Investigative analysis tool for physics textbook representations: The force diagram case study

Alessandro Pietro Ventura, Francesco Longo, Maria Peressi, and Valentina Bologna
Physics Department, University of Trieste, Via Alfonso Valerio, 2, 34127 Trieste TS, Italy

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Recent advancements in educational research have underscored the significance of multiple representations (MR) in teaching practices, supporting student learning outcomes. Despite this progress, to our knowledge, two issues remain open: currently, there is no established tool for evaluating the quality of representations used in learning materials, and there is no comprehensive research on how such representations are integrated into teaching resources, especially in school textbooks. Our study aims to address these gaps by developing an analytical tool for categorizing representations in physics learning materials. We designed a grid for the analysis based on the theoretical underpinnings of MR, cognitive load theory, and modes of reasoning subframeworks. This grid helps identify MR features: building parameters, functionalities, and levels of optimization for learning environment usage, thus establishing the insights necessary to improve and enhance the quality of the representations. We also tested and applied this tool to force diagrams (FDs), which are commonly used in helping students learn mechanics, examining these representations across selected chapters from widely used high school physics textbooks. In this paper, we provide the results of our investigation of the current state of FDs in these textbooks as a compare-and-contrast case study. At the same time, we aim to offer insights into the effectiveness and potential areas for improvement in deploying MR for teaching-learning purposes.

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I. INTRODUCTION

Over 30 years of physics education research have demonstrated the paramount role of multiple representations (MR) in scaffolding students' physics learning [1–7]. Their use finds its origin in the early 1990s, based firstly on previous work devoted to overcoming undergraduate students' problem-solving difficulties [2,8–10]. MR use in concept knowledge building has spread, resulting in many examples of their effectiveness in a broad range of different physics topics such as work and energy processes [3,11], mechanics [12,13], or electric current [14]. Indeed, MR have been recognized as a powerful tool for reasoning [1,6,10,15–17], supporting scientific talk [18–20], and solving regular [15,21–23] or Jeopardy problems [4]. The plurality of their use undoubtedly confirms how MR promote conceptual understanding [13,16], make physics more accessible to learners [24], and support math and physics interplay and joint integration [25–27]. The representations have an epistemic role in science, and different representations match different kinds of scientific explanations. Therefore, understanding how to coordinate MR can lead to a more authentic learning approach where studying physics also means doing physics [10,24,28]. Helping and supporting students in their learning process [29–31], MR are extensively recommended in teaching practices [6,10,32–37]. Clearly, (as stated in Ref. [16] page 12) "representations provide an empowering learning tool that might support students' understanding of physics concepts, and it is impossible to learn physics without representations."

Despite this recognized role in cognitive and epistemological perspectives and teaching-learning processes, future research needs to address some gaps. Even though there's plenty of evidence on the effectiveness of MR as a whole [6,37], to our knowledge, the features that could make a representation effective are much less discussed. Are there some rules for graphical representations that make them more effective compared to similar representations that do not follow these rules? This aspect might be particularly relevant when focusing on low-achieving students or those affected by specific learning disabilities [38,39] that could benefit from these hypothetical rules to enhance representations' effectiveness. Then, there is limited research regarding how teachers use representations, how their use of representations affects their teaching and students' learning, and, most importantly, how MR are used in textbooks [16,23,30,40]. This is important, considering that textbooks are widely used in physics courses across various levels of instruction. Physics

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textbooks support students in developing fluency in switching from one form of representation to another and overcoming their difficulties when engaged in MR use [31,41]. Also, students in their work tend to use the same representations that are presented to them in their textbooks, and this plays a significant role in their process of knowledge development [42]. Nonetheless, the level of consistency and the role of multiplicity in the representations' format within textbooks have not yet been investigated.

The goal of the present work is to contribute to and partially fill these gaps by producing a public tool ideal for analyzing representations used in physics textbooks, therefore answering the question: "What tools can be used to objectively analyze visual representations in teaching materials?" We also tested said tool, applying it to high school physics textbooks with the following research question in mind: "What are the functions, roles, and visual features of the force diagrams used in physics textbooks?" This study aims to evaluate to what extent force diagrams (FDs) are used in physics textbooks to meet learning goals, focusing on the following working steps:

(1) investigating theoretical frameworks for designing an analysis tool for representations used in physics textbooks;
(2) developing the analysis tool based on the chosen framework;
(3) testing the analysis tool in the case study of force diagram representation;
(4) discussing and evaluating the analysis tool for future work and research.

We provide an analysis tool for investigating the use of representations in physics textbooks, recognizing their paramount roles in helping students develop reasoning skills and conceptual understanding. Our investigation also attempts to evaluate how visual features of a representation affect cognitive load processes. Physics textbooks are the first scientific texts students encounter in their instruction. Overcoming difficulties in "reading representations" and preparing textbooks with "new" features should lead to the adoption of physics textbooks that satisfy inclusive conditions, promoting a Universal Design for Learning [43].

II. TOOL DESIGN

Given that our goal is to design the rubrics with which we can assess and categorize the use of representations in teaching materials and learning processes, our first step was to search for the appropriate theoretical frameworks necessary to extensively describe visual representations (VR). It should be noted that in the work about multiple representations cited before, more specifically within the Investigative Science Learning Environment (ISLE) approach [44] (more on that on Sec. III C), some rubrics to assess use of representation are already available. However, those rubrics are meant to evaluate scientific abilities' development in students, therefore, those regarding multiple representations are also focused mainly on how they are employed in students' work rather than on their structural and visual features. Our work is complementary and aims to establish a rubric for fine grid analysis on the representation itself and is meant to be used primarily in textbooks and in educational materials analysis. However, the two works are connected, as we believe that our analysis tool could help in better identifying which section of the original rubrics should be taken into account for a certain task.

We drew upon several theoretical frameworks to analyze as extensively as possible how representations are used in textbooks. In this section, we discuss our useful findings from literature on visual representations (Sec. II A) and then show how we put them together in a comprehensive analysis tool (Sec. II B).

A. Theoretical frameworks

1. Multiple representations

From the MR framework, we extrapolated a categorization of the possible functions that representations may fulfill referencing the functional taxonomy developed by Ainsworth [1]. Representations are generally divided into two macrocategories: internal and external. The former refers to the mental models that the brain builds in the course of reasoning and that evolve with learning. In the process of knowledge building, consolidating a concept means producing an internal representation [45] that is ready to be recalled in other contexts, such as in problem solving. Text and formulas are often abstract and symbolic in nature, whereas the mind relates better to more visual, figurelike representations. The brain processes images, or referents [46], and associates them with abstract concepts. In the course of reasoning and during the construction of new ideas, the mind recalls these referents [7,45]. Given their intangible nature, those are not subjects of our analysis.

On the other hand, external representations are concrete and, therefore, in nature, also visible to other individuals, such as pictures, texts, graphs, symbols, etc. [1,16], and they will be the focus of our study. They can take on different functions in the context of learning related to the individual's use of this tool. Thus, identifying a representation's function cannot be done *a priori*, but one must consider the dependence on the user: their level of knowledge and skills and how the mental construction of concepts takes place in their minds [1,5]. However, in teaching, trying and anticipating students' thoughts and behaviors is an everyday practice. We cannot analyze a representation without its context, its target audience and their level of conceptual understanding of the discipline, but a teacher does know their classes and students and could make a valid guess. Also in textbook analysis, one assumes that the "previous knowledge of the students" is based on the previous chapters of the book. So even though

we are stating that establishing a representation function objectively is difficult because of all the subtle nuances one should take into account that might add a level of subjectivity, we do think it is worth the effort. Trying to reach objectivity, even without fully accomplishing it, develops metacognition on teaching practices so we think it useful to pursue our research.

External representations serve four main functions in learning situations:

(1) to complement (representations present complementary information and support complementary cognitive processes [1]);
(2) to constrain (representations reduces (mis)interpretations of other unfamiliar representations of a physical concept, therefore assisting students [14]);
(3) to construct (multiple representations of the same situation support students in understanding abstract concepts and establishing relations among them [1,14]);
(4) to evaluate (different representations bridge abstract words with mathematical relations [10,11], making explicit what is implicit and therefore gaining an evaluative purpose. Students checking for consistency between representations can assess their work [7,13,47]).

The main functions' components and their specificity are detailed in Fig. 1.

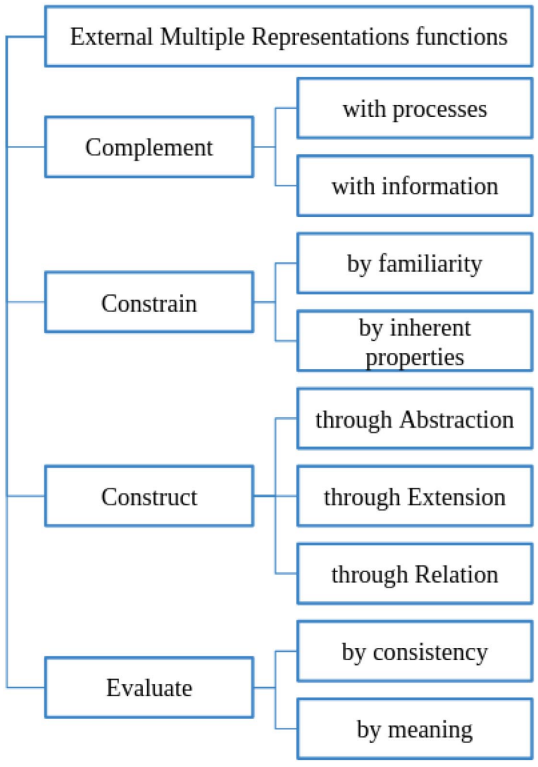


FIG. 1. External multiple representations (EMR) functions [1,5,7,10,11,13,17,47].

2. Role in text and reasoning

In this section, we will be focusing on VR and on their role within the textbook. We will explore the relation between the text and the pictures and also the way in which VR are involved in scientific reasoning.

The goal of a textbook should be to draw on scientific, didactic, and pedagogical elements to develop explanations that make scientific topics and related content more accessible to the reader. There is evidence that the student's meaning-making process is affected not only by the student's prior experience but also by the explanations provided by the textbook [48]. Therefore, textbooks should aim to be as accessible as possible to scaffold students in the sense-making process, meaning being capable of making students recall basic scientific concepts to independently construct the readers' own explanation through scientific reasoning.

We want our students to reason like experts, and scientists often use various semiotic resources while developing their explanations or models, ranging from text and mathematical expressions to pictures and sketches, to give greater structure, validity, and accessibility to their reasoning [18]. A textbook, as does a teacher, acts as a bridge between this knowledge and skills possessed by expert physicists and the knowledge and scientific skills of the readers. If the goals of a physics course include teaching students the modes and the ways of thinking by which physicists operate, then the reasoning practices promoted by the textbook must go beyond the verbal and algebraic mathematical component alone, focusing also on the importance of VR. They constitute a fundamental epistemological aspect in the delineation of "thinking and acting like a physicist" [49,50], and therefore it is necessary to expand the analysis of scientific reasoning to VR as well [51] and to take into account the fact that the reader is faced with the need to interpret not only verbal components but also visual ones in order to understand the reasoning used by the textbook author. We should not work under the assumption that images speak by themselves; their connection to the text is essential [52–55], and we should not take for granted the idea of students interpreting it as intended by the author [56].

To consider all these implications, we used as a reference theoretical framework a study conducted by Triantafillou *et al.* [40,57], where they analyzed written text as a sequence of modes of reasoning: "the logical act created by a part of the text of a thematic unit." In their work, the authors also proposed a categorization of the possible connections between VR and text and between VR and reasoning, summarized in Fig. 2. We used this categorization as a reference framework for further developing our tool of textbook analysis.

We identified eight categories concerning the role of VR in reasoning across two dimensions: (1) the relation of VR with the text and (2) the role of VR in the reasoning

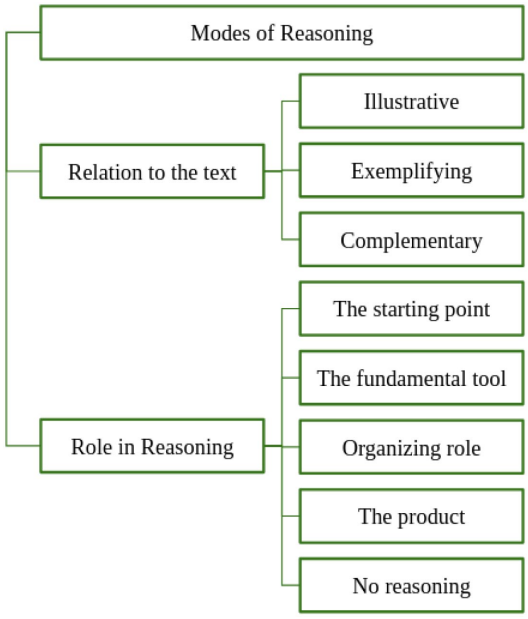


FIG. 2. Categorization of VR in relation to the text and their role in reasoning [40,57].

process. Regarding the first dimension, we have the following types:

- Illustrative: The VR and/or its caption illustrates a situation in a familiar context, mentioned also in the text.
- Exemplifying or explicative: The VR and/or its caption introduces a new, usually a more advanced, contextual application or example or explains a reasoning process.
- Complementary: The content of the VR and/or its caption complements the reasoning by adding new information not mentioned in the main text.

Under the second dimension, we distinguish five categories (one more than the ones proposed by Ref. [40]:

- The starting point: The reasoning starts from the specific VR and is further developed.
- The fundamental tool: The reader needs to use the VR throughout the reasoning process.
- Organizing role: The VR functions as an organizational tool of reasoning.
- The product: The VR functions as the final product of the developed reasoning.
- No reasoning: There is no explicit reasoning associated with the VR, which seems to be used without context.

3. Cognitive load theory

Finally, we focus on how students elaborate content when it is presented in the form of an image. With this purpose, we rely on Mayer's findings and his multimedia learning theory [58] that states that where there is a copresence of text and images, learning occurs more effectively than with the presence of text alone. However, it is necessary to take into account the cognitive capacity of a student who needs to interpret the multimedia language [58,59]. The analysis of a cognitive load framework finds its foundation in the working memory model [60] and, in particular, in the study of its limits [61,62]. The three key principles at the base of the cognitive load framework are:

- The human mind processes information through two distinct channels, one visual and one auditory or verbal [58,63].
- The processing capacity of the human mind is limited, i.e., it is not possible to process infinite amounts of information simultaneously [64].
- Meaningful learning only occurs if the learner plays an active role. That is, knowledge construction happens when the mind is active in the processes of selecting, organizing, and integrating concepts [58].

The latter condition is highly dependent on the cognitive load associated with the learning material. If too much information needs to be processed, then the short-term memory "has no room" to process it. This results in a lack of transition to long-term memory and therefore less effective learning [65–67].

The literature divides cognitive load into three categories [59,68], but we will refer exclusively to extraneous cognitive load, which we relabeled cognitive overload (CO) for convenience. Cognitive overload occurs in the presentation of the content, particularly because of those details that do not contribute to learning, such as inappropriate instructions, decorative images, or descriptions not relevant to the concept to be learned, and confusing displays of content. All of these engage the learner's mind, leaving less space for important concepts. This is the only component of the cognitive load that is under the control of a teacher and of those producing the learning material.

To answer our research questions, we decided to apply to the examined texts an existing analysis grid that was used in a similar context [69]. Nurjanah and Retnowati analyzed the cognitive overload of a mathematics text by drawing up a grid summarizing all possible sources of overload found in the text and then applying it to six teaching units. Compared to the original study, we assigned a name to each category of overload sources for convenience:

- Split-attention, Overabundance: The image is presented together with other information that should be processed simultaneously but serves a separate purpose.
- Split-attention, Physical Separation: The image should be processed along with other pieces of information that are spatially separated from the text layout.
- Redundancy of information: The image contains information that has already been presented and is

easily accessible. (It is important to emphasize the accessibility feature. Partial redundancy can act as a catalyst for the learning process; here, we are analyzing the presence of repeated information without any modification or additional component, simply overloading the mind that has to process it).
- Lack of signaling, Highlighting: There is a lack of visual code or highlighting important concepts in the image.
- Lack of signaling, Guide: The image is not accompanied by the information needed to interpret and use it to carry out the required activity.
- Lack of signaling, Formalism: A new formalism in the image, which has not been properly introduced to the reader.
- Incoherence, Noninherence: The image presents information not relevant to what is to be taught or addressed by the text.
- Incoherence, Wall of text: Too much textual information is present in addition to an image meant to speak, not verbally.
- Incoherence, Opacity: The image lacks clarity.
- Misprints: A misprint may affect the ability of a reader to understand and interpret the image.

Unlike Nurjanah and Retnowati's study [69], our study did not focus on the textbook as a whole but exclusively on the visual representations. This approach led to some complications in the application of their rubric, which prompted us to carry out a further analysis, thus expanding the framework. A saturation of instances of overabundance led us to make a distinction between different kinds of overlapping representations present in the cognitively overloaded pictures that we here name and explain:
- Photographic—a photo is overlapped onto the representation
- Realistic and schematic—a sketch or a realistic drawing is overlapped on the representation
- Mathematic—an equation, a graph, or some digits are overlapped on the representation
- Vector Arrows—vectors such as velocity or fields are overlapped on the representation (this is considered a source of cognitive overload because, as we later explained, we focus our analysis on force diagrams only; see Sec. III A)

Our supplementary list of possible overlapping representations is shown in Fig. 3, along with the sources of cognitive overload mentioned in the original study [69], renamed by us for practicality.

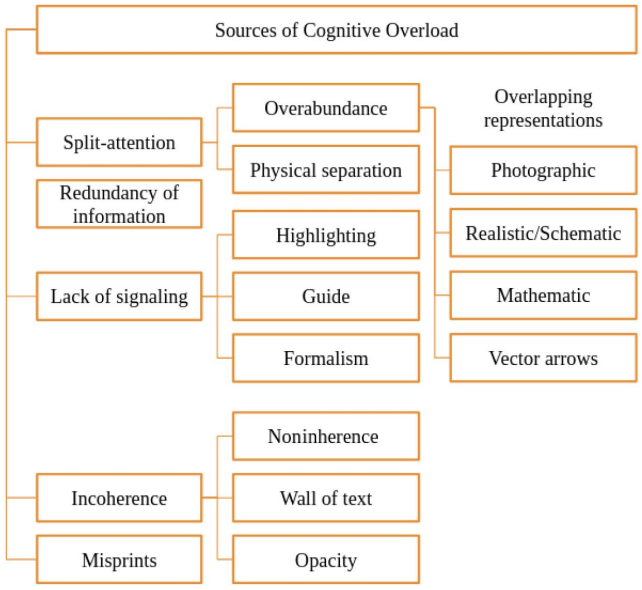


FIG. 3. Categorization of sources of cognitive overload (from Ref. [69] but renamed) and an additional list of possible overlapping representations.

B. Nature of the tool

1. Description of the tool

The combination of the reference frameworks discussed above led us to design an investigative analysis tool that could be applied to the visual representations found in physics textbooks. The tool is shown in Fig. 4 and consists of three sections appropriately color-coded.
- The blue section refers to multiple representations (Fig. 1).
- The green section refers to visual representations (Fig. 2).
- The orange section refers to the sources of cognitive overload (Fig. 3).

The table in Fig. 4 works as a checkbox tool to help researchers and teachers identify the main characteristics of a representation in terms of its function, role, and the possible presence of sources of cognitive overload. It is critical to specify that none of the shown categories should be considered mutually exclusive: a representation could be carrying out both a complementary and a constraining function, as it could be playing both an illustrative and an exemplifying role. However, while utilizing this tool, it is optimal to focus on the main feature of the representation chosen and resort to multiple ticks per section only when necessary. Correct use of this analysis tool requires checking at least one box for: external multiple representations functions, modes of reasoning—relation between image and text, and modes of reasoning—relation between image and reasoning. Not all visual representations present sources of CO; in that case, the orange row should be left empty; otherwise, one should tick as many boxes as sources of CO. The last rows of the table serve as a tool for further analysis in the case of an instance of overabundance: one should identify and count the number of different kinds of visual representations overlapping in the picture, thus obtaining an estimate of the severity of the overabundance identified. As mentioned before, this tool

Representation	External Multiple Representations Functions						
	Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction	Extension	Relation
	Relation to the Text			Relation to the Reasoning			
	Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool	Organizing Role	Final product	No Reasoning
	Sources of Cognitive Overload								
	Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints
	If overabundance is present: **Overlapping Representations**									
	Photographic		Realistic/Schematic		Mathematic		Vector arrows		**Total**	

FIG. 4. The investigative analysis tool for physics textbook representations.

can be used alongside the rubrics developed for the assessment of scientific abilities in the ISLE approach [44]. As teachers, if we are able to identify the function or the relation to reasoning of a certain representation, we can more easily pinpoint exactly which rubrics we should give to students in their assessment. For example, if a representation has a complementary function and is the starting point of the reasoning process, we will use the rubric "A1: Is able to extract the information from representation correctly" (from Ref. [70] page 4–9) when assessing students' work; if the representation has a constraining function and is the final product of the reasoning, we will use the rubric "A2: Is able to construct new representations from previous representations."

2. Target audience and uses

The investigative analysis tool that we developed, Fig. 4, was designed with a categorizing purpose in mind. Therefore, it is targeted toward researchers, and here is a list of possible uses of the tool:

- In physics education research: The tool could be used to compare teaching materials or textbooks with the intent of highlighting specific features, functions, and roles of representations. Students' and teachers' performances could be studied in correlation with the above characteristics, and the application of these tables intends to lead toward a more objective, analytic, and reproducible analysis.
- In teacher training: Researchers could analyze the representations teachers intend to use and coach them toward a deeper understanding of the representations' role, purpose, and key design features. This process could result in better teachers' metacognition toward their own practices, as teachers could also learn how to use this tool to address their own teaching styles in the classroom. The investigative tool uncovers functional aspects of the representations that are left unsaid, pinpointing which ones could be addressed more explicitly to help achieve comprehension in certain students with specific needs.
- In textbook writing: Researchers reviewing drafts of new textbooks for authors and editors could, through the use of the grid, better understand and visualize the features of the representation they intend to use, purposefully asking themselves whether the pictures chosen actually address the role and function that they expect them to address. They could also apply the tool to check for consistency within the textbook, highlighting instances in which an unintended implicit change in formalism could lead to students' difficulties and point out the critical steps to the publishers.

Overall, the tool is specifically designed to answer research questions. It can be applied whenever researchers, authors, or teachers are in need of unveiling the functions, roles, and visual features of the representations they are planning to use. In the following sections, we discuss an actual application of the tool on Italian textbooks' representations, along with the most notable case studies.

III. TOOL APPLICATION

A. The choice of the force diagram

There are many noteworthy representations, and it would be interesting to test our tool on most of them to see if different trends appear or if our tool is more effective on some of them than others. However, that requires time and resources that we do not have. Therefore, aiming at greater effectiveness, we chose not to devote our attention to all possible types of representations, for now, but rather to analyze only one of them: the FD. We justify this choice with the belief that this representation is one of the most important in secondary education (i.e., high school-level instruction) within the context of knowledge building in physics [7].

The difficulties that students experience while conceptualizing forces in physics have long been studied [71–74], so much so that there is a fairly well-established categorization of their intuitive ideas or alternative conceptions and how these affect their learning [75–79]. Hypotheses about the sources of these alternative ideas about force are widely discussed in the literature [80], and it is not the purpose of this study to explore them in depth. However, it is worth noting that these difficulties do not exclusively afflict students with little interest in the discipline but are registered among a large number of those enrolled in physics and engineering college courses and who therefore showed interest and at the same time also high performance in the subject during high school [81]. Exacerbating these difficulties is the lack of ability in making FDs that are helpful in problem solving. It was found [82] that, if the prompt is not made explicit, only 15% of students consistently use FDs as part of their solving strategy, while the majority of subjects use it only sporadically. Moreover, only half of the subjects showed that they can draw the diagram correctly often enough to derive a positive effect from it. This is a key factor, as where there is a correctly executed FD, it is more likely that the solution to the problem is correct [7,13], but should the representation be incorrect or incomplete, the results turn out to be worse than in its absence.

While we cannot establish *a priori* which representations are more effective [6,37], these studies show us a first step in that direction, at least for FDs, as it seems necessary for textbooks to provide the appropriate representational strategies to obtain a diagram that is at least "correct." Here we are defining that term correct as "consistent with the represented phenomenon." To avoid any sort of misunderstanding, we are therefore talking about all those diagrams without obvious fallacies, like wrong numbers, dimensions, or direction of vector arrows, that are representing the physical model adequately. For example, an FD describing the physical system as a rigid body is not consistent with the model of a pointlike object, or an FD that shows a null sum of forces is not consistent with an accelerating physical system. We think that, to achieve evidence on the effectiveness of representations in the future, our tool should aim to assess this kind of "correctness and consistency" first. This goal finds its justification again within the framework of multiple representations, as it is described in the following subsection.

1. Applying the tool on FD

In the tool presented previously (Fig. 4), there is a function of MR that has not been addressed yet: the evaluation function, which requires a bit of discussion. The evaluation function of a representation is not normally associated with its presentation inside a textbook. The evaluative purpose originates from the representation's users in problem-solving practices. While producing a representation, the students check for meaning and consistency between different representations and, in return, can keep track of the correctness of their work [47].

Evaluation			
Consistency		Meaning	
FD shows only forces and not other vectors		Description of the physical system	
Forces are applied to the pointlike object			
No other representations overlapped			
FD shows all forces at play		Force represented as interaction	
All forces shown are acting on the selected			
Components or sum (if present) are color-coded			
Total			

FIG. 5. An additional component of the tool to analyze the evaluation function, meant explicitly for FDs.

We assume that a textbook that promotes consistency in a representation format encourages the user to exploit this MR function. The same could be said for a textbook that explicitly shows the connection between the representation and the scientific principles the representation is based on (we will refer to this connection as "meaning" from now on). Therefore, when analyzing a textbook, one should try to assess these two features of the representation studied, using features that are specific to said representation. The result should resemble the table we produced, which is shown in Fig. 5.

This additional table must be specifically designed to take into account the features of the individual representation type that is being examined. It could not have been presented previously when we offered a more general analysis approach that was applicable to all kinds of representations. In this study, we designed the tool with the intent to check for consistency and meaning exclusively in FDs. To do so, we extrapolated from College Physics—Explore and Apply, 2nd Edition by Etkina, Planinsic, and Van Heuvelen [83], the rules to construct an FD based on literature research in physics education [3,7,13], and obtained the list of features shown in Fig. 5 under the category "consistency." For each representation, one should count how many of the "rules" are correctly followed; the higher the score, ideally the more correct the FD is in terms of consistency. Intuitively, there are some features that seem to be more important than others and should therefore have more weight in the analysis. So, as the tool is right now, two representations with the same score are not necessarily equal in quality. There is not enough evidence in the literature, though, to justify the tool

having different score weights for each of these features, so we decided to keep it as is and assume that on average a representation with a higher score is generally better than one with a lower score. We remind the reader here that we are considering the ideal and better representation shown in the reference textbook [83], as it is based on years of research on representations in physics education.

It should be stated that here we are talking about consistency not only inside the textbook itself but also within the physical models and theories that support the use of the FD as a tool, such as the pointlike object model and Newtonian dynamics. As previously stated, the FD must always be associated with a certain purpose, encouraging its use as a reasoning tool and keeping it from becoming just a visual accessory with a decorative purpose. We, therefore, established two aspects that should always be present alongside the FD. These are the descriptions of the physical system taken into consideration and the conceptualization of force as a physical quantity characterizing an interaction between two entities. The tool works as a checkbox for the presence of these features.

Combining Fig. 4 with Fig. 5, we effectively obtained a fine grid categorization tool for visual representations and force diagrams particularly. Ideally, though we would like a series of rubrics to gauge the quality of the representations and predict their effectiveness, but, as we stated before, we are not able, to our knowledge, to specifically pinpoint which representations are more effective. To conduct studies on the effectiveness of different representations, though, it is essential being able to group them in different categories according to their structural, visual, and functional parameters; therefore, we certainly believe that our tool could be a step in the right direction toward the mentioned objective.

It could be argued that the step we are taking might also be slightly more significant. There is in fact evidence that teaching materials in general are more effective when their cognitive load is reduced [85,86] and also that consistency and meaning association within the use of representations leads to improvements in problem-solving strategies [3,7,13] and self-evaluation abilities [47]. We therefore strongly believe that those representations that present fewer sources of cognitive overload and higher scores of consistency and meaning association will also be the most effective in terms of students' learning. However, without further studies on the effectiveness of specific representations, this can only be described as a prediction rather than a scientific claim.

Even said so, we can still define a textbook as "effective in presenting its content" if it presents multiple representations comprehensively, with all their different roles and functions and with minimal cognitive overload [85] and work under the assumption that a textbook that is effective in presenting content also makes students' learning more effective [31,42]. Therefore, even if our tool is objectively just a categorization tool, we believe that it could help researchers make some claims also on textbooks' effectiveness.

2. Example of tool use in the analysis of FD

Here, we provide two examples of how the tool works. In the first one, we applied the tool to the book we mentioned before (College Physics—Explore and Apply, 2nd Edition [83]), as it is the one we used to develop the tool itself. The second example is instead to show the application of the tool on another textbook: La fisica di Cutnell e Johnson [84] (Italian adaptation of Cutnell and Johnson physics edited by Zanichelli), which is part of our data sample, as described later in Sec. III C.

- Example 1: In Fig. 6, we see the analysis conducted with our table of the FD shown in College Physics—Explore and Apply, 2nd Edition [83], chapter 3.1, page 52, figure 3.2 c. The goal of the textbook here is to teach students how to represent interactions. The reader is invited to imagine holding a bowling ball with one hand and a volleyball with the other. Each ball is considered a system, and students must reflect on what objects are interacting with said systems.

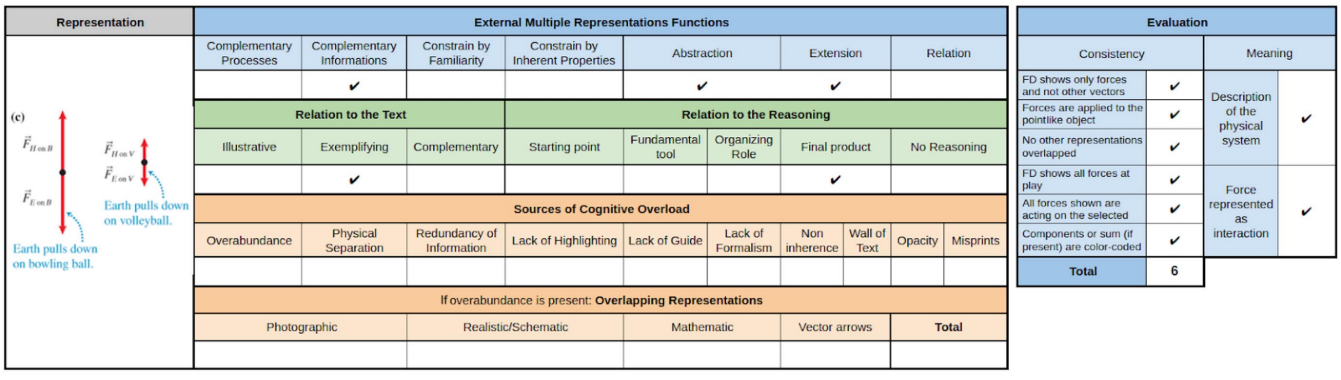


FIG. 6. Analysis of an FD extrapolated from College Physics—Explore and Apply, 2nd Edition [83], chapter 3.1, figure 3.2 c, page 52.

The textbook then explains how to correctly draw the arrows. The FDs show the forces exerted on a bowling ball compared to the ones exerted on a volleyball while being supported by a girl's hands. The diagram gives additional information to the reader, and therefore, it contains a complementary function. It is also the final step of a guided abstraction procedure and a copy of the previous image with further details added. We can, therefore, say that it helps construct a deeper understanding through abstraction and extension. This representation explains the content of the text; therefore, it has an exemplifying role, and given that it is carefully constructed through reasoning, it has a role as a product of the reasoning process. The picture does not have sources of CO. Finally, this FD totals the maximum score in terms of consistency. It is associated with a description of the physical system, and the force is presented as an interaction.

- Example 2: Figure 7 is the analysis conducted with our table on an FD found in La fisica di Cutnell e Johnson [84] (Italian adaptation of Cutnell and Johnson physics edited by Zanichelli), chapter Meccanica e Termodinamica (Mechanics and Thermodynamics), page 57. The diagrams describe the forces at play in a two-pulley system in which a load is lifted by an unidentified agent. The FD constrains the interpretation of the physical concept through its inherent properties and tries to construct a deeper understanding through the relation between the adjacent diagrams. The consistency score is low because the forces are not correctly exerted on the pointlike object, a schematic representation overlaps with the FD, not all the forces are shown, and it is unclear which system is selected. The text also lacks a description of the physical system, and the force is treated here as an agent rather than an interaction. Its role in the text and reasoning is the same as in Example 1. Given the overlapping of different representations, there is a source of cognitive overload, namely, overabundance. Further analysis reveals the overlapping of one different representation: the realistic and schematic one.

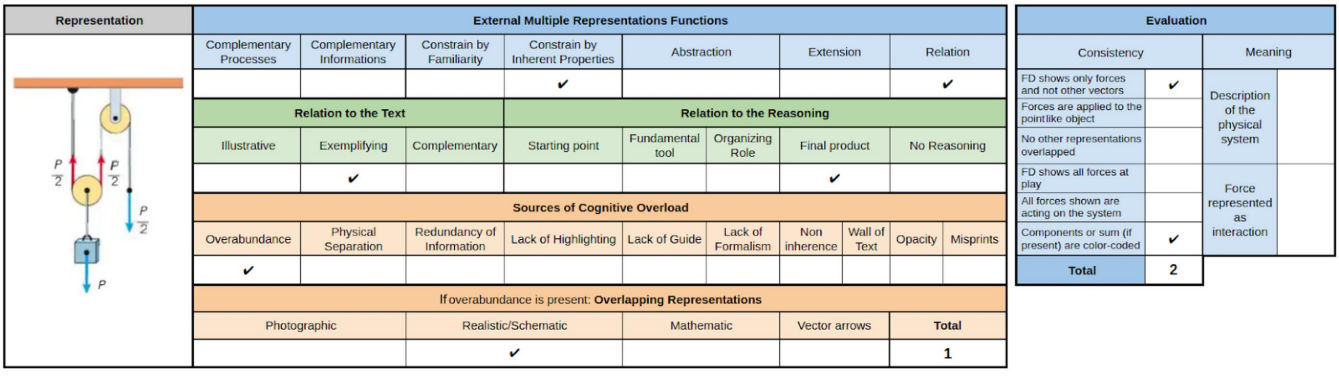
B. Research design and data analysis

For this study, we opted for a comparative contrast approach [87,88] between a reference textbook and three sample ones chosen within the widely adopted (further details on the choice of textbooks are in the following subsection). We selected each textbook chapter with the following criteria:

- The content topic of the chapters has to be comparable between different textbooks.
- The chosen chapters must allow us to evaluate the consistency and coherence of the use of FDs within the same textbook.

These factors led us to choose two chapters per book: one on Newtonian dynamics, in which the FD is introduced, and the other one on electric forces, which is much later in the curriculum but still heavily focused on forces. We analyzed all parts in each chapter, including exercises, searching for all the representations of force used. We analyzed selected FDs with the designed tool, as was shown in the previous examples in Figs. 6 and 7. We chose to consider all representations in which there is a vector representation of the forces involved as a part of an FD, since any effort to develop a more restrictive criterion would have led to the exclusion of too many representations within the selected pool of data.

Textbooks were coded collaboratively by the research team, with coders discussing the application of the tool to each representation and reaching agreement through consensus discussion. Because coding was conducted collaboratively rather than independently, we did not calculate a formal interrater reliability statistic, and that is a limitation of our research that should be addressed in future studies. However, our goal was to produce a tool strongly anchored



Representation

External Multiple Representations Functions

Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction	Extension	Relation
			✔			✔

Relation to the Text			Relation to the Reasoning				
Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool	Organizing Role	Final product	No Reasoning
	✔					✔	

Sources of Cognitive Overload

Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints
✔									

If overabundance is present: **Overlapping Representations**

Photographic	Realistic/Schematic	Mathematic	Vector arrows	**Total**
	✔			**1**

Evaluation

Consistency		Meaning	
FD shows only forces and not other vectors	✔	Description of the physical system	
Forces are applied to the pointlike object			
No other representations overlapped			
FD shows all forces at play		Force represented as interaction	
All forces shown are acting on the system			
Components or sum (if present) are color-coded	✔		
Total	**2**		

FIG. 7. Analysis of an FD extrapolated from La fisica di Cutnell e Johnson [84], chapter Meccanica e Termodinamica (Mechanics and Thermodynamics), page 57.

to the literature of multiple representations and therefore think that it has value as is, even though it could receive further validation with added statistical rigor. We applied the tool to each FD identified and noted their defining features. We then organized the data into tables and charts relative to each chapter. We compared the collectively analyzed data of each chapter with the other chapters of the same textbook and with similar chapters of the other books of the sample. We later extracted and reported here the most significant case studies to prompt further discussion and possible revisions.

C. Reference textbook and data sample

In order to perform the analysis, we first chose to set a physics reference textbook. The goal was to identify a textbook based on extensive research in the field of physics education and particularly in the use of representations. The identified textbook served as a term of comparison for the other texts taken into consideration. The comparison with an existing textbook has the advantage of explicitly demonstrating the practical application of concepts discussed in this study. We opted for the previously mentioned College Physics—Explore and Apply, 2nd Edition by Etkina, Planinsic e Van Heuvelen [83] (CP from now on). The above aims to implement the ISLEapproach at the algebra-based level of introductory physics courses [24,70,89] and is the result of years of extensive research in physics education and the use of multiple representations [3,4,44,89,90]. To go deep into the research that supports and describes this approach is beyond the purpose of this study; however, it must be duly noted that the ISLE approach sees physical representations as the bridge between abstract words and mathematics. Quoting the instructor guide of the College Physics, page 0–6, [90] "The goal of this bridge is to help our students create concrete referents in their minds when they reason about abstract ideas—the same purpose behind placing a picture of an apple next to the word 'apple' when teaching a child that word." The strict relation between research and this textbook should be a plausible justification for our selection; however, since this study focuses only on FDs, it is necessary to evaluate the CP approach with respect to this representation.

The textbook dedicates a chapter (3) on Newtonian Mechanics in which, after a brief introduction, there is an entire chapter section devoted to describing and representing interactions. Quoting the instructor guide, page 3–1, [90]: "In this chapter, students learn how to explain why objects move the way they do and add a new representation to their toolbox—a force diagram." Therefore, in this section, the FD is presented along with a detailed guide to its construction and use. Special care is taken in the steps the student must follow to make the diagram, emphasizing the physical concepts and assumptions underlying the representation. Another key point distinguishing this text from many others is the clear emphasis on the concept of force as a physical quantity describing the interaction between two objects rather than as an agent. In addition, the book explicitly requests not to overlap the diagram and the sketch of the physical situation, putting them alongside and therefore separating two representations that play a distinct role but equally allowing for parallel and simultaneous analysis of them, thus avoiding particular forms of Cognitive Overload such as overabundance. The last reason for choosing CP as a reference is the consistency with which the force diagram is used. The representation is introduced at the beginning of the book and remains a key tool used in subsequent chapters. Moreover, the FD is often accompanied by other representations (such as motion diagrams, for example) in order to promote an overall understanding of physical concepts by taking advantage of the presence of invariants among the different tools.

In addition to the reference text, a sample of three textbooks was selected among the most widely adopted (they together cover 75% of the market as of June 2026). Even though the study took place in Italy and we're referencing the Italian versions of these books, it's worth noticing that two out of three of the selections are American textbooks that went through adaptations for Italian schools and are also adopted in other countries. In particular, our interest was directed toward texts for the "Liceo Scientifico" (science-focused high school) since most college students pursuing physics and engineering majors come from those schools. The sample textbooks are the following:

- La fisica di Cutnell e Johnson [84] (Italian adaptation of Cutnell and Johnson physics [91] edited by Zanichelli);
- Il nuovo Amaldi per i licei scientifici.blu [92];
- Il Walker—Corso di Fisica [93] (Italian adaptation of Physics by James S. Walker [94] edited by Sanoma).

One might argue that the reference book and sample ones are meant for different audiences with different education levels. However, the reference text, College Physics—Explore and Apply, 2nd Edition, is meant not only for a college-level education but also for any algebra-based general physics course and is currently widely used in high schools in the United States. Italian high school textbooks are also mainly algebra-based, with some exceptions in specific topics of the last year of instruction, in which calculus is introduced, but these are not taken into account in this study. Therefore, we can conclude that the books we analyzed are comparable, and they all target the same audience in terms of scientific reasoning skills. The summary of the collected data for each textbook is shown in Fig. 8 (for clarity: M is used for chapters about "mechanics," while EF is used for chapters about "electrical forces").

Title of the book	Volume	Chapter	#FD	Referred in this paper as:
College Physics Explore and Apply, 2nd Edition	/	3-Newtonian Mechanics	32	Reference Txtb—M
		17-Electric Charge, Force and Energy	23	Reference Txtb—EF
La fisica di Cutnell e Johnson Meccanica e Termodinamica	1	LA DINAMICA	24	Txtb 1—M
La fisica di Cutnell e Johnson Onde Campo elettrico e magnetico	2	11-FORZE ELETTRICHE E CAMPI ELETTRICI	16	Txtb 1—EF
Il nuovo Amaldi per i licei scientifici.blu Le misure, l'equilibrio, il moto,il calore, la luce	1	2-I principi della dinamica e la relatività galileiana	23	Txtb 2—M
Il nuovo Amaldi per i licei scientifici.blu Onde Campo elettrico e magnetico	2	14-La carica elettrica e la legge di Coulomb	12	Txtb 2—EF
Il Walker Corso di Fisica	1	2-La dinamica newtoniana	49	Txtb 3—M
Il Walker Corso di Fisica	2	12-Forze e campi elettrici	13	Txtb 3—EF
Total of analyzed FDs:				192

FIG. 8. Chosen textbooks and data sample.

IV. CASE STUDY: FD IN PHYSICS TEXTBOOKS

A. Most notable examples

As a reference for what we consider the best possible force diagrams, we propose the one visible in Fig. 9.

In this section of the reference book, the authors explain the rules for constructing an effective FD, which we later translated into the checklist shown in Fig. 5. It is important to notice the separation between the sketch, whose purpose is to give a concrete idea of the physical situation subjected to the analysis, and the FD, which is instead a tool to operate with forces. The link between the two representations resides in circling the system, which is then turned into a pointlike object in the FD. The application of the tool to said FD is shown in Fig. 10, and it is important to note the lack of sources of CO.

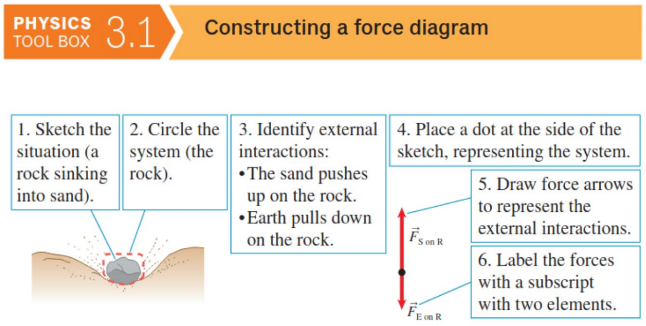


FIG. 9. How to construct a FD, extrapolated from College PhysicsExplore and Apply, 2nd Edition [83], chapter 3.1, physics tool box 3.1, page 54.

The following are instead examples of FD that present some of the most commonly found flaws. It should be noted that in each example, the FD is not drawn on a pointlike object, but rather on the sketch of the physical situation, which is a form of cognitive overload. To avoid excessive repetition, the focus will now be only on the unique features of each FD:

- Figure 11: The analyzed image shows the forces exerted on a sled that is being pushed from above at an angle. The FD presents an overabundance of representations as a redundancy of information: the expression of the force is stated in the text, the diagram, and also its caption. It is not clear what the core concept of the first image is. Thus, the lack of highlighting and what is presented here, friction, is not inherent to the logical thread that the textbook was unraveling, as if this small section was glued later on. Finally, there is opacity around the implicit rules followed to construct this FD. Why is the force exerted on the child, but immediately after it is exerted on the sled? What is the system? Why are some forces exerted on the sled that is treated as a pointlike object, but the friction force is not?
- Figure 12: The analyzed diagram describes the forces at play on a generic object placed over an inclined surface. Here, only the gravitational pull is drawn on the pointlike object, while the normal force and friction are exerted at an arbitrary point on the base of the object. This choice and the use of the same colour create an illusion of a curving vector.

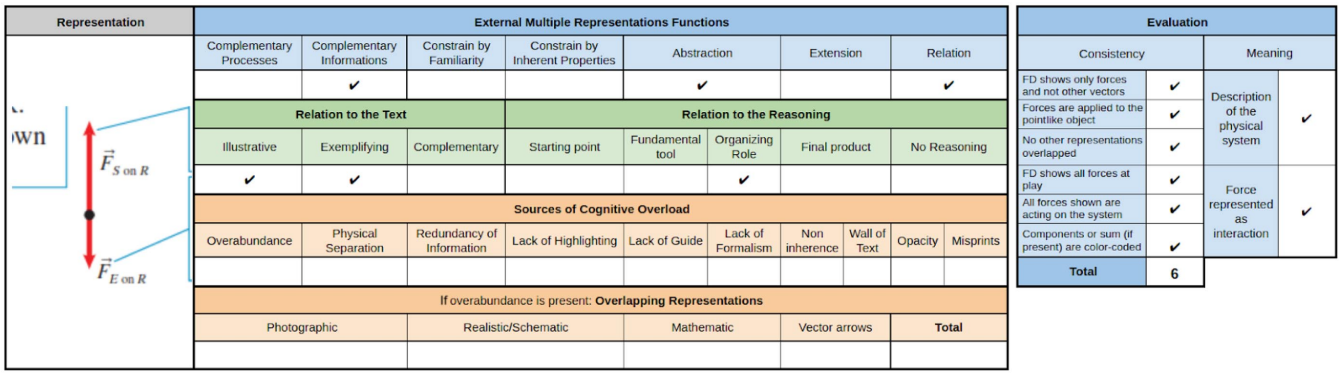
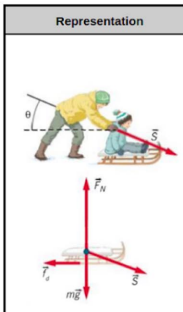


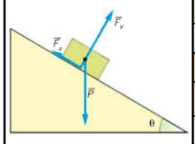
FIG. 10. Analysis of an FD extrapolated from College Physics—Explore and Apply, 2nd Edition [83], chapter 3.1, physics tool box 3.1, page 54.



Representation	External Multiple Representations Functions									
	Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction		Extension	Relation		
		✔		✔						
	Relation to the Text			Relation to the Reasoning						
	Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool		Organizing Role	Final product	No Reasoning	
			✔						✔	
	Sources of Cognitive Overload									
	Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints
	✔		✔	✔			✔		✔	
	If overabundance is present: **Overlapping Representations**									
	Photographic		Realistic/Schematic		Mathematic		Vector arrows		**Total**	
			✔						1	

Evaluation			
Consistency		Meaning	
FD shows only forces and not other vectors	✔	Description of the physical system	
Forces are applied to the pointlike object			
No other representations overlapped			
FD shows all forces at play		Force represented as interaction	
All forces shown are acting on the system	✔		
Components or sum (if present) are color-coded	✔		
Total	3		

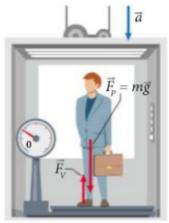
FIG. 11. Analysis of an FD extracted from La fisica di Cutnell e Johnson [84], chapter Meccanica e Termodinamica (Mechanics and Thermodynamics), Fig. 1, page 46.



Representation	External Multiple Representations Functions									
	Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction		Extension	Relation		
	✔			✔						
	Relation to the Text			Relation to the Reasoning						
	Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool		Organizing Role	Final product	No Reasoning	
	✔								✔	
	Sources of Cognitive Overload									
	Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints
	✔								✔	
	If overabundance is present: **Overlapping Representations**									
	Photographic		Realistic/Schematic		Mathematic		Vector arrows		**Total**	
			✔						1	

Evaluation			
Consistency		Meaning	
FD shows only forces and not other vectors	✔	Description of the physical system	
Forces are applied to the pointlike object			
No other representations overlapped			
FD shows all forces at play	✔	Force represented as interaction	
All forces shown are acting on the system	✔		
Components or sum (if present) are color-coded	✔		
Total	4		

FIG. 12. Analysis of an FD extracted from La fisica di Cutnell e Johnson [84], chapter Meccanica e Termodinamica (Mechanics and Thermodynamics), page 54.

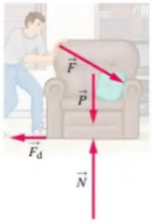


Representation	External Multiple Representations Functions									
	Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction		Extension	Relation		
								✔		
	Relation to the Text			Relation to the Reasoning						
	Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool		Organizing Role	Final product	No Reasoning	
	✔						✔			
	Sources of Cognitive Overload									
	Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints
	✔								✔	
	If overabundance is present: **Overlapping Representations**									
	Photographic		Realistic/Schematic		Mathematic		Vector arrows		**Total**	
			✔		✔		✔		3	

Evaluation			
Consistency		Meaning	
FD shows only forces and not other vectors		Description of the physical system	✔
Forces are applied to the pointlike object			
No other representations overlapped			
FD shows all forces at play	✔	Force represented as interaction	
All forces shown are acting on the system	✔		
Components or sum (if present) are color-coded	✔		
Total	3		

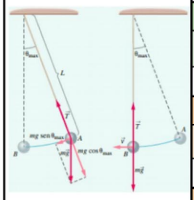
FIG. 13. Analysis of an FD extracted from Il nuovo Amaldi per i licei scientifici.blu [92], chapter 2.4, page 60.

- Figure 13: The analyzed image represents forces and acceleration inside an elevator. In this FD, it is not clear what the system is, but mainly, there is a problem with the overabundance of information conveyed with a single image. The student is supposed to read the analogue hand shown while also deciphering the correlation between the acceleration shown above the elevator and the forces shown inside, which are shown both as vectors and as a mathematical equation.
- Figure 14: The analyzed diagram describes the forces at play when a man pushes on a chair from above. It appears at a glance that the forces here represented are simply all over the place, but the book then proceeds to treat the chair as a pointlike object rather



Representation	External Multiple Representations Functions										Evaluation			
	Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction		Extension	Relation			Consistency		Meaning	
	✔							✔			FD shows only forces and not other vectors	✔	Description of the physical system	✔
	Relation to the Text			Relation to the Reasoning							Forces are applied to the pointlike object			
	Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool	Organizing Role	Final product	No Reasoning			No other representations overlapped			
		✔		✔							FD shows all forces at play	✔	Force represented as interaction	
	Sources of Cognitive Overload										All forces shown are acting on the system	✔		
	Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints	Components or sum (if present) are color-coded	✔		
	✔								✔		Total	4		
	If overabundance is present: **Overlapping Representations**													
	Photographic		Realistic/Schematic		Mathematic		Vector arrows		Total					
			✔						1					

FIG. 14. Analysis of an FD extrapolated from Il Walker—Corso di Fisica [93], chapter 2.2, figure 7a, page 84.



Representation	External Multiple Representations Functions										Evaluation			
	Complementary Processes	Complementary Informations	Constrain by Familiarity	Constrain by Inherent Properties	Abstraction		Extension	Relation			Consistency		Meaning	
		✔									FD shows only forces and not other vectors		Description of the physical system	
	Relation to the Text			Relation to the Reasoning							Forces are applied to the pointlike object	✔		
	Illustrative	Exemplifying	Complementary	Starting point	Fundamental tool	Organizing Role	Final product	No Reasoning			No other representations overlapped			
		✔		✔							FD shows all forces at play	✔	Force represented as interaction	
	Sources of Cognitive Overload										All forces shown are acting on the system	✔		
	Overabundance	Physical Separation	Redundancy of Information	Lack of Highlighting	Lack of Guide	Lack of Formalism	Non inherence	Wall of Text	Opacity	Misprints	Components or sum (if present) are color-coded	✔		
	✔									✔	Total	4		
	If overabundance is present: **Overlapping Representations**													
	Photographic		Realistic/Schematic		Mathematic		Vector arrows		Total					
			✔				✔		2					

FIG. 15. Analysis of an FD extracted from Il Walker—Corso di Fisica [93], chapter 2, exercise 69, page 122.

than a rigid body. The confusion between two different formalisms generates opacity.

- Figure 15: The analyzed diagrams show the forces and movement of an ideal pendulum consisting of a sphere attached to a weightless string. This is an example of the confusion that could emerge from a distraction within colour coding. Here, the same colour represents both the components of the gravitational pull (on the left) and the velocity (on the right).

B. Results

As shown in Fig. 16, even though we selected two chapters that both heavily deal with forces, we registered a visible reduction in the use of FDs across all sampled textbooks. Given that the results of the rest of the analysis are significantly similar between the chapters on mechanics and the ones on electrical forces, we will now focus only on comparing the textbooks with each other rather than analyzing the differences within chapters of the same one. With this purpose in mind, to achieve better clarity, all of the results shown in Figs. 17–19 come from the analysis of mechanics chapters only.

The EMR function analysis shows us a unique functional spectrum for each book in Fig. 17. However, it

Textbook - Chapter (Mechanics or Electrical Forces)	Number of figures	Number of FD	Percentage of FD
Reference Txtb—M	95	32	34%
Reference Txtb—EF	99	23	23%
Txtb 1—M	46	24	52%
Txtb 1—EF	87	16	18%
Txtb 2—M	47	23	49%
Txtb 2—EF	87	12	14%
Txtb 3—M	96	49	51%
Txtb 3—EF	116	13	11%

FIG. 16. Frequency of FDs within chapters' figures. See also Fig. 8 for extended titles of books and chapters.

should be noted that there is a remarkable difference in the construct deeper understanding category, with these functions utilized less frequently in the sample textbooks compared to the reference one. For example in the analyzed textbooks only 8%, 35%, and 4% of FDs in the chapter perform the construct through abstraction

Results are in (# of FD with said external MR function) divided by (total of FD in the chapter) expressed as %

External MR function	Reference Txtb—M	Txtb 1—M	Txtb 2—M	Txtb 3—M
Complement with processes	31%	67%	52%	50%
Complement with information	63%	29%	26%	22%
Constrain by Familiarity	0%	0%	4%	8%
Constrain by Inherent Properties	0%	38%	17%	14%
Construction through Abstraction	59%	8%	35%	4%
Construction through Extension	34%	13%	4%	4%
Construction through Relation	53%	21%	35%	33%
Description of the physical system	88%	58%	43%	51%
Force represented as interaction	97%	8%	43%	10%
Consistency (Avg. Score)	5.8	3	4	4

Key: 0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

FIG. 17. Results of external multiple representations function analysis on the chapter about mechanics. We remind here that the percentages do not add up to 100 because the mentioned categories are not mutually exclusive.

Results are in (# of FD with said relationship) divided by (total of FD in the chapter) expressed as %

Relationship with text	Reference Txtb—M	Txtb 1—M	Txtb 2—M	Txtb 3—M
Illustrative	44%	33%	48%	29%
Exemplifying	41%	33%	26%	45%
Complementary	38%	33%	26%	29%
Relationship with reasoning	**Reference Txtb—M**	**Txtb 1—M**	**Txtb 2—M**	**Txtb 3—M**
The starting point	6%	42%	22%	47%
The fundamental tool	44%	21%	44%	14%
Organizing Role	28%	21%	13%	18%
The product	22%	4%	0%	4%
No reasoning	0%	17%	17%	16%

Key: 0% 10% 20% 30% 40% 50%

FIG. 18. Results of modes of reasoning role in text and in reasoning analysis on the chapter about mechanics.

Results are in (# of FD with said sources of CO) divided by (total of FD in the chapter) expressed as %

Sources of cognitive overload	Reference Txtb—M	Txtb 1—M	Txtb 2—M	Txtb 3—M
0	84%	8%	4%	10%
1	16%	42%	26%	47%
More than 1	**0%**	**50%**	**70%**	**43%**

Key: 0% 20% 50% 100%

FIG. 19. Results of sources of cognitive overload analysis on the chapter about mechanics.

function; compared with the reference one in which 59% of FDs in the chapter perform the same function (Fig. 17). Similar are the results for the construct through extension function and the construct through relation function: respectively, 13%, 4%, 4% in the sampled textbooks versus 34% in the reference one and 21%, 35%, 33% in the sampled versus 53% in the reference. We also recorded fewer instances of the association of meaning to the FD (58%–43%–51% in the sampled books vs 88% in the reference textbook) and the description of force as a quantity characterizing interaction (8%–43%–10% in the sampled vs 97% in the reference). The consistency row at the end of Fig. 17 shows the average total scores of consistency across the books. Said scores were lower on the sample (3–4–4) than in the Ref. [5.8], as expected (given the definition of the rubric itself, see Sec. III A 1). The scale goes from 0 to 6. Our analysis of the role of the visual representation in reasoning (see Fig. 18) highlighted that rarely is the FD presented in the sample textbooks as a product of reasoning. In the sampled textbooks, only 4%, 0%, or again 4% of the FDs in the chapter are shown as a product of scientific reasoning, compared to the 22% of FDs in the chapter of the reference textbook. Most importantly, in the sampled textbooks there is a significant portion, around 17%, of the FDs of the chapter that are presented like "axioms," without contextualizing them or accompanying them with the reasoning process. In the reference textbook, there is no such occurrence. Finally, we also revealed that in many force diagrams of the sampled textbooks, more than one source of cognitive overload is present (see Fig. 19). In the sampled textbooks, respectively, 50%, 70%, and 43% of the FDs in the chapter are overloaded with more than one source, while in the reference textbook there is no such occurrence. The most common sources of cognitive overload are Opacity and overabundance; in particular, the realisticand schematic representations emerged as the most frequently overlapped on the FD, but there are also cases in which as many as three different kinds of representations are superimposed (see Fig. 13).

V. DISCUSSION

As mentioned before, the aim of the previous analysis is to test the tool developed in the course of this research. We are confident that this tool can be effectively utilized for a range of scientific representations but, for the scope of our study, we focused on analyzing force diagrams as presented in high school physics textbooks. We hope to further address this research in the future or that others will be able to pick up our work and apply it to different kinds of representations. After selecting a specific representation, we were able to refine the tool by adding table specifically designed to analyze also the evaluative function of FDs (Fig. 5). The goal of our research was to answer the following question: "What are the functions, roles, and visual features of the force diagrams used in physics textbooks?" We opted for a comparative contrast

approach, analyzing all force representations in selected chapters of three Italian high school textbooks and comparing the results with those obtained from a reference textbook. We chose the reference textbook based on its strong connection to research in physics education and in particular the research toward the use of multiple representations [3,4,44,89,90]. We analyzed 192 FDs across the selected chapters (Fig. 8).

The results (Fig. 16–19) show that FDs in high school textbooks are rarely used as tools for deeper understanding, but rather as an accessory to mathematical expressions. This idea is reinforced by the fact that those textbooks also lack consistency in their use and rarely reflect on the meaning of the representation itself (Fig. 17). FDs are often built without explicit or fixed rules, rendering them opaque and therefore more cognitively challenging, as beginners must spend resources to understand what rules are being followed each time. Forces are presented as agents rather than as interactions and are often mixed with velocities or accelerations (Fig. 17), factors that we believe could reasonably hinder the overcoming of alternative conceptions.

Furthermore, the role analysis (Fig. 18) revealed that FDs are mainly presented at the beginning of the reasoning, ready-made, or, worse, they are not associated with any scientific reasoning at all, but rather presented as axioms. This may discourage students from using force diagrams because these textbooks do not demonstrate how to actually develop one or how it emerges from physical concepts. The diagrams are shown only as something that is useful if specifically requested or already made.

Finally, our analysis also highlights that most diagrams in Italian textbooks are overloaded with overlapping representations (Fig. 19). These textbooks convey the idea that the more details you add to an image, the more it is clear and understandable. This idea obviously clashes with the working memory theory, which leads us to believe that unnecessary details only obstruct students' comprehension [85]. We are not saying that physics problems should be approached without a graphical representation of the physical system, but that the sketch of the system and the force diagrams are two distinct representations with two different purposes and should therefore be kept separate, as is clearly done in the reference textbook. This separation can allow the students to process each representation according to its purpose. We strongly believe that choices like this one, together with clear rules and consistency, could make grasping physical concepts overall less mentally demanding.

VI. CONCLUSION

Multiple representations have been the subject of literature research for years, but as stated by Munfaridah in their overview [16], there are still some gaps that need to be addressed. For example: (from Ref. [16], page 12) "The literature does not provide evidence about what forms or combinations of forms of representations are more effective than others in physics learning for specific groups of students (for instance, low achievers)" and also (Ref. [16], page 12) "No studies were found that examine: (a) the use (if any) of MR in university textbooks and how those are used by the students; (b) the ways in which the use of MR might support students' development of analogical reasoning; and (c) the use of contemporary technology applications, as for example, augmented and virtual reality in visualizing MR in physics education." It is our belief that to address these gaps, a tool to analyze MR as objectively as possible is necessary, for example, to discriminate between different forms and uses or different kinds (in terms of MR use) of textbooks. We aimed to develop this tool as the goal of our research.

After a careful literature analysis with a goal of extracting the defining features of graphical representations, we focused mainly on three frameworks: multiple representations, cognitive load theory, and modes of reasoning. Based on those, we developed a table (Fig. 4) that can be applied to any graphical representation in teaching resources to evaluate its defining features: MR function, role in text and in reasoning, and cognitive load. It is worth pointing out that the generic table does not address the evaluative function of MR [4,7,13] because studying this function requires specific knowledge of the representation used, including its rules and meaning; therefore, it is our opinion that it cannot be done with an analysis tool meant for any representation. We then decided to focus on textbook analysis to test our tool, and we analyzed the use of force diagrams within four different textbooks; the results of this analysis are discussed in the previous chapters ("Sec. IV B—Results" and "Sec. V—Discussion"). The tool proved to be effective in discriminating apparently similar representations, giving us better hindsight of the defining features of each textbook, and therefore we considered the test successful and replicable.

This research presents some limitations. We based our tool partly on the cognitive load theory of working memory and decided which representations were easy to process and which weren't. However, testing the effect of representations with different cognitive load on students is necessary before being able to make any stronger claims. Also, in our work we did not provide an interrater reliability analysis. Our goal was to produce a tool based on literature, and we did not implement independent triangulation during its testing, opting instead for collaborative discussions among researchers until agreement was reached in sample analysis. Finally, we focused only on three commonly used textbooks (+1 reference textbook) in Italy, even though many of those books are adaptations for Italian secondary education of originally American college-level textbooks (three over four). A wider sample of

textbooks in their original version would have been ideal and beneficial to our analysis to support our results.

Regardless of these limitations, which are worth knowing, our tool still has some noteworthy applications. As we said at the beginning, high school textbooks are among the first scientific textbooks students encounter in their instruction, and they can have a significant influence on how they learn. The scientific practices promoted by the textbooks are usually the ones followed by students, but also by teachers with less experience. If we want students to develop authentic scientific practices and sound scientific reasoning, multiple representations are essential tools that should be implemented in every textbook [4,10,16,26,70]. This implementation must also be thoughtful. Each representation should be utilized, keeping in mind what its function and role should be within the text and within the scientific reasoning. Also, aiming for a universal learning design, one that speaks to different kinds of people, with different strengths, the books should be as accessible as possible. While our tool is meant for categorization and not evaluation, so it cannot explicitly pinpoint which books are "better," it can still be used both by teachers and researchers as a means to discriminate different textbooks according to their use of MR. With this enhanced categorization ability, teachers could be more mindful in their decision of which book should be adopted for their class or for specific students. For example, if they have some students with learning disabilities, they could consciously choose books that are more consistent, make meaning associations explicit, and present less cognitive overload in representations, and actually see if it benefits the students involved. It could also be used by authors of teaching materials in the phase of development to expand their MR use in all their functions, leading to a more conscious learning design.

We established that there is a gap [16,95,96] between the extensive research done in physics teaching, particularly in MR, and in teaching practices and materials, and this tool could help us understand which elements are closest to physics teaching research, therefore helping us improve the situation. Future work should focus on establishing interrater reliability of the tool and applying on a larger sample of textbooks in their original version, but also to different kinds of representation rather than just on FDs. It is also important to collect data on students' performance and understanding using different kinds of representation so that it can be cross-referenced with data collected using the tool on them. This could lead toward a better understanding regarding the effectiveness of specific representations in relation to their structural, visual and functional parameters. It would also be of interest to further pursue research on learning disabilities, trying to correlate improvement with certain kinds of representations. Overall, we believe that our work is a step toward a more thoughtful implementation of multiple representations in physics learning and teaching, and we invite anyone working in the field to explore, use, and develop the investigative analysis tool we have developed.

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DATA AVAILABILITY

The data that support the findings of this article are openly available on Zenodo; see Ref. [97].

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