Effect of scale comprehension on student reasoning about astronomical phenomena. I. The apparent sizes of Moon and Sun and Solar Eclipses

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This paper is the first in a two-part series that investigates students' reasoning on astronomical phenomena. We interviewed $N = 25$ last-year high school students to uncover their reasoning on three observable phenomena, two of which will be discussed in this paper. The first phenomenon was the apparent sizes of the Moon and the Sun (as seen from Earth) and potential causes for their variation. The second phenomenon regarded the event of a solar eclipse and the difference between total and annular eclipses. The interviews were held in light of the students' understanding of spatial scales in the Earth-Moon-Sun system. Based on their answers on a prior online survey assessing students' estimates of these scales, a personal to-scale model of the Earth-Moon-Sun system was provided for every student. The students selected for interviews were those who showed a substantial underestimation of these scales. Students were repetitively encouraged to illustrate their reasoning on both phenomena by using the scale model or by providing a drawing. For both discussed astronomical phenomena, we encountered several alternative explanations that—as far as we know—were not previously documented in the research literature. Many of these relied on an inaccurate comprehension of the involved scales. Although for some students these explanations were in line with the scale model on the interview table representing their substantial underestimations, others did not seem to take spatial scales into account when making suggestions on the causes for these astronomical phenomena. The implications of these findings are discussed.

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I. INTRODUCTION

All astronomical observations and measurements depend heavily on the vast distances between the observer and the object(s) of observation. One example is the simple perception of the apparent sizes of the Moon and the Sun. By apparent size, we refer to visual size as seen from Earth (that is, how large they look in our sky) or, equivalently, to angular size. It can be assumed that students observe these apparent sizes on a very regular basis, although they might do so without much reflection. Also in school curricula, the mechanism that explains differences in apparent sizes receives little attention. However, these apparent sizes are highly relevant in the context of several astronomical events, one of which is a solar eclipse. During an eclipse, the apparent sizes of the Moon and the Sun have very concrete implications for what we observe. Whether the Moon can fully cover the Sun, as seen from Earth, depends crucially on their apparent sizes, which in turn depend on the combination of the physical sizes of both bodies and their respective distances from Earth.

Both the general phenomenon of apparent Moon and Sun sizes and the specific event of a solar eclipse are central to this work. While students' understanding of the latter is fairly well documented (see, e.g., [1–4]), students' estimates of the apparent sizes of celestial bodies and of possible variations in those sizes are not. In fact, the literature on students' estimates and comprehension of these sizes is surprisingly limited. While this was certainly one incentive for conducting the current research, another was the explicit relevance of spatial scales to both solar

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eclipses and apparent sizes. Several studies have shown that students generally lack an accurate understanding of astronomical scales [5–10]. Yet, consensus exists that the concept of scale is essential for a thorough understanding of many astronomical phenomena [11,12]. Despite the concerning image sketched by these two findings, few studies have investigated the impact of inaccurate scale comprehension on the understanding of specific astronomical phenomena.

The objective of this work is thus twofold:

1. To document students' reasoning when asked about the (variation in) apparent Moon and Sun sizes, and about (different types of) solar eclipses.
2. To investigate how (if at all) students use their comprehension of the involved spatial scales in the Earth-Moon-Sun system to substantiate their reasoning.

While the current work covers the astronomical topics of apparent Moon and Sun sizes and solar eclipses, the companion paper [13], which focuses on Moon phases holds the same objectives.

In Sec. II, we provide a concise overview of the existing literature on students' understanding of the apparent sizes of celestial bodies, solar eclipses, and astronomical scales. We discuss how the first two are influenced by scale comprehension and how this comprehension might be impeded by common visual representations. In Sec. III, we elaborate more thoroughly on the methods used in this work. These include a (brief) overview of the online survey, an elaboration on how the data from this survey were used to select a subset of students for the interviews, a description of the interview protocol, and information on the data analysis procedure. The main results are reported in Sec. IV, and a discussion of these results is presented in Sec. V.

II. BACKGROUND

A. Apparent sizes

The first observable phenomenon is that of the apparent sizes of the Moon and the Sun. Several studies have examined people's estimates of the physical sizes—rather than the apparent sizes—of celestial bodies. For instance, Rajpaul *et al.* [14] used ranking tasks to show that many middle school students believe planets to be bigger than stars and galaxies smaller than solar systems. Bakas and Mikropoulos [15] found that only 65 out of 102 secondary school students (63.7%) selected the correct multiple-choice option for the relative sizes of the Sun and the Earth. Bryce and Blown [16] reported that roughly one third of Chinese preschool children were unsure about how to rank the Moon, Earth, and Sun in order of increasing size, but they added that this percentage decreased with age.

Compared to the literature on the physical sizes of astronomical bodies, studies investigating estimations of apparent sizes are rather scarce. Moreover, focus is often directed specifically to the Moon illusion phenomenon and not to apparent sizes of celestial bodies in general. The Moon illusion is the well-known psychological effect that the Moon (and, likewise, also the Sun and stellar constellations) appears larger when seen low above the horizon than when highly elevated in the sky. For instance, Holway and Boring [17] investigated the Moon illusion by establishing a positive relation between apparent brightness and apparent size, which was later supported by, e.g., Robinson [18]. More recently, Galili *et al.* [19] report that most of their participants, ranging from tenth graders to graduate students, were familiar with the phenomenon of the Moon illusion. However, the proposed explanations were diverse; some involved a physical approach of the Moon, and others suggested effects of atmospheric refraction of light. Some students argued that it was a mere illusionary effect but could not explain it further.

According to Francis *et al.* [20], there is no direct way to assess apparent sizes, because "when forced to produce a stimulus that matches the perceived size of the Moon, participants tend to match the visual angle of the Moon, which of course does not correspond to the perceived size," a statement based on the results of Solhkhah and Olbach [21]. However, these authors use the term apparent size to refer to the subjective perception of the Moon, not its objective angular size. Solhkhah and Olbach explain that "a person does not appear to shrink as he recedes from us, nor does a building grow in apparent size as we approach it." In this interpretation, of course, the Moon illusion causes its apparent size to effectively change over time. This interpretation of the term apparent size is mostly used by psychologists, while astronomers and physicists generally use apparent size as a synonym for true angular size [22]. As mentioned, we adopt the latter view and thus reject the previous statements by Francis *et al.* [20] and Solhkhah and Olbach [21].

The apparent Moon size is generally overestimated by planetarium visitors [23]. This was investigated by Carbon in the planetarium of Nuremberg, where participants experienced a realistically sized Moon projection of 0.5° as "massively too small." As a result, planetaria often include a "show sized Moon" in their dome presentations, which enlarges the apparent Moon size by a factor up to 2.5. The authors hypothesize that although spectators are unaware of the spatial limitations when looking at the planetarium's Moon, the registered distance seems to take this limitation into account; thus, the perceived Moon is a tiny bright disc [23]. In this interpretation, registered distance refers to Kaufman and Rock's "automatic, nonconscious process for computing distance" [24], which can be small even in the complete darkness of a planetarium dome. Additionally, no variation in apparent Moon size depending on elevation was reported in the planetarium setting of Carbon's study [23]. This is contrary to the results of, for instance, Suzuki [25], who reported a size discrepancy comparable to the Moon illusion in their planetarium environment.

No other literature was found that investigates specifically people's estimates of the apparent Moon and/or Sun sizes. Likewise, we could not retrieve any prior studies on students' ideas about the variation in these apparent sizes. For completeness, we add that both the Moon and the Sun have an average apparent size of roughly 0.5° in the sky. Due to its elliptical orbit around the Earth, the apparent Moon size varies by roughly 14% over the course of a month. The eccentricity of the Earth's orbit is much smaller, causing the apparent Sun size to vary by only 3% approximately. This variation is too small to be perceptible to the naked eye. There are no other causes for real (nonillusional) variation in the apparent sizes of the Moon or the Sun. This work aims to investigate whether students are aware of these apparent size variations and how they explain them. As such, we provide a contribution to this largely unexplored topic of research in astronomy education.

B. Solar eclipses

The second phenomenon discussed during the interviews is that of solar eclipses. As a solar eclipse is a very spectacular event, and the mechanism behind it is not so difficult to grasp, one might assume that students have little difficulty understanding the nature of solar eclipses [4]. However, previous research shows that alternative conceptions about eclipses are widespread and resistant to change, both among students and adults [26].

One of the most common alternative conceptions about solar eclipses is that the Moon must be in its full phase for one to occur. This was first found by Zeilik *et al.* [27] and later by Trumper [7,28] and others [29,30]. However, it is certainly not the only documented alternative conception about solar eclipses. For instance, LoPresto and Murrell [31] reported that more than half of their participants believed that during a solar eclipse, the Sun emits more harmful radiation than usual. On the other hand, an Indonesian study investigated the argumentation profiles of physics undergraduate students in the context of the annular solar eclipse of June 2020 [3], using both questionnaires and interviews as evaluation methods. They found that the participants had a "fairly good knowledge and understanding related to the phenomena of annular solar eclipses."

Like these Indonesian authors, many researchers in astronomy education have anticipated an upcoming solar eclipse event both by assessing conceptual knowledge and by testing interventions to improve that knowledge. For example, in light of the upcoming "Great American Eclipse" of 2017, Slater and Gelderman [4] discussed the conceptual hurdles students encounter while learning about eclipses. These included an understanding of Moon phases, the involved scales, and the inclination of the Moon's orbit with respect to the ecliptic. The latter two elements are crucial to explain why eclipses do not occur every month but instead are rather rare. Purwati *et al.* [32] responded to the widespread media attention for the 2016 total eclipse in Indonesia by investigating a potential correlation between people's exposure to media reports and their level of eclipse understanding but found no such correlation.

Practical hands-on activities, involving physical scaled models of the Moon and the Earth and a light bulb to represent the Sun [33,34], often show promising results in enhancing student understanding of solar eclipses. Other instructional interventions include the application of a context-based learning approach [35], visiting a NASA Challenger center [2], a Socratic dialogue-based teaching approach [36], kinesthetic classroom lessons [37], and video-immersive planetarium lessons [37].

As physical scaled models inevitably have practical limitations, other studies have investigated the potential of virtual tools to test and enhance students' understanding of solar eclipses. For example, Keating *et al.* [38] integrated 3D computer modeling into a virtual solar system course. The effect on students' comprehension of several astronomical concepts was assessed using pre- and post-tests. The authors motivate their study by stating that the nature of the Moon's orbit and Earth's orbit is inherently and crucially three dimensional so that representations in 2D formats are inevitably insufficient for eclipses. Their results show that computer modeling can be a powerful tool in enhancing student conceptualization for a range of astronomical phenomena, including solar eclipses. In a Japanese study [39], several ICT applications were used to improve elementary students' understanding of solar eclipses. These included a classical lecture style (in which a projector was used to display Earth- and space-based diagrams and slides, with little student interaction), a tablet device with simulated animations (including 360° videos, which students could pause and rewind), and a model-based learning approach (in which the Earth was projected on a 2-m wide balloon, the Sun was modeled by a projector, and a miniature Moon was moved around to simulate eclipses). Results showed that learning gains were smallest for the classical lecture style and largest for the tablet application. Turkish researchers [40] used augmented reality (AR) to create concept maps and enhance conceptual learning of solar eclipses. Observing relationships between concepts in AR (the Moon moving between the Earth and the Sun is given as an example of such a relationship) helps students understand these relationships. The authors further claim that AR not only facilitates the construction of such concept maps and their components but also increases the students' sense of enjoyment and curiosity while learning about this topic.

Testa *et al.* [41] studied the topic of solar eclipses in the context of the big idea of celestial motion [42], under which a wide range of astronomical phenomena is categorized, from seasons to Moon phases. They report two main

cognitive abilities to be important while learning about these astronomical phenomena: spatial thinking (as also found by, e.g., Kattner *et al.* [37]) and logical reasoning. Spatial thinking formed the main barrier for students' understanding of solar eclipses, as it requires them to "put together pieces of information from the motions of three celestial bodies (Sun, Earth, and Moon)" [41]. In a later study of the same research group [43], the use and impact of visual representations to explain solar eclipses were evaluated. They argue that many astronomy textbook images display features that may interfere with the identification of the relevant factors underlying the phenomena to be illustrated. For example, diagrams illustrating the Moon phases tend to align the Moon's orbit with the ecliptic, causing confusion between a full (new) Moon and a lunar (solar) eclipse [44]. As such, the difficulty of reading and interpreting these images may explain the persistence of misconceptions [43].

A solid comprehension of the involved sizes and scales has been identified as another important barrier to teaching and understanding eclipses [45]. However, many students lack such accurate mental scale models. Barnett and Morran [2] suggest that students develop a mental model of the Earth-Moon-Sun system that they use to explain astronomical phenomena such as eclipses. This is much in agreement with Cole *et al.* [46]: "When students learn about scientific phenomena that are too small or too vast to see with the naked eye, they use their perceptual and cognitive abilities to form internal mental models and to parse external representations, including diagrams, charts, and graphs describing these phenomena."

If students were to gain a deeper understanding of these phenomena, the concept of scale thus emerges as an important conceptual tool. The notion that a reasonable understanding of relative sizes, distances, and motions is essential to gain an articulate understanding of astronomical phenomena has also been suggested by Vosniadou [47] and Treagust and Smith [48]. In this work, we investigate empirically whether and how students use their comprehension of scales in the solar system to explain the phenomena of apparent sizes and solar eclipses.

C. Scale comprehension

While the main topics of interest in this work are observable astronomical phenomena, student explanations are viewed in the context of scale comprehension. A good understanding of sizes and distances in astronomy is important for several reasons. First, the concept of size and scale is among the "Big Ideas in astronomy education [11,42], and it is known to be an interdisciplinary threshold concept [12], essential in understanding a range of scientific content on a deeper level. In the conclusion of their review of astronomy education research literature between 1974 and 2008, Lelliott and Rollnick state that size and scale are "crucial to so much of astronomy," and that "not only is this concept underresearched, but it is undertaught" [11].

Second, scale comprehension is closely related to spatial thinking skills [46]. In turn, spatial thinking proficiency has been shown to positively correlate with many topics in science, technology, engineering, and mathematics disciplines, as shown by multiple longitudinal studies covering many years and large sample sizes [49,50]. More specifically, it has been demonstrated that spatial thinking is a successful predictor of symbolic calculation ability in mathematics [51], problem-solving skills in chemistry [52], interpreting graphs of force and acceleration in physics [53], and logical thinking [54]. It thus seems plausible that training students to become more proficient in (astronomical) scale reasoning would have beneficial implications for a wide range of science fields, although no direct causal relation has been explicitly established.

Despite its importance, the literature suggests that most students have a poor understanding of size and scale in the solar system and beyond. A general finding is that students tend to systematically underestimate distances in space, such as the distance between the Earth and the Moon [8], between the Earth and the Sun [5,6,8], and between the Sun and a close star [5–7]. In these studies, students had to express their scale estimates primarily through multiple-choice surveys. The provided set of options often consists of large numbers in units of kilometers or miles, and several studies have shown that students have difficulties grasping those numbers [55–58]. Miller and Brewer [6] therefore note that "this method confounds the students' intuitive conceptions about astronomical distances with their ability to understand large numbers." They further note that students might tend to choose the largest provided option, whatever that may be, because they only remember the assessed distance to be "surprisingly large." However, similar research using ranking tasks instead of multiple-choice questionnaires depicts an equally poor comprehension of astronomical scales. For instance, Trumper showed that among his participants, almost half of the university students [28] and preservice teachers [7] deemed Pluto to be further away from Earth than the stars. Similar work by Lightman and Sadler [9] found that less than half of the 1414 high school students in their study could correctly rank "space shuttle in orbit," "planets," and "stars" in increasing distance from Earth. From these findings, the authors infer that students may base their evaluation of distances on visual cues, without considering that bright stars could be much further away than the dimmer planets. As a last example, both Serttaş and Türkoğlu [59] and Bitzenbauer *et al.* [60] reported that many students (from seventh grade up to college) believe the Sun to be the largest and/or hottest star in our universe.

Students' understanding of astronomical scales is usually not facilitated by the visual representations displayed in textbooks and online. On the contrary, Taylor and

Grundstrom [61] investigated the scale accuracy of the Earth-Moon system in 35 sixth-grade students' drawings, 44 representations on governmental websites, and 30 illustrations in 8 middle school textbooks. They found that in all three sources of imagery, the median of the Moon size (relative to Earth) was significantly higher than its scientific value, and the median of the Earth-Moon distance (relative to the Earth's size) was significantly lower than its actual magnitude. The median of this distance was even 23 times smaller than its scientific value for the textbook images and 14 times smaller in website illustrations. It is hardly surprising then, that in the sixth graders' drawings this distance was typically 16 times too small. The authors advocate for more accurate diagrams in terms of sizes and distances, and they suggest that schematic representations should mention explicitly when an image is not to scale. Testa *et al.* [62] investigated the effectiveness of astronomy textbook images in guiding students toward understanding certain phenomena such as eclipses, seasons, and Moon phases. They argue that these images sometimes hinder rather than favor students' understanding. One core issue of the considered images was that they tend to exaggerate certain iconic features (e.g., the eccentricities of the Moon's orbit and/or Earth's orbit), thereby losing accuracy in terms of scale. Of course, scale accuracy is not always feasible when portraying astronomical systems so that schematic depictions become inevitable. This should not be regarded as an issue by itself, as long as the user is pointed explicitly to the schematic nature of the illustration.

Few studies have specifically investigated a potential link between astronomical scale comprehension and the explanation of phenomena such as solar eclipses. Fanetti [63] assessed college students' scale estimates on the Earth-Moon system and their ideas on the cause of Moon phases. It was hypothesized that students who suggested Moon phases to be caused by the shadow of the Earth would be more likely to make the Moon very large and place it close to the Earth. However, they conclude that "this research did not show a significant connection between students' explanation of lunar phases and their scale model of the Earth-Moon system." Rudmann [64] used written tests and structured interviews to probe university students' spatial ability, knowledge of astronomy, and ability to solve astronomy problems. They found a weakly significant correlation between the participants' spatial ability and their causal model used to explain the seasons. Participants who explained the temperature difference between the seasons in terms of the tilt in the Earth's rotational axis scored significantly better on spatial ability than students who explained the seasons in terms of a varying distance between the Earth and the Sun. The author concludes that "spatial abilities may be a factor that limits or restricts a student's ability to instantiate scientific knowledge." A similar study by Türk [65] probed the spatial thinking skills, astronomical achievements, and attitudes toward astronomy of 280 preservice teachers in Turkey. The author reports a strong and significant correlation between the teachers' spatial thinking abilities and their achievements in astronomy.

III. METHODOLOGY

In this study, 25 semistructured interviews were conducted with students in their last year of high school (age 16–18). Twenty-two students were going to schools in Flanders, Belgium, while the remaining 3 students went to school in Sweden. During these interviews, first the apparent sizes and solar eclipses were discussed, and thereafter the discussion turned to Moon phases. As such, the results from the companion paper [13] stem from the same interviews as the results of this paper. All Belgian students learned about the origin and evolution of the solar system and the Earth, about the rotation and revolution of the Earth and the Moon, and about Moon phases during a (limited) number of geography classes, most often in the year prior to the year of participation in this study. The Swedish students were introduced to these topics earlier and in a more distributed manner across primary and secondary education, although the national curricula describe them in less detail.

As the objective of these interviews was to investigate if and how students use their scale comprehension of the solar system to explain certain phenomena, this scale comprehension first was assessed by means of an interactive online survey, administered several days to weeks prior to the interview. Students were specifically selected for interviews based on their answers in the online survey. During these interviews, a personalized to-scale model of the Earth-Moon-Sun system was constructed and placed on the table for every student. Therefore, the students selected for an interview were those for whom this was feasible. In practice, this consideration resulted in selecting students who underestimated most (if not all) of the relative sizes and distances in the survey. This selection criterion was not only pragmatic but had also two empirical motivations. First, as shown in our previous work [10], most students drastically underestimate the spatial scales in the solar system. Therefore, the students selected for interviews in this work do not represent outliers, but rather represent a well-populated profile of scale models. Second, it can be expected that students with poor spatial skills are more likely to hold alternative ideas on astronomical phenomena [54,64,65]. It would make intuitive sense that this also applies to scale comprehension specifically. For instance, to students who think that the Moon is very close to Earth, it would be logical that its apparent size changes between observers on different locations on Earth. Since we aim to investigate the role of spatial scales in such alternative explanations, this was an interesting group of students to focus on. In every class where we conducted the online survey, a sufficient number of students (at least two) could

be selected for the interviews. In the next section, we will elaborate more on the used methodology in both the survey and the interviews.

A. Student selection—online survey

The online survey was conducted with a total of 249 students in their last year of high school, between 16 and 18 years old ($\mu = 17.1; \sigma = 0.6$). The data from 211 participants were collected in seven different schools across Flanders, Belgium, and the remaining 38 participants attended schools in Sweden. No specific selection criteria were applied at this point. Students were informed about the content of the test at the beginning of the lecture, and they were reassured that their answers were completely anonymous and would in no way affect their grades. A written consent form was signed by both the participants and the researcher after clarification of its contents was provided. The students were also informed that the test may be followed by an interview at some later time. The online test was easily accessible through an URL link that students could copy onto their (school) laptops. They could take as much time as needed to complete the test, which on average took roughly 25 min.

The online survey served as a selection tool for the subsequent interviews. It probed the students' estimates of certain relevant distances and sizes in the solar neighborhood. These estimates were asked in a relative sense. For instance, students were not asked to estimate the Sun-Earth distance in isolation, but rather to evaluate how much larger it is compared to the Earth-Moon distance. Similarly, the sizes of the Sun and the Moon were compared to that of the Earth. At the start of the survey, these distances and sizes were individually ranked by the students from smallest to largest, and the formulation of the subsequent questions was adjusted accordingly.

After every answer, students were provided a customized visualization of their previous answer. For instance, when students indicated that the Earth's radius was 10 times that of the Moon, they saw two discs of exactly that size contrast. An example of such a visualization is shown in Fig. 1. For distance-related questions, there was an analog visualization involving line segments with specific lengths based on the student answer. Students could then either agree with this visualization—indicating that it was a realistic representation to scale in their opinion—or disagree and reconsider their previous answer. In the latter case, a new answer would also be visualized, and the process of evaluating the representation would start over until the student eventually agrees with the visualization. Both the initial and final answers of the students were saved.

After every question, students were also asked to make a self-assessment of certainty of their answers. This was done with a quasicontinuous sliding bar, where the far left represented "very unsure" and the far right "very sure."

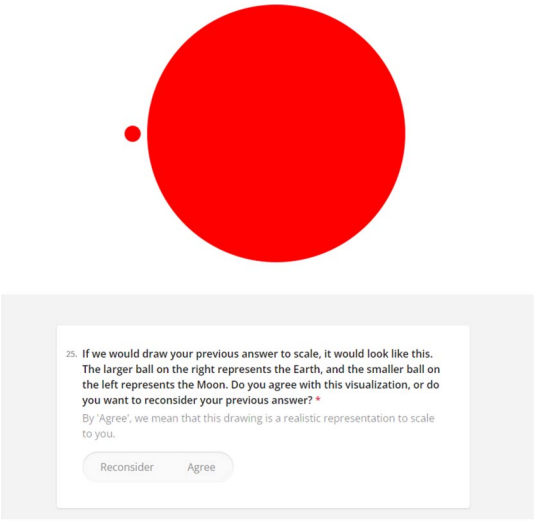


FIG. 1. Example of a customized visualization after a size-related question on the online survey. Translated to English for inclusion in this publication.

Alternatively, students could indicate that their previous answer was a completely random guess, in which case no indication of certainty with the sliding bar was required. This online survey is to a large extent identical to the one used by the authors in a previous study [10], in which the precise questions and visualizations are described in more detail.

In addition to the scale questions, two multiple-choice questions on astronomical phenomena were included, which were not part of the previous version of the survey [10]. One asked for the reason for seasons, and the other for the cause of Moon phases. The formulation of the questions and the set of provided options were based on the AMoSS test [66] and the Lunar Phases Concept Inventory [67,68], for the seasons and Moon phases questions, respectively. The motivation to include these questions was not to fully probe students' reasoning on these phenomena, but rather to distinguish which students were susceptible to common misconceptions about these topics. For instance, when students indicated that seasons were caused by a varying distance between the Earth and the Sun, we hypothesized that they might also apply scale-related arguments for other astronomical phenomena. Therefore, the answers to these questions were regarded as a secondary selection criterion, after the scale estimates. That is, if two students made very similar underestimations in the scale questions but only one of them assigned the cause for seasons to a varying distance between the Earth and the Sun, then that student was selected for an interview.

B. Individual interviews

The interviews with Belgian students were conducted in Dutch, so all interview excerpts from these interviews were translated to English before inclusion in this work.

The interviews with Swedish students were conducted in English. In this section, we describe the interview procedure for each of the discussed phenomena. For the full interview protocol, we refer the reader to Appendix B.

1. Scale model

A personalized, to-scale model of the Earth-Moon-Sun system was constructed for every interviewed student, based on their answers in the online survey. In the construction of the scale model, only students' final answers were used—that is, those entered after seeing and agreeing with the interactive visualizations. This scale model was placed on the table between the interviewer and the student before the student entered the room. At the start of the interviews, students were pointed to the scale model and informed that the relative sizes and distances were retrieved from their survey answers. They were encouraged to make any adjustments to the model they deemed necessary by replacing and/or resizing the celestial bodies. After agreeing with the model, students were assured that additional adjustments could still be made at any time throughout the interview. During the subsequent discussion on astronomical phenomena, students were repeatedly encouraged to use the scale model to illustrate their reasoning.

To illustrate what such a scale model looked like, we include Fig. 2. This figure shows a reconstruction of Fien's scale model, in which both the relative sizes of the celestial bodies and the distances between them are based on her survey answers. At the start of her interview, Fien increased the distance between the Earth and the Sun from 12 cm to roughly 20 cm before agreeing with the scale model.

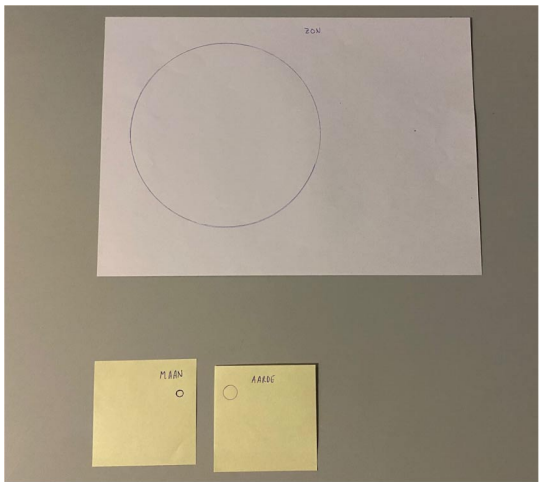


FIG. 2. Reconstruction of Fien's scale model, based on her survey answers and used during her interview. The large circle on the A4 paper represents the Sun, the circle on the post-it note on the lower right represents the Earth, and the circle on the post-it note on the left represents the Moon.

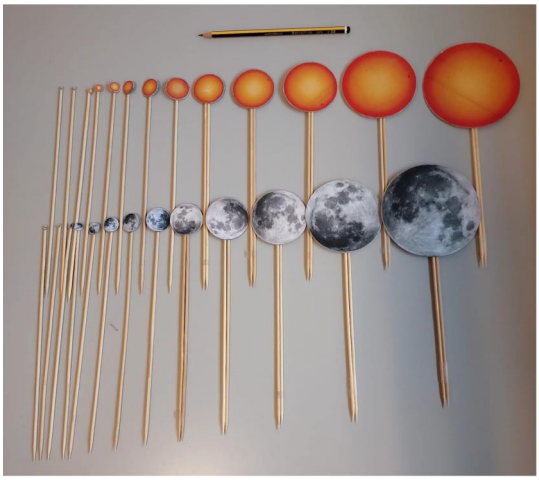


FIG. 3. Discs on sticks to assess students' estimates of the apparent sizes of the Moon and the Sun. A pencil is shown for scale.

2. Apparent sizes

The first astronomical phenomenon discussed during the interviews was the apparent sizes of the Moon and the Sun. Participants were pointed to a number of Moon (Sun) discs on sticks with varying sizes, as shown in Fig. 3. They then estimated the apparent size of the Moon (Sun) by choosing one disc that, when held at arms' length, best resembled the apparent size of the Moon (Sun) as seen in the sky. The correct option for both the Moon and the Sun would be the third smallest disc, which had a diameter of 5.2 mm and subtended an angle of 0.5° when held at a distance of 60 cm. Of course, the participants' arm lengths could deviate somewhat from this generalized value. However, the differences between the disc sizes were made large enough to prevent any other disc from ever becoming the better option.

Once the students had chosen a Moon (Sun) disc, they were asked whether they thought the chosen size was consistent with the scale model on the table. More specifically, they were asked if people on the model Earth would perceive the model Moon (Sun) as equally large in their sky as the chosen disc. This is equivalent to comparing the angle subtended by the model Moon (Sun) as seen from the model Earth to that subtended by the respective disc when held at arm's length. This question was added to determine whether students realized that those two variables should (in principle) be equal. If students indicated that those angles did not match, they were encouraged to either adjust the scale model or reconsider the chosen disc.

Thereafter, participants were questioned about potential causes for variation in the apparent sizes of the Moon and the Sun. If they indicated that the Moon (Sun) does not always look the same size, they were encouraged to provide an explanation for this variation. They could do so in

words, by interacting with the scale model on the table and/or by making a sketch on a blank piece of paper. Students were asked to give an indication of the amount of variation (if any) by pointing out a range of discs. As a concluding question for this part of the interview, participants were asked if they actively remembered the Moon (Sun) appearing surprisingly large or small at some instance. If so, then the interviewer asked where the Moon (Sun) was in the sky at that time. These questions were added to investigate whether students were aware of the Moon illusion effect.

3. Solar eclipses

The discussion on solar eclipses started with an open-ended question to explain this phenomenon. Regardless of the answer, students were again encouraged to illustrate their reasoning by interacting with the scale model and/or by making a drawing. Following this introductory question, participants were shown two pictures of an annular and a total solar eclipse (see Fig. 4). They were asked to describe the differences between the pictures and to think of what could possibly cause these differences.

C. Data analysis

All interviews were transcribed verbatim, based on the video and audio recordings. The participants' names were replaced by pseudonyms in the transcriptions (and thus also in this article). A coding scheme was developed for each astronomical phenomenon separately. The codes contained information about the different elements present in the students' explanations and their scientific correctness. These schemes were then tested against newly conducted interviews and refined when necessary. This process continued iteratively until it converged and all new data could logically be assigned to a code in the constructed scheme. The coding schemes are shown in Appendix A, together with information on their structure and contents. Thereafter, the assigned codes were compared both over several questions for one participant and across one question over several participants. As such, recurring themes and argumentations could be uncovered. All explanations, gestures, and drawings were evaluated in light of the personal scale model to determine whether students used this model to guide their thoughts.

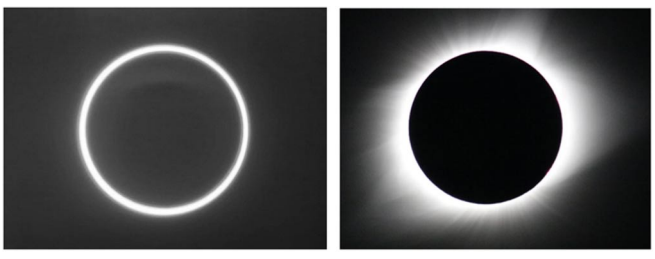


FIG. 4. Two pictures of an annular (left) and total (right) solar eclipse, which were shown to the students during the interviews. Sources: [69,70].

The coding of all participants was performed by the first author of this work, while an additional coding of five participants (20% of the total dataset) was done by one of the co-authors to calculate interrater reliability scores. We obtained an overall Cohen's κ score of 0.82. Since the set of possible codes differed between questions (see Appendix A), this κ score was calculated by taking the mean of the question-specific κ values. While the obtained score indicates a very good agreement between the raters, it should be taken into account that the limited number of cross-coded participants can potentially cause it to be imprecise. To limit the error on κ, codes that were never assigned by either of the raters were excluded from the calculation. Additionally, we report an overall percentage agreement of roughly 95% and a mean Jaccard score of 0.89, indicating that, on average, the raters overlapped on 89% of the assigned codes. Together, these values guarantee a high reliability of the coding procedure.

IV. RESULTS

A. Online survey

The results of the online survey were very similar to those found in our previous study [10], in which the survey was administered with $N = 201$ Belgian students. As in the previous assessment [10], the participants of this study generally underestimated all relative distances. For size-related questions, there were comparable amounts of overestimation and underestimation. Slightly more than one quarter (26%) of all answers were altered after seeing the personalized visualization. Approximately 36% of all answers were pure guesses, while the certainty scores of the nonguesses roughly followed a uniform distribution. As for the ranking questions at the beginning of the survey, a little more than half of the participants (58%) ranked both the relative sizes and distances correctly. This task proved slightly easier for the sizes (73%) than for the distances (65%).

As explained in Sec. III, only students who severely underestimated the distances and sizes in the survey were selected for an interview. Due to the general underestimation of the astronomical scales assessed in the survey, it was possible to select some students for an interview from every class. We quantify this statement by specifying the precise selection criteria. First, we wanted the Sun-Earth-Moon system to fit on the interview table. The average available space was roughly 2 m, and the minimal size of any circle (in practice, of the Moon) in the scale model was set to 2 mm. That implies that the ratio of the entire system's size compared to the Moon's size could be a factor of 1000 at most. This was the case for 101 students in our sample, or just over 40%.

As a second criterion, we required interviewees to have correctly answered the size-related ranking question of the survey. That is, they correctly ranked the Moon, Earth, and

Sun in increasing size. For students who answered this ranking question incorrectly, the survey would later ask how much smaller the Sun is compared to the Earth, for instance. We argue that these students lacked the most fundamental content knowledge in astronomy, and therefore, they were not considered suitable for participation. This second criterion reduced the number of potential interviewees to 80, or roughly 32%. Finally, for the same reason, we required interview participants to have ranked the distance "Earth-Moon" as smaller than "Sun-Earth." As a result, the final number of potential interview participants became 75, or approximately 30% of our total survey data pool. The number of students interviewed per class ranged from two to five.

B. Apparent sizes

1. Size of the Moon and Sun

The students' ideas about the apparent sizes of the Moon and the Sun could be inferred in two ways: from the chosen Moon or Sun discs and from the scale model on the table. Considering the chosen discs first, it quickly became clear that most students drastically overestimated the apparent sizes of both celestial bodies. During the first few interviews, the two largest Moon/Sun discs in Fig. 3 were not crafted yet. They were added later to avoid a ceiling effect, since several students in those first interviews opted for the third largest disc (which was the largest disc at that time). This is a radical overestimation, since the correct option for both the Moon and the Sun was the third smallest disc.

At first glance, the scale models of the interviewed students were consistent with the chosen discs, in the sense that they also reflected strongly exaggerated apparent sizes. However, this was to a large extent determined by the design of the study, since students were selected for interviews based on their low scale estimates. These estimates cause the Earth, Moon, and Sun to be very close together, and this almost inevitably leads to an overestimation of the apparent sizes of the Moon and the Sun.

In Fig. 5, the estimates made for the apparent size of the Sun (as inferred from the personalized scale models and as indicated by the chosen Sun discs) are displayed. To better understand this figure, we consider the example of Fien. Fien stated that the apparent size of the Sun could be anything between the third and fourth largest Sun discs (when held at arm's length). Approximating Fien's arm length as 60 cm, those discs subtend an angle between 4.5° and 5.7°. In Fien's scale model, shown in Fig. 2, the Sun had a diameter of 15 cm and was positioned at a distance of roughly 20 cm from the Earth (after Fien moved it further from Earth at the beginning of the interview). This amounts to a subtended angle of approximately 43°.

Given the practical limitations of these calculations, the values in Fig. 5 should be taken with a grain of salt. This is reflected by an applied standard error of 10% on both the x and y axes. Error bars on the disc angles can be larger, since students sometimes indicated a range of discs as

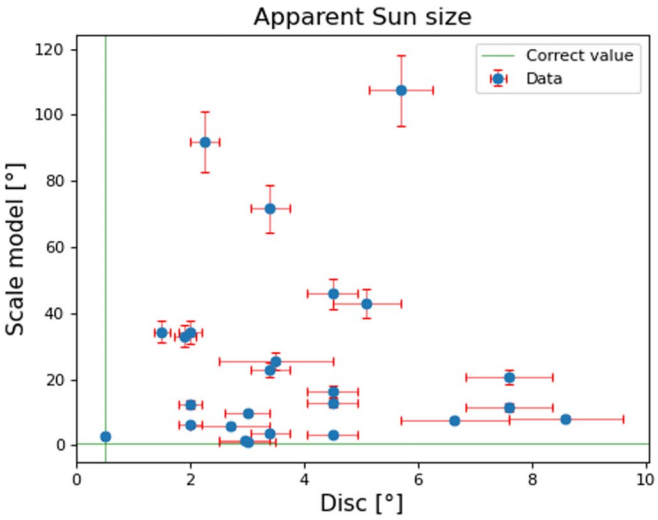


FIG. 5. Student estimates of the apparent size of the Sun, made both through the scale model and the chosen discs.

appropriate (as Fien did). The horizontal and vertical green lines indicate the correct visual angle of the Sun. The results for the apparent size of the Moon are not shown but were very similar.

Two things can be noted from Fig. 5. First, it is clear that there are very large differences between the apparent size estimates made in terms of the chosen discs and in terms of the scale models. The data point near the upper left corner of Fig. 5, for instance, represents a student who chose a Sun disc subtending an angle slightly larger than 2°, while the Sun in the student's scale model subtended an angle of roughly 90°. Second, there is no relation between the apparent Sun sizes according to the scale model and according to the chosen disc. That is, students for which the Sun looks larger in their scale model do not systematically choose larger discs. We reflect further on these findings in the discussion.

2. Variation models

When asked whether the (full) Moon and Sun sometimes look larger than at other times, roughly one third of the students answered that they did not think so. They argued that both celestial bodies appear the same size at any time. Four students indicated that the Moon varies in apparent size but could not explain that variation. Among the students who provided an explanation for varying Moon and/or Sun sizes, several groups could be identified.

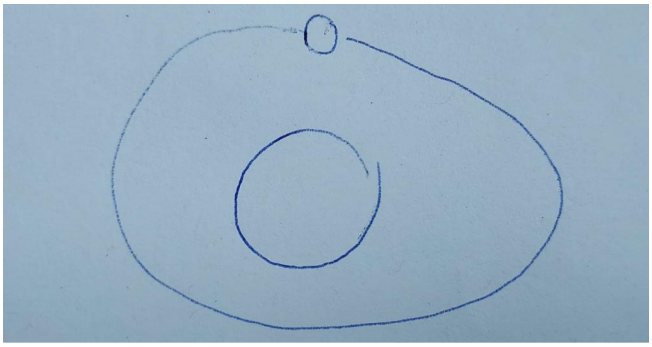
Optical models.

A first group of explanations was based on luminosity. The core idea of these arguments was that a brighter observed object looks larger. Most students in this group did not provide more specification and simply stated that the Sun (Moon) appears larger when it emits (reflects) more light. A few students added that this could be observed, for

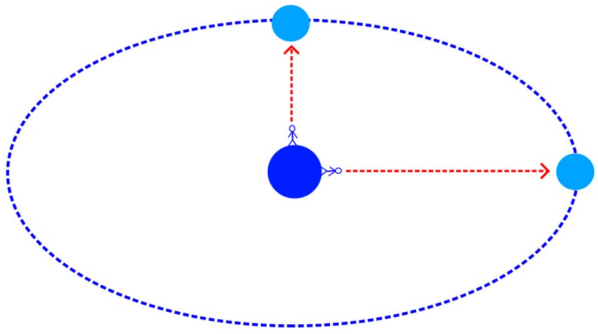
example, by comparing observations with and without clouds (partly) covering the bodies. Two students claimed that the Moon appears larger as more of its surface is visible from Earth. It was not entirely clear whether they referred to an enlargement of the apparent radius of the Moon or to the visible surface itself.

Geometric, elliptical models.

A second group of students explained the variation in apparent Moon and Sun sizes in terms of the ellipticity of the Moon's orbit around the Earth and of the Earth's orbit around the Sun. This ellipticity causes the Moon (Earth) to sometimes be physically closer to the Earth (Sun) than at other times, thus causing variation in apparent size. Students in this group argued that when the Moon (Earth) is positioned at perigee (perihelion), the Moon (Sun) would look noticeably larger than when the Moon (Earth) is in apogee (aphelion). This is correct for the Moon, although students generally overestimated the effect. This overestimation was determined by asking the students whether the variation was large enough for different Moon discs to be preferable at different times, to which several students answered affirmatively. These students pointed out two or three discs, usually corresponding to a variation in apparent Moon size of roughly 1° or more. For the Sun, this argument was considered incorrect because the described effect is imperceptible by the naked eye. This reasoning is illustrated for the Moon and Sun sizes in Figs. 6 and 7, respectively.



(a) Original drawing from Mila



(b) Reproduced illustration

FIG. 6. Variation in apparent size of the Moon due to the ellipticity of its orbit around the Earth, as drawn by Mila (a) and as reproduced by the authors (b).

In her drawing of Fig. 6(a), Mila drew only one Moon in orbit, while the illustration in Fig. 6(b) shows two Moons to clarify the idea. However, it was beyond doubt that Mila referred to the ellipticity of the Moon's orbit, as she stated that "[the Moon] has like a different orbit [gestures an elliptical shape in the air with her finger], it doesn't have a round orbit around the Earth, right? […] I know that it doesn't go perfectly round, I think" while making her drawing. She then indicated that when the Moon is closer in its orbit, it looks bigger.

The idea that seasons are caused by a varying distance to the Sun is a well-documented alternative conception in astronomy [7,11,29,31,71–73]. However, the idea that this varying distance would also cause the apparent size of the Sun to noticeably change is not, as far as we know, previously documented in the literature. The interconnectedness of these two ideas was neatly formulated by Louisa while drawing Fig. 7(a):

L: In my mind it [the Sun's apparent size] has always stayed the same, but I think that maybe… maybe it's smaller in winter? I mean, it seems logical to me that it's smaller in the winter, because we have a kind of elliptical orbit, I think.

[…]

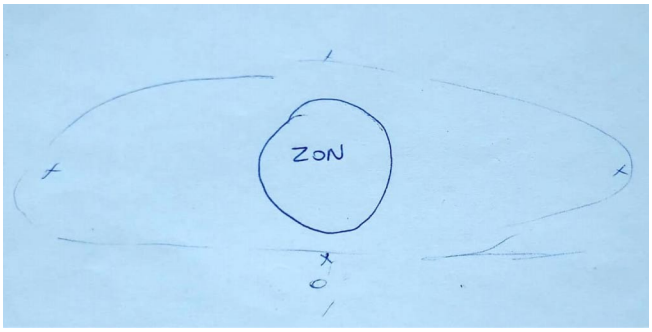
L: [Makes the drawing in Fig. 7(a)] and then here [indicates the two points in the orbit close to the Sun] there will be more like warm temperatures, like summer and such, and then there [indicates the two points in the orbit further from the Sun] you have more of the colder, stuff…

I: Yeah, and how come [the Sun] looks larger at one moment than at another?

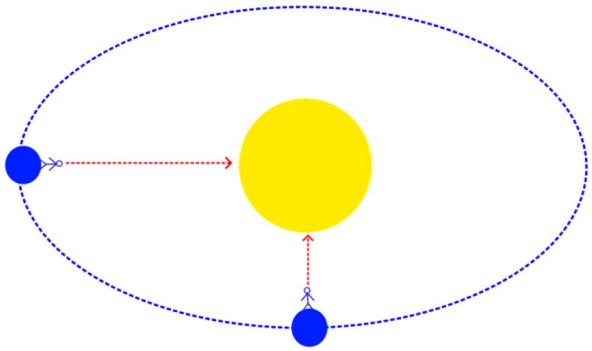
[…]

L: Here [points at one of the points in the orbit close to the Sun] it is larger because we're just closer to the Sun then.

Remarkably, two students reversed this reasoning for the Moon, stating that in winter the Moon would be closer to Earth and, therefore, look larger than during summer. This became apparent during the interview with Christopher, while he produced the drawings in Fig. 8. He argues that the orbit of the Moon around the Earth is different during summer than during winter. The exact difference in the shape of the orbit is not entirely clear, but Christopher concludes by stating that in winter the Moon is closer and thus looks larger. He explains his thoughts in the following excerpt:



(a) Original drawing from Louisa



(b) Reproduced illustration

FIG. 7. Variation in apparent size of the Sun due to the ellipticity of Earth's orbit, as drawn by Louisa (a) and as reproduced by the authors (b).

C: So you have Earth here [starts making the drawing marked as "1" in Fig. 8] and then it might move and it would go like this [draws an elliptical shape around Earth].
I: Yeah?
C: But then I think that's maybe on summer, then maybe on winter it might go this [starts making the drawing marked as "2" in Fig. 8]… They might go.. bit larger out here and smaller closer here I think. This is also very, just, not that detailed, but I think it might be… orbiting closer to Earth, depending on […] season, depending on season, as […] its orbit around Earth.
I: Okay. So do you actually actively remember that, that when in winter you look at the moon, it seems larger than in summer?
C: Hmm, that's not something I think of, that's just something I think… I when I think of it now, I think it's… first one, it sounds pretty reasonable, and then I also think it's, when I'm trying to remember, I feel like in winter it looks larger than in summer, but it might not be the size of the moon, I think it might be that the moon is closer and that might be a good reason for why.

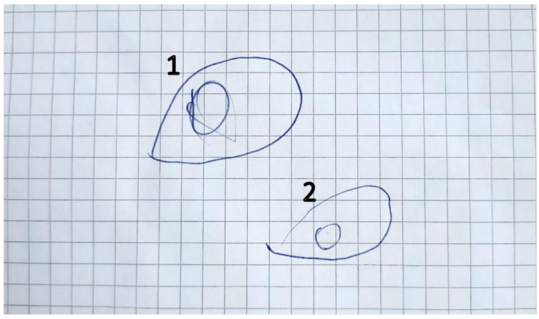


FIG. 8. Drawings made by Christopher while explaining variation in the apparent Moon size.

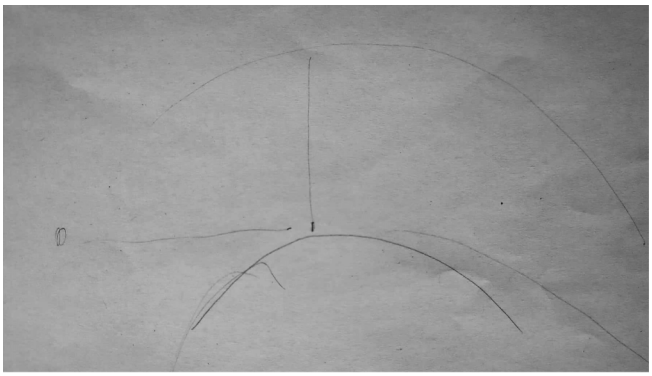
Geometric, nonelliptical models.

The third group assigns the variation in apparent sizes to a range of geometrical causes, without relying on the eccentricity of the involved orbits. While the elliptical models relied on assumptions about how much the distance between two celestial bodies can vary over the course of an orbit, the nonelliptical models are based on assumptions about the relative magnitudes of several distances and/or sizes. This makes these nonelliptical models particularly interesting for this work.

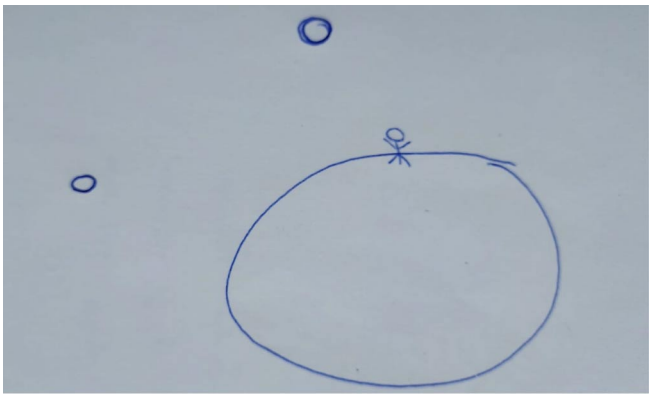
As a first example, we found that several students used geometrical arguments to suggest that the Moon looks smaller when it is low above the horizon than when it is high in the sky, because in the former case, it is further away. This model is illustrated in Fig. 9. Ben, who made the drawing in Fig. 9(a), states that the Moon looks smaller in the morning and in the evening. He then explains his thoughts in the following excerpt:

B: Yeah because [the Moon] orbits around the Earth right, and we stay roughly in the same place, so the distance around… becomes slightly larger right?
I: Could you try and draw what you mean by that? When does the distance become larger?
B: [starts making the drawing in Fig. 9(a)] Eh, if we're standing at a certain point here… [draws someone on the Earth]
I: Yes?
B: And then the Moon will first be here [draws the Moon on the left, low above the horizon], and this distance is larger than when it's here [draws the Moon directly above the person].

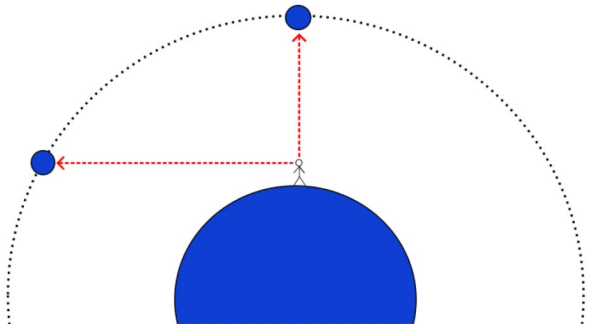
Note that Ben implicitly assumes that the Moon is low above the horizon in the morning and in the evening, and high in the sky in between those times, analogous to the Sun. For completeness, we note that the model in Fig. 9 is not strictly incorrect, but rather it is totally irrelevant in light of the involved scales. The apparent size ratio between the two Moons drawn in Fig. 9 is in reality roughly 1.02 [20] or half a minute of arc, which is presumably below the



(a) Original drawing from Ben



(b) Original drawing from Pascale



(c) Reproduced illustration

FIG. 9. Variation in apparent size of the Moon depending on where in the sky it is observed, as illustrated by (a) Ben and (b) Pascale, and as (c) reproduced by the authors.

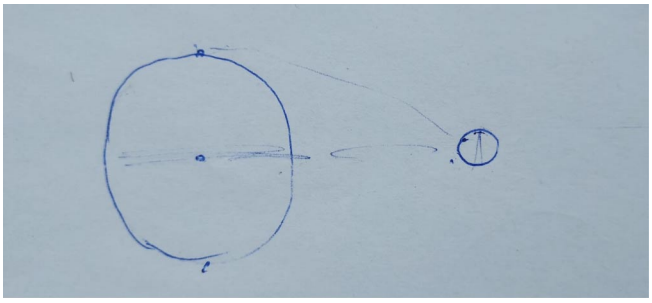
threshold of discrimination [22]. It is interesting that this effect is opposite to the Moon illusion, which makes the Moon appear larger when it is low above the horizon. Out of the 25 participants, only one described the Moon illusion, although she made no mention of that term and could not explain it in depth. The model described in Fig. 9, on the other hand, was identified by six students. One particularly interesting case was that of Pascale, who first described the Moon illusion but completely changed her mind when making the drawing in Fig. 9(b). What follows is a short fragment of this interview:

P: If the Moon is higher, I would take that one [a smaller Moon disc], if the Moon is lower, like closer to the horizon, I would take the larger one.
I: Okay, so you're saying, the lower above the horizon, the larger the Moon looks?
P: Yes.
I: Yeah, okay, and do you have any idea why that would be?
P: Uhm, that's maybe a bit of a weird reasoning, but maybe because it's like closer?
I: Okay, could you try and either show that with this [scale] model [on the table] or draw that on the paper? How would that work?
P: Yeah [starts making the drawing of Fig. 9(b)] So situation one is when the Moon is high up.
I: Yeah?
P: Uhm, so this is the Earth, here is a little man. Uhm, what did I say, no I think I'm going to turn it around, that if the Sun, no the Moon is higher up, that it will be larger than when it is here [low above the horizon].
I: Uhu, and why is that?
P: Then that one [the Moon low above the horizon] is smaller, because it is further away.

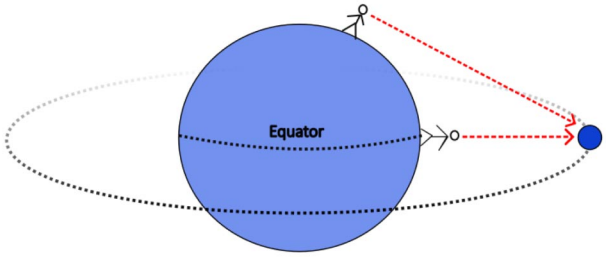
The mentioned number of occurrences (six students for the model in Fig. 9, one student for the Moon illusion) refers only to the variation in apparent Moon size. For the Sun, only one student claimed it to be larger when high in the sky, and three others described the actual "Sun illusion." Some students did not really explain why the reasoning of Fig. 9 applied to the Moon and not to the Sun, but others realized that the involved scales must play an important role in the relevance of the model. Pascale stated that "the distance between the Earth and the Moon, and the distance between the Earth and the Sun, is an important thing to be able to see that size," but struggled to formulate her thoughts more precisely. Fabio, who also drew the model in Fig. 9 for the Moon, explained that this effect did not matter for the Sun, "because it's so.. big… Well I don't think you'll really notice that difference, […] because it's so big and far away."

As a variation to the previous model, some students argued that the apparent size of the Moon would change with respect to the geographical position of the observer. More specifically, the Moon would look larger if one approaches the equator and would decrease in size if one stands closer to the poles. This idea is illustrated in Fig. 10. One of the students explaining this idea was Mark:

I: Do other people on Earth see other sizes then?
M: Yeah… oh wait, actually, I think if you stand on the top [points at the model earth], you know it's… It's a bit more, you know, further away compared to other parts. So I think the sizes would, would kind of like change.



(a) Original drawing from Louisa



(b) Reproduced illustration

FIG. 10. Variation in apparent size of the Moon depending on the geographical position of the observer, as (a) illustrated by Louisa and as (b) reproduced by the authors.

I: So if I'm on the equator like, right here [points at what would be the equator on the model earth], it would be different?
M: Yeah.

Following this fragment, Mark picked out two differently sized Moon discs and argued that the larger one would be seen by an equatorial observer and the smaller one by an observer closer to the poles. While Mark did not provide a drawing to visualize his reasoning, Louisa seemed to follow a similar train of thought when producing Fig. 10(a). However, she clearly had difficulties explaining the differences in apparent Moon size between the two observers:

L: Uhm… I have, like it seems logical to me that there is a difference [between the two observers], but I'm not quite sure whether one is now larger or the other is now smaller. Sorry, yeah.[…]
L: Yeah, it seems logical to me that this one [pointing at the equator in the drawing of Fig. 10(a)] sees a larger Moon than this one [pointing at the pole], but I'm not sure.
I: Why would that seem logical to you?
L: Because that [on the equator] is like straight-on, and the other is more slanted, but that could totally be a wrong line of reasoning, so…

Note that by suggesting this idea, these students implicitly assume the Moon's orbit to be aligned with the equatorial plane. An illustration of this idea is provided in Fig. 10(b).

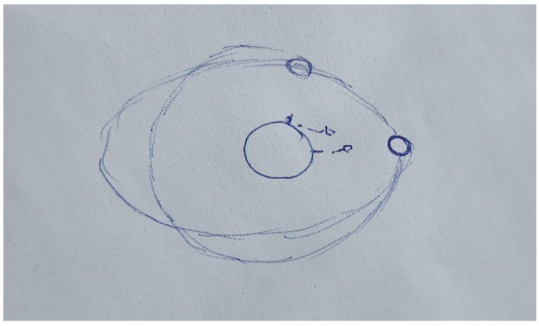


FIG. 11. Fabio's drawing to explain variation in apparent Moon size.

Mark and Louisa were the only participants to make the above argument. Similar to the model in Fig. 9, this argument was made only for the apparent Moon size and not for the Sun. Like Pascale and Fabio did for the model in Fig. 9, Mark also reasoned that the involved scales prohibit this model from being applicable to the Sun. He argued that "that has a lot to do with the… the length, or like the distance between the Sun and the Earth. That it doesn't change, comparatively, between the placements on the Earth, like the distance between the Earth and Sun here, and the Earth at the equator, and let's say, well, the, the south pole… matters a lot less when it comes to the Sun compared to looking at the Moon, yeah."

Of course, the above-described groups—and the individual models within a group—are not mutually exclusive. Some students used a combination of two or more models to explain the variation in apparent sizes of the Moon and the Sun. A good example of this is Fabio, who claims that the Moon looks larger when seen low above the horizon (as mentioned before), but also adds that there is a secondary effect of the elliptical Moon's orbit. His illustrative drawing is shown in Fig. 11.

Another interesting case where a combination of variation models is considered is the interview with Adam. Adam used the elliptical model (Fig. 7) to explain variation in apparent Sun size, and he knew that the Earth is in perihelion during winter (in the Northern Hemisphere). However, he also knew that the Earth's rotation axis is tilted, which causes the Northern Hemisphere to be physically further away from the Sun during winter. This is another well-known reasoning that students tend to use to explain seasons [11,74]. Adam was challenged to weigh the relative impact of those two effects on the perception of the Sun. This is clear from the following excerpt:

A: […] for us, the Sun seems larger in the summer than in the winter, because we're facing toward it then, even though I'd say—it's a bit tricky, because

you've got, like, I think, you've got the fact that the Earth moves like this [makes a gesture with his hand of a tilted Earth axis that shifts orientation], but also that in summer we're actually further away from the Earth, so…

I: From the Earth?

A: Uh, from the Sun, excuse me. So we're looking at it from a better, well, sharper, straighter perspective, but we're also further away from it.

I: Right, so in summer we're further from the Sun due to the elliptical orbit, and in winter we're closer?

A: Yeah, that's right, but in winter we're tilted away and, yeah, kind of tilted backward, so to speak.

I: So which of those effects do you think has a stronger influence?

A: Uhm, I think the elliptical orbit does, yeah. So then we do actually see it larger in the winter than in the summer.

Remarkably, even this distance model involving the tilt of the Earth to explain the apparent Sun size was applied analogously to the Moon by one student. Noah stated that "during the summer, the Earth is like tilted [gestures]. […] And then we're like, yeah closer to the Sun I believe? And during the winter it's tilted in the other direction then, and then we're also further from the Sun, so I think you could apply the same thing to the Moon." However, the interview with Noah was rather turbulent, as he changed his mind many times and made several statements that were internally contradictory. This made it difficult for the interviewer to keep Noah on topic. Noah also assigned potential variation in the apparent Moon size to Earth's deviation from a perfect sphere. Two short fragments in which Noah talks about this are given here:

I: Are there specific reasons why you would think that the Moon could change size in the sky?

N: Uhm, our Earth is like not completely spherical, and it's like a bit deformed, so…

I: The Earth itself, the shape of the Earth?

N: Yes.

[…]

N: I think it can change a little, since the Earth is not completely spherical.

I: But when you say 'not completely spherical', do you mean that the Earth does not look like this [draws a circle] but rather has such a shape [draws an oval]? Or do you mean that the Earth is tilted and sometimes tilts like this and sometimes like that [draws a circle with a diagonal line through it to illustrate the tilted axis]. Which do you mean by 'not completely spherical'?

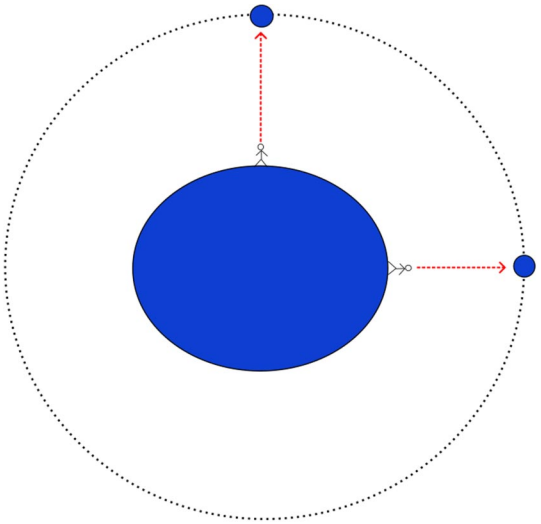


FIG. 12. Illustration of Noah's idea of a nonspherical Earth, causing variation in the apparent Moon size.

N: Yeah, not that much, but… I think our Earth is something like that, I think [points at the drawn oval shape].

I: Okay, and where is the North pole? Are the North and South poles there, or are the North and South poles… [points at several positions on the oval]

N: There and there. [points at flattened sides of the oval]

I: Here en here, yeah. And do you mean that if you're here [at what should be the equator], you're then closer to the Moon and therefore larger, or..?

N: Yeah, I think something like that.

Unfortunately, Noah did not provide a drawing to illustrate his thoughts, even after explicit encouragement to do so. We acknowledge some degree of suggestiveness in the above interview excerpt, but nonetheless, we interpret that Noah refers to the flattening of the Earth due to its rotation. This rotation causes the equatorial radius of the Earth to be slightly larger than the polar radius. Noah overestimated this effect, causing an equatorial observer to be physically closer to the Moon—and therefore to see a larger Moon—than a polar observer. This effect is illustrated in Fig. 12.

To conclude this section, we display the number of occurrences of every model in Table I. This table clearly shows that most of the above-described models were not one-time occurrences but rather resurfaced for several independent students.

C. Solar eclipses

1. General comprehension

Most students were familiar with the concept of a solar eclipse. Eighteen students arrived at the explanation that the

TABLE I. Number of occurrences for every model of apparent size variation. Note that these numbers amount to more than the number of participants, since the models are not mutually exclusive.

Model	Moon	Sun
Optical	**7**	**4**
Geometric, elliptical	**10**	**12**
Geometric, nonelliptical	**9**	**3**
Depending on elevation in the sky	6	1
Depending on observer's geographical position	2	0
Depending on season due to Earth's tilted axis	1	2
Nonspherical Earth	1	0
Moon/Sun illusion	**2**	**3**
No variation/No explanatory model	**9**	**7**

Moon is between the Earth and the Sun by themselves, without help from the interviewer. They were given code II in the coding scheme of Table IV. The other seven students either could not provide any explanation or only arrived at the correct explanation after being aided by the interviewer (e.g., by showing them one of the pictures in Fig. 4). These seven students were given code I in Table IV. As mentioned, all students were then shown the two pictures of Fig. 4 (regardless of their previous answer) to initiate the discussion on the two types of eclipses.

2. Total versus annular eclipses

Three students could not formulate any hypothesis on the cause for the difference between the two solar eclipse pictures. The answers provided by the other students could be classified in the same way as the answers to the questions on apparent size variation. To all original drawings and reproduced illustrations of this section, we added the tags E, M, and S to clearly indicate Earth, Moon, and Sun, respectively.

Optical models.

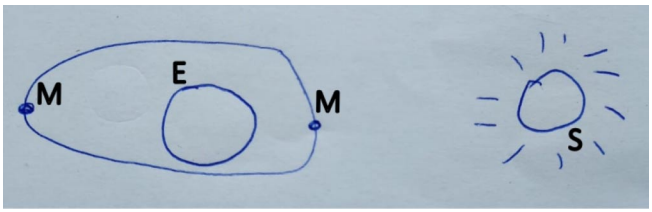
A number of students assigned the difference between the two types of solar eclipses to an optical cause. The main idea in these models is that the amount of light reaching the eye (or the camera) must differ between the two pictures, leading to different observations. However, the reason for the variable light intensity differed between the members of this group. For example, two students suggested that there could be more clouds in one picture than in the other. One other student, Dirk, seemed confident that the picture of the annular eclipse must have been taken with protective eclipse goggles, since the "bright radiation" (the corona) seen in the total eclipse picture had been eliminated. Dirk argued that the Sun is still visible behind the Moon in both pictures, but for the total eclipse, it is barely distinguishable from the emitted radiation since that picture was taken without eclipse goggles. Emily also believed that the Sun should always look larger than the Moon, but she assigned the difference between the pictures to the effect of "zooming". The edge of the Sun must always be visible behind the Moon, she argues, but sometimes we can only see it when sufficiently zooming in. Therefore, when one would zoom in on the picture of the total eclipse, a picture of an annular eclipse would be obtained. As a last example, Noah realized that the "radiation" in the picture of the total eclipse is the Sun's corona. Since the corona was not visible in the picture of the annular eclipse, he concluded that the Sun must be "less active" at the moment that picture was taken. We interpret this as indicating that Noah refers to the solar cycle, where an eclipse during maximum would look like a total eclipse and during minimum like an annular eclipse.

Geometric, elliptical models.

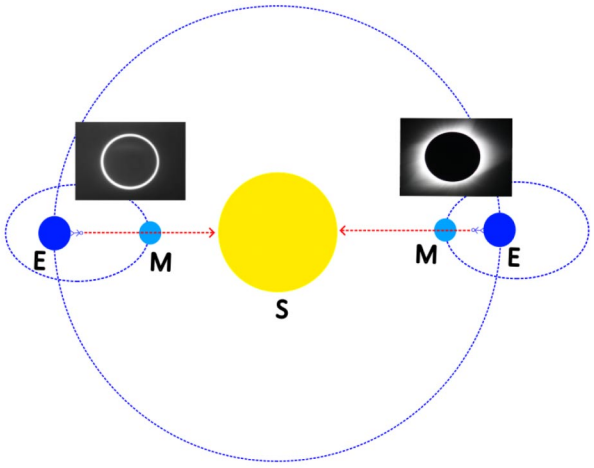
Many students who followed an elliptical reasoning during the discussion on apparent sizes fell back on the same argument when considering the two types of eclipses. Some of them realized that the apparent size of the Moon changes between the pictures and that a total and annular eclipse should correspond to its position in perigee and apogee, respectively. For completeness, we stress that this is actually the correct answer to this question. An illustration of this model is given in Fig. 13. Out of the ten students who explained the variation in apparent Moon size in terms of its elliptical orbit, eight suggested this variation to be (one of) the cause(s) for the difference between the two eclipse pictures. Three of those explicitly referred to the elliptical orbit again; the others rather spoke in terms of the Moon being "closer at one instance and further at another."

A more atypical reasoning was that of Pascale, who previously claimed the apparent Moon size to depend only on its place in the sky (as described in Sec. IV B), but now seemed to remember the ellipticity of the Moon's orbit. What follows is an excerpt of the interview with Pascale, while she made the drawing of Fig. 13(a) to explain the difference between the two pictures in Fig. 4.

P: I think that's again the distance between, uhm, the Moon and the Earth.
I: Okay, and is that this narrative then [points at the previously made drawing of Fig. 9(b)], or is it something else?
P: Uhm, no there is a long distance from the Earth and a short distance from the Earth, I think.
I: Of the Moon or of the Sun?
P: Between the Earth and the Moon, and I would say this [pointing at the total eclipse] is the short distance.
I: Okay?
P: That's aphelion and perihelion or something, and then this [pointing at the annular eclipse] is the long distance, so then it's smaller and the Sun sticks out a little.



(a) Original drawing from Pascale



(b) Reproduced illustration

FIG. 13. Explanation for the difference between total and annular solar eclipses based on the eccentricity of the Moon's orbit, as (a) drawn by Pascale and (b) reproduced by the authors.

I: Okay, could you maybe draw how the Moon moves, or what you mean exactly by that short and long distance?

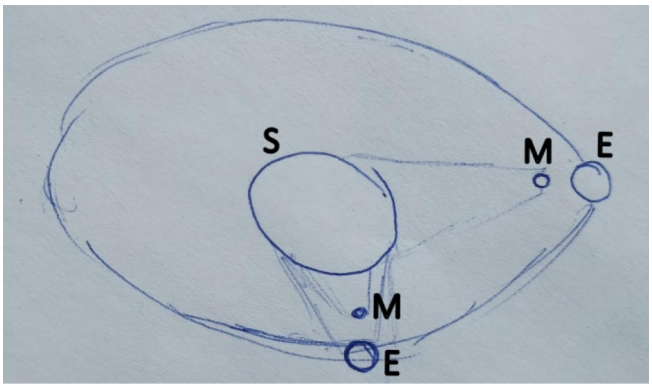
P: So this is the Earth [starts drawing], and then that's let's say the motion of the Moon around. This is then the one, the shorter distance [indicating the right point in Fig. 13(a)] and that is then the longer distance [indicating the left point in Fig. 13(a)].

I: Yeah, so which photo will I see from where then?

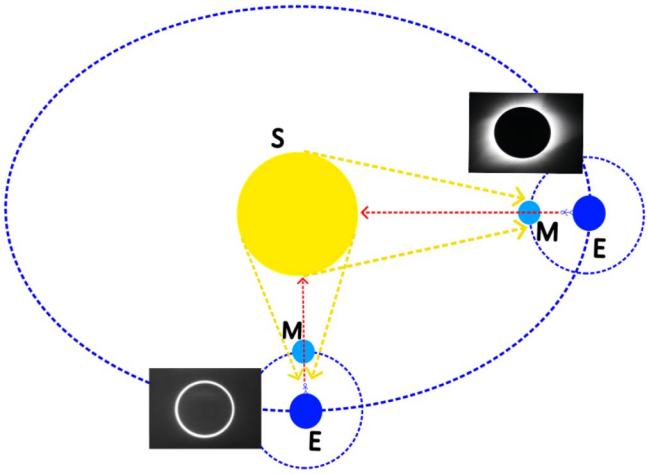
P: This [total eclipse] is the short distance, that [annular eclipse] is the longer distance.

Note that we drew a circular Earth's orbit in the illustration of Fig. 13(b), while Pascale did not draw the Earth's orbit in Fig. 13(a). However, during the discussion on the apparent Sun size earlier in the interview, she had stated: "I think that [the apparent Sun size] is always the same, because the Sun is always I think more or less at the same distance from the Earth." While this statement does not necessarily imply a circular Earth's orbit, we argue that it is reasonable to assume that Pascale imagined it to be.

An analogous argument was made by several students with respect to the ellipticity of the Earth's orbit. Out of the 12 students who suggested that the ellipticity of the Earth's orbit causes a variation in apparent Sun size, 6 explicitly mentioned this elliptic orbit again when asked to explain



(a) Original drawing from Fabio



(b) Reproduced illustration

FIG. 14. Explanation for the difference between total and annular solar eclipses based on the eccentricity of the Earth's orbit, as (a) drawn by Fabio and (b) reproduced by the authors.

the two eclipse pictures. One good example of a student in this group is Fabio, who made the drawing in Fig. 14(a). He explains his reasoning in the following excerpt:

I: Yeah, and so the Moon is always large enough to cover the Sun then [during an eclipse], or is that not always the case?

F: Uhm, I don't think it's always large enough, for example when we're here [indicates the Earth at the bottom position in Fig. 14(a)], that we would still see the sides of the Sun past the Moon [draws the Sun rays past both sides of the Moon].

[…]

F: I think here [pointing at the picture of the annular eclipse] you can actually still see the Sun, and with the other you can only see the shine of the Sun and not the Sun itself. So then the Moon really fully covers the Sun.

[…]

F: […] Uhm, yeah, the only explanation I could have for that is that from here [indicates the Earth position at the right of Fig. 14(a)] we're further from the Sun so that [the Moon] then will cover the whole Sun, but from here [pointing at the Earth position at the bottom of Fig. 14(a)] that the edges of the Sun will still like, come through.
I: So it's rather the Sun that becomes larger between the two pictures?
F: Yes.
I: Rather than the Moon?
F: I think we get closer to the Sun while the Moon is still about as far.

While Fabio did not draw the shape of the Moon's orbit in Fig. 14(a), we included it in the illustration of Fig. 14(b). Based on the last line of the previous excerpt, we drew the Moon's orbit as circular. However, as discussed before (see Fig. 11), Fabio actually imagines the Moon's orbit to be elliptical. It is unclear whether he forgot about this when discussing the two types of solar eclipses, or he deemed the ellipticity of the Moon's orbit to be irrelevant.

Geometric, nonelliptical models.

Both Louisa and Mark, who explained variation in apparent Moon size in terms of the geographical position of the observer (Fig. 10), again fell back on this idea when considering the two types of solar eclipses. Unfortunately, neither of them provided a clear drawing. However, Mark formulated his reasoning as follows:

I: And so, what changes between the two pictures [points at both pictures in Fig. 4]? Is it the Moon or is it the Sun or is it both, or is there something else?
M: I'd like to say, well, obviously both [points at his previously drawn sketches of the orbits of the Moon and the Earth]. I think, I think it has a lot more to do with the Moon than it has to do with the Sun.
I: Okay. And do you think that when you fix positions, you could sometimes see either of these observations? Or are these observations linked to positions?
M: Well, since the Moon size would be in correlation to where you are on Earth, while looking at the Moon, the, the eclipse would look differently depending on where you would be on the Earth.
I: So it can be one single eclipse and some people see this [points at circular eclipse] and other people see that [points at total eclipse]?
M: Yeah.

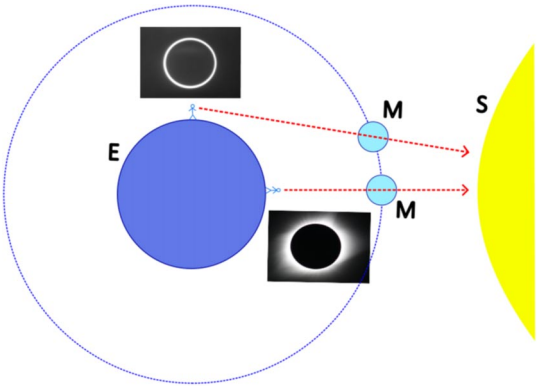


FIG. 15. Illustration of how two observers at different geographic positions could simultaneously see two different types of eclipses (made by the authors).

When we consider this excerpt together with Mark's reasoning on variation in the apparent Moon size (see Sec. IV B), we interpret his model as illustrated in Fig. 15. Since the polar observer is physically farther from the Moon than the equatorial observer, the former will see a smaller Moon. Therefore, the polar observer could witness an annular solar eclipse at the same time that the equatorial observer sees a total eclipse.

In total, five students (Mark, Louisa, Celine, Sam, and Ben) suggested that the observers' locations could (partly) explain the difference between the two eclipse pictures. Although Celine, Sam, and Ben did not follow the model of Fig. 10 to explain variation in apparent Moon size, they all argued that the Moon should look larger high in the sky than low above the horizon (Fig. 9).

It should be added that the Ben's explanation differed slightly from the other four students. According to Ben, different positions on Earth would impact the observed solar eclipse because the alignment between the observer, the Moon, and the Sun would change. For Ben, it is a matter of perspective rather than a change in distance to the Moon. He explained his thoughts in the following fragment:

I: Could you try to describe or maybe explain what would be the differences between those two pictures?
B: I think this one [pointing at the annular eclipse] is perpendicular.
I: Okay?
B: Well, I mean that, the point where they're really like perpendicular to that position [indicates the Earth-Moon-Sun axis in the scale model]. And this one [pointing at the total eclipse] is more to the sides.
I: What do you mean by more to the side?
B: For example, slightly more to the right or the left, like from this point [pointing at positions on Earth in the scale model that are slightly deviating from the Earth-Moon-Sun axis].

TABLE II. Number of occurrences for every model of total versus annular solar eclipses. Note that these numbers amount to more than the number of participants, since the models are not mutually exclusive.

Model	Number of occurrences
Optical	**5**
Geometric, elliptical	**8**
Ellipticity of Moon's orbit	3
Ellipticity of Earth's orbit	6
Geometric, nonelliptical	**6**
Observers' geographic position dependence	5
Earth's position dependence	1
Varying apparent Moon size—not specified	10
Varying apparent Sun size—not specified	5

I: Not perfectly behind the Moon, you mean?
B: Yes.
I: Yeah, a little bit slanted?
B: Yes, but on Earth that's, on our scale that's a pretty, I think, large number of kilometers.

To conclude, Henry suggested that the difference is not caused by the position of the observer on Earth, but by the position of the Earth itself. In case of a total eclipse, he argues that the Earth, Moon, and Sun should be perfectly aligned. During an annular eclipse, on the other hand, the Moon might be "a little bit under or above the Earth-Sun axis.

Table II gives an overview of the different explanations for the two types of solar eclipses. The bottom two rows represent students who argued that the Moon and/or Sun must change apparent size between the two pictures but could not explain further how that works. Students who vaguely stated that the Moon and/or Sun can sometimes be "closer" to Earth than at other times (but did not explicitly mention elliptical orbits) are also included in this group.

V. DISCUSSION

In this work, $N = 25$ students were interviewed about two astronomical phenomena: the apparent sizes of the Moon and the Sun and two types of solar eclipses. Throughout the interview, students were encouraged to use a personalized scale model of the Earth-Moon-Sun system on the interview table to illustrate their reasoning. For both phenomena, we uncovered explanations (or models) that—as far as we know—were not previously documented in the research literature. This applied especially to the models in the "Geometric, nonelliptical" category of Tables I and II. These models all rely on skewed ideas of the spatial scales involved in the Earth-Moon-Sun system. When analyzing the drawings in Secs. IV B 2 and IV C 2 more closely, one can derive that the skewed scale assumption often concerns the size of the Earth relative to the distance to the Moon. Students overestimate the size of the Earth—or, equivalently, underestimate the distance to the Moon—and this erroneous estimate gives rise to a range of causal principles for both phenomena. For apparent Moon size variation, this is best illustrated by Figs. 9 and 10. Here, the Moon is drawn extremely close to the Earth, both in the original student drawings and in the authors' reconstructions.

These models are not considered incorrect because they are illogical, but rather because the effects they describe become totally irrelevant when considering the true spatial scales of the Earth-Moon system. However, as the personalized scale models on the interview table showed, many students believed the Moon to be much closer to Earth than it actually is. Therefore, in light of the skewed spatial proportions of these scale models, the described mechanisms for apparent size variation became actually very relevant to these students. For example, in Louisa's scale model, the Earth's diameter couldfit only 1.5 times in the distance between the Earth and the Moon. This is very comparable to the proportions in her drawing in Fig. 10(a), and if it were accurate, the described variation in apparent Moon size would indeed be considerable. The same holds for Fabio, who described the apparent Moon size variation model of Fig. 9 and whose scale model indicated that there are only two Earth diameters between the Earth and the Moon. Although these students fail at making accurate estimates of spatial scales, they seem to be able to use their scale model to compare several distances (e.g., observer to apex-Moon versus observer to horizon-Moon) and to deduce plausible differences between two observations.

Remarkably, several students were aware that their models' relevance depended heavily on the involved scales. This was indirectly suggested by the much smaller number of students proposing the models of Figs. 9 and 10 for the Sun than for the Moon. Moreover, it was explicitly mentioned for both models by Pascale and Mark, respectively. These students were able to verbalize why they thought these models were applicable to the Moon, but not to the Sun. They both argued that the much larger distance to the Sun compared to that to the Moon causes this differentiation. This suggests that, if these students had an accurate understanding of the spatial scales in the Earth-Moon system, they would also refrain from applying these apparent size variation models to the Moon.

However, not all students suggesting the geometric, nonelliptical models had equally skewed scale models of the Earth-Moon-Sun system. Some students proposed apparent size variation models that should not be of any importance when simultaneously considering their scale models. For example, Noah suggested that the deviation of the Earth's shape from a perfect circle causes variation in apparent Moon size between several observers, even

though in his scale model, the Moon was positioned at a distance of roughly 20 Earth diameters. This shows that the argument of the previous paragraph does not apply to all students. Like Noah, some students built arguments to explain phenomena without taking the involved scales into account to evaluate their plausibility. This is much in agreement with Rudmann [64], who assessed university students' knowledge of astronomy and their ability to solve astronomy problems. He found that some students tended to solve problems incorrectly given their own causal beliefs, as the answers they offer to these problems were inconsistent with their own understanding of the underlying phenomenon. We hypothesize that these students did not take into account their conceptual knowledge of astronomy when trying to solve the problems, as they analogously did not take into account their scale model to answer the questions in our interviews.

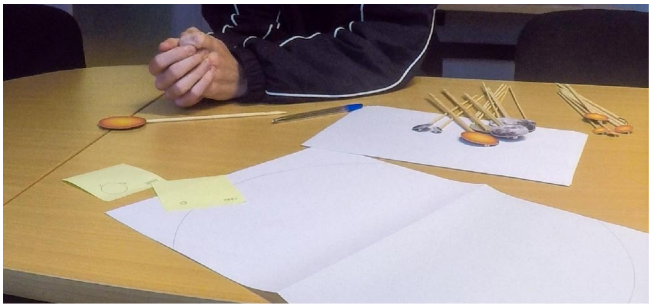
When we consider the pattern of student answers across different questions, we notice that most students tend to use the same model to explain different phenomena. For instance, when students explain variation in apparent Moon or Sun size in terms of the ellipticity of the Moon's or Earth's orbit (Figs. 6 or 7, respectively), they tend to also use this model when explaining the difference between total and annular solar eclipses (Figs. 13 or 14). The same argument holds for students with geometric, nonelliptical models. All five students who explained the two types of solar eclipses in terms of the position of the observer (Fig. 15) previously argued that the apparent Moon size could vary either depending on its elevation in the sky (Fig. 9) or depending on the position of the observer (Fig. 10).

However, there are exceptions to this trend, as some students did not reuse their previous arguments. Some participants even made inconsistent or contradictory statements. For instance, as shown in Fig. 11, Fabio explained variation in apparent Moon size in terms of the Moon's eccentric orbit and its elevation in the sky. Yet, when discussing the two types of solar eclipses, he assigns the difference between the pictures solely to the variation in apparent Sun size, which is caused by Earth's eccentric orbit. He states that "I think we're closer to the Sun [for an annular eclipse compared to a total eclipse], while the Moon is still at roughly the same distance." Another example of inconsistency was found in the interview of Sam. Sam's chosen Moon disc was larger than the chosen Sun disc, but she later indicated that she thought the Sun always looked larger in the sky than the Moon. For a few students, their tendency to contradict previous statements was so pervasive that it complicated the interview itself and the analysis afterward. The best example here was Noah, who seemed to change his mind constantly, formulating arguments and refuting them instantly again, thus switching between several models within the course of one interview topic.

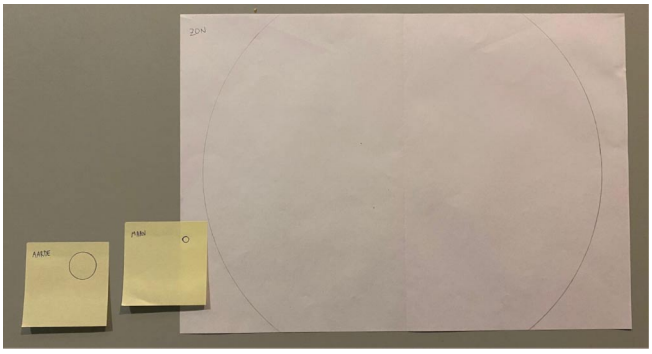
Although some students could interpret and reason with the distances in their (inaccurate) scale model, leading to models like the ones in Figs. 9 and 10, this competence generally broke down when considering subtended angles. Asking to mentally adopt the perspective of the Earth in their scale model and imagining the apparent sizes of the Moon and the Sun proved too difficult a task for almost all participants in this study. This is shown in Fig. 5, which displays large discrepancies between the apparent Sun sizes as indicated by the scale model and by the chosen Sun discs. The results in Fig. 5 showed no correlation between the two measures, indicating that students with a larger angle subtended by the Sun in their scale model did not systematically choose larger Sun discs. However, this finding should plausibly be attributed to the particular selection of interviewed students. As we selected students with substantially underestimated spatial scales, we indirectly chose those students with large subtended angles by the Moon and the Sun in their scale models (since both bodies are placed much too close to the Earth). We thus restricted the domain of the apparent size variable in the scale models, thereby potentially erasing a correlation in the broader population.

The inability to interpret subtended angles was also demonstrated during the discussion on solar eclipses, where some students did not realize that the angular size of the Sun in their scale model was drastically larger than that of the Moon. A good example was Gregory, who was familiar with the concept of a solar eclipse and demonstrated this knowledge by placing the Moon in between the Earth and the Sun. However, as shown in Fig. 16, the assumed sizes and distances in his scale model clearly prohibit a total solar eclipse to ever occur. Figure 16(a) shows Gregory's scale model during the interview, and Fig. 16(b) shows the scale model as reconstructed by the authors. From this scale model, it is obvious that the tiny Moon could never fully cover the Sun to create a solar eclipse (as seen from Earth). However, Gregory does not realize this and he is not explicitly pointed to it by the interviewer. This result closely aligns with the findings of Plummer *et al.* [75], who demonstrated that the spatial skill of perspective-taking is highly instructive for a thorough understanding of astronomical phenomena such as Moon phases and seasons.

Besides the discrepancy of apparent Sun size between the scale model and the chosen discs, Fig. 5 also shows a general overestimation of this size for both modes of assessment. Although not shown, the same result was found for the apparent Moon size. This finding is in line with the previously discussed work of Carbon [23], who documented how planetaria often use an artificially enlarged Moon in their dome presentations because visitors otherwise perceive the Moon as too small. Similar to the planetarium setting, participants in this study might have taken the "registered distance" [24] to the Moon disc into account when estimating its apparent size. The average



(a) Screenshot from Gregory's interview



(b) Reconstruction of Gregory's scale model

FIG. 16. Gregory's scale model, both (a) in a screenshot during the interview and (b) in a reconstruction that was *a posteriori* made by the researchers.

overestimation of apparent sizes in this study was considerably larger than the factor of 2.5 reported by Carbon [23], suggesting that a decrease in registered distance leads to a more drastic overestimation.

People's tendency to overestimate small angles is also well known in the field of psychophysics. Generally, the subtense of acute angles is overestimated and that of obtuse angles underestimated (while parallel and orthogonal lines are evaluated most accurately) [76–79]. Some researchers attribute these errors to the inherent ambiguity of any 2D retinal image of a 3D real-world angle [80,81]. Simplistically stated, an observed small angle in 2D often represents a (slightly) larger 3D angle, once the distance factor is taken into account. This psychophysical mechanism could be another factor, besides registered distance, that causes the observed overestimations of apparent Moon and Sun sizes. However, the participants in the cited literature usually evaluate the angles subtended by two lines, not by real physical or astronomical objects. Moreover, the described error reaches a maximum around 45° and is minimal around the much smaller angle of 0.5° subtended by the Moon and the Sun. Therefore, its impact on our results is presumably limited.

When considering the geometric, elliptical models for variation in apparent Sun and Moon sizes, we note an interesting parallel between this phenomenon and the cause for seasons. As mentioned, it has been widely shown [7,11,29,31,71–73] that students tend to assign the seasons to a varying distance to the Sun, which in turn is explained by the elliptical orbit of the Earth. In this work, we show that students also infer a varying apparent Sun size from this elliptical orbit. We interpret that both alternative conceptions originate from the same erroneous scale assumption: an overestimation of how much the distance between the Earth and the Sun varies over a year. A very similar argument can be made for the alternative distance theory for seasons, which claims that the Earth's tilted rotation axis brings one hemisphere physically closer to the Sun, and that this smaller distance causes summer in that hemisphere [11,74]. Multiple students in this study formulated the same argumentation to explain a variation in apparent Sun size. Here, the underlying scale assumption of both alternative conceptions concerns an overestimation of the Earth's size relative to the distance to the Sun (or vice versa, an underestimation of the distance to the Sun with respect to the Earth's size). Remarkably, both these arguments were extrapolated to the Moon by some students. While the Moon actually approaches and recedes from Earth in its monthly orbit, its effect on the variation in apparent Moon size was generally overestimated by the students. This reflects how they also overestimate the amount of variation in the distance between the Earth and the Moon over the course of a month. The one student applying the alternative distance model to the Moon illustrates how students can occasionally draw too close to a parallel between the Moon and the Sun.

It is worth highlighting that many of the explanatory models described in this work were not previously documented in the research literature. Due to the lack of studies investigating apparent size estimates, this was *a priori* true for the models described in Sec. IV B. One particularly interesting model is that of Fig. 9, which suggests that the Moon appears larger when seen highly elevated than when low above the horizon. Our interest in this model stems from two factors. First, it occurred surprisingly frequently. While the number of participants is definitely too low to perform serious statistics, we note that this model was certainly not a unique case. Instead, it resurfaced multiple times, in several schools (although not with any of the three Swedish students). This suggests that this idea might be widespread in the considered population—that is, among last-year high school students with limited astronomy background. Second, this model describes the exact opposite effect of the Moon illusion. This makes it very unlikely that the students actually experienced the described effect before, suggesting instead that this idea was constructed on the spot. This was explicitly observed during the interview of Pascale, who first described the Moon illusion but completely turned around the argument when asked to draw what she meant. The number of students describing the Moon illusion was very low overall, indicating that students in general are not familiar with this effect. This is

in contrast to the findings of Galili *et al*. [19], who report that the majority of their high school participants showed familiarity with the Moon illusion phenomenon. However, in one of the two high school classes in their study, 25% of the students indicated that the Moon looks enlarged when observed near the zenith compared to low above the horizon. Therefore, an awareness of the Moon illusion might have been dominant among their participants, but the opposite belief is nonetheless represented as well.

Of course, all these findings need to be interpreted in light of the fact that participants in this study were not randomly selected. They were specifically chosen based on their flawed understanding of spatial scales in the Sun-Earth-Moon system. Consequently, the findings of this work can not simply be extrapolated to the general population of high school students. However, as we have shown in previous work [10], the majority of high school students tend to underestimate these spatial scales. Therefore, the participants of this study were not extreme outliers in terms of these estimates; rather, they represent a well-populated profile of scale models. We thus argue that many explanations described in this work can be expected to resurface in future studies with random, sufficiently large samples of students. Nonetheless, this should be verified in future work, meaning that future studies should investigate whether similar explanations for astronomical phenomena are suggested by students who do not (drastically) underestimate astronomical spatial scales and, in particular, by those students with an accurate understanding of these scales. Also, the range of astronomical phenomena should not be restricted to apparent sizes and solar eclipses alone, since many other phenomena are equally related to spatial scales.

VI. CONCLUSION

This work demonstrated that students use a wide variety of explanations for the astronomical phenomena of apparent Moon and Sun sizes and solar eclipses. Often, the alternative reasoning followed by students was underpinned by skewed ideas of the spatial scales in the solar system. As such, we conclude that students' conceptions of the spatial scales in astronomy can impact their interpretation of astronomical phenomena. On multiple occasions, students were revealed to be aware that their arguments depended on scale assumptions. This suggests that more attention to scale concepts in astronomy classrooms could benefit students' conceptual knowledge of several astronomical phenomena. Of course, these findings were not without exceptions, as some students did not seem to take scale assumptions into account for their explanations. Moreover, all findings of this work are susceptible to the severe bias applied in the selection procedure for the interviews. Only students of a particular scale profile were interviewed (i.e., students who underestimated the spatial scales in the online survey), but there is reason to assume that this profile is widely present among high school students. Nonetheless, future research should investigate whether the explanations described in this work are also offered by students who do not (or to a lesser degree) underestimate the spatial scales in the solar system. Moreover, future work should not be limited to the two astronomical phenomena investigated in this work, as similar reasoning patterns could be expected for other phenomena as well.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because of legal restrictions preventing unrestricted public distribution. The data are available from the authors upon reasonable request.

APPENDIX A: CODING SCHEMES

The coding of student answers was performed at the level of the different astronomical phenomena. No coding with smaller grain sizes (e.g., on the level of individual sentences) was performed. In this appendix, we elaborate on the coding procedure for both phenomena discussed in this work.

1. Apparent size variation

For variation in apparent sizes, a hierarchical scheme was constructed that logically classified students' explanatory models. As such, similar but slightly different models were close to one another in the scheme and were assigned related codes. On the first level of the hierarchical structure, students were distinguished based on whether they provided any explanation for apparent Moon or Sun size variation. The three main groups were indicated with roman numbers: I for students who thought no variation existed, II for students who said there is variation but could not provide an explanation, and III for students with explanations. On a second level, students with explanations were subdivided based on the nature of their explanation. These were indicated with capital roman letters: A for geometric models, B for comparative ones, C for optical models, and D for models that were either not classifiable in the former three groups or that were unclear.

Both the elliptical and nonelliptical models in this work were classified under group III. A., only to be subdivided later on. In group III. B., we classified explanations based on comparing the size of the Moon or Sun with that of other objects in the field of view, for instance to trees. Also classified in this group was the Moon (Sun) illusion, since most theories explain this illusionary effect by a

TABLE III. Coding scheme for apparent size variation.

Variation?	Explanation	Specification -	#Discs
I. No variation	/	/	1
II. Variation without explanation	/	/	1/2/3…
III. Variation with explanation	III.A. Geometrical	III.A.1. Elevation dependence: higher = larger	1/2/3…
		III.A.2. Position dependence: equatorial > polar	1/2/3…
		III.A.3. Flattened Earth	1/2/3…
		III.A.4. Nonspecified varying distance Earth-Moon/Sun	1/2/3…
		III.A.5. Elliptical Moon's orbit/Earth's orbit	1/2/3…
		III.A.6. Varying distance due to tilted Earth axis	1/2/3…
	III.B. Comparative	III.B.1. Other objects in field of view	1/2/3…
		III.B.2. Moon/Sun illusion	1/2/3…
	III.C. Optical	III.C.1. Larger = brighter, no specification	1/2/3…
		III.C.2. Cloud influence	1/2/3…
		III.C.3. Phase dependence	1/2/3…
	III.D. Others/unclear		1/2/3…

subconscious comparison with the distance features on the horizon [82–84]. Group III. C. consisted of students who stated that the Moon or Sun looks larger when it reflects or emits more light. Distinctions were made between statements with no more specification and more detailed ones, for instance, those claiming that the amount of light emitted or reflected is determined by the amount of clouds in the sky. These individual models were coded by Arabic numbers.

At the end of every code, a second Arabic number was added, separated by the rest of the code by a hyphen. This ending number reflected the amount of discs on sticks over which the apparent Moon or Sun size could vary according to the student. As such, it reflected the extent of the variation. Logically, codes that started with I. (indicating no variation) ended with the number 1. Codes starting with II. or III. could end with any natural number. Examples of codes were: II.-2, III.A.2-1, III.C.3.-4, or I.-1. The full coding scheme is shown in Table III.

2. Solar eclipses

In the analysis of the discussion on solar eclipses, students' answers were captured in a single code. To this end, their explanations for a solar eclipse, in general, were combined to those about the difference between total and annular eclipses. The answers to the initial open-ended question on solar eclipses were divided into three groups: I. students who could not provide any explanation, II. students who explained correctly that a solar eclipse occurs when the Moon is in between the Earth and the Sun, and III. students who provided an alternative explanation. For the explanations on the difference between total and annular eclipses, five groups were distinguished: A. students who could not formulate an explanation, B. students who referred to optical arguments, C. students making a geometric reasoning, D. students who could see how the apparent size of the Moon and/or Sun must change, but did not specify further, and E. students with other explanations. The optical models in group B were quite diverse, which is why this group is further subdivided into four specifications. The geometric models in group C contained both elliptical and nonelliptical models. For the codes C.1. and D., a further specification was made on whether the argument concerned the elliptical orbit of the Moon or the Earth (for code C.1.) or a varying apparent size of the Moon or the Sun (for code D.).

When students did not know what a solar eclipse was, they were aided by the interviewer, who gave hints or sometimes showed one of the two pictures as a clue. Thereafter, students were always shown the two pictures of a total and annular eclipse, regardless of their former answers. As such, even students with code I. to the initial question were assigned an additional code for the subsequent part of the discussion. Examples of codes were I.D., II.B.1., and II.C.3. The complete coding scheme is shown in Table IV.

3. Data analysis

After every interview, detailed verbatim transcriptions were made. Using both the transcriptions and the original interview data, students were assigned one or several codes per phenomenon from the respective schemes. Especially after the first couple of interviews, the constructed schemes were still subject to change. When new, unforeseen explanations were found, or new variations to already uncovered explanations, new codes could be added to the schemes, or their structure could be altered. As more interviews were conducted, these additions or alterations to the coding schemes became more rare. In his book on *Qualitative Content Analysis*, Mayring refers to this technique as inductive category formation [85]. Often, students were assigned more than one code, for instance, when they

TABLE IV. Coding scheme for solar eclipses.

Eclipse explanation	Total versus annular eclipse explanation	Specification
I. No explanation	A. No explanation	/
	B. Optical	B.1. Effect of zooming
		B.2. Impact of Sun cycle
II. Moon in between Earth and Sun		B.3. Effect of clouds
		B.4. Effect of filter/eclipse goggles
	C. Geometric	C.1. Elliptical orbit: M/E
		C.2. Position dependence
III. Other explanation		C.3. Varying configuration E-M-S positions
	D. Nonspecified apparent size variation: M/S	
	E. Other explanations	

made both geometric and optical arguments for apparent Moon or Sun size variation. When students provided a certain explanation but later on changed their minds and adopted a different reasoning, both codes were assigned to their answers.

When all student answers were assigned one or several codes per phenomenon, the answers within one coding group were regarded collectively. By repetitively reading the transcripts and watching the video recordings, it was checked whether these students really expressed the same ideas. Special attention was given to how students used the scale model to explain their ideas, to the gestures they made, and to the drawings they produced. If the explanations were seen as identical, no changes were made to the codes. The researcher then selected one or a few suitable students of this group (based on unambiguous formulation used in the explanation and clear drawings) to describe the respective model in this work. If not all explanations of one code were deemed identical, some student explanations could potentially be reallocated. Alternatively, these differing explanations could occasionally be kept within the same code group, but then the variation between their interpretations was nuanced in the text of this work.

APPENDIX B: INTERVIEW PROTOCOL

In this appendix, we include the interview protocol used during all interviews of this study. The protocol is provided in English, but we note that the interviews with students in Belgium were conducted in Dutch. As the interviews were semistructured, additional follow-up questions could differ between participants. In the following, **I** refers to the interviewer, and **S** refers to the student. Bullet points indicate several possible responses of the student, to which the interviewer reacted differently.

1. Introduction

I: In the next twenty minutes or so, I will ask you some questions about how you think our solar system and the surrounding space look like, as well as about a few astronomical phenomena. We are purely interested in what you think. You can take as much time as you need to answer, and there are no right or wrong answers as long as you share your own thoughts. I would like to ask you to think out loud as much as possible throughout the entire interview. That can be difficult, but I will remind you if I notice that you are thinking without saying anything. Does that sound okay to you?

S: […]

I: Great, thank you.

2. Reviewing the scale model

I: As you can see, I have placed a scale model of the space around Earth on the table. This circle represents the Earth, this one the Moon, and this one the Sun. The relative sizes of these three objects and the distances between them are based on the answers you provided in the online survey a while ago. In that survey, you saw a visualization for each question and had to indicate whether you agreed with it. Do you remember that? Here, we will do something similar: Now that you see everything together, do you agree with this scale model of the Earth-Moon-Sun system? Or would you like to change anything, such as making objects bigger/smaller or moving them?

S: […]

I: Okay, if at any point during the interview you want to make additional changes, that is still possible.

3. Apparent Moon size

I: The next thing we will discuss is how large the Moon appears in the sky. I have brought with me different discs of a full Moon, with different sizes [showing the Moon discs from Fig. 3]. Your task is to imagine how large a full Moon in the sky looks like, and to compare that with these discs by holding them at arm's length. Then, try to choose the Moon disc that you think best represents the

size you would see if you went outside tonight and saw a full Moon. Do you understand the question? You can try as many discs as you want.

S: [Chooses a Moon disc.]

I: Okay, great. Let's compare that with the scale model here on the table. Imagine that you are standing on this model Earth and you're looking at this model Moon. This Moon now has a certain size and is at a specific distance, which we just established. Do you think that this model Moon would look the same size when seen from the scale model Earth as the disc you just chose?

•S: Yes, I think so.

I: Okay, great.

•S: No, I think it would look smaller/larger.

I: Okay. Now, if we want the Moon disc to match your scale model, they should in principle look the same, right? So we would either need to adjust the model or choose a different Moon disc. What do you think we should do?

I: Do you think a full Moon always looks the same size? Or can it sometimes appear larger or smaller?

•S: I think the Moon always looks the same size.

I: Okay, great.

•S: I think it can differ.

I: Okay, and is that variation such that you would sometimes prefer a different disc than the one you just chose? Or is the variation not that drastic?

I: What causes this variation? Why does the Moon sometimes appear larger or smaller? Can you demonstrate this by making a drawing or using the scale model on the table?

I: Do you remember certain instances where you looked at the Moon and thought: now it looks remarkably large or remarkably small? If so, can you tell me when and where that was? Do you have any idea why the Moon looked so large/small at that moment?

4. Apparent Sun size

I: Now, we will do the same exercise, but with the Sun instead of the Moon. I also have brought all these Sun discs with varying sizes, as you can see [shows the Sun discs in Fig. 3]. Again, you can try any number of discs by holding them at arm's length, and then try to choose one Sun disc that you think represents the correct size, meaning the size you would see if you looked at the Sun on a sunny day—with protective sunglasses. Do you understand the question?

[Repeat the protocol as described above for the apparent Moon size, but now for the Sun. The student chooses a Sun disc, we compare it with the scale model on the table, and then, we ask if the apparent size of the Sun can vary and by how much/over how many discs. If so, we ask about the cause for this variation. Conclude by asking whether the student remembers seeing the Sun surprisingly large or small, and if so, when and where that was.]

5. Solar eclipses

I: Let's talk about something else now. Do you know what a solar eclipse is? Can you explain what happens during a solar eclipse?

•S: No, I don't know what a solar eclipse is.

I: Maybe this can help: this is a picture of a solar eclipse [shows one of the two pictures of Fig. 4]. Do you have any idea what a solar eclipse is now?

•S: Yes, a solar eclipse is when […].

I: Can you make a drawing of that or show it with the scale model?

I: I have actually brought two pictures of a solar eclipse [now showing both pictures of Fig. 4]. So it's the same phenomenon, but a different observation. As you can see, they clearly look different. Can you try to describe the differences between these pictures?

I: How do you think these differences come to be? Something must have changed between the two images to cause the differences–what do you think that might be?

The interview continues with the topic of Moon phases, which is described in the companion paper [13]. Thereafter, the student is thanked for his participation and the interview is ended.

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