

Students' experience of cultural differences between mathematics and physics

Jeffrey M. Rabin^{*}

Mathematics Department, UCSD, La Jolla, California 92093, USA

Andrew Meyertholen[†] and Brian Shotwell[‡]

Physics Department, UCSD, La Jolla, California 92093, USA



(Received 5 September 2024; revised 20 January 2026; accepted 9 February 2026; published 6 March 2026)

How students use mathematics in their physics classes has been studied extensively in the physics education literature. In addition to specific mathematical methods in specific physics contexts, possible effects of more general “cultural” differences between the two disciplines have also been explored. However, there has been little examination of students’ own awareness and interpretations of these differences. We explore the undergraduate student experience of these cultural contrasts, and how students think they affect their learning. Through a qualitative exploratory study, including surveys and interviews with students double majoring in mathematics and physics (or majoring in one and minoring in the other), we investigate students’ awareness of distinct pedagogical approaches, mathematical justifications, and organization of concepts in mathematics versus physics classes. We find that students do recognize and navigate these cultural differences, often employing specific coping strategies. We identify specific themes from our data and comment on how students feel that these themes impact their learning. We suggest that increased faculty and student awareness of the identified differences in educational practice could facilitate knowledge transfer between mathematics and physics.

DOI: [10.1103/n43-nstx](https://doi.org/10.1103/n43-nstx)

I. INTRODUCTION

Philosophy is written in this grand book, the universe, which lies continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to comprehend a single word of it.—Galileo, *The Assayer*, 1623

Before we consider Galileo’s demonstrations, it seems necessary to prove how far from the truth are those who wish to prove natural facts by means of mathematical reasoning, among whom, if I am not mistaken, is Galileo ... Therefore

anyone who thinks he can prove natural properties with mathematical reasoning is simply demented, for the two sciences are very different.—Vincenzo di Grazia, 1613.

Since at least the time of Galileo, it has been commonplace to observe that mathematics is the language of science, particularly physics. Mathematical proficiency is necessary for students to understand physics deeply, and the physics education research literature has extensively investigated how they use their mathematical knowledge in physics [1]. Much of this literature is highly specific in terms of both mathematical methods and physical applications, studying, for example, students’ use of integration in electrostatics [2], partial derivatives in thermodynamics [3], or linear algebra in quantum mechanics [4]. These studies can be conceptualized in terms of mathematical modeling, transfer of knowledge, and even conceptual blending. Certainly, differences in how “the same” material is presented and used in physics versus mathematics courses can affect students’ ability to transfer their knowledge of one to the other. However, there have also been more general studies of the relationship between physics and mathematics, for example, the differences between the “dialects” of mathematics spoken in each field. Redish and Kuo [5] describe a number of differences between these dialects, for example, that “loading physical meaning onto symbols does work for physicists and leads to differences

^{*}Contact author: jabin@ucsd.edu

[†]Contact author: ameyertholen@physics.ucsd.edu

[‡]Contact author: bshotwell@physics.ucsd.edu

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article’s title, journal citation, and DOI.

in how physicists and mathematicians interpret equations.” An anecdotal example is “Corinne’s shibboleth” (attributed to Corinne Manogue [6]), which allegedly discriminates well between mathematics and physics faculty: if the function $f(x, y)$ is defined as $x^2 + y^2$, then what is $f(r, \theta)$? Physicists are said to answer r^2 , while mathematicians answer $r^2 + \theta^2$, showing that they interpret symbols and use function notation quite differently. This literature suggests ways in which this language barrier may impact students’ learning to use mathematics in physics contexts. Redish [7] is an introduction to seven subsequent short articles he wrote to disseminate these ideas to practicing physics teachers to support their students to “think about physics with math instead of just calculating.” Our conception of such “cultural” differences between mathematics and physics is quite broad, including not only these distinct dialects, but also how and to what extent mathematical claims are justified in each field, how approximations are used, how new concepts are introduced, and differences in pedagogy that may be common to instructors within each field but differ between them. All these features go beyond specific topics like integration or electrostatics, but may affect students’ ability to “transfer” or apply their mathematical knowledge in the physics context. They form a sort of background to the teaching and learning of individual topics.

From Redish and Kuo [5], we cite two broad categories of cultural differences between mathematics and physics:

- A. Physicists load meaning onto symbols in ways that mathematicians do not, which leads to differences in how physicists and mathematicians interpret equations.
- B. Physicists and mathematicians have different goals for the use of math.

In addition to Corinne’s shibboleth, Karam *et al.* [8] provide further examples in category A. First, physical quantities have units attached, while mathematical ones usually do not. Product or quotient quantities have corresponding units, and physics equations must be dimensionally correct. This incorporates a significant amount of proportional reasoning in physics automatically. Second, functional dependence may not be made explicit in physics. An equation may connect multiple physical quantities, any of which could be “dependent” or “independent” depending on the physical context. Ohm’s law $V = IR$ serves as a simple example.

Under category B, mathematicians process formal syntax in a largely self-contained way, while physicists constantly draw on particular physical processes and/or applications. Physicists are constantly modeling (physical system $\rightarrow$ mathematical representation) and interpreting (mathematical representation $\rightarrow$ physical system). These modeling and interpreting steps are intimately tangled, and it may not be clear to students how to disentangle them. This category also includes the more “relaxed” standards of proof found

in physics, where physical insight, rather than rigorous proof of general theorems from *a priori* axioms, is the more common goal.

As an example of both categories A and B, López-Gay *et al.* [9] describe multiple conceptions of differentials that appear when calculus is used in mathematics and in physics. While mathematicians might treat the differential as a formal instrument with no meaning in itself, physicists will often view the differential as a linear approximation, or an infinitesimal one, the difference between these being deliberately ambiguous. For instance, in electrostatics, dq is a small/infinitesimal quantity of charge with units, a physical entity we can algebraically manipulate (category A). A physicist’s goal in using differentials (category B) may be to conveniently set up an integral for the total electric field based on its physical meaning as a superposition of infinitesimal elements while avoiding the explicit consideration of Riemann sums or limits. The conceptualization of the differential (and the need for differential calculus in the first place) is not obvious to students, and so they overemphasize “processing” steps at the expense of understanding the physical context. Students in this study predominantly felt that they were expected to use calculus in this way but not to understand it.

In addition to the work cited above, we have drawn on other papers addressing the cultural aspects of mathematics as applied to physics. Uhden *et al.* [10] distinguish between the technical skills of mathematics that are used for computation or derivation, and the structural skills that “are related to the capacity of employing mathematical knowledge for structuring physical situations.” These authors provide a model for students’ use of mathematics in physics that recognizes structural skills in addition to procedural ones and recognizes that physical problems are not simply “translated into mathematics” as a naive modeling approach would suggest but rather that the math and the physics are inseparable at every stage. Palmgren and Rasa [11] give examples of the roles of mathematics in physics drawn from the history of quantum mechanics. They argue that math is much more than a tool for problem solving; rather, “mathematics contributes to representations of physical information that in turn serve as a basis for further reasoning and modification.” To our knowledge, however, no research has investigated whether students themselves are aware of such cultural differences between physics and mathematics, or how they think such differences may affect their own learning.

As detailed in the next section, we addressed this with a group of undergraduate students who were either double majors in mathematics and physics or majoring in one of these and minoring in the other. Our data come from an email survey they completed, followed by individual interviews with a subset of them.

Our study is exploratory and designed to elicit the experiences of our participant group of double majors/minors

and may not generalize to broader populations of physics or mathematics majors. We hope it provides a foundation for future research with such populations.

Our research questions were the following:

1. There are known cultural differences in how physics courses and mathematics courses handle mathematics instruction. Are the students in our study aware of these differences?
2. To what extent do students perceive that these and other cultural differences affect their learning of mathematics and physics, and how do they manage these differences?
3. What do students cite as major differences in the presentation of “common” mathematical content between their physics and mathematics courses? What specific classroom examples do they offer?

Some previous empirical studies have used questionnaires and interviews to explore students' experiences of differences between mathematics and physics, but with a more specific focus and/or different target populations than ours. de Ataíde and Greca [12] asked undergraduate students in a thermodynamics course about the role of mathematics in physics (for example, which basic concepts of mathematics are needed to understand the concepts of thermodynamics?) and explored correlations between students' problem-solving strategies (categorized as operational mathematics, conceptualization, or mathematical reasoning) and their epistemic views of math in physics (as tool, translator, or structure). de Winter and Airey [13] asked preservice physics teachers in England about the role of mathematics in the learning of physics at both university and school level. Responses were summarized in three themes: mathematics is necessary for success in physics; success in mathematics can mask a lack of conceptual understanding in physics; and there can be a disconnect between physical and mathematical aspects of a course. Kapucu [14] asked Turkish preservice elementary math and science teachers about their conceptions of mathematics, physics, their relationship, and the learning of each, but it appears that participants had only majored in these subjects in high school. Responses were rather general (for example, “physics is primarily based on calculations, so requires math”) and coarsely classified as “fragmented” versus “cohesive” and “lower level” versus “higher level” conceptions. Schermerhorn *et al.* [15] were specifically focused on undergraduate students in a spins-first quantum mechanics course at three universities and asked whether the mathematics helped students understand the physics concepts, whether the reverse was true, and which was more challenging. Some student comments resonate with those of our subjects, for example, “The physics makes the math easier to visualize and the math provides a base for physical intuition. For example, the properties of wave functions helped me understand Fourier transforms” (Ref. [15], p. 534).

Our understanding of what constitute “cultural differences” between mathematics and physics is based on our own experience and the literature cited above and is operationalized by our choice of survey and interview questions. We also provided opportunities for students to bring up any experiences they considered relevant in this context, which we may not have anticipated. Our view of cultural differences is also aligned with the findings of Ref. [16] which include the disciplines' use of terminology, notation, calculus concepts, modeling, and justification. The term “disciplinary cultures,” as used, for example, in Ref. [17], is largely distinct and refers to the organization of disciplines within the university setting, including the roles of faculty and students within departments, issues of status, power, and equity, and the administrative organization of classes.

II. DESIGN OF STUDY

The authors are two physicists and one mathematician with a postgraduate physics background. They teach at a large, public university on the west coast of the United States, which has approximately 400 physics majors and 2400 math majors. The study focused on students who were math and physics double majors at the authors' home institution, our assumption being that they would have enough advanced course background in each subject to have experienced a variety of these differences. In particular, these students have taken upper-division coursework beyond the introductory sequences in each discipline, including proof-based mathematics beyond calculus. To increase the number of participants, we also included those majoring in one subject and minoring in the other. An email was sent to all such eligible students, offering a \$20 gift card in exchange for completing our survey.

We distributed the survey to all participants, and after an initial analysis of the responses, we selected a subgroup for individual interviews. The interviews were designed to (i) further explore survey responses that the authors found particularly interesting, and (ii) follow up on unexpected themes that emerged from student comments. Our survey questions were quite broad and included the following areas: the role that students think mathematics plays in physics, differences in their experiences in math versus physics courses, stereotypes of mathematicians and physicists, how proof or justification differs in the two subjects, and whether mathematics has been a help or a hindrance in their understanding of physics. Based on our own experience and prior research literature, we asked about specific mathematical topics that we expected to differ between the fields or create learning difficulties for students, for example, Fourier transforms, infinitesimals, linear algebra, and the Dirac delta function. We expected, for example, that the common use of infinitesimal quantities such as dx or dW in physics would be an obstacle for students in

applying calculus as they learned it in mathematics courses. We also included Corinne’s shibboleth as one question, since we have not seen published data on it.

In designing the survey questions, we wanted students to describe experiences in both mathematics and physics classes, both positive and negative. Beyond simply comparing these, however, our main interest was in how students use (or transfer) their mathematical knowledge in the physics context. To that end, we included two questions (4 and 5 on the survey) which might be interpreted as disparaging physicists’ treatment of mathematics. We still felt that it was important to ask these questions, and that the rest of the survey and interview questions were even-handed enough not to bias our results.

After agreeing on the survey questions, the authors sent out invitations to all math/physics double majors at our institution and also to those students majoring in one subject and minoring in the other. We sent out 36 emails, offering each student a gift card to complete the ~ 1 h survey. A total of 24 students responded and were interested in participating in the study. The survey questions are given in Appendix A.

At this point, we numbered students roughly in the order they responded (numbered 1–24). In addition, the students were given a letter code to indicate their major combination: “D” for double majors, “M” for math majors with physics minors, and “P” for physics majors with math minors.¹

The authors were able to collect 20 completed written responses to the surveys. After analyzing the surveys individually, we met multiple times to discuss the responses, flagging significant quotes and potential themes to explore in subsequent interviews.

We extended an interview invitation to those 13 students whose responses included experiences and opinions that we wanted to follow up on. Of these 13 students, 11 agreed to participate in an interview with one of the authors (with audio recorded). Interviewed students were compensated with another gift card.

Interviews were designed [18] to be semistructured, based on a general framework of questions that we asked each student, but we also followed up on specific statements made during the interview. Also, we followed up on statements made in the student’s survey responses. If a student mentioned a specific mathematical topic in their written survey (see Appendix A, question 6), then we asked them in the interview to describe their learning experience in math and physics classes involving the topic. We split up the students randomly among the three authors and each

author interviewed three to four students. The interview questions are given in Appendix B.

After completing the interviews, the authors transcribed each of their interviews from the audio recordings. All interview transcripts were then read by all authors and analyzed thematically [19]. We created and shared summary notes and memos. We met repeatedly to discuss each interview in depth, highlighting quotes that stood out to us, or themes that seemed to come up multiple times.² We were able to categorize the main results into four principal themes, which we discuss in detail in Secs. IVA–IVD. Themes reflected comments and viewpoints that came up multiple times in multiple contexts. Since this is an exploratory study, we have also included points made by just one or two students that might be more broadly confirmed in future work.

The following table summarizes students’ participation at various stages of the study:

Stage	No. of D	No. of M	No. of P	Total students
Request for participation	13	8	15	36
Surveys sent out via email	10	3	11	24
Surveys received/analyzed	8	2	10	20
Interview invitations extended	4	2	7	13
Interviews conducted/analyzed	4	1	6	11

These numbers are small in absolute terms, which is a limitation of our study. Note, however, that 36 is the total size of the target population at our institution. Relative to this, we analyzed surveys from 55% of the population and interviewed 30% of them, which are high response rates. As noted above, the total number of physics majors is on the order of 400.

III. SUMMARY OF FINDINGS IN SPECIFIC MATH TOPIC AREAS

Before presenting the major themes we identified in our data, we summarize student comments about some specific mathematical topics we asked them about. Many of these topics will appear again within those themes, which will provide broader context for the students’ observations.

A. Infinitesimals

We anticipated that the common use of infinitesimal quantities in physics would conflict with students’ prior understandings of calculus from their mathematics courses. Physicists happily work with infinitesimal amounts of charge, dq , or displacement, dx , whereas mathematics

¹Although we did not select on the basis of class standing, all 20 students who completed the survey had JR/SR standing, and all 11 interviewees had SR standing. These interviewees were a mixture of third-year students (who had enough credits for SR standing) and fourth-year students nearing graduation.

²We also reviewed the survey answers after the interviews; we did not find anything mentioned in the “noninterviewee” survey responses that was not also represented in the interviews.

courses treat calculus using limits and often explicitly deny that infinitesimal or differential quantities are individually meaningful. However, our students did not identify this as a major obstacle. Some of them recognized physics infinitesimals as an indirect way to reason about limits or small finite increments, which could be rigorously reformulated in those terms if desired. Although they were not bothered by the supposed *infinitesimal* nature of these quantities, they did express some confusion over the rules for *manipulating* them, which they included in their catalog of mysterious physics “tricks.” For example, one student realized that canceling two infinitesimals would solve their homework problem but struggled to justify this manipulation. Others were unsure when differentials could be separated from derivatives and manipulated independently or when it was permissible to change partial derivatives to total derivatives and vice versa. Cancellations that look intuitive in single-variable calculus, for example in the chain rule, appear less so when partial derivatives are involved.

I know that if I cancel them then [infinitesimals], then I would be able to get an answer pretty easily, but there's a reason that I was stuck on this problem for an hour and it wasn't that I didn't see that I could cancel them, it was that I kept looking at it and saying, well why can I do that? (P1)

B. Fourier analysis

Another topic that we expected the students to have observations on was Fourier analysis. This subject is not generally discussed in the introductory calculus/linear algebra/differential equations sequence but is crucial to an introductory physics curriculum. Fourier analysis may be encountered later in (nonrequired) mathematics courses such as partial differential equations or numerical methods. Our physics majors are introduced to the Fourier transform in the optics portion of the advanced first-year physics sequence, but they find the treatment of it there too brief. Some were unfamiliar with the complex exponential function e^{ikx} and cited the differences between this notation and the alternative sines and cosines as a challenge. Some pointed out that the Fourier transform is defined as an improper integral but is rarely evaluated directly from that definition. Rather, tricks such as completing the square are employed. Many students mentioned their physics course on mathematical methods as being the first place they were given enough time to fully learn and appreciate Fourier analysis.

C. Delta functions

Another topic we specifically asked students about was the Dirac delta function. Like Fourier analysis, the delta function is not discussed in the first-year math sequences (or perhaps anywhere in their math coursework). Most

students were willing to accept it in the context of physics applications despite feeling that they did not understand it deeply as a mathematical object. They knew the rules for manipulating it and could do so despite wondering how these were justified. One student wondered how $a\delta(x)$ can differ from $\delta(x)$ if both are infinite at $x = 0$ and zero elsewhere. A mathematics major who had seen the delta function in quantum mechanics took the view that since quantum mechanics violates our macroscopic intuitions anyway, it makes sense that its mathematical tools such as the delta function would also violate them. Several students mentioned the integral form of the delta function, $\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{ikx}$, specifically as something that seemed “like magic.”

D. Corinne's shibboleth

Dray and Manogue [6] propose that the following question will distinguish physicists from mathematicians:

One of your colleagues is measuring the temperature of a plate of metal placed above an outlet pipe that emits cool air. The result can be well described in Cartesian coordinates by the function

$$T(x, y) = k(x^2 + y^2)$$

where k is a constant. If you were asked to give the following function, what would you write?

$$T(r, \theta) = ?$$

The physicist, considering $T(x, y)$ to be a function representing the physical temperature at a point in space, would infer that $T(r, \theta)$ is the same temperature function but expressed in 2D polar coordinates instead of 2D Cartesian coordinates. The mathematician, instead, would consider the function $T: \mathbb{R}^2 \rightarrow \mathbb{R}$ as a mathematical object independent of the dummy variables used to represent the two real number inputs. As a result, the physicist will answer $T(r, \theta) = kr^2$, and the mathematician will answer $T(r, \theta) = k(r^2 + \theta^2)$.

In our version of the prompt, below, we omitted the physical context for the function, hypothesizing that this would remove potential bias toward the “physicist” response. In addition, we explicitly suggested that there might be other possible answers given by a physicist or mathematician to encourage students to apply multiple perspectives to the problem:

Suppose that $f(x, y) = x^2 + y^2$. What is $f(r, \theta)$? Do you think that either a mathematician or a physicist might give a different answer than you did? What answer, and why?

Surprisingly, *all* students gave the physicist's response (or a variant of it³), and only one recognized that a mathematician might possibly give a different answer because of an assumption made in the physicist's response:

It's possible someone who doesn't study physics would be confused by/not understand the abuse of notation implied here, where the actual function f is a different mapping depending on what symbols (coordinates) we use. They might instead treat the question "What is $f(r, \theta)$ " as a request to directly substitute the symbols " r " and " θ " into the original function. (D4)

Student D4 mentioned that the mathematician's response is the more "rigorous" one and clarified that in his view, the physics interpretation is an abuse of notation. This student also mentioned that he had thought about this issue before through discussions with a close friend who is a math major.

Redish and Kuo [5] identify that physicists load meaning onto variables in a way that mathematicians normally do not. We confirmed a strong version of this in our study with our version of Corinne's shibboleth—someone giving the physicist's response is clearly assigning meaning to the coordinates (x, y) and (r, θ) , even though the identification of these variables as Cartesian and polar coordinates was never given in the problem statement. In the interviews, we tried to probe the students' answers a bit more, asking if their answer would have changed if we instead asked about the function $f(u, v) = u^2 + v^2$ or $f(p, q) = p^2 + q^2$ (and then asked what $f(r, \theta)$ would be in that case). It seemed that the interviewees had already decided the question had to do with changing coordinates and still gave the answer of $f(r, \theta) = r^2$. Alternatively, they said they would not know how to answer without knowing the meaning of the new variables.

We do not find it surprising that undergraduate students associate definite meanings with variables in this way. What is surprising is that even mathematics majors do not suggest the mathematician's interpretation of function notation when prompted for an alternative.

E. The role of mathematics in physics

We asked our interviewees for their views on the role that mathematics plays within physics. The most common response was that mathematics is a tool for solving problems and understanding the physical world. For some students, the term "tool" includes the implication that physicists do not always need to delve into or justify how the tool works; for their purposes, it is enough that it does work and the underlying reasons can be left to

mathematicians. A related view, echoing Galileo, was that math is a language for describing the world. Physics needs a language that is more precise and more adapted to logical reasoning and calculation than English, and mathematics fills this need.

More detailed responses suggested that mathematics provides a general framework that physicists can then fill in with boundary conditions, physical parameters, or other "limitations" that describe the specific laws of physics and material properties applicable to a given problem.

Physics is just applied math with concrete, like, limitations ... like oh, the ball is not going to go outside of the room ... we'll never, when doing physics, ... you'll never consider something outside of a third dimensional realm. (P20)

In cosmology, the Einstein equations and the Friedmann equations are all just math, the physics is when you then say, okay, well this is the, this is our idea of what the dark matter content is, and this is our idea of the equation of state of the matter, and we plug that into the stuff that's been mathed out for us and then we get, you know, we get the scale factor throughout the expansion history of the universe. (P1)

IV. MAJOR THEMES FROM OUR DATA

In the following subsections, we present the four major themes we identified in our data, supported with quotes from the student interviews.

A. Major theme 1: The nature of the subjects

A major theme in our data is that students perceive mathematics to be deductive, standardized, and highly structured, whereas physics is inductive, eclectic, and flexible. They expressed these views very broadly through their comments on the nature of proofs or arguments presented in each subject, the presentation of other material in the classroom and in textbooks, the homework and exam problems posed in each class, and the ways in which instructors respond to questions from students.

A broad summary, in our words but based on student comments, is as follows. The presentation of mathematics is often structured by the definition-theorem-proof format, even when "motivation" or "application" are added to this template. Physics is more likely to proceed inductively from examples of phenomena to generalization and theory. When a physics class requires a new mathematical concept or technique, it is likely to be presented briefly in the form of an equation or rules and then applied without a deep justification or motivation. Students recognize that mathematics is abstract and not necessarily connected to the physical world, whereas physics must be so connected even

³Two students said that $f(r, \theta) = r$, which we consider closer to the physicist's answer than the mathematician's answer.

at its most abstract. Physics is about explaining or predicting real phenomena, while mathematics is “about” itself. In that sense, physics is a blend of theory and experiment, while mathematics is “all theory.”

1. The nature of proofs/arguments in each subject

Students perceived mathematical proofs as highly structured, with the hypotheses and conclusion announced at the start and all the logic spelled out explicitly. Specific named methods such as mathematical induction may form the proof structure, and named theorems may be quoted in support of particular steps. A physics “proof” is more free form and can contain jumps and gaps; it looks more like a calculation than a piece of reasoning, and the result may appear suddenly as a surprise. (One student gave the example of the speed of light emerging suddenly from a calculation based on Maxwell’s equations.) The steps typically follow by standard computational tools from algebra and calculus without further overt justification. Students find mathematical proofs to be more wordy, the words serving to explain the reasoning. In contrast, we would add, when words appear in a physics proof, they may interrupt the flow of the calculation or proof to introduce a physically motivated approximation, which does not have a rigorous justification, or perhaps to dismiss a case as “unphysical” rather than mathematically disproved. Unlike a mathematical proof, a physics proof may not be self-contained, and students feel that they have to ask additional questions after class if they want to understand the transitions between steps. Students also mentioned that mathematical proofs may be more rigid in terms of how terminology or symbols must be used, or how the logic must be sequenced, while physics proofs are more flexible.

Once I go through a mathematical proof, ... as long as I understand the intermediate steps, I end up with, like, I know why this result works inside and out. And, whereas physics proofs sometimes require a little bit more, in some ways, trust. (D22)

Whereas in physics sometimes if you look at the notes for your proofs there’s not—they have to make a jump but I’m not entirely sure how they got there, then I have to go talk to them, like how do you get from here to here? (M9)

Proofs in math class are more like a written paragraph type of thing versus in physics ... it is all just math like algebra and calculus ... and a lot of the proof in physics is in the math rather than in the spoken words. (P20)

These comments are consistent with the view of a student in [5], who said (p. 579), “[In math] it just all makes sense

to me, because there’s a reason it works, and it’s just one reason. It’s not like in physics really where there’s so many different cases like I said before. In math, if I understand the proofs of why it’s that way, and then, I’m comfortable using that equation.”

2. How topics are presented in class

Students felt that topics in mathematics classes are developed thoroughly through the definition/theorem/proof structure so that their properties are built up gradually and fully justified. In contrast, mathematics in physics classes was seen as a means to an end, and new techniques may be simply presented and then exemplified through applications. Students recognized that the main focus is on the physics, while the math is simply a tool which can be accepted without rigorous justification if it works. The rules for calculating with it take precedence over complete definitions or derivations.

My math classes have been, they have, like a familiar structure ... they have some motivation, they have some definitions, a theorem, and a proof ... Whereas in physics I feel like it’s kind of been more all over the place. Like the transitions between when somebody’s giving like a heuristic argument or an actual proof of what they’re trying to show, it’s a bit blurry. (D3)

It’s also a lot easier to take notes in a math class for that reason ... I can see what each thing is defined as, what the main theorem is, and how it’s proved. Whereas in physics I think things are a bit at least, in my experience, a bit more, like haphazard. (D3)

In physics the words that you would be using in that are generally the physical explanation of the phenomena where you are kind of saying like this is roughly how the equation matches up to the phenomena, while in math you’re not trying necessarily to match up the equation to a phenomenon, you’re simply trying to explain the equation and how it works. (M9)

[Math topics are presented as definitions, theorems, proofs, but] in the physics class I feel like often we’re given an equation maybe and then we jump right into uses of it, like we’re given the Fourier series and then we jump right into applying it. (P1)

3. Nature of homework and exams in each subject

We will have more to say about assignments in each subject when we discuss student perceptions of tricks in physics. However, students find math assignments more

predictable than those in physics, particularly in terms of the expectations for what correct or complete solutions should include. The stricter standards of justification in mathematics and the presence of physically motivated approximations or shortcuts in physics contribute to this.

If I have my math homework and my physics homework on my desk, and someone comes by and looks at it [and asks] “Oh, what class is this?” I’m like, ... if it looks like an essay, then it’s math. If it’s math, then it’s physics. (D5)

I can get full credit on the assignment even though I don’t know why the thing I just applied works in this instance ... you can BS the physics assignment and still get 100 on it, even though you don’t understand why the stuff you’re writing works ... I can explain the steps and still not understand why they work, versus in a math problem, in a math homework assignment, if you BS the homework assignment you’re not getting 100 on it ... because you’re going to be missing some crucial steps in your proof, and you know you’re missing those crucial steps, and when a proof is complete you feel it. (P1)

The midterms and the finals in math classes are so much more standardized, like I feel like I know exactly what I’m getting into in a math exam even if it’s with a whole new professor. Like with all the math exams of the [calculus] series, they kind of felt the exact same, and then now that I took a couple proof-based math classes, those kind of exams all felt the exact same too. Versus the physics exams it kind of feels like there’s like a massive variance, like a swing between them ... my grades in math exams are very consistent like I feel like I get pretty much the same score on every math exam even in different classes, versus in physics classes, like I’ve gotten anything from above a 100 to below a 20 in physics classes and it’s not because I’m studying any differently, just because the exams vary so much. (P1)

4. Responses to student questions

Some students felt that their mathematics professors were more likely to give complete or satisfactory answers to “why” questions. Physics professors might not know the answers to technical mathematical questions or might deemphasize the importance of such issues relative to developing physical understanding.

In physics, generally, ... we use it because it works, not because it’s like, we never, like, prove anything ... If I ask some question, or if I hear

somebody else ask a question, like related to the math, ... like “How are we able to use this? What does it mean?” If it’s, like too math, like “jargony,” in general the professor seemed to be like, “It doesn’t really matter, like don’t think about it ... you just have to know that it works.” (P11)

B. Major theme 2: Physics tricks

Another major theme from our data focuses on categorizing ways in which students perceive mathematics, as it is used in physics classes, to appear unclear, sloppy, hand-wavy, etc. In general, we will call such sources of confusion physics tricks, as did our students.

We separate these tricks into two categories: “techniques” and “approximations.” Roughly speaking, the former are mathematical objects, structures, or manipulations that can be mathematically justified or further explained, although sometimes this is not clear to students and may be omitted by their instructors. It is mostly students’ unfamiliarity with the techniques or mathematical subject matter that leads to their confusion. The latter, approximations, involve approximating exact expressions, “erasing” information (our term), and reducing general expressions to specific cases, sometimes as a shortcut to get around difficult math. These often require physical arguments specific to the physics problem at hand.

It is worth noting that students’ lack of conceptual understanding of the material, and their lack of intuition about what the math is representing, can contribute to their confusion for both types of tricks. Also, we note that students became more comfortable with the use of many tricks over time, after initial discomfort when first encountering them.

1. Mathematical techniques

Some of the mathematical tricks that students mentioned can be classified as mathematical techniques, manipulations, or entirely new topics. Generally, these are things that could be encountered in a math class, as they do not rely on physical arguments or approximations.

For the most part, students were okay with an abuse of notation or somewhat “sloppy” mathematical methods, so long as they had some exposure to the mathematical topic. For example, the authors anticipated more complaints about the interpretation of differentials in physics classes (e.g., treating dq in electromagnetism as a small amount of charge), but largely this was okay with students. A few students said that they would have liked to have learned a bit more about differentials in math courses, but that they picked up what they needed in physics (especially upper-division electromagnetism). Another topic was the order of taking integrals or derivatives—a few students who had a bit more math background mentioned that physics instructors tended not to check the conditions justifying

the interchange of the order of integrals/derivatives/summations, but largely this did not affect their ability to follow along in the courses. When asked a follow-up question about some steps of proofs not being justified in physics classes, one double major said:

Uhh, they went like slow enough, whereas, like, if they did something, I could just kind of see why it worked out. Like, even if he/she didn't justify it like, I can just like, think, "Oh, this, works," so I didn't really ... there's nothing that really stood out to me. (D3)

One topic that confused several interviewees was Fourier analysis (Fourier series and transforms)⁴:

I would say the Fourier transform ... It was probably, I mean, it's definitely like the most foreign piece of math. (D5)

There seemed to be a lot of factors contributing to this: the topic was new when they were introduced to it in physics, it was introduced for a specific purpose (to discuss optics) and not discussed in full generality, and homework exercises tended to focus on particular manipulations that were important in performing calculations, but did not focus on conceptual understanding of Fourier analysis in general. It was these calculations in particular that were almost uniformly called tricks by the students—they did not know why they were doing what they were doing, didn't know the "rules," in effect, and did not know what tricks were allowed or justified. This was especially pronounced when the students reflected on their homework assignments in these courses:

The homework for [the upper-division math methods course in the physics department] was, was really complex and, and very difficult and it required some creative manipulations of the Fourier transform and of complex numbers that like, like we had to manipulate it in interesting ways, in order to do that you need to understand exactly what the, the Fourier transform is and what these operations are doing, and without really understanding exactly what they're doing it, it just felt like these problems were kind of magic and it's like, oh look, there's this trick to solve it but you know tricks aren't helpful if you

don't understand the operation that they're, that they're getting around. (P1)

I personally felt like there was a lot of like tricks that are used in the Fourier transforms that are just kind of well-known and you just have to look up ... I didn't know they were tricks ... (P11)

A similar sentiment was expressed at the upper-division level with linear algebra in quantum mechanics. Almost uniformly, students mentioned that the one quarter of lower-division linear algebra that they had in a math class was insufficient preparation for upper-division quantum mechanics. The lower-division course never focused on abstract (or infinite- dimensional) vector spaces, nor used the field of complex numbers, nor discussed hermitian and unitary operators. As a result, there was a similar feeling of being overwhelmed in quantum mechanics as with Fourier analysis, as the students were trying to learn the math along with the particular physics applications, with new notation on top of it all:

... in linear algebra you have a matrix and eigenvalues and corresponding eigenvectors but I feel like that sort of calculation, or like I guess that calculation algorithm type thing that you would do in linear algebra is not at all how it was done in quantum mechanics. And that's definitely more of a notation thing in physics where you use the Dirac notation ... I feel like in physics they assumed like, "oh they learned this in, like linear algebra," but obviously we didn't learn Dirac notation in linear algebra. (P20)

There were a few other mathematical topics mentioned, but not as often as the above two. These include symmetry arguments (for evaluating integrals or determining the direction of vector fields) and solutions to differential equations. The authors suspect that these issues were not as universally cited because the students in general knew the physical context and/or had some familiarity with the mathematical techniques required.

2. Approximations

Students brought up approximations in a few different contexts, with Taylor series being the most common. In general, students expressed discomfort working with an approximate expression if they are aware (or believe) that an exact expression exists, if they do not see how the approximation will help solve the problem at hand, or if they do not think they are developing tools or skills that will generalize and help them tackle other problems on their own. A related issue was mathematical "shortcuts" taken during lecture: going from general expressions to specific cases, which is a different kind of erasing information.

⁴Most of the interviewed students took a five-quarter honors sequence of introductory physics, which goes into a little more detail than the standard sequence intended for engineering majors. In this honors sequence, Fourier transforms are brought up in either the fourth or fifth quarter, first with optics and later with quantum mechanics.

For example, regarding back-of-the-envelope calculations, one student expressed discomfort when a professor would plug in numbers, but not their exact values. An example might be canceling out π in the numerator with 3 in the denominator of an expression when an instructor is estimating numerical values. Students felt a disconnect between doing “real” (“precise” or “exact,” in their eyes), difficult physics problems, and being cavalier when getting a final result:

[The professor] always liked to mention, like back-of-the-envelope calculations. And [laughs] I remember a lot of people really were kind of upset by that. He/she liked to just approximate a lot of stuff very quickly, and I remember people were kind of like ... people didn't really like that at first ... You're like, “why can't we just, why don't we just use like, the most exact answer?” But then later on, I realize it's like, it's really like, negligible. It's like there's no point in including all the extra stuff. (P20)

Many students mentioned Taylor series when the topic of approximations came up. One student, a double major who had already taken several advanced mathematics courses, gave the following anecdote:

An example that I've thought about where it's like, physics kind of like sweeps a bit of stuff under the rug. I think for like, you know like the pendulum swinging? Umm, they make an approximation where the angle is small. So normally when like you Taylor expand it you have a bunch of trig terms, but they just do like $\sin \theta$ is approximately θ , and it seems like, you know, as somebody coming from math, there's a whole lot of other information in those Taylor expansion terms ... hypothetically, if I were to ask a professor like, “What do you do about those other terms, like isn't that part of the physics?” And they say like ‘no, it only matters that the angle is small, and we can make this approximation.’ It's like, well that's not satisfying to somebody like, likes the full solution, I guess. (D3)

This student actually mentioned that their math background was a hindrance in physics problems—they are not used to discarding information in a seemingly arbitrary manner. What are the rules behind when students are allowed to linearize equations? When should we keep the second-order terms? Why are we not trying to quantify the error when throwing away these terms? Should the error not accumulate over time in a problem like the pendulum? And, if so, why are we ignoring this?

When asked later if there was an exact answer to that problem, the student replied:

I think, well I'm pretty sure there is an exact answer. I think I knew that, because like the differential equation isn't that hard, like $x'' = \sin x$. (D3)

This confusion is probably not too surprising, as a 5–10 s quick remark in lecture to the effect of “this equation is not solvable analytically” or “a complete solution involves elliptic integrals” can easily be missed. Instructors may say that advanced physics classes cover things in more detail, implicitly promising that a more exact treatment exists and awaits them in a more advanced course.

Another student mentioned that if such approximations were to be made in a math class, then there would be more rigorous analysis of error bounds:

I've taken real analysis, when you do real analysis and you're giving the approximation functions for equations in that class like the Taylor expansions, they of course give you the error and they prove why the error works and they are like, this is how we minimize the error to make sure that there is as little error as possible. We weren't given anything like that [in the physics course] ... (M9)

When students mentioned Taylor expansions in interviews, they cited negative experiences of homework problems where they were told explicitly to Taylor expand some expression to a certain order. Many said that Taylor series appeared often and seemed important, and that they wished they had more introduction to it in their classes. However, only one gave the sentiment that Taylor series/expansions can be a useful tool to analyze physics problems.

C. Major theme 3: When are preparatory math courses helpful?

Our next theme centers on how physics students perceive their preparatory math courses. In general, the student feedback that we collected seems to support the claim that these math courses are always at least somewhat helpful, but some appear to be more helpful than others. While not a cultural difference *per se*, this addresses our research questions of how students navigate or coordinate differences between their mathematics and physics classes.

For most physics majors, their first college math courses are the introductory calculus sequence. At our institution, this sequence is a series of five one-quarter courses. Based on our interviews, students taking introductory physics

seem to universally appreciate introductory calculus and feel it prepares them well for introductory physics. For example:

And so learning, being able to learn, like, something in [vector calculus] like a path integral or a surface integral, and then like a week and a half later it comes up for the first time in [electromagnetism], it made it a lot easier ... so then obviously learning it first in math made it easier to apply in physics but seeing a very concrete physical solution and in a way having a whole class dedicated to concrete physical solutions made [vector calculus] a bit easier to kind of digest. (P20)

In the quote above, vector calculus seems to be especially helpful for electricity and magnetism, as one might expect. In math classes in particular, there does seem to be a tension between physical application and theoretical exposition. Whereas the introductory calculus series seems primarily interested in applications and building mathematical skills, a good math program is invested in fostering proof-writing ability and thus some courses end up spending time in this pursuit. A subtheme that we noted was how the balance of time spent by a math instructor on physical applications versus more pure mathematical theory affected student learning. There seems to be student consensus that a more “proofy” course is less helpful when it comes to the learning of physics. For example, we have a quote comparing the complex analysis course with the “Introduction to Proof” course in the mathematics department:

Like when you multiply, the angle just kind of goes around. And when you divide, the angle goes back the other way. That kind of stuff. When you multiply, it just increases. And then, the visualization of umm, the, what was it, the branch cut and stuff, where you know, it becomes like a spiral, and it keeps going around. I was like, “Wow, this is, this is really cool.” It was just really interesting to me. But then [Intro to Proof] was like a lot of these symbols, symbols. You have to write a proof in this way, this way, and I was like, it wasn't as interesting. (P12)

For some physics majors, it would seem that connecting math to real life and/or being able to visualize math is part of the allure of being a physics major. Furthermore, student quotes express similar feelings regarding other courses such as real analysis, typically a very proof-heavy course. In a further example, a student comments that upper-level math courses that are taught with physical applications in mind or that balances the proof-based approach with more applications can be very beneficial, and in particular heaps praise on complex analysis, which has many applications

useful to upper-level physics and, while being proof-based, is often taught with an eye toward applications.

So, I think that math for us really helped me understand the complex numbers that are used in physics. Similarly, as I mentioned before, many physics courses just give you this, and you have to apply them immediately after you know this fact. Sometimes you don't fully understand the mathematical foundation of it. So, I think [complex analysis] really provided you a good foundation for understanding those complex numbers and how to use them. (P23)

We also found student comments suggesting that there are some cases where the content of a particular math course does not line up well with what physics students say they need, even if it is more application based. As an example, the Taylor series is a standard component of an introductory calculus series and an essential piece of a physics curriculum. However, as crucial as this is in physics, some students feel that it plays more of a minor role in introductory calculus.

... Oh, another one would be Taylor series for sure, because my [calculus course] for some reason we got behind schedule and so Taylor series we spent only week 10 on it with like the understanding that it would play a very minor role on the final and so I didn't really care much about, about Taylor series, not realizing that Taylor series would be one of the most important things I would like, return to time and time again in physics ... (P1)

Another place where there seems to be misalignment is in quantum mechanics. A solid upper-division course on quantum mechanics involves many topics from linear algebra, and many physics programs require a course on linear algebra from the mathematics department. However, these prerequisite courses are often not taught with the goals of quantum mechanics in mind. Quantum mechanics uses complex vector spaces, Hermitian operators, and commutators, topics that a typical introductory linear algebra course will not discuss. Many students expressed frustration with this mismatch and with how difficult it can be to fully comprehend these topics upon encountering them for the first time.

Further complicating things is that, according to students, some physics faculty are unaware of this misalignment. Several students noted that instructors would sometimes make comments, such as, “as you probably learned in your linear algebra course,” regarding advanced topics that most linear algebra courses do not cover.

Additionally, there were several student comments suggesting that a more advanced linear algebra course could be

a better match for the expectations of a quantum mechanics course.

Obviously the number one useful thing was eigenvalues, eigenvectors, eigenfunctions and, and I feel like that was, that was maybe something that was more reinforced in [advanced linear algebra] then, and we, we kind of talked about eigenstuff in [basic linear algebra] but, but maybe in [basic linear algebra] it felt a little more arbitrary like, like I, I told my Dad that we were doing something called like, eigenvalues and he was like, oh yeah I use eigenvalues all the time in my work. (P1)

Another subtheme was the utility of so-called trading zones, an anthropological term for spaces where members of different cultures meet to trade goods and ideas that was adapted by Peter Galison [20] to describe physicists interacting with other cultures, such as mathematicians and engineers, in the development of new technologies. Trading zones often develop their own languages that blend those of the cultures that interact there. Most physics departments, including ours, offer a mathematical methods course, in which mathematical topics are presented using more physical language and applications. We view such courses as trading zones that use blended math and physics language and offer an opportunity to really focus on the physical applications and visualizations of the math topics and their connection to real-life systems. Furthermore, many math topics essential to physics are not discussed in typical introductory math courses. Examples include partial differential equations, Fourier analysis, complex analysis, and advanced linear algebra. From student comments, it seems that having this space, this trading zone, to more fully explore these mathematical topics is valuable and appreciated.

I recently took mathematical physics [math methods] last quarter and I very much enjoyed that class because it was a very like nice blend, I think, between, you know, physics and actual solving math stuff and I think, you know, we covered Fourier transforms, you know, maybe not as rigorously as you might in a math class but I think it was relatively satisfying. We covered Fourier transforms and complex analysis and we did use a little bit of linear algebra, you know, but you know find out eigenvalues of like a linear, linear system and I thought that was very satisfying. (D4)

As discussed, one topic rarely covered in introductory math courses that seemed problematic for our students is the Fourier transform. Even though this is typically included in a mathematical methods course, it is so

important that it is often introduced earlier in the physics curriculum. When this happens, it seems that many students feel that there is not enough time to develop a good understanding of it.

D. Major theme 4: When do physical contexts for math help understanding?

Physical examples can often be extremely helpful in bridging the gap between mathematics and physics. The final main theme that arose was a question of timing: when do physical contexts for math help understanding? Which is more helpful, seeing the math before, after, or at the same time as the physical context? The students had much to say about this. We even had several cases of students referencing a single class example, involving matrices, as being extremely valuable.

Many student comments promoted the idea that learning math first in a math class and then applying it in a physics class works well when the class is aligned well with the needs of the physics topic, as with the introductory calculus sequence and first-year physics. There were also several comments suggesting that when this does not occur, and the math topic is introduced in a physics class right before its application, it can lead to problems. There was consensus that this way of presenting the material often does not give enough time for students to fully process the new math topic, making a discussion of the physical application difficult. For example, here is a quote relating to the introduction of Lagrangians and functionals in a junior-level classical mechanics course.

It was in [classical mechanics], we were presented with the Euler-Lagrange equation, and that, that seemed ... it seemed a little fast to me how it was like functionals. I didn't know that was a thing before that, and then we just have this thing like the action. Minimizing action. I was like, okay, I guess, I'll ... I mean, I didn't really understand, you know, the exact derivations, and in ... intuitions behind everything. But then you could still use the Lagrangian, and use all the equations, and then, you know, get the equation of motion. But then, I kind of just start with that. And I'm not sure ... I wasn't exactly sure how the ... how that even tied into, you know, getting those equations of motion. (P12)

Furthermore, a student directly discusses this situation regarding the introduction of Fourier transforms in a first-year quantum theory course. As mentioned, Fourier transforms are not generally presented in the typical math courses that physics majors are required to take. Here again, we see a student describing not having enough time to sufficiently digest the new mathematics to apply it.

I think the ideal way to do it would be learn it from math professor, and then learn it from a physics professor. (Interviewer: Why is that?) 'Cause, I mean, I think like things are always gonna be more complicated when you don't have, like, context around them. But at the same time, at least from my experience with both, like having at least some sort of baseline foundation of just the math part of it, and then being able to apply it. (D5)

There also seems to be evidence that providing physical examples along with the math theory can be an effective way to teach, provided that students are given enough time to begin to master the material. In the upper-level complex analysis course, a mixture of theory and applications seemed to work well.

For [complex analysis] I took the class taught by Professor X, and he gave us a lot of examples in physics. I think that bridges like well from complex numbers in math to complex applications in physics. I feel like I took [complex analysis] because of my major, so I think most physics students won't take that course. I think the complex numbers are only told by that course. No other lower physics courses teach you thoroughly about complex numbers. (P23)

It seems that the balance and sequencing of time spent on new mathematical tools and physical applications is pedagogically important. There were several comments stating that little or no physical application was also not helpful.

Student comments would seem to support the idea that what works best for the learning of mathematics needed for physics is a mathematical style presentation with plenty of physical applications included. It does not seem to be ideal when the inverse occurs—that is, a physics presentation with mathematics topics sprinkled in. However, what seems to be the most successful approach, according to student comments, is when the math is presented first in a mathematics course and then the topic is used in a subsequent physics course.

Finally, well-timed and key physical applications can prove very effective. There was one key example, regarding a ring of masses and springs solved with linear algebra techniques, that really resonated with many of our students. It seemed that this type of “aha” moment could be very powerful.

It was like, basically just starting with like a couple ... coupled oscillators and then extending it to like, n of them, like n of them in a ring, and then as n goes to infinity, and we were just talking about how you can write out like the force matrix or you can write out the symmetry matrix, which

is equivalent, and then you can like find the eigenvalues of that. And like the way that they kind of presented eigenvalue and eigenvectors, and some of the linear algebra, I thought, was like really helpful, because it was like right in the context. They like, put it all into context, because everyone's taking [basic linear algebra]. Everyone kind of understands the ideas, but putting it in the practice was really helpful ... (D5)

V. DISCUSSION

A. Research questions

Our first research question asked, “There are known “cultural” differences in how physics courses and mathematics courses handle mathematics instruction. Are the students in our study aware of these differences?” We focused on two broad categories of differences identified in prior work:

- A. Physicists load meaning onto symbols in ways that mathematicians do not, which leads to differences in how physicists and mathematicians interpret equations.
- B. Physicists and mathematicians have different goals for the use of math.

Overall, students did not seem aware of the type A differences. For example, they did not recognize that mathematicians and physicists would likely interpret Corinne's shibboleth differently. Our interviews further support this, with students often describing physics classes as disjointed, unclear, or even “haphazard.” As noted in Sec. IV B 1, students reported that notation was often confusing—particularly in topics such as Fourier analysis and linear algebra in quantum mechanics. Several even described being able to complete their physics homework without fully understanding the steps they were writing down.

Students did seem to demonstrate awareness of type B differences, though they were not always certain about the exact goals of instruction. Evidence of this trend is first seen in the survey data. For instance, responses to question 8 indicate that most students associate physics with a stronger emphasis on intuition. Similarly, as discussed in Sec. III E, students observed that mathematics is taught differently in the two disciplines because instructors have different aims: in physics, math primarily serves as a tool to better illustrate or understand physical phenomena.

This trend also appears in the interviews, particularly in themes 1 and 2. In Sec. IV A, students commented on the contrasting organizational structures (e.g., deductive versus inductive approaches) and on the different levels of rigor in proofs. In Sec. IV A 2, student M9 points out that in physics you are trying to explain phenomena while in math you are simply “trying to explain the equation.” Similarly, in theme 2 (Sec. IV B), students noted the comparatively

“sloppy” presentation of math in physics classes, sometimes describing it as relying on tricks. Student P20’s comments in Sec. IV B 2 make the point that, in their experience, approximations are handled more rigorously by mathematicians.

Our second research question asked, “To what extent do students perceive that these and other cultural differences affect their learning of mathematics and physics, and how do they manage these differences?”

Students had much to say about the impact of cultural differences on their learning. In Sec. IV A 1, many expressed frustration with the quality of proofs presented in physics courses. In Sec. IV A 2, several noted that the more regimented structure of mathematics lectures made them easier to follow and take notes in, which in turn helped them prepare for assessments such as homework and exams. By contrast, the less predictable, more *ad hoc* presentation style in physics often made coursework harder to navigate. In Sec. IV B 1, students described the tricks used in physics instruction as a barrier to understanding. For example, students P1 and P11 reported that the explanations they received for Fourier analysis were unsatisfactory and led to confusion. Likewise, many wished for a more formal treatment of approximations than what they encountered in physics courses.

Survey responses reinforced these themes. In question 7, which asked how differences in justification standards affect learning, about 35% of respondents reported that the lack of rigor in physics hindered their learning, while 25% felt it had no impact, and 30% viewed the exposure to two standards as beneficial. Question 4 produced a similar trend: 75% of students recalled a derivation in a physics class that they felt was not rigorous enough. In question 10, 35% reported experiencing an unsatisfactory mathematical explanation from a physics instructor, while none reported dissatisfaction with explanations provided by math instructors about physics-related topics.

Regarding coping mechanisms, students described several approaches. Some simply accepted the techniques, as in the case of Dirac delta functions, while others tried to seek extra resources either online or in more advanced math courses.

Our third research question asked, “What do students cite as major differences in the presentation of “common” mathematical content between their physics and mathematics courses? What specific classroom examples do they offer?”

Beyond the notational issue of the reversed roles of the spherical coordinates θ and ϕ between mathematics and physics, students noted the casual use of approximations in physics for which error bounds would be provided in mathematics. Techniques, say for working with Fourier transforms, that would be developed deductively in mathematics are more likely to simply be introduced and illustrated with examples in physics, leading students to

view them as tricks. Calculus manipulations are more likely to be carried out with differentials in physics than in math. Linear algebra in quantum mechanics is presented in the unfamiliar Dirac notation.

B. Emergent themes

Our analysis of the students’ surveys and interviews is summarized by our four major themes:

1. Students perceive physics classes as inductive, eclectic, and flexible, whereas they perceive mathematics classes as deductive, standardized, and highly structured.
2. Students were frustrated by so-called physics tricks—mathematical techniques or approximations used in physics classes that appear unclear, unjustified, unmotivated, handwavy, etc.
3. Students cite the introductory calculus sequence as being helpful for physics, and more formal, upper-division, proof-based math classes as less so. Trading-zone courses, which might include upper-division applied math classes (in the math department) as well as math-methods physics classes (in the physics department), helped clear up several topics.
4. Students thought they benefit more from initial exposure to a mathematical topic in the mathematical “style” (theorem-definition-proof), and only later applying it to a physical context after a general introduction. Learning new mathematical ideas simultaneously with new physical applications (e.g., quantum mechanics using Hilbert spaces) can easily be overwhelming.

Our analysis of Corinne’s shibboleth confirmed Redish and Kuo’s claim [5] that physicists and undergraduates load meaning onto symbols in a way that mathematicians do not. One finding that resonated with the authors was that students were uncomfortable with “erasing information” (our term). In our data, this meant making approximations in physics (where an exact answer is not required, relevant, or possible). However, we have seen similar student reactions in math classes when manipulating inequalities in analysis: they tend to want to find the optimal bounds and are uncomfortable settling for weaker ones. This may be due to the fact that it is difficult to anticipate the goal of the instructors’ seemingly arbitrary manipulations; students are more focused on individual manipulations rather than the overall big picture objective. It is also a general communication norm to make the most informative statements that you are able to.

Some of the results surprised us as well. First, students by-and-large were not very bothered by manipulations of infinitesimals in physics classes (for example, treating dy/dx as a ratio of two small but physically meaningful quantities). In addition, the authors did not anticipate how often Fourier analysis came up as an unsettling and

confusing topic. However, perhaps this has to do with the specific presentation of this topic at our university and is not a universal experience.

C. Limitations of study

Our study has several limitations. Although we contacted every student at our institution having a double major, or major plus minor, in mathematics and physics, our sample size was small: 20 surveys and 11 interviews. We only interviewed one student with a math major and physics minor. Our study could be repeated at a larger university, or at several, making a more quantitative analysis possible. Although we felt that double majors would be best able to compare the cultures of mathematics and physics, one could conduct a similar study with physics majors exclusively inasmuch as they are a larger population and physics education research is particularly concerned with improving the curriculum for them. The students we interviewed all had senior standing at our institution and therefore might have taken many of the same classes with the same instructors. Some of our findings may reflect characteristics of those instructors rather than the courses or the curriculum. Finally, we are on the quarter system, which may imply a faster pace for covering material than semesters provide. A one-quarter linear algebra course taken by physics students may not reach topics relevant for quantum mechanics that could be included in a semester. Our first quarter of complex variables usually cannot reach the residue theorem.

D. Pedagogical recommendations

Our study involved a small number of students at a single institution, so we should be cautious in making pedagogical recommendations based on our limited data. We venture to suggest some which might be supported by further investigation. First, and perhaps most importantly, we suggest making efforts to raise awareness of these cultural differences for both faculty and students. For example, it would be helpful for physics faculty to be more familiar with what particular mathematical topics students seem to have trouble with (e.g., Fourier transforms and Hilbert spaces.) Being more conscious of these issues could lead instructors to be more prepared and proactive when dealing with these topics, for example looking more closely at what content the typical linear algebra course at their institution covers or taking more time to introduce the Fourier transform. Furthermore, these types of conversations might reveal blind spots in the curriculum that could be fixed. For instance, knowing that the Taylor series is taught at the end of the semester at your institution could lead one to spend a little longer on the mathematical background when first discussing this topic.

In much the same way that an instructor might explain the rationale behind new research-based teaching techniques they are using, we feel it might be beneficial to

explicitly discuss these cultural differences, especially in a physics class, as the students seem to be less satisfied with mathematical explanations given by physics instructors. As our interviews indicate that students are cognizant of the differences between math and physics classes, a discussion of why each discipline has developed its characteristic culture could be enlightening. These discussions could perhaps even help to start a dialog. There seem to be times when a more mathematically inclined student is turned off by aspects of the physics teaching culture. We feel the first step in easing this tension is talking about it openly. There is even the opportunity to make light of some of these differences.

Several students suggested that it would be beneficial for physics instructors to offer extra resources, perhaps textbooks, articles, or web-based materials, for students who want more details or proofs about new or more advanced mathematical topics. We feel that this is an excellent suggestion. Perhaps the physics education community could create such resources that are specific to the needs of physics students.

There also seem to be some common issues that we imagine many physics departments would share. For example, being aware that approximations and tricks can prove troubling for students could lead instructors to rethink how they approach these topics, such as spending more time on why an approximation is needed or helpful, or even trying to take a more mathematical approach and provide rigorous error bounds for an approximation. We were surprised by some of the resistance we found to what we felt were standard physical approximations, such as the small-angle approximation for a pendulum. We imagine that, even if math majors are more troubled by them than physics majors, most students would benefit from a more thorough analysis of some of these approximations and tricks.

Finally, it is possible that physics departments might want to reevaluate their curriculum, perhaps even surveying majors in a similar manner as we did in this study. For example, perhaps they will find that modern quantum mechanics has reached a point where the traditional linear algebra course is no longer sufficient. Maybe there are small changes that could be made to alleviate some of the tension that arises when a new math topic is introduced. Perhaps better coordination between required sequences of math and physics courses is needed. Again, we believe that just being more mindful would be a great first step. It is difficult to remember when one was a student and how foreign some of these topics may have seemed at a first encounter.

APPENDIX A: SURVEY QUESTIONS AND RESULTS

The survey consisted of the following questions. A quick summary of the results directly follows each question.

There were no questions where there was a clear distinction of answers by major (other than question 1, of course). That is, there were no questions for which double majors responded significantly differently from physics majors with a math minor (or from math majors with a physics minor). However, our sample size is probably too small for this to be meaningful.

1. What is your major (and minor, if any)?

(All students were double majors or one major and one minor in math and physics, by design.)

2. If you chose one major/minor first, and added the second later, what was the reason you chose to add the second?

Most students said that this was done out of interest and/or that the minor subject complements the major subject.

3. Suppose that $f(x, y) = x^2 + y^2$. What is $f(r, \theta)$? Do you think that either a mathematician or a physicist might give a different answer than you did? What answer, and why?

Only one student (D4) was able to identify that the answer *could* be $r^2 + \theta^2$ (the student provided both the “physicist” and the “mathematician” response). Seventeen others gave the physicist’s response, stating that the answer is r^2 . Two students said $f(r, \theta) = r$, with one adding, “A physicist would probably not be concerned with the square on the radius.”

4. Have you been dissatisfied with an explanation, derivation, or proof given in a physics class because you felt it was not mathematically precise or rigorous enough? Can you give examples?

15 out of 20 students cited at least some dissatisfaction, especially with Fourier analysis, Hilbert spaces, and differential equations. In addition, students did not enjoy the “hand-waviness” of mathematical manipulations and/or approximations made in physics classes. Others were satisfied with physics classes not spending too much time on the math.

5. Have you had difficulty understanding mathematical techniques presented in a physics class because the presentation or notation differed from what you had seen in previous math classes? Can you give examples?

A great majority of students either said “no” or cited the difference in which variable was used to represent the polar angle in spherical coordinates (θ versus ϕ) in physics versus math courses. Nearly all of the students who cited the polar angle notational difference said that they got used to this.

6. Have you found any of the following specific topics challenging to learn about or use because of differences in how they are presented in math versus physics? Conversely, are there instances in which

your understanding of a topic was improved by having both a physics and a math perspective on it?

- (a) The Dirac delta function (quantum mechanics, electromagnetism)?
- (b) Infinitesimal quantities or vectors such as dx and $\vec{ds}$, used extensively in physics but not so much in math?
- (c) Concepts and terminology from linear algebra and matrix theory, such as basis, linear operator, eigenvalues?
- (d) Calculations involving derivatives or integrals of functions?
- (e) Calculations involving infinite series?
- (f) Methods for solving or understanding differential equations?
- (g) Fourier series and Fourier transforms?
- (h) Dimensions or units of physical quantities, emphasized in physics but often ignored in math?
- (i) 3D vector calculus (div/grad/curl, Stokes’ theorem, etc.)?

Students cited the Dirac delta function and Fourier analysis as topics they never saw presented in math classes; they had some difficulties with the topics as they were presented in physics. Several also mentioned that infinitesimals were not treated rigorously in math courses, but that they were used enough in physics that it would have been beneficial to have seen them more in math. For the most part, students said their understanding was improved by having seen 3D vector calculus and differential equations in both subjects, and nearly everyone agreed that their ability to calculate derivatives and integrals was improved by exposure in both subjects. Responses about linear algebra were mixed, with some saying that the math exposure was helpful for upper-division quantum mechanics, while others said that their math exposure was not sufficient. Not much was said about infinite series or dimensions/units.

7. Have the different standards of proof/explanation/justification/evidence accepted in physics versus math classes had effects (positive or negative) on your learning, or your understanding of the material, in those classes?

(Numbers below indicate the number of responses in each category.)

(7) Yes: proofs and justifications are more rigorous in math classes than in physics classes, which has had a negative effect on my understanding in physics classes due to lack of clarity; (6) Yes: the two complement one another nicely, which has had a positive effect on my learning; (5) No: I recognize there are differences, but I don’t think this has affected my learning overall. Other answers were either not clear or a blend of these.

8. Along with the definitions, equations, laws, theorems, and so forth taught in physics and math courses, instructors and textbooks also try to convey intuitions about these topics (“how you should think about them or visualize them”) that are appropriate to each subject. Have you noticed differences between the intuitions taught or expected in math versus physics and have any of these differences had effects (positive or negative) on your learning?

A few students said that they did not notice a difference, but most said that intuition is more emphasized in physics classes. Students varied in which they preferred (math providing a better framework, but physics providing more concrete examples to visualize). A few mentioned that math and physics have different patterns of reasoning: math classes start fully general and only later provide examples or specific applications, while physics classes start with specific cases to build intuition, and then try to generalize.

9. Have you experienced explicit comments from either physics or mathematics instructors that are critical of how some topics are presented in the other discipline? Can you give examples?

(8) Minor or only playful joking comments; (12) No. Most examples involved physics instructors saying something to the effect of “mathematicians wouldn’t like me doing this, but ...” (not explicitly critical of the other discipline).

10. Have you asked questions of your mathematics or physics instructors about topics from the other field, and been unsatisfied with their responses? Can you give examples?

(0) Yes, from math instructors; (7) Yes, from physics instructors; (13) No. Topics where the physics professor’s response was unsatisfying: math behind special relativity (for example, Lorentz/Poincaré groups), various group and set theory concepts, expectation of an operator, special cases of integrals, bracket notation in quantum mechanics or probability, divergence and curl in electromagnetism.

11. What suggestions would you give to physics or mathematics faculty about how to help students better understand material that overlaps or is common to both subject areas?

Suggestions to math professors: give real-world and physics applications.

Suggestions to physics professors: be more organized with the presentation of material, be more clear/precise/rigorous with the math.

Suggestions to both: be clear about differences in notation and conventions (for example, factors of 2π

in Fourier transforms), provide resources to students to better understand math/physics connections.

12. When the presentation of a topic in a lecture or a textbook is different between your math and physics classes, do you have strategies for reconciling them or coping with the differences?

(6) Use online resources; (5) Try to understand the differences; (5) Ask a professor; (3) Keep them separate; (3) Has not been an issue; (2) Give the math understanding priority.

APPENDIX B: INTERVIEW QUESTIONS

We used the following working script of interview questions during our interviews:

1. How would you describe the role that mathematics plays within physics, in general? How would you describe the role it has played in your own physics courses? Describe instances where math has helped your understanding of physics concepts. Describe instances where it has been an obstacle to your understanding.
2. How does a physics class “feel” different from a math class? Particularly in the ways that math is presented and used?
3. What do you mean by calling a math or physics proof or problem solution “handwavey”? Can you give examples? How do you react to such proofs/solutions? [If they used this word in the survey responses.]
4. Think of some examples where the same mathematics topic has been presented or used in both your math and your physics classes. Describe cases in which it was helpful for your learning to have both perspectives on that topic. Describe cases in which the different perspectives made the topic harder to learn. [This can lead to the specific subject-area questions given below, if students mention that topic.]
5. What kinds of questions (from students, or by the instructor) seem expected, encouraged, or discouraged in math and in physics classes?
6. Mathematicians have been teased for refusing to solve an equation until they can prove it has a solution, or not using an infinite series until they have proved that it converges. On the other hand, physicists have been teased for not caring whether a series converges when they make use of it. Is this consistent with your experience of math and physics professors?
7. In the survey question about the function $f(x, y) = x^2 + y^2$ asking for $f(r, \theta)$ instead, would you have thought about the problem differently if it said $f(u, v) = u^2 + v^2$ instead? Would your answer for $f(r, \theta)$ have changed? Are variables like x, y, u, v used differently in mathematics than in physics?

8. Can you expand more on (...)? [student-specific survey questions.]
9. Please illustrate (...) with an example? [student-specific survey questions.]

We also drew on the following subject-specific questions:

1. (Fourier transform): How were you introduced to the Fourier transform in your mathematics classes? How were you introduced to it in your physics classes? In what ways was it useful to have two different perspectives on it? In what ways was it confusing? Has the use of the F.T. in later physics or math courses improved your understanding? Has it raised further questions for you? What differences have you noticed between the math and physics versions and uses of the F.T.?
2. (Linear algebra and quantum mechanics): Was your understanding of linear algebra (matrices, eigenvectors) from math classes helpful when those ideas were applied in your quantum mechanics classes? In what ways? In what ways were those aspects of quantum mechanics different from your prior mathematical understanding? How did this affect your learning? [If they limit their answers to particles moving in potentials, which involve

infinite-dimensional vector spaces, suggest that they think instead about spin systems, which are closer to linear algebra as presented in math courses.]

3. (Infinitesimals): Applications of calculus in physics often involve reasoning with infinitesimal quantities (dx , dQ , $d\mathbf{s}$). Did your math calculus courses prepare you for this? Are you comfortable with both the mathematical and physical styles of doing calculus? How do you see them as related, or different?
4. (Dirac delta function): Have you seen the Dirac delta function in physics, mathematics, or both? If both, how was it presented differently in each subject? In what ways do you feel that you understand it? In what ways is it confusing?
5. (Proof): What differences have you seen between the types of proof (or justification) provided in your math courses versus your physics courses? Do you feel that either (or both) styles of proof give you a solid sense that you understand why the claims are true? Or do they leave you unconvinced to some extent?
6. (Units): Another student told us that quantities with units attached (meters, grams, newtons, etc.) are very important in physics and less so in math. Have you experienced that as an important difference?

-
- [1] M. D. Caballero, B. R. Wilcox, L. Doughty, and S. J. Pollock, Unpacking students' use of mathematics in upper-division physics: Where do we go from here?, *Eur. J. Phys.* **36**, 065004 (2015).
 - [2] L. Doughty, E. McLoughlin, and P. van Kampen, What integration cues, and what cues integration in intermediate electromagnetism, *Am. J. Phys.* **82**, 1093 (2014).
 - [3] M. B. Kustusch, D. Roundy, T. Dray, and C. A. Manogue, Partial derivative games in thermodynamics: A cognitive task analysis, *Phys. Rev. ST Phys. Educ. Res.* **10**, 010101 (2014).
 - [4] K. Serbin and M. Wawro, The inextricability of students' mathematical and physical reasoning in quantum mechanics problems, *Int. J. Res. Undergrad. Math. Educ.* **10**, 57 (2024).
 - [5] E. Redish and E. Kuo, Language of physics, language of math: Disciplinary culture and dynamic epistemology, *Sci. Educ.* **24**, 561 (2015).
 - [6] T. Dray and C. Manogue, Vector calculus bridge project website, Available at <http://www.math.oregonstate.edu/bridge/ideas/functions> (2002) [accessed February 15, 2018].
 - [7] E. F. Redish, Using math in physics: Overview, *Phys. Teach.* **59**, 314 (2021).
 - [8] R. Karam, O. Uhden, and D. Höttecke, The "math as prerequisite" illusion: Historical considerations and implications for physics teaching, in *Mathematics in Physics Education*, edited by G. Pospiech *et al.* (Springer Nature, Switzerland, 2019), Chap. 2, pp. 37–52.
 - [9] R. López-Gay, J. Martínez Sáez, and J. Martínez Torregrosa, Obstacles to mathematization in physics: The case of the differential, *Sci. Educ.* **24**, 591 (2015).
 - [10] O. Uhden, R. Karam, M. Pietrocola, and G. Pospiech, Modelling mathematical reasoning in physics education, *Sci. Educ.* **21**, 485 (2012).
 - [11] E. Palmgren and T. Rasa, Modelling roles of mathematics in physics, *Sci. Educ.* **33**, 365 (2024).
 - [12] A. Ataíde and I. Greca, Epistemic views of the relationship between physics and mathematics: Its influence on the approach of undergraduate students to problem solving, *Sci. Educ.* **22**, 1405 (2012).
 - [13] J. de Winter and J. Airey, Pre-service physics teachers' developing views on the role of mathematics in the teaching and learning of physics, *Phys. Educ.* **57**, 065007 (2022).
 - [14] S. Kapucu, University students' conceptions of the relationship between mathematics and physics and the relationship between mathematics and physics learning, *J. Baltic Sci. Educ.* **13**, 622 (2014).
 - [15] B. P. Schermerhorn, A. Villanor, D. D. Agunos, H. Sadaghiani, G. Passante, and S. Pollock, Student perceptions of math-physics interactions throughout spins-first

- quantum mechanics, *presented at PER Conf. 2019, Provo, UT*, [10.1119/perc.2019.pr.Schermerhorn](https://doi.org/10.1119/perc.2019.pr.Schermerhorn).
- [16] J. M. Rabin, A. Burgasser, T. J. Bussey, J. Eggers, S. M. Lo, S. Seethaler, L. Stevens, and H. Weizman, Interdisciplinary conversations in STEM education: Can faculty understand each other better than their students do?, *Int. J. STEM Educ.* **8**, 11 (2021).
- [17] D. L. Reinholz, R. L. Matz, R. Cole, and N. Apkarian, STEM is not a monolith: A preliminary analysis of variations in STEM disciplinary cultures and implications for change, *CBE Life Sci. Educ.* **18**, 12 (2019).
- [18] E. Knott, A. H. Rao, K. Summers, and C. Teeger, Interviews in the social sciences, *Nat. Rev. Methods Primers* **2**, 73 (2022).
- [19] G. Terry, N. Hayfield, V. Clarke, and V. Braun, Thematic analysis, in *The SAGE Handbook of Qualitative Research in Psychology*, edited by C. Willig and W. Rogers (SAGE Publications Ltd, London, 2017), Chap. 2.
- [20] P. Galison, Trading with the enemy, in *Trading Zones and International Expertise: Creating New Kinds of Collaboration*, edited by M. E. Gorman (The MIT Press, Cambridge, England, 2010), Vol. 3, pp. 25–52.