicting responses on the harmonic oscillator question (Q4).

1. Hierarchical logistic regression model predicting correct responses on Q3

To examine predictors of students' success on the trigonometric derivative question (Q3), we fit a generalized linear mixed-effects model (GLMM) with a binomial distribution and logit link function. The model included fixed effects for field of study, GPA, mechanics and calculus scores, and academic year. The country was modeled as a random intercept to account for clustering by the national education system, and the model was fitted

on data from 610 students who provided complete responses on all relevant variables.

The variance of the random intercept for country was moderate (SD = 0.35), suggesting some baseline differences between countries, although most of the within-country variation was still present. Importantly, the model's structure allows for more accurate estimation of fixed effects by accounting for nested data.

As shown in Table VII, engineering students were significantly more likely to answer Q3 correctly than life sciences students (odds ratio—OR = 4.22, $p < 0.001$), as were mathematics/computer science students (OR = 3.52, $p = 0.006$) and physics students (OR = 2.63, $p = 0.004$). GPA, mechanics, calculus, and academic year were not significant predictors. These findings support the conclusion that conceptual understanding of trigonometric derivatives varies by discipline more than by academic background or national context.

G. Hierarchical logistic regression model predicting correct responses on Q4

To analyze student performance on the harmonic oscillator expression (Q4), we used the same GLMM structure as for Q3, with country as a random intercept and fixed effects for field of study, GPA, mechanics and calculus, and academic year. The model was fit to data from 610 students who provided complete responses on all relevant variables.

Unlike Q3, the model detected essentially *no variation* between countries (random effect variance ≈0). This boundary solution indicates that the baseline probability of success on Q4 was statistically indistinguishable across national groups. As such, the inclusion of country as a random effect had no measurable contribution to the model, but was retained for consistency with Q3.

As shown in Table VII, the results indicate that engineering students (OR = 1.85, $p = 0.028$) and students with a GPA > 95 (OR = 2.34, $p = 0.010$) were significantly more likely to respond correctly. No significant effects were found for physics or mathematics majors, mechanics or calculus scores, or academic year.

The absence of cross-national variance reinforces the central claim of this paper: that the misunderstanding surrounding angular units in physics is not country-specific, but represents a global blind spot in mathematical reasoning across educational systems.

H. Linear mixed-effects regression predicting confidence level

To examine predictors of students' confidence levels, we conducted a linear mixed-effects regression analysis with 610 students from 4 countries who provided complete responses across all measured variables. The model treated students' self-reported confidence (1–5 scale) as the dependent variable, with country as a random intercept to account for hierarchical clustering within national samples.

Fixed effects included field of study, academic performance indicators (GPA, mechanics, and calculus scores), academic level, and response accuracy on the harmonic oscillator problem (Q4), and presence of explicit uncertainty expressions. The model intercept indicated a baseline confidence level of 2.98 for the reference group (life sciences students). Random-effect decomposition revealed minimal between-country variation (SD = 0.27) relative to substantial within-country individual differences (SD = 0.96), confirming that confidence patterns were primarily driven by individual rather than national factors.

The model revealed that students majoring in physics ($\beta = 0.63$, $p < 0.001$) and engineering ($\beta = 0.50$, $p < 0.001$) reported significantly higher confidence than life sciences students. Mathematics and computer science students did not differ significantly. The strongest predictor of reduced confidence was the presence of explicit uncertainty statements ($\beta = -1.10$, $p < 0.001$). Additionally, students who selected *radians* for Q4 were more confident than those who chose *degrees* ($\beta = 0.47$, $p = 0.006$), and students who selected *both* were marginally more confident ($\beta = 0.34$, $p = 0.052$).

No significant effects were found for GPA, mechanics, or calculus scores, or academic year. The full regression

TABLE VII. Hierarchical logistic regression predicting correct responses.

	Trigonometric derivatives (Q3)				Harmonic oscillator expression (Q4)			
	OR	95% CI		p	OR	95% CI		p
		Lower	Upper			Lower	Upper	
Physics vs life sciences	2.63	1.36	5.09	0.004	0.88	0.53	1.48	0.634
Engineering vs life sciences	4.22	2.20	8.10	<0.001	1.85	1.07	3.22	0.028
Math and CS vs life sciences	3.52	1.44	8.59	0.006	1.46	0.71	2.98	0.303
GPA 95+ vs below 95	1.27	0.65	2.48	0.492	2.34	1.23	4.48	0.010
Mechanics 95+ vs below 95	1.47	0.87	2.49	0.153	0.93	0.57	1.51	0.759
Calculus 95+ vs below 95	1.45	0.93	2.25	0.098	1.14	0.77	1.70	0.509
Bachelor's years 1–3 vs 4+	1.46	0.76	2.80	0.256	1.21	0.77	1.91	0.410
Response confidence level	...	...	...	...	1.29	1.10	1.51	0.002

TABLE VIII. Hierarchical linear regression predicting confidence level.

Variable	β	p
Physics vs life sciences	0.63	<0.001
Engineering vs life sciences	0.50	<0.001
Math and CS vs life sciences	0.30	0.106
GPA 95+ vs lower	−0.09	0.55
Mechanics 95+ vs lower	0.14	0.208
Calculus 95+ vs lower	0.07	0.450
Undergrad (1–3 BA only) vs advanced studies	−0.07	0.642
Explicit uncertainty vs other explanations	−1.10	<0.001
Q4 radians vs degrees	0.47	0.006
Q4 both correct vs degrees	0.34	0.052

results are provided in Table VIII. These results reflect broader patterns of reasoning and confidence explored in the main discussion.

V. DISCUSSION

This study investigated a global blind spot in understanding trigonometric derivatives and their implications for physics education. The analysis focused on students' understanding of the distinction between radians and degrees, their ability to apply this knowledge to physical systems such as harmonic oscillators, the associations between confidence and performance, and the prevalence of these issues across different educational systems and countries. Below, we address each research question and discuss the broader implications.

A. Do students correctly identify the derivatives of trigonometric functions in radians versus degrees?

The results demonstrate a significant gap in students' understanding of the conditions under which trigonometric derivatives are valid in an abstract or mathematical context. Only 26.3% of participants correctly identified that the known derivative of a trigonometric function is valid exclusively in radians. The majority (70.7%) incorrectly assumed that both radians and degrees are equally valid, indicating a widespread misunderstanding of the mathematical distinction. This error is critical, as it reflects a failure to recognize the scaling factor required when differentiating in degrees.

B. Do students understand the difference between radians and degrees in physical systems such as a harmonic oscillator?

Approximately 59% of participants correctly identified that the standard harmonic oscillator equation assumes angle measurements in radians. However, a substantial proportion (34.7%) believed that both radians and degrees are valid. This misunderstanding is particularly problematic in physics, where incorrect assumptions about units can lead to significant errors in modeling and interpretation.

It is important to evaluate the results of these two research questions in the context of previous studies conducted within a similar framework, where the ability of students to comprehend and transfer mathematical and physical concepts was investigated. These studies reveal common patterns of difficulty, along with some possible differences in their outcomes, offering valuable insights into the interplay between mathematics and physics.

Beichner [27] examined students' understanding of kinematics graphs (position-time and velocity-time) using a dedicated assessment tool (TUG-K). The findings revealed that students struggled to connect slopes with their physical meaning, such as velocity. Physical graphs introduced confusion, as students found it challenging to translate mathematical understanding into physical concepts.

Christensen and Thompson [28] focused on students' ability to interpret derivatives and slopes in a purely mathematical context, without introducing physical elements. It revealed that many students lacked a deep understanding of the relationship between the derivative and the slope, even in a mathematical context. Difficulties in physics often stem from a lack of foundational understanding of derivatives in mathematics.

Carli *et al.* [10] assessed students' ability to solve isomorphic questions in mathematics and physics, showing that students performed better in purely mathematical questions than in physical ones. This was attributed to the cognitive bridge required to transfer knowledge between the two domains. Physical contexts impose an additional cognitive demand that many students struggle to manage effectively.

However, our results showed that students were more successful in identifying the necessity of using radians in physical questions than in mathematical ones. This finding highlights that students, within a physical context, demonstrated a higher recognition rate for the dependency on radians.

These findings may be due to grounding the problem with a physical anchor or representation commonly found in physics contexts. Students' familiarity with the particular representations associated with the harmonic oscillator may have allowed them to recognize that commonly used expressions [for example, ω, $x(t) = A\cos(\omega t + \phi)$] are valid only when trigonometric functions are expressed in radians. This suggests that, in some cases, physical context provided a support structure for students to correctly identify the relevant mathematical framework. This pattern aligns with the concept of anchor equations in the literature, where familiar physics equations serve as cognitive anchors linking physical ideas with mathematical structure [32]. In our case, the harmonic oscillator equation plays this role, helping students recognize the need for radians within a familiar physical context. However, this improved

recognition does not necessarily reflect a deep mathematical understanding. Students may well have relied on their memory of familiar physical expressions rather than systematically analyzing the difference between radians and degrees.

In light of Gifford and Finkelstein's [22,23] framework on mathematical sensemaking (MSM), our findings may reflect a distinction in how students engage with mathematical concepts in different contexts. Specifically, in the mathematical setting, students must apply derivatives as a tool to make sense of another mathematical object [e.g., differentiating $\cos(\theta)$], a process categorized as MSM-M (mathematical sensemaking in mathematics). In contrast, in the physics context, the physical system (the harmonic oscillator) may have served as a tool to interpret derivatives, aligning with PSM-M (physical sensemaking in mathematics). This shift in reasoning suggests that students may perceive these as fundamentally different types of problems, activating distinct cognitive resources.

We acknowledge that the order of the questions (mathematical first, followed by physical) may have influenced students' responses. In this study, all questionnaires were administered in the same sequence, and we did not test for reversed order effects. Future research could explore whether altering the sequence of mathematical and physical questions impacts students' reasoning patterns, as has been done in previous studies [10].

C. Are there significant differences in understanding between countries and educational systems?

Cross-national analysis using mixed-effects logistic regression ($N = 610$ students, 4 countries) revealed distinct patterns for the two mathematical concepts. For the harmonic oscillator question (Q4), the random intercept variance was effectively zero, indicating no baseline differences between countries in angular unit comprehension. Error rates remained consistently elevated across all educational systems (38.1%–45.6%), suggesting universal difficulty regardless of national pedagogical approaches.

The trigonometric derivatives question (Q3) showed moderate between-country variation (SD = 0.35), demonstrating that educational systems influence performance on this concept. However, error rates remained substantial across all countries (68.7%–81.7%), indicating that while statistical differences exist, the fundamental challenge transcends national boundaries.

D. What is the correlation between students' confidence levels and the correctness of their answers?

One of the most intriguing aspects of our findings is the relationship between students' confidence levels and their conceptual understanding. While traditional assessment methods might categorize incorrect responses uniformly as evidence of misunderstanding, a more nuanced analysis of students' confidence levels and reasoning patterns reveals a complex landscape of conceptual difficulties. This complexity becomes particularly evident when viewed through the theoretical lens of confidence-based assessment.

In analyzing students' responses and confidence levels, it is crucial to distinguish between two fundamentally different types of learning difficulties: knowledge gaps and alternative conceptions. Hasan *et al.* [33] proposed a theoretical framework that differentiates between these through the Certainty of Response Index (CRI), where incorrect answers accompanied by low confidence indicate knowledge gaps, while incorrect answers with high confidence suggest the presence of alternative conceptions—robust cognitive structures that differ from accepted scientific understanding.

Our findings align with this theoretical framework in several significant ways. Students who incorrectly identified trigonometric derivatives as valid in both radians and degrees demonstrated high confidence levels ($M = 3.76, \mathrm{SD} = 1.08$) statistically indistinguishable from those who answered correctly ($M = 3.78, \mathrm{SD} = 1.09$). According to the model of Hasan *et al.*, this pattern is characteristic of alternative conceptions rather than mere knowledge deficiency.

Further support for this theoretical framework emerges from the analysis of students' explanations. Those providing meaningless reasoning exhibited the lowest confidence levels ($M = 3.48$, $\mathrm{SD} = 1.2$), while students offering meaningful explanations, whether correct or incorrect, displayed significantly higher confidence ($M = 4.04$, $\mathrm{SD} = 0.94$ and $M = 3.94, \mathrm{SD} = 0.96$, respectively). This pattern aligns with the CRI framework's distinction between knowledge gaps (manifested in low confidence) and alternative conceptions (exhibited through high confidence regardless of correctness).

These findings indicate that students' difficulty in understanding the role of radians in trigonometric derivatives stems from a strong commitment to the equivalence of degrees and radians, rather than a simple knowledge gap. The consistently high confidence levels among students who believe in the mathematical equivalence of radians and degrees, coupled with their ability to provide meaningful (albeit incorrect) justifications, suggest a deeply ingrained alternative conceptual framework. The persistence of this pattern across four distinct educational systems and cultural contexts further underscores the fundamental nature of this alternative conception in students' mathematical reasoning.

This confidence-maintenance effect is not limited to trigonometric derivatives. A similar pattern has been observed in mechanics education, where alternative conceptions persist despite students' strong confidence in their reasoning. Research on students' understanding of force and motion has shown that misconceptions in mechanics are often accompanied by high confidence, reinforcing the idea that such errors are not merely the result of missing

knowledge but rather stem from deeply ingrained alternative conceptions [34].

E. Additional considerations regarding confidence levels

The overall confidence level reported was moderate ($M = 3.72, \mathrm{SD} = 1.11$). One possible explanation for this finding is that participants were required to justify their answers before rating their confidence. Previous research in mathematical problem solving suggests that requiring people to provide explanations before making confidence judgments tends to lower reported confidence levels, although the general tendency towards overestimation persists [35]. This interpretation is further supported by our hierarchical linear regression analysis, which identifies explicit uncertainty statements as the most significant predictor of reduced confidence. Specifically, participants who explicitly expressed uncertainty exhibited a substantial decrease in confidence ($\beta = -1.06$, $p < 0.001$), with this predictor demonstrating both the strongest statistical significance and the highest absolute β value in the model. This suggests that the requirement for verbal justification played a central role in moderating confidence levels across the sample.

Finally, it is important to consider which factors did not significantly influence confidence levels. While students' field of study was a significant predictor of performance, it also played a role in confidence levels, with students in physics and engineering showing higher confidence than those in life sciences. However, the academic year did not exhibit a strong effect on confidence in this context. This suggests that, while disciplinary background contributes to confidence, general academic experience does not necessarily lead to greater self-assurance in conceptual physics questions. These differences may reflect not only curricular emphases but also deeper cultural commitments within each discipline regarding the role of formal representations and unit conventions.

F. How do students' explanation patterns relate to answer accuracy and confidence?

The analysis of the students' justifications revealed three distinct reasoning patterns: (i) *Correct reasoning*, based on differentiation or dimensional analysis; (ii) *Incorrect but structured reasoning*, where the students argued that the trigonometric derivatives are valid in both degrees and radians; and (iii) *Incoherent or authority-based reasoning*, lacking conceptual justification.

The most common response (25%) was *incorrect but well-articulated reasoning*, in which students claimed that trigonometric derivatives are equally valid regardless of angular units. This structured yet incorrect reasoning represents a cognitive "blind spot," where students do not recognize the need for consistency between angular units and differentiation rules.

Additionally, 50% of students provided explanations that lacked conceptual depth or relied on authority (e.g., "this is how it was taught"), while 10% explicitly expressed uncertainty regarding their answer.

These findings suggest that the root cause of this misconception is a combination of a deeply ingrained alternative conception alongside a lack of formal knowledge. Together, these factors contribute to students' difficulty in distinguishing between valid and invalid applications of trigonometric derivatives.

G. How does an academic discipline influence students' responses

To evaluate the impact of academic discipline on students' performance in both questions, we rely on the hierarchical logistic regression models presented above. These analyses reveal a consistent advantage for engineering students, who were significantly more likely to answer both the trigonometric derivative question (Q3) and the harmonic oscillator question (Q4) correctly. Students in mathematics, computer science, and physics showed a significant advantage over life science students only in the mathematical question (Q3). In the physics-based question (Q4), none of these disciplines demonstrated a statistically significant advantage over life sciences. Similar patterns were observed in the supplementary correlation analyses by academic discipline (see Appendix B), providing additional support for these findings, despite the absence of statistical controls in those analyses.

Interestingly, an analysis of confidence levels provides a different perspective. Although engineering and physics students expressed greater confidence in their responses, their actual success rates differed. This suggests that engineering students' confidence may be supported by their superior performance, while physics students may feel confident due to their familiarity with the subject rather than explicit proficiency in the mathematical formalism.

These findings may reflect differences in how disciplines approach mathematical reasoning in physics-related contexts. Engineering curricula often emphasize structured mathematical problem solving and applied differential equations, which could contribute to stronger performance in these tasks. Physics education, by contrast, frequently prioritizes conceptual reasoning and physical intuition over explicit mathematical formalism, which may explain why physics students did not outperform life sciences students in this domain.

A possible factor contributing to the higher success rates of engineering students compared to physics students may be the demanding nature of admissions to engineering programs. In many countries, including India, the United States, and Israel, admission to engineering studies is significantly more competitive than admission to physics and life sciences departments. As a result, engineering students in our sample may have undergone a more

rigorous selection process, which could explain their higher performance in this study.

Further research is needed to explore how curricular differences influence students' ability to apply formal mathematical reasoning in physics-related problems. A comparative study across disciplines, focusing on how mathematical concepts are integrated into physics education, could provide deeper insights into the extent to which instructional approaches shape both conceptual understanding and computational proficiency.

H. How well do grades in foundational courses predict students' answers?

A key assumption in physics education is that success in foundational courses such as mechanics and calculus should correlate with the student's ability to apply fundamental mathematical tools correctly in physical contexts.

We rely on the hierarchical logistic regression models presented above, which indicate that this assumption does not hold: self-reported grades in mechanics and calculus were not significant predictors of success in Q3 or Q4. These analyses confirm that neither mechanics nor calculus grades were statistically significant predictors of success. The only academic performance variable that showed a significant effect was a high overall GPA (95+), which predicted success in Q4, but not in Q3.

To further examine this relationship, see the supplementary analyses in Appendix B, which present the correlations between students' grades in foundational courses and their performance on Q3 and Q4. These findings, while not showing significant direct relationships, provide additional context that aligns with the results from the hierarchical logistic regression, reinforcing the interpretation of the main analysis.

This result suggests that even students with strong technical backgrounds in mechanics and calculus struggle to correctly identify and apply trigonometric derivatives, reinforcing the cognitive blind spot hypothesis. The ability to manipulate mathematical expressions in a computational setting does not necessarily translate to a conceptual understanding of their underlying constraints and applicability in physical scenarios. These findings align with previous research in physics education research (PER), including Mazur [36], who demonstrated that algorithmic expertise does not ensure conceptual understanding.

VI. CONCLUSION

This study reveals a blind spot in the students' understanding of trigonometric derivatives. Surprisingly, the difficulty in distinguishing between radians and degrees is consistent across four different countries and various educational systems, indicating a fundamental challenge in mathematics and physics education. While most students struggle to identify the necessity of radians in pure mathematical contexts, they perform better when encountering the same concept in concrete physical situations. This finding of a physical anchor effect suggests that concrete physical contexts can serve as a cognitive bridge or a touchstone) for abstract mathematical concepts [32,37]. However, the fact that approximately 40% of students make mistakes even in physical contexts points to an urgent need to modify the way this topic is taught in physics courses.

These findings have significant implications for physics and mathematics instruction. We recommend that calculus and introductory physics courses incorporate dedicated instruction time for trigonometric derivatives, emphasizing the fundamental principles underlying their distinct behavior in radians versus degrees. This approach should integrate multiple evidence-based, interactive pedagogical strategies, e.g., visual demonstrations comparing trigonometric functions plotted in both units on the same graph, dynamic simulations illustrating their different rates of change, and structured exercises comparing derivatives at various angles. [38,39] Given students' extensive prior experience with degrees in early education, the transition to radians in higher education requires explicit guidance and clear demonstration of its necessity. At the same time, this approach can build on Arons' notion of "concept first name after"—beginning with conceptual understanding and moving to more formal representations subsequently [40]. Arons' teaching methods should extend beyond computational success to include assessment tools that evaluate students' conceptual understanding, requiring them to articulate why radians are necessary rather than merely performing mechanical calculations. The integration of physical examples and contexts in both mathematical instruction and practice, starting from the early stages of teaching derivatives, will enable students to recognize inherently where and why radians are essential.

This study has several limitations to consider. Our sample, drawn exclusively from leading research universities, may represent students who are academically better prepared than average. Additionally, while the study included four major countries, there may be other countries where students' understanding patterns of this topic differ, particularly in nations with substantially different educational systems or curricula from those examined. Furthermore, the interpretation of certain findings, such as confidence levels in responses, can be culturally dependent, and cross-cultural comparisons should be approached with caution when dealing with subjective measures collected in different languages. These limitations not only contextualize our findings but also point to promising directions for future investigation.

Future research could focus on developing and testing educational interventions that emphasize the connection between trigonometric derivatives and the use of radians. Specifically, studies might examine the effectiveness of

incorporating visual demonstrations of rates of change in teaching this topic. Furthermore, extending this research to other countries could provide deeper insight into the universality of this phenomenon and identify successful teaching approaches in different educational systems. Moreover, it would be valuable to explore whether the physical anchor effect can be harnessed as a pedagogical tool, specifically whether instruction beginning with physical examples can enhance students' mathematical understanding in more abstract domains as well, following Arons' suggestion [40].

This study reveals a widespread conceptual failure in understanding trigonometric derivatives: most students do not recognize that these derivatives are valid only in radians, although they identify this better in physical contexts. This finding points to a significant gap between mathematics and physics instruction that requires changes in teaching methods. Without such a change, students will continue to memorize formulas without understanding their deeper meaning.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [41].

APPENDIX A: ADDITIONAL TABLES AND DATA

TABLE IX. The distribution of responses for the correct case code for trigonometric derivatives by country. Percentages and absolute counts are provided for each response category.

	Radians	Both	Degrees	Total
Israel	31.3% (85)	63.2% (172)	5.5% (15)	(272)
United States	18.3% (30)	81.7% (134)	0.0% (0)	(164)
China	29.6% (55)	68.8% (128)	1.6% (3)	(186)
India	21.8% (32)	74.8% (110)	3.4% (5)	(147)
Total	26.3% (202)	70.7% (544)	3.0% (23)	(769)

TABLE X. The distribution of responses for the correct case code for harmonic oscillator expression by country. Percentages and absolute counts are provided for each response category.

	Radians	Both	Degrees	Total
Israel	62.9% (171)	30.5% (83)	6.6% (18)	(272)
United States	56.1% (92)	36.0% (59)	7.9% (13)	(164)
China	59.7% (111)	37.1% (69)	3.2% (6)	(186)
India	54.4% (80)	38.1% (56)	7.5% (11)	(147)
Total	59.0% (454)	34.7% (267)	6.3% (48)	(769)

TABLE XI. The distribution of respondent types by country.

Country	Consistent Correct	Correct Q3 only	Correct Q4 only	Consistent Incorrect	Total
Israel	26.1% (71)	5.1% (14)	36.8% (100)	32.0% (87)	(272)
United States	15.2% (25)	3.0% (5)	40.9% (67)	40.9% (67)	(164)
China	21.0% (39)	8.6% (16)	38.7% (72)	31.7% (59)	(186)
India	15.0% (22)	6.8% (10)	39.4% (58)	38.8% (57)	(147)
Total	20.4% (157)	5.9% (45)	38.6% (297)	35.1% (270)	(769)

APPENDIX B: SUPPLEMENTARY ANALYSES: CORRELATIONS BETWEEN FOUNDATIONAL COURSES, ACADEMIC DEPARTMENT, AND STUDENT SUCCESS

In this appendix, we provide supplementary analyses that offer additional context to the main results. Specifically, we present the correlations between students' grades in foundational courses (Calculus) and their performance on the trigonometric derivatives (Q3) question, and between students' grades in Mechanics and their performance on the harmonic oscillator (Q4) question. Additionally, we explore the relationship between academic department (field of study) and student success in answering both questions.

TABLE XII. The distribution of respondent types by country.

Country	Meaningful reasoning	Meaningless reasoning	Misguided reasoning	Total
Israel	21.3% (58)	60.3% (164)	18.4% (50)	(272)
United States	26.2% (43)	41.5% (68)	32.3% (53)	(164)
China	20.4% (38)	55.4% (103)	24.2% (45)	(186)
India	27.9% (41)	47.6% (70)	24.5% (36)	(147)
Total	23.4% (180)	52.7% (405)	23.9% (184)	(769)

TABLE XIII. Correctness of the students' responses for trigonometric derivatives (Q3) by grouped calculus score in all countries.

	Israel		USA		China		India	
Calculus score	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct
Low (0–79)	78.9% (30)	21.1% (8)	84.6% (11)	15.4% (2)	74.1% (43)	25.9% (15)	86.7% (13)	13.3% (2)
Medium (80–94)	73.9% (85)	26.1% (30)	85.7% (60)	14.3% (10)	67.6% (71)	32.4% (34)	78.6% (66)	21.4% (18)
High (95–100)	59.8% (70)	40.2% (47)	78.6% (44)	21.4% (12)	73.9% (17)	26.1% (6)	70.6% (12)	29.4% (5)
χ^2 (2)	7.56		1.14		0.91		1.22	
p	0.02		0.56		0.63		0.54	

These analyses are intended to complement the main results of the study, providing further insights into how students' academic backgrounds and fields of study may impact their ability to succeed in the given tasks. The correlations are discussed in light of the hierarchical logistic regression findings, supporting the interpretation of the main analyses while also offering additional perspectives on the factors influencing student performance.

1. The correlation between calculus grades and students' answers to derivatives

We examined whether students' self-reported scores in Calculus are correlated with the correctness of their answers in the Trigonometric Derivatives question (Q3).

Results showed that no significant association was found between the grouped calculus score and the correctness of the answers in the United States $[\chi^2(2) = 1.14, p = 0.56]$, China $[\chi^2(2) = 0.91, p = 0.63]$, or India $[\chi^2(2) = 1.22, p = 0.54]$. However, a significant association was found in Israel $[\chi^2(2) = 7.56, p = 0.02]$, suggesting that higher calculus scores were associated with improved performance in this country only.

These findings are summarized in Table XIII, which presents the distribution of correct and incorrect answers by grouped calculus score categories across all four countries.

2. The correlation between mechanics grades and students' answers to harmonic oscillator questions

We examined whether students' self-reported scores in Mechanics are correlated with the correctness of their answers in the Harmonic Oscillator Expression question (Q4).

Results showed that no significant association was found between the grouped mechanics score and correctness in Israel $[\chi^2(2) = 2.89, p = 0.23]$, the United States $[\chi^2(2) = 1.95, p = 0.37]$, or China $[\chi^2(2) = 2.77, p = 0.25]$. However, a significant association was found in India $[\chi^2(2) = 6.08, p = 0.05]$, suggesting that higher mechanics scores were associated with improved performance in this country only.

These findings are summarized in Table XIV, which presents the distribution of correct and incorrect answers by grouped mechanics score categories across all four countries.

3. The relationship between academic departments and students' answers

We examined whether the academic department (field of study) is associated with the students' answers to Q3 (correctness of identifying derivatives) and Q4 (correctness of identifying the correct condition for a harmonic oscillator).

Results showed a significant association between the academic department and the correctness of Q3$[\chi^2(4) = 22.26, p < 0.001]$ with the highest rate of correct responses among Engineering (39.3%), and the lowest rate of correct responses among chemistry, life sciences, or neuroscience (17.1%).

Results showed a significant association between the academic department and the correctness of the answers of Q4$[\chi^2(4) = 13.72, p < 0.001]$ with the highest rate of

TABLE XIV. Correctness of the students' responses for the harmonic oscillator expression (Q4) by grouped mechanics score in all countries.

	Israel		United States		China		India	
Mechanics score	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect	Correct
Low (0–79)	39.4% (28)	60.6% (43)	51.9% (14)	48.1% (13)	51.5% (17)	48.5% (16)	66.7% (14)	33.3% (7)
Medium (80–94)	31.7% (39)	68.3% (84)	44.2% (34)	55.8% (43)	36.4% (40)	63.6% (70)	40.4% (36)	59.6% (53)
High (95–100)	43.2% (32)	56.8% (42)	35.0% (14)	65.0% (26)	33.3% (7)	66.7% (14)	27.3% (3)	72.7% (8)
χ^2 (2)	2.89		1.95		2.77		6.08	
p	0.23		0.37		0.25		0.05	

TABLE XV. Correctness of the students' responses by academic department.

	Trigonometric derivatives question		Harmonic oscillator expression question	
Academic department	Incorrect	Correct	Incorrect	Correct
Physics	77.2% (298)	22.8% (88)	45.6% (176)	54.4% (210)
Engineering	60.7% (102)	39.3% (66)	30.4% (51)	69.6% (117)
Math or computer science	71.0% (44)	29.0% (18)	33.9% (21)	66.1% (41)
Chemistry, life sciences, or neuroscience	82.9% (92)	17.1% (19)	45.9% (51)	54.1% (60)
Other	75.6% (31)	24.4% (10)	39.0% (16)	61.0% (25)
χ^2 (4)	22.26		13.72	
p	<0.001		<0.001	

correct responses among engineering (69.6%) and math and computer science (66.1%), while the lowest rate of correct responses among physics and any other field (54.4%) and chemistry, life sciences, or neuroscience (54.1%). As shown in Table XV, the rates of correct answers differ significantly across academic departments.

APPENDIX C: FULL DEMOGRAPHIC QUESTIONNAIRE

Participants were also asked to complete a demographic section to allow analysis of responses across academic and personal backgrounds. Below is the full demographic questionnaire as presented to participants:

Q7: Your age. (Open-ended question)

Q8: With which of the following do you identify? Male/female/nonbinary or other/prefer not to say.

Q9: Degree you are currently seeking: bachelor's/master's/doctoral/postdoctoral or other.

Q10: Which year are you currently enrolled in: 1/2/3/4/5/6.

Q11: Your major field(s) of study (select all that apply): Physics/mathematics/engineering/chemistry/computer science/life sciences/Other.

Q12: Your current GPA (estimated is fine) within your major (e.g., 3.3). (Open-ended question)

Q13: Country or state where you went to high school. (Open-ended question)

Q14: Overall score in mechanics/University Physics I (e.g., Phys1110)—percentage, if applicable: 0–59/60–79/80–89/90–94/95–100/N/A or don't know.

Q15: Overall score in calculus I (e.g., Math1300)—percentage, if applicable: 0–59/60–79/80–89/90–94/95–100/N/A or don't know.

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