

Investigation of preservice physics teacher's conceptual changes using the quantum interactive learning tutorials in Korea

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As global interest in quantum mechanics grows, its inherent complexities continue to pose challenges for effective instruction. This study investigates the impact of Quantum Interactive Learning Tutorials (QuILTs) on enhancing conceptual understanding of quantum mechanics among preservice physics teachers in Korea. The QuILTs, consisting of pretests, tutorial worksheets, and post-tests, targeted the following five key concepts: possible wave functions, bound/scattering states, time evolution of wave functions, quantum measurement, and the uncertainty principle. To assess conceptual change, the Quantum Mechanics Survey (QMS) was administered to 20 preservice teachers before and after the completion of QuILTs instruction to assess conceptual change. Additionally, embedded quizzes and interviews provided deeper insights into persistent misconceptions. Results indicated a moderate to substantial overall improvement, with a Hake gain of 0.54. Significant gains were observed in understanding possible wave functions and time evolution, although challenges remained in areas such as normalization of free particle wave functions and interpretation of scattering states. Misconceptions related to quantum measurement and the uncertainty principle showed partial improvement. The findings demonstrate the effectiveness of QuILTs in enhancing conceptual understanding among preservice teachers, suggesting that research-based, concept-focused instructional strategies can support future educators in mastering complex quantum mechanics concepts. These results offer valuable insights into the development of teacher preparation curricula and instructional strategies in Korea.

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I. INTRODUCTION

Quantum mechanics explains numerous phenomena in modern science and technology, offering a framework for understanding the microscopic world. With applications such as quantum computing and cryptography, the need for public comprehension of quantum mechanics has grown. Consequently, it has been integrated into science curricula worldwide, from high school in countries like South Korea [1] and Germany [2] to middle school in the Netherlands [3] and France [4]. However, the abstract nature of quantum mechanics presents significant learning challenges, as its principles often contradict everyday experiences and

require sophisticated mathematical formalism [5]. These barriers hinder students' mastery of core concepts [6,7] and prompt educators to explore strategies for improving instruction [8,9].

Efforts to address these challenges have led to the development of diagnostic tools, including the Quantum Mechanics Visualization Instrument (QMVI) [10], the Quantum Mechanics Conceptual Survey (QMCS) [9], and the Quantum Mechanics Survey (QMS) [11], which assess learners' conceptual understanding and help identify persistent misconceptions. Similarly, instructional interventions such as Quantum Interactive Learning Tutorials (QuILTs) and diagnostic assessments like QMS aim to enhance comprehension through research-based, interactive modules and targeted evaluations [12–14].

These tools are grounded in conceptual change theory, which has been a foundational framework in science education since the 1980s, particularly following the influential model proposed by Posner *et al.* [15]. Since then, extensive research has examined the mechanisms of

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conceptual change and how instruction can facilitate it. QuILTs are designed within this framework, aiming to facilitate the replacement of naïve or scientifically inaccurate conceptions through cognitive conflict, reflection, and guided inquiry. By engaging learners with counterintuitive quantum phenomena through visualization and structured exploration, QuILTs help students construct more scientifically accurate mental models. While numerous studies have identified common misconceptions in quantum mechanics—such as the belief that particles cannot exist in regions where their total energy is less than the potential energy, that all wave functions must be sinusoidal, or that wave functions always evolve into stationary states [9,11,16]—these misconceptions have primarily been examined in studies involving undergraduate students in Western contexts. Despite their critical role in shaping future quantum instruction, preservice physics teachers have received limited attention in existing research—particularly in non-Western contexts such as Korea. This study addresses this gap by examining how Korean preservice teachers conceptualize quantum principles and how their misconceptions evolve through scaffolded instruction using QuILTs. In Korea, growing interest in quantum mechanics education has prompted studies on conceptual understanding among high school students and undergraduates [17,18]. However, little research exists on how preservice physics teachers—future educators shaping the next generation—grasp these concepts or address persistent misconceptions. These challenges underscore the need for targeted research to improve understanding in specific areas, such as the uncertainty principle and time evolution of wave functions, which are essential for teaching quantum mechanics effectively.

To evaluate conceptual change resulting from instruction, this study adopts normalized gain [19], a widely used metric in physics education research. Its limitations have been noted, including potential bias toward students with higher pretest scores [20], and influence from latent variables such as scientific reasoning ability or standardized test performance [21]. Nonetheless, recent work [22] supports the continued use of normalized gain, showing strong correlations with scientific reasoning even under non-normal score distributions. Accordingly, this study uses normalized gain as the primary indicator of conceptual change, interpreted with caution and complemented by qualitative interview data.

To address these gaps, this study investigates how Korean preservice physics teachers conceptualize quantum mechanics and evaluates the impact of QuILTs on resolving misconceptions. By situating this research within an international context, the study provides insights into effective strategies for enhancing teacher preparation programs in quantum mechanics. The research questions guiding this study are as follows:

1. What is the level of quantum mechanics conceptual understanding among Korean preservice physics teachers?

2. How does the conceptual understanding of quantum mechanics change for Korean preservice physics teachers following instruction using QuILTs?

II. METHODS

A. Design

This study employed a mixed-methods design with an explanatory sequential approach, in which quantitative data from the Quantum Mechanics Survey (QMS) and QuILTs assessments were first analyzed to identify patterns in conceptual change. These results then informed the qualitative phase, which involved semistructured interviews to explore participants' reasoning processes and learning mechanisms in greater depth. This design enabled both a broad measurement of learning outcomes and a nuanced interpretation of how conceptual change occurred.

B. Procedure

Each instructional session included a pretest (QuILTs test A), a guided tutorial using QuILTs worksheets, and a post-test (QuILTs test B). These sessions were conducted weekly over 16 weeks. After completing all sessions, participants took a postinstruction QMS to measure overall conceptual change. In addition, 30-min individual interviews were conducted with 11 participants to explore their experiences and responses to QuILTs assessments. This multistep process provided insights into both immediate and longitudinal impacts of the instructional intervention.

C. Participants

This study involved 20 preservice physics teachers enrolled in a physics education program at a university in South Korea. All participants were in their third or fourth year of study and were taking the course “*Quantum Mechanics for Secondary Physics Education*,” which is specifically designed to prepare future secondary-level physics teachers. The QuILTs were integrated into the curriculum over a 4-week instructional period.

Prior to this course, all participants had completed at least one semester-long introductory quantum mechanics course in their second year, covering fundamental topics such as the Schrödinger equation, the uncertainty principle, and basic potential well problems. Thus, they shared a relatively uniform academic background in both physics and science education.

The cohort consisted of 12 male and 8 female students. Since all participants were pursuing a nationally accredited teaching credential, the content and structure of their coursework were standardized, minimizing variations in their prior exposure to quantum mechanics.

To contextualize participants' performance, they were also asked to self-assess their prior understanding of quantum mechanics. Based on these self-assessments, 5% of participants reported a high level of understanding,

50% rated their understanding as moderate, and 45% rated it as low. This variation in self-perceived knowledge provided a useful baseline for evaluating the effectiveness of QuILTs across a range of learner profiles and for ensuring the broader applicability of the findings.

In South Korea, preservice physics teachers are trained in specialized colleges of education rather than in general university programs. This institutional structure differs from that of many Western countries, where physics majors and preservice teachers often share coursework. Consequently, direct comparisons between preservice teachers and nonteaching-track physics students within a single institution are not feasible in Korea. This unique training environment should be taken into consideration when interpreting the findings of this study.

D. Data analysis

Conceptual acquisition was quantified using the Hake gain (g), calculated from pretest and post-test scores of both QMS and QuILTs. The Hake gain formula is

$$\text{Hake gain } \langle g \rangle = \frac{S_{\text{post}} - S_{\text{pre}}}{100 - S_{\text{pre}}}$$

Values greater than 0.7 indicate high improvement, 0.3 to 0.7 moderate improvement, and below 0.3 low improvement. Additionally, interview transcripts were thematically analyzed to identify persistent misconceptions and uncover qualitative aspects of conceptual change. This mixed-methods approach provided a robust framework for interpreting both numerical and narrative data. For the qualitative component, we adopted a thematic analysis approach (Braun and Clarke [23]). Semistructured interviews with 11 participants were transcribed and analyzed to explore their reasoning and experiences with quantum concepts. Three researchers independently performed open coding on the transcripts to identify patterns related to conceptual understanding, misconceptions, and the effects of the QuILTs intervention. After the initial coding, the researchers compared their codes and collaboratively discussed overarching themes. Through several rounds of discussion and refinement, key themes were finalized. Disagreements were resolved through consensus, and efforts were made to ensure credibility and interpretive validity by grounding the themes closely in participants' language and context.

E. Instrumentation

The Quantum Mechanics Survey (QMS) is a 31-item diagnostic tool that evaluates conceptual understanding across wave functions, quantum measurement, time evolution, and the uncertainty principle [7]. It was administered as a pretest to establish baseline understanding and a post-test to assess overall conceptual change. The Quantum Interactive Learning Tutorials (QuILTs) consisted of 14 modules addressing specific misconceptions through

TABLE I. QuILTs and assessment by concept.

QuILTs concept	No. of Items	Type of assessment tool	
		Pretest	Post-test
Possible wave function	5	Test A	Test A B
Bound and scattering states	10	Test A	Test A B
Measurement	7	Test A	
Time evolution of wave functions	6	Test A	Test B
Uncertainty principle	4	Test A	Test B

guided problem-solving [24]. Each module included pre- and post-tests (test A and test B) to measure conceptual improvement within targeted quantum mechanics topics. The structure of the QuILTs assessments utilized in this research is detailed in Table I. These tools were selected to ensure alignment with the study's focus on both breadth and depth of conceptual understanding.

III. RESEARCH RESULTS

To assess the impact of QuILTs on conceptual changes in quantum mechanics, we analyzed data from the QMS, QuILTs tests A and B, and student interviews.

A. Conceptual analysis of QMS results

The QMS assesses conceptual understanding through pre- and post-test items across eight domains: possible wave functions, bound/scattering states, measurement, expectation values, time dependence of the wave function, stationary/nonstationary states, the role of the Hamiltonian, and other topics. In this study, instruction based on QuILTs focused on five key concepts: possible wave functions, bound/scattering states, measurement, expectation values, and time dependence of the wave function. Additionally, one item from the “other” category of the QMS was reclassified under the uncertainty principle for this analysis. The pre- and post-test results for 20 prospective physics teachers are summarized in Table II.

The five quantum mechanics concepts addressed by QuILTs showed notable improvements in accuracy, with scores rising from a minimum of 27% on the pretest to over 57% on the post-test. The Hake gain values further indicated a medium effect size, exceeding 0.41. The areas with the most significant improvements are highlighted in bold. The magnitude of conceptual change was greatest in the following order: possible wave functions, uncertainty principle, measurement, time dependence of the wave function, and bound/scattering states.

The QMS items evaluated various quantum mechanics concepts. For the concepts directly addressed by QuILTs—such as wave functions, stationary/non-stationary states, and the role of the Hamiltonian—the Hake gain values of 0.37 and 0.31 suggest moderate to substantial effects.

TABLE II. Analysis of responses to individual QMS items.

Quantum mechanics concept	No. of Items	Pretest (%)	Post-test (%)	Hake gain
Possible wave function	5	36	78	0.66
Bound and scattering states	5	27	57	0.41
Measurement	9	40	71	0.52
Expectation value	7	48	54	0.12
Stationary state/nonstationary state	8	30	56	0.37
Role of the Hamiltonian	3	25	48	0.31
Time dependence of the wave function	7	40	68	0.46
Uncertainty principle	1	30	75	0.64
Other	2	33	35	0.04
Average		35	70	0.54

However, other items outside the focus of the QuILTs intervention did not show significant changes in students' understanding of quantum mechanics concepts.

B. Analysis of conceptual changes in five areas of quantum mechanics

1. Possible wave functions

Analysis of responses to individual QMS items. There are five QMS items that assess the concept of possible wave functions. The pretest, post-test, and Hake gain values for each item are presented in Table III. The analysis shows that correctness rates improved across all items. Notably, the Hake gain values for Q1, Q14, and Q16 exceeded 0.83, indicating highly effective conceptual change.

Analysis of responses to QuILTs test items. Among the QuILTs test items, there are five questions addressing the concept of possible wave functions. The pretest and post-test accuracy rates for each question are presented in Table IV.

For Q1, Q2, and Q3, the post-test accuracy rates exceeded 80%. However, Q4 and Q5 showed either low post-test accuracy rates or little difference between the pre- and post-test results. For Q3, as shown in Table V, only

TABLE III. Analysis of QMS responses on possible wave functions.

Item contents	Pretest (%)	Post-test (%)	Hake gain
Q1 Possible wave functions for a given potential	60	95	0.88
Q30	20	70	0.63
Q6 Meaning of superposed wave functions	15	40	0.29
Q14	40	90	0.83
Q16 Criteria for valid wave functions	45	95	0.91

TABLE IV. Analysis of QuILTs assessment results for possible wave function items.

No.	Item contents	Pretest (%)	Post-test (%)
Q1	Possible wave functions for a given potential	80	100
Q2	Possible wave function conditions: continuity and boundaries	50	100
Q3	Possible wave function condition: Infinite (finite) potential well	10	80
Q4	Normalization of free particle wave functions	10	40
Q5	Plotting wave functions in infinite (finite) potential wells	65	63

10% of the preservice teachers selected the correct answer on the pretest. However, upon analyzing the reasoning behind their responses, it was found that even those who selected the correct answer did not base their choice on scientifically valid reasoning. The remaining 80% of the students considered the linear superposition of stationary state wave functions and boundary conditions, but failed to account for the continuity of the derivative at the boundary. Following the QuILTs sessions, over 80% of the students answered correctly in the post-test item addressing the finite potential well system. This suggests that the conceptual gaps regarding boundary conditions and derivative continuity observed in the pre-test were resolved after the QuILTs instruction. However, for those who still chose incorrect answers on the post-test, it was evident that while they understood the general conditions for possible wave functions, they struggled to apply these concepts accurately to the wave functions themselves.

The analysis of Q4 focused on students' understanding of the wave functions of a free particle. The pretest results, summarized in Table VI, revealed that while most students recognized that eigenstate wave functions for a free particle are not normalized, they struggled to apply this concept effectively. Post-test results following the implementation of QuILTs demonstrated an increase in correct responses, suggesting improved understanding. However, some misconceptions regarding normalization conditions persisted, underscoring the necessity of further instructional emphasis on this fundamental concept.

Q5 involved a qualitative analysis of wave functions drawn by students. The pretest results revealed that students found it more difficult to sketch possible wavefunctions for a finite potential well compared to an infinite potential well, and several types of misconceptions were identified, as shown in Table VII. Misconceptions regarding boundary conditions and the continuity of wave functions were prominent. For instance, 5% of students incorrectly believed that wave functions for a finite potential well could not extend outside the well, similar to an infinite potential well.

TABLE V. Analysis of QuILTs assessment results for Q3.

Response type	Pretest		Post-test	
	Percent (%)	Student responses	Percent (%)	Student responses
Correct, response correct	...	...	80	• The wave function and its derivative must be continuous across all regions. • In (I) and (II), the derivative is not continuous at $x = 0$, $x = a$. • In (IV), the wave function does not converge at infinity. • The wave function and its derivative are continuous and satisfy the boundary conditions.
Correct, response incorrect	10	t must satisfy the Schrödinger equation: $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi = E\Psi$ Thus, the wave function must either be of the form $\sin x$ or e^x	...	...
Incorrect, response incorrect	5	This is valid because the wave function can take the form $\sin \frac{n\pi x}{2}$	5	A wave function is valid if it is continuous and differentiable at $x = 0$, $x = a$.
	5	A time-independent wave function can only correspond to a single state.	5	The wave function is continuous, its derivative is continuous, and the boundary conditions are satisfied.
	80	• It is possible because it can be represented as a superposition of stationary states. • The wave function can be transformed into trigonometric functions using a Fourier transform. If the boundary conditions are satisfied and the derivative is continuous, it can be expressed as a linear combination of sine functions. • The boundary conditions are met, and the wave function must exist only within the well. Since it is a sine function, this is valid.	5	It can be normalized and expressed as a superposition of eigenfunctions.

Consequently, these students depicted wave functions that either terminated abruptly or failed to asymptotically approach zero at infinity. Approximately, 10% of students sketched wave functions for an infinite potential well that exhibited discontinuities in their derivatives at the boundaries. This reflects a misunderstanding of the appropriate boundary conditions governing such systems. Around 15% of students demonstrated misconceptions about normalization. These students sketched scattering state wave functions for a finite potential well without considering the normalization requirement and ignored the continuity of the derivative at when representing the scattering state. Such errors indicate difficulty in interpreting the graphical representation of potential wells. Singh *et al.* [25] reported that more than 60% of students struggle to distinguish between stationary and nonstationary states, with some incorrectly attributing time dependence to a decaying exponential. This study's findings align with such observations, revealing similar challenges among Korean preservice teachers. These parallels suggest that such misconceptions are widespread across

diverse educational contexts in quantum mechanics instruction. Addressing these persistent challenges requires targeted instructional interventions, particularly those emphasizing boundary conditions, normalization concepts, and the interpretation of potential well diagrams. Third, 15% of students displayed misconceptions about normalization conditions. They drew scattering state wave functions for the finite potential well while ignoring the requirement for normalization and neglected the continuity of the derivative at $x = 0$ when representing the scattering state. This indicates that students face challenges in interpreting the meaning of the potential well graph, consistent with previous findings that students often perceive the potential well as an external object [26].

After the QuILTs sessions, the post-test results showed that students had a better understanding of boundary conditions and the continuity of derivatives. Moreover, 90% of the preservice teachers were able to distinguish between the wave functions for finite and infinite potential wells. However, 55% of the preservice teachers still

TABLE VI. Analysis of QuILTs assessment results for Q4.

Response type	Pretest		Post-test	
	Percent (%)	Student responses	Percent (%)	Student responses
Correct, response correct	10	• Since it is not normalized, Ae^{ikx} alone is not a valid wave function. • It is not possible because the wave function is not normalized.	40	• Although it cannot be normalized, $\psi(x) = \int_{-\infty}^{+\infty} \phi(k)e^{-i(kx - \frac{Et}{\hbar})} dk$ can be expressed in this form. • It is not a valid wave function because it cannot be normalized. • It is invalid because it does not approach zero at infinity.
Incorrect, response incorrect	55	• It represents a traveling wave in free space. • While it cannot be normalized, it still represents a valid wave function for a wave oscillating in free space. • Despite the lack of normalization, it is a possible wave function because it is a solution derived from the corresponding equation, even though A cannot be determined without normalization.	40	• It is valid because it can be expressed as a superposition of eigenfunctions. • It is a valid wavefunction despite being non-normalizable. • It is valid because the wave function and its derivative are continuous. • It satisfies the boundary conditions and is valid as it is a smooth function.
No response	35	• It cannot be normalized. • I am unsure whether it is a valid wave function.	20	• It is not normalized, and the probability density is interpreted as being uniform across all space. • I am unsure whether it is a valid wave function.

TABLE VII. Analysis of QuILTs assessment results for Q5.

Response type	Pretest		Post-test	
	Percent (%)	Student responses	Percent (%)	Student Responses
Correct	40	Accurately depicted both the infinite and finite potential well wave functions.	45	Correctly depicted the wave functions and satisfied all conditions.
Partially correct	50	Only drew the wavefunction for the infinite potential well.	45	Accurately drew and clearly explained the differences between the wave functions for the infinite and finite potential wells.
	5	Only drew the wavefunction for the finite potential well.		
Incorrect	5	Failed to draw any of the wave functions.	10	Failed to explain the differences between the possible wave functions for an infinite and finite potential well and was unable to depict nonstationary states.

struggled with representing nonstationary states. Examples of student responses are shown in Fig. 1. The response on the left was classified as correct, as it clearly distinguished between the wave functions for finite (and infinite) potential wells and successfully represented the possible wave functions for a nonstationary state. On the other hand, the response on the right was classified as partially correct, as it failed to accurately depict a nonstationary state. These results suggest that while conceptual changes occurred regarding the conditions for possible wave functions, some

students still had difficulty representing nonstationary states as superpositions of wave functions. Therefore, even after the QuILTs sessions, a subset of preservice teachers continued to experience challenges with the normalization conditions and the representation of nonstationary wave functions.

Analysis of interview data from students. The following presents the results from individual interviews conducted with preservice teachers after the implementation of QuILTs.

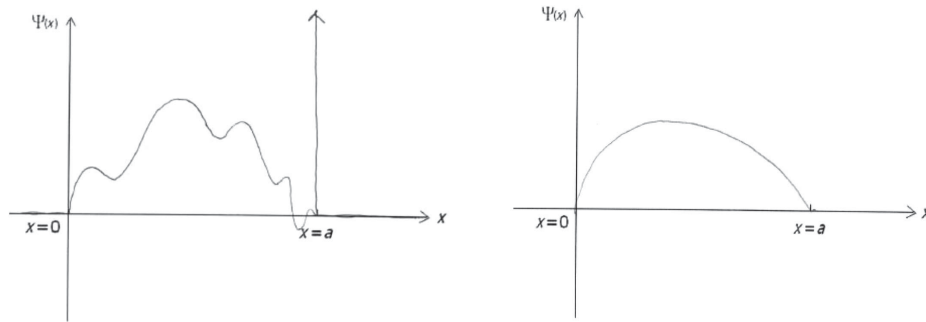


FIG. 1. Examples of responses to QuILTs Q5 on possible wave functions.

Preservice teacher A: I used to think only stationary states could represent valid wave functions, but the tutorial clarified that even nonperiodic superpositions can be valid.

Preservice teacher M: My understanding was superficial; I thought only the sine wave solution was valid. Learning about boundary conditions highlighted the importance of continuity.

Preservice teacher S: I misunderstood wave function conditions, believing discontinuities in the first derivative were acceptable. This study helped me correct that misconception.

Preservice teachers A and M exhibited the misconception that only specific forms of wave functions, such as stationary state wave functions or sine waves, were valid. However, through the learning process, they realized that possible wave functions can be constructed as superpositions of eigenfunctions. In the case of preservice teacher S, the initial belief was that a wave function could be valid even if the continuity condition of the derivative was not satisfied, based on the example of the delta function. However, their understanding shifted, reflecting a conceptual change regarding the importance of continuity conditions for valid wave functions.

Preservice teacher I: I learned that any wave function shape is possible if it's a linear combination of eigenstates that satisfies boundary conditions. This clarified the importance of eigenstates in understanding wave functions, a concept I previously found confusing.

Additionally, as seen with preservice teacher I, the preexisting concept of the conditions for possible wave functions became more refined. Preservice teacher I engaged in self-reflection, exploring how possible wave

functions can be constructed through the superposition of eigenfunctions, and addressing conditions such as continuity and the continuity of the derivative. This process of self-inquiry enabled the teacher to connect prior knowledge with new insights, thereby refining their understanding of wave function properties.

2. Bound and scattering states

Analysis of responses to individual QMS items. Among the QMS items, five questions assess the concept of bound and scattering states. The pretest, posttest, and Hake gain values for each item are shown in Table VIII.

The analysis revealed an improvement in accuracy across all items. Notably, the Hake gain values for Q19, Q24, and Q31 exceeded 0.44, indicating moderate to significant conceptual change. In particular, Q19 showed a notable increase in accuracy, reflecting substantial gains in understanding.

Analysis of responses to QuILTs test items. Among the QuILTs assessment items, ten questions address the concept of bound and scattering states in wave functions. The pretest and post-test accuracy rates for each item are shown in Table IX.

Post-test accuracy rates improved following the QuILT sessions. Analysis of Q1, Q2, and Q4 indicated that

TABLE VIII. Analysis of QMS responses on bound and scattering states.

No.	Item contents	Pretest (%)	Post-test (%)	Hake gain
Q18	Understanding of the wave function for bound and scattering states	35	45	0.15
Q19	Determination of whether a	20	55	0.44
Q31	given particle is in a bound or scattering state based on its position	40	80	0.67
Q24	Evaluation of whether a bound or scattering state is permitted for a given potential	15	70	0.65

TABLE IX. Analysis of QuILTs assessment results for bound and scattering items.

No.	Item contents	Pretest (%)	Post-test (%)
Q1	Distinguishing between classical and quantum states of particles	47	78
Q2	Indicating energy for bound and scattering states	72	95
Q3	Transmission and reflection of quantum particles in scattering states	56	80
Q4	Differentiating between bound and scattering states	50	90
Q5	Quantum tunneling effect	61	75
Q6	Directional preference of scattering state wave functions in symmetric potentials	22	63
Q7	Sketching the wave function in a finite potential well with different potentials on each side	39	73
Q8	Drawing the wave function for a scattering state	32	96
Q9	Drawing the scattering state wave function for a particle incident from $x = -\infty$	24	75
Q10	Sketching the wave function for a system with an infinite potential on one side	27	88

preservice teachers were initially confused about the distinction between bound and scattering states, with some mistakenly believing that both could coexist for a single energy eigenstate. After instruction, their responses reflected more scientifically accurate conceptions. The accuracy rate for Q6 also increased significantly, from 22% to 63%.

As shown in Table X, 50% of participants selected the correct answer on the pretest item related to wave function representation, yet only 11% provided scientifically valid reasoning. This suggests that while many could classify states based on particle energy, their understanding of the corresponding wave function was limited.

Prior research has shown that problems assuming a single direction of incidence may reinforce misconceptions about directionality in quantum systems [27]. Consistent with this, 22% of participants incorrectly believed that the direction of incidence determines the eigenfunction and energy eigenvalue.

To address this issue, the instructional materials incorporated examples featuring incidence from multiple directions and encouraged students to articulate their reasoning. After the QuILT sessions, 90% of participants answered the post-test item correctly, and 50% provided accurate explanations—indicating substantial conceptual improvement.

Despite these gains, the results suggest that some students continued to struggle with interpreting the form of wave functions in scattering scenarios, understanding directional symmetry, and distinguishing between bound and scattering states. These findings indicate that while overall conceptual understanding improved, certain aspects of quantum wave behavior remain challenging.

The analysis of four questions involving the drawing of wave functions for bound and scattering states revealed effective conceptual change across all items. Additionally, four types of misconceptions were identified. First, students struggled to understand how the relationship between particle energy and potential affects the shape of the wave function. Second, a misconception about the reference axis of the wave function was observed. When asked to draw the wave function for a scattering state, 88% of preservice teachers incorrectly shifted the vertical axis of the wave function instead of having it oscillate about the axis where $(\Psi(0) = 0)$, a finding consistent with Zhu and Singh [24]. Third, a misconception regarding wavelength was evident. Most students, with the exception of 33%, either depicted the wavelength as longer in the region $a < x < b$ or faced difficulties in reasoning about the wavelength. Fourth, there was a misconception about amplitude. None of the preservice teachers accurately represented the amplitude. Many assumed intuitively that the amplitude was larger in the region $a < x < b$ as seen in Fig. 2 or depicted the amplitude as uniform across different regions as in Fig. 3. This finding is consistent with prior research [27], which reported that students often believe increasing the width or depth of a potential well increases the particle's energy, and that the amplitude or wavelength of the wave function scales proportionally with the potential. Q9 focused on the wave function for a scattering state where the particle is incident from $x = -\infty$. The wave function should be asymmetric since the particle originates from the left side of the potential. On the pretest, only 6% of students correctly drew an asymmetric wave function, but this increased to 45% in the post-test. This suggests that students continued to struggle with the concept of directional preference in scattering state wave functions, similar to the findings from Q6. Q10 asked students to draw the wave function of a particle bound in a potential well with an infinite potential on one side. On the pretest, only 11% of students correctly depicted the wave function as exponentially decaying near $x = 0$ and correctly represented $\varphi(a)$ at $x = a$. However, 33% of preservice teachers incorrectly drew the wave function as existing at $x = a$, indicating a misunderstanding of the infinite potential.

Analysis of interview data from students. The following presents the individual interview responses of preservice teachers and the analysis results after the application of the QuILTs focused on the concept of bound and scattering states.

TABLE X. Analysis of QuILTs assessment results for Q6.

Response type	Pretest		Post-test	
	Percent (%)	Student responses	Percent (%)	Student responses
Correct with accurate explanation			50	• When the particle is incident from the left, the reflected wave results in a smaller amplitude on the transmitted right-hand side. However, if the direction of incidence is not specified, the wave function is symmetric. • In the case of a bound electron gaining energy in an atom, the wave function has no directional preference, and thus, symmetry is possible. If there is no directional preference, the wave function must be symmetric.
Correct with incomplete explanation	11	When calculating the transmission and reflection coefficients, the direction of incidence from the left or right must be considered. However, in other cases, the wave function can be drawn symmetrically.		
Correct with incorrect explanation	39	Incorrect drawing or lacking explanation.	40	• A reflected wave is present, typically making the wave function asymmetric, but at certain specific times, it can become symmetric. The incident and reflected waves can interfere to form a standing wave. • The drawing is incorrect.
Incorrect with incorrect explanation	22	• In a scattering state, an initial incident state is required. • A particle is incident from one side, resulting in both a reflected wave and a transmitted wave. • As the wave passes through the potential well, it loses energy, causing the amplitude of the transmitted wave function to decrease	10	When the wave is incident from one side, it is asymmetric. Both are possible.
	22	The wave function can be symmetric in some cases and asymmetric in others.		

Preservice teacher A: This was the most significant realization for me. I initially thought that as long as a particle is confined in a potential, it automatically becomes a bound state. However, I learned that the distinction between bound and scattering states is defined by the behavior of the wavefunction as $x \rightarrow \pm\infty$, which I wasn't aware of before.

Preservice teacher S: I was a bit confused about the cases when the potential energy is higher than the particle's total energy E . For instance, when tunneling occurs, or when the potential barrier is lower than the particle's

energy, the wave function oscillates. I had some difficulty understanding this, but I was able to clarify it through the lessons.

First, misconceptions held by preservice teachers regarding the definitions of bound and scattering states, as well as their differentiation based on potential, were addressed through the development of scientifically accurate concepts. Preservice teacher A initially misunderstood the identification of bound states, while preservice teacher S was unclear about the form of the wave function in scattering states. Both teachers demonstrated conceptual change following the intervention. Domert *et al.* [28] found that some students believed only the peaks of a wave above a barrier could pass through, indicating that a limited

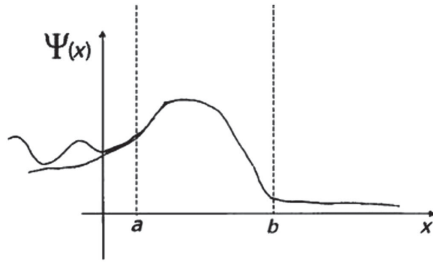


FIG. 2. Misconceptions about the wavefunction axis, wavelength, and amplitude.

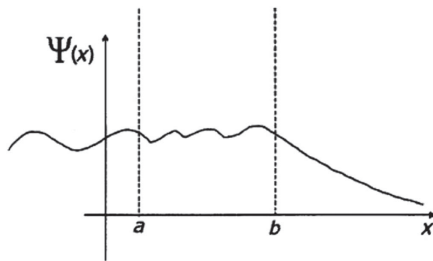


FIG. 3. Misconceptions about the wavefunction axis and amplitude.

understanding of probability can hinder proper comprehension of scattering and tunneling. Similar misconceptions were observed among students in this study.

Pre-service Teacher I: Drawing concepts like frequency and wavelength helped me understand the abstract nature of wave functions. It clarified the link between wavelength and momentum in de Broglie waves and how to interpret probability density.

Pre-service Teacher S: The semiclassical explanation of wave functions, relating k and momentum, clarified how particle probability varies with k value. After this, I corrected my initial drawing based on the scientific concept.

Preservice teacher Q: I hadn't drawn wave functions before, as I was focused on equations. Initially, I misplaced the graph above the axis, but later learned how wave function shape varies with energy and potential differences.

Second, preservice teachers were able to represent bound and scattering state wave functions through the qualitative process of drawing wave functions, which allowed them to correct their misconceptions and develop scientifically accurate concepts. While the teachers were generally familiar with the mathematical expressions of wave

functions, they found the activity of drawing wave functions unfamiliar, and several misconceptions emerged. However, through the QuILTs sessions, their understanding of key wave function concepts—such as shape, wavelength, and amplitude—was refined and clarified.

Preservice Teacher Q: I understood how energy determines bound or scattering states in a potential well, but struggled with whether the electron “must be emitted” and how to draw the wave function without emission. This uncertainty made it challenging, especially in understanding directional preference.

Third, there was ongoing confusion regarding the concept of directional preference in bound and scattering states. Although preservice teacher Q engaged with the QuILTs lessons on directional preference, they continued to struggle with understanding and accurately representing this concept. This suggests that more explicit and focused instruction is needed to address the directional preference in bound and scattering states.

Preservice teacher I: Simulations were the most helpful. They allowed me to independently explore and verify solutions to questions beyond the examples in textbooks, especially for bound and scattering states in complex wells.

Preservice teacher T: I knew the equations for bound and scattering states but didn't fully grasp their graphical representations. The simulations helped me see the actual shapes of these wave functions, enhancing my understanding.

Finally, as highlighted in the interviews with preservice teachers I and T, many participants expressed positive opinions regarding the benefits of using simulations. By comparing their predictions of the wave functions for bound and scattering states in a given system with the results from simulations, the preservice teachers experienced cognitive conflict, which led to conceptual revisions. Additionally, as seen in the case of preservice teacher T, interactive simulations allowed them to manipulate conditions directly and observe the resulting wave functions, which enhanced their intuitive understanding and appeared to be an effective tool in fostering conceptual change.

3. Time evolution of wave functions

Analysis of responses to individual QMS items. Among the QMS items, seven questions assess the concept of time

TABLE XI. Analysis of QMS responses on time evolution of wave functions.

No.	Item contents	Pretest (%)	Post-test (%)	Hake gain
Q3	Probability density at time t	75	95	0.80
Q4		30	70	0.57
Q6	Relationship between wave functions in a superposition of energy eigenstates and the Schrödinger equation	15	40	0.29
Q15	Time evolution of the wave function following a sudden change in potential energy	20	50	0.38
Q17	Understanding the time	45	70	0.45
Q22	dependence of position eigenstates in relation to measurement and experimentally estimating possible energy probability distributions	70	95	0.83
Q29	Relevance of the Hamiltonian's role in the time dependence of the wave function	25	55	0.40

evolution of the wave function. The pretest, post-test, and Hake gain values for each item are presented in Table XI.

The analysis of Q3 showed that preservice teachers who initially struggled with the concept developed a better understanding of how to represent the probability density of a stationary state at time t through the QuILTs sessions. Consistent with prior research [10,29], many held the misconception that probability density remains constant over time. After instruction, their understanding of the time-dependent Schrödinger equation and its implications for nonstationary states improved. Additionally, the analysis of Q4 demonstrated that the preservice teachers corrected their misconceptions regarding the time dependence of the probability density when a wave function is given as a superposition of eigenfunctions, shifting toward a more scientifically accurate concept. However, some students still believed that a single measurement determines the probability density, suggesting that further instructional strategies are needed to address this persistent misconception.

The pretest analysis of Q6 revealed that 85% of preservice teachers held the misconception that the superposed wave function $\Psi(x, 0) = \sum_n A_n \psi(x)$ satisfies the time-independent Schrödinger equation. Although the correct response rate improved to 40% after the QuILTs sessions, many participants continued to struggle with understanding the physical meaning of the wave function and its relationship to the Schrödinger equation. Similarly, analysis of Q29 showed that 45% of participants failed to recognize that the Hamiltonian governs the time evolution

of a quantum system. This finding echoes results from previous studies [10,25,29–31], which have consistently reported that many students mistakenly believe that all wave functions must satisfy the time-independent Schrödinger equation, regardless of context—a robust and persistent misconception. A closer examination of Korean preservice teachers' responses revealed that 50% believed the Hamiltonian determines the form of the position eigenfunction. However, following instruction with QuILTs, conceptual improvement was observed in some participants. This trend is consistent with earlier findings [11], which suggest that such misconceptions stem from a fundamental misunderstanding that the eigenstates of physically observable operators are necessarily stationary states.

Analysis of responses to QuILTs test items. The QuILTs assessment includes seven items related to the time evolution of the wave function. Table XII presents the pre- and post-test accuracy rates for each item.

The analysis of the QuILTs assessment results revealed an increase in accuracy across all items, with significant conceptual change observed in Q1 and Q2. The low pretest accuracy rates for Q3, Q4, and Q6 indicated the presence of misconceptions about the wave function among preservice teachers. Specifically, only 11% of preservice teachers answered Q3 correctly, and 17% answered Q4 correctly, suggesting difficulty in understanding the time-dependent expression of possible wave functions in the superposition of stationary states. For Q3 and Q4, when the initial wave function is given in a superposition, preservice teachers encountered two main challenges. First, they struggled with the mathematical representation and coefficients. In the pretest, 39% of preservice teachers recognized that the initial wave function must be decomposed into stationary state forms with phase factors, but they could not clearly express the coefficients. Second, 28% of preservice teachers mistakenly applied a single phase factor to the entire initial wave function to represent its time dependence. While this misconception was largely corrected after the QuILTs sessions, 53% of preservice teachers still failed to clearly describe the coefficients in the post-test. Q5 and Q7

TABLE XII. Analysis of QuILTs assessment results for time evolution of wave functions.

No.	Item contents	Pretest (%)	Post-test (%)
Q1	Infinite square potential well $\Psi(x, t)$	72	89
Q2		72	100
Q3		11	42
Q4		17	58
Q5	Simple harmonic oscillator potential $\Psi(x, t)$	50	74
Q6		11	42
Q7		50	74

TABLE XIII. Analysis of QuILTs assessment results for Q6.

Response types	Response content	Pretest (%)	Post-test (%)
Correct	Correctly expressed $\Psi(x, t)$	11	42
Incorrect	Expressed with a single phase factor	44	11
	Mathematical expression error	6	31
	No response	39	16

assessed conceptual change in understanding the wave function for a simple harmonic oscillator potential well. Compared to Q1, where the quantum system involved an infinite potential well, Q5 showed that 33% of preservice teachers struggled to describe the stationary state wave function accurately when the potential was switched to that of a simple harmonic oscillator. This suggests that the representation of the wave function in the harmonic oscillator potential was unfamiliar to many preservice teachers. Q6 further revealed misconceptions about the wave function in a simple harmonic oscillator potential well. Table XIII categorizes the responses of preservice teachers for Q6.

In the pretest, 44% of preservice teachers demonstrated the misconception of expressing $\Psi(x, t)$ with a single phase factor, with only 11% responding correctly. This misconception likely stems from the belief that the given wave function in Q6 represents a stationary state.

Figures 4 and 5 illustrate examples of conceptual change in response to Q6. Preservice teacher G's pretest response showed an interpretation of the initial wave function as a ground state wave function in a harmonic oscillator potential, of the form $\Psi(x) = Ae^{-\alpha x^2}$, with a horizontal shift. The preservice teacher mistakenly believed it to be identical to the ground state wave function. In the pretest, 33% of preservice teachers held the same misconception. Following the QuILTs sessions, most preservice teachers (except for 11%) came to understand that the given initial wave function is constructed as a superposition

$$\Psi(x,t) = Ae^{-\alpha(x-x_0)^2} e^{-iEt/\hbar} \quad \text{Traslation.}$$

$x_0 = \frac{\hbar}{2m\omega}$

FIG. 4. Pre-test responses to Q6.

$$\Psi(x,t) = \sum_{n=0}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}$$

$$c_n = \int \psi_n^*(x) A e^{-\alpha(x-x_0)^2} dx$$

FIG. 5. Post-test responses to Q6.

of stationary state wave functions. However, 31% still struggled with the correct mathematical expression of $\Psi(x, t)$, indicating that more time and practice are required to fully internalize the scientific concept.

Analysis of interview data from students. The individual interview responses and analysis of preservice teachers' understanding of the concept of time evolution in wave functions revealed that they gained a deeper understanding of the meaning of the time-dependent Schrödinger equation and the expression of time evolution in wave functions. Additionally, a conceptual shift regarding the time evolution of wave functions was observed.

Preservice teacher A: I initially thought adding $e^{-iEt/\hbar}$, always yielded a stationary state, but now I understand it sometimes requires different expressions for time evolution.

Preservice teacher I: I used to just attach $e^{-iEt/\hbar}$ without fully understanding its application in time evolution. I realized it must be applied to each eigenfunction for accuracy.

Pre-service teacher M: I didn't grasp that $e^{-iEt/\hbar}$ should be added to each stationary state individually. I was accustomed to time-independent solutions and didn't connect them to time dependence until realizing that combining stationary states requires the time term for evolving wave functions.

Like preservice teachers A and M, the majority of participants corrected the misconception that all wave functions possess a single-phase factor. Furthermore, preservice teachers, such as teacher I, refined their understanding of the scientific concepts they had previously learned and expressed difficulties related to the concept of time evolution, which were identified through the interviews.

Pre-service Teacher G: I still don't think I fully understand time evolution. It's hard to explain clearly, but I feel confused every time I try to approach it. Whether it's time-independent or time-dependent, or how exactly it changes... I don't think I answered correctly on the post-test either.

Preservice teacher J: At the beginning, it was okay, but later it got a bit difficult. At first, there wasn't much to think about, but as it progressed, there were more factors to consider, and that made it confusing. I'd say I

- understood about 70%. When I first encountered it, my understanding was probably below 50%.
- Pre-service teacher K: I made a lot of mistakes on the post-test. That's why I said I needed more time to process. I wanted to organize my thoughts, but since I hadn't fully done that, my answers ended up being all over the place.
- Preservice teacher Q: It's hard for me to understand mathematically. The number of equations increased a lot, and I still don't really understand why the sum and integral overlap in certain cases.

Preservice teachers demonstrated only a superficial understanding of the time evolution of the wave function, with responses from preservice teachers G and Q indicating ongoing confusion regarding its meaning or difficulty with mathematical representation. Additionally, as seen in the responses from preservice teachers J and K, there were individual differences in understanding achieved through QuILTs instruction. This suggests that prior knowledge of quantum mechanics concepts is essential for grasping the time evolution of the wave function and that individual learning support may be necessary, especially given difficulties with concepts such as wave function superposition [16]. Consistent with previous studies [29,30], the findings of this study also suggest that many preservice teachers held the misconception that a superposition of eigenstates would eventually reduce to a single eigenstate or could be represented by a single phase factor. While the QuILT sessions facilitated some degree of conceptual change, their understanding of the dynamic evolution of time-dependent quantum states remained limited.

4. Quantum measurement

Analysis of responses to individual QMS items. The QMS includes nine items designed to evaluate the concept of quantum measurement, with pretest, post-test results, and Hake gain values summarized in Table XIV.

The analysis of these items revealed the following: First, significant improvements were observed in conceptual understanding related to the probability of finding a particle at an arbitrary position and the probability of measuring specific energy levels. For instance, Q5 examines the probability of finding a particle between x and $x + dx$ where 25% of preservice teachers responded using $\int_x^{x+dx} x |\psi_1(x)|^2 dx$. This indicates a misconception that an operator corresponding to an observable must be explicitly included in the probability expression. The same misconception was evident in Q11, which asks about the probability of measuring energy En . Here, 40% of preservice

TABLE XIV. Analysis of QMS responses on quantum measurement.

No.	Item contents	Pretest (%)	Post-test (%)	Hake gain
Q5	Probability of finding a particle at an arbitrary position	45	85	0.73
Q7	Energy measurement result for a superposition state	40	95	0.92
Q8		70	95	0.83
Q22		70	95	0.83
Q11	Probability of measuring a specific energy at an arbitrary position	35	80	0.69
Q13	Continuous measurement	20	35	0.19
Q17	State of the wave function after position measurement	45	70	0.45
Q20	Probability of locating a particle in a simple harmonic oscillator potential system	30	40	0.14
Q28	Distinguishing between stationary states and eigenstates of other physical observables	5	45	0.42

teachers responded with $|\int_0^a \psi_n^*(x) \hat{H} \Psi(x, 0) dx|^2$ failing to recognize that the energy measurement probability should be calculated by projecting the wave function onto the corresponding energy eigenstate, i.e., $|\int_0^a \psi_n^*(x) \hat{H} \Psi(x, 0) dx|^2$. Instead, they incorrectly assumed that the Hamiltonian operator must appear explicitly in the probability expression.

Following QuILTs instruction, these misconceptions were addressed and transformed into scientifically accurate concepts. Both Q5 and Q11 showed post-test accuracy rates exceeding 80%, demonstrating the effectiveness of QuILTs in fostering conceptual change. Second, significant conceptual improvements were observed regarding the post-measurement state of the wave function and the measurable energy values. Results from Q7 and Q22 revealed that preservice teachers held the misconception that the wave function or energy would revert to its initial state after collapse and a long passage of time. In Q8, many preservice teachers incorrectly believed that all possible energy values En , beyond the measurable values, could also be obtained. After QuILTs instruction, these misconceptions were corrected, with post-test accuracy rates exceeding 95%. Third, conceptual changes regarding the wave function state following position measurement were also observed, though to a lesser extent. Analysis of Q17 showed that preservice teachers often believed the wave function would revert to its initial state over time after a position measurement or failed to understand that the position eigenstate is represented as a delta function. While these misconceptions were reduced after QuILTs instruction, 20% of preservice teachers still believed the wave function would return to its initial state over time, indicating that understanding position measurement concepts remains more

challenging compared to energy measurement. Fourth, preservice teachers demonstrated confusion regarding the probability of finding a particle in a harmonic oscillator potential. Analysis of Q20 revealed that preservice teachers held the misconception that a particle cannot exist in the $E < V(x)$ region, with low levels of understanding both before and after QuILTs instruction. Zhu and Singh [11] attributed this misconception to students' prior experiences with the infinite potential well. Additional contributing factors include a lack of understanding of quantum tunneling and difficulty in transferring concepts between classical and quantum frameworks, as observed in recent undergraduate studies [27]. Post-test results indicated that approximately 20% of preservice teachers misunderstood the correspondence principle, believing that for high-energy stationary states, the probability peaks near the classical turning points, while for the ground state, the probability is highest at the center. Additionally, preservice teachers exhibited limited understanding of wave functions in potentials beyond infinite and finite wells. They also struggled to connect the shape of a wave function with the probability of particle localization, reflecting difficulties in reasoning about the relationship between wave function form and measurement probabilities. Fifth, preservice teachers demonstrated difficulties in understanding the concept of sequential measurements. Q13 assesses the possible energy values that can be measured immediately after determining the position of a particle initially described by a wave function in the superposition of the ground state and the first excited state. In the pretest, only 30% of preservice teachers correctly responded that one of the energy eigenvalues (E_n) could be measured, indicating an understanding that after a position measurement, the wave function collapses into a delta function, which is a superposition of multiple energy eigenstates. However, 40% of preservice teachers incorrectly believed that only the initial wave function's energy eigenvalues (E_1 and E_2) could be measured, suggesting a failure to comprehend that the wave function collapses into a position eigenstate after a position measurement. After QuILTs instruction, the accuracy rate increased to 35%, but 30% of preservice teachers still maintained the misconception that only the energy eigenvalues associated with the initial wave function could be measured. Additionally, while some preservice teachers understood that the wave function collapses into a position eigenstate, a new misconception emerged postinstruction: that any energy value in $E_n = \sum_{n=1}^{\infty} c_n E_n$ could be measured. This indicates that confusion about the concept of sequential measurements persists. Results from Q28 revealed that many preservice teachers held the misconception that the eigenstates of a measurable observable correspond to stationary states. While QuILTs instruction led to some improvement, 50% of preservice teachers continued to believe that the eigenstates of a measurable observable are equivalent to stationary states. This finding

suggests that the distinction between eigenstates and stationary states was not fully established.

The combined results from QMS suggest significant conceptual changes in the understanding of measurement probability expressions, postmeasurement wave function states, and possible measurement outcomes. While most preservice teachers refined their understanding of energy measurement concepts, their understanding of position measurement concepts remained relatively lower, despite some improvement. In the case of sequential measurements, many preservice teachers continued to struggle, with approximately half failing to distinguish between the eigenstates of a measurable observable and stationary states.

Analysis of responses to QuILTs test items. The QuILTs assessment included six items focused on the concept of quantum measurement, with pretest and post-test accuracy rates for each item presented in Table XV.

In the pretest for Q1, only 17% of preservice teachers correctly responded that the position, energy, and momentum of a particle described by an initial wave function are not well defined. After QuILTs instruction, the post-test accuracy increased to 26%. A detailed analysis revealed that while 89% of preservice teachers selected the correct answer, they were unable to provide scientifically accurate explanations. This indicates that preservice teachers struggled to clearly distinguish between stationary states and the eigenstates of physical observables, reflecting a persistent misconception. The analysis of Q2 and Q4 revealed conceptual changes regarding the possible energy values during energy measurement. In the Q2 pretest, 67% of preservice teachers responded correctly. After QuILTs instruction, 95% of preservice teachers provided scientifically accurate explanations for the superposition state in an infinite square potential well in the Q4 post-test. Similar findings were reported in international research [13], where a high proportion of students, similar to Korean preservice physics teachers, accurately determined energy values. However, some students struggled with energy calculations

TABLE XV. Analysis of QuILTs assessment results for quantum measurement.

No.	Item contents	Pretest (%)	Post-test (%)
Q1	Eigenstates of physical observables and stationary states	17	26
Q2	Energy measurement process	67	79
Q3	Qualitative sketch of the wave function post-measurement	44	95
Q4	Probability calculation for energy measurement	50	95
Q5	Consecutive measurements (position followed by position)	31	68
Q6	Consecutive measurements (energy followed by energy/position)	56	61

due to their inability to utilize the given time information effectively. The post-test results for Q2 and Q3 following QuILTs instruction demonstrate that preservice teachers refined their understanding of the wave function after quantum measurement. In the Q2 post-test, 79% of preservice teachers correctly understood that the wave function collapses into a stationary state after an energy measurement. In the Q3 post-test, 95% of preservice teachers accurately described that immediately after measuring energy E_2 , the system's wave function collapses into the eigenstate $\psi_2(x)$, and they were able to represent this concept through illustrations. These results indicate that most preservice teachers developed a scientifically accurate understanding of the wave function's state following energy measurement. Additionally, conceptual changes regarding the wave function's state after position measurement were also observed. Through the analysis of Q4 and Q6, shifts in preservice teachers' conceptual understanding regarding the calculation of measurement probabilities for physical observables and their implications were identified. In the pretest for Q4, 50% of preservice teachers utilized the projection method to measure energy E_2 in the $\psi_2(x)$ state. However, the remaining respondents encountered difficulties in applying this approach. Following the implementation of the QuILTs instructional strategy, the Q4 post-test revealed that 63% of preservice teachers successfully employed the projection method to calculate the probability of position measurement, indicating a conceptual shift. Additionally, the post-test results for Q6 demonstrated that

95% of preservice teachers acquired the understanding that the wave function collapses into an energy eigenfunction after an energy measurement and that these energy eigenfunctions correspond to stationary states. Furthermore, the Q5 post-test showed that 58% of preservice teachers comprehended the scientific concept that the wave function's form and position eigenfunctions are time-dependent after a position measurement followed by an elapsed time before a subsequent measurement. However, 42% of the respondents remained unable to clearly articulate the reasoning related to consecutive position measurements, indicating persistent misconceptions or confusion in this area. The results of Q5 and Q6 effectively highlight misconceptions and conceptual shifts regarding sequential measurements. In the Q5 pretest, as shown in Table XVI, 45% of preservice teachers responded correctly, but only 6% provided accurate explanations for their answers. Additionally, 11% offered incomplete explanations, while 28% either omitted explanations or provided incorrect ones. Furthermore, 55% of preservice teachers held the misconception that the wave function collapses into a fixed position after a position measurement. Q6, which asked whether two energy measurements at different times yield identical results, revealed in the pretest that only 44% of preservice teachers answered correctly. Moreover, 66% of preservice teachers demonstrated consistent but incorrect reasoning across Q5 and Q6, indicating a lack of understanding of the differences between energy eigenstates and position eigenstates. Following the implementation of

TABLE XVI. Analysis of QuILTs assessment results for Q5.

Response type	Pretest		Post-test	
	Percent (%)	Student responses	Percent (%)	Student responses
Correct	6	The delta-shaped wave function spreads gradually over time	58	- When a measurement is made, the position function becomes a delta function, which can be represented as a linear superposition of energy eigenfunctions. As time passes, the wave function changes, and the probabilities alter. - After collapsing to the position eigenstate, the wave function evolves over time.
Correct, explanation insufficient	11	If we wait longer, the result will not be the same because the state has changed	...	...
Correct, response Incorrect	28	Since position is probabilistic, the result cannot be the same. The Dirac delta function does not spread over time	21	No explanation provided
Incorrect	55	- Measuring the particle's position collapses the wave function, so the result will be the same as before. - At the moment of measurement, the wave function collapses, so only a specific value can be measured	21	- The probability distribution inside the well remains uniform. - The probability p is given by $p = \psi_2(x)\psi_2(x)dx$, and it remains constant.

QuILTs instruction, the Q6 post-test results showed that 95% of preservice teachers understood that after an energy measurement, the system collapses into an energy eigenstate and that energy eigenstates correspond to stationary states. In the Q5 post-test, 58% of preservice teachers demonstrated a scientific understanding that, after a position measurement, the wave function and position eigenstates are time-dependent. However, 42% of preservice teachers still struggled to articulate a clear explanation or experienced confusion regarding consecutive position measurements, suggesting persistent conceptual challenges in this area.

Through QuILTs, preservice teachers demonstrated conceptual shifts regarding the possible energy values during energy measurements and the probability of measuring a specific energy. They also developed the ability to calculate position measurement probabilities using the projection method. Furthermore, conceptual changes were observed in their understanding of the wave function's state after measurements. Nearly, all preservice teachers scientifically understood the wave function's state after an energy measurement, and many could also scientifically explain the wave function's state after a position measurement. Regarding sequential measurements, many preservice teachers initially struggled to distinguish between the scenarios of measuring energy twice and measuring position twice. However, QuILTs instruction helped improve their understanding to a more scientific level. Nevertheless, some preservice teachers continued to face difficulties, either failing to clearly explain the results of sequential measurements or experiencing confusion.

Analysis of interview data from students. The following presents the content of individual interviews with preservice teachers and the analysis results conducted after the QuILTs instruction on the concept of quantum measurement via the wave function.

Preservice teacher M: I didn't fully understand what a stationary state was. I wondered if every operator has its own stationary state or if the Dirac delta function could be one. I even asked my professor about it.

Preservice teacher Q: I thought that once you measured something, like energy or position, the wavefunction would stay fixed at that value. But now I realize energy stays fixed, while position spreads out again after measurement. That was new to me.

First, conceptual changes regarding stationary states and the wave function's state after measurement were observed, along with an improved understanding of sequential measurements. Preservice teacher M refined their understanding by distinguishing between eigenstates of operators

and stationary states. Preservice teacher Q initially held the misconception that the measurement value and wave function remain fixed after measurement but was able to explain this concept more accurately through scenarios involving energy and position measurements. Second, the use of simulations proved highly effective in facilitating the understanding of measurement concepts. Most preservice teachers experienced general confusion about the concept of measurement prior to the QuILTs instruction. However, interviews with preservice teachers C and S revealed that the visual representation provided by simulations greatly contributed to their conceptual understanding.

Preservice teacher C: I had never studied the measurement process separately, and textbooks didn't specifically cover it either, so I didn't think much about it. I only understood the concept of collapse but couldn't calculate the wave function's form. Seeing the simulation was really helpful.

Preservice teacher S: When a position is measured, the wave function collapses and then evolves. Watching it happen on the computer was fascinating. I used to think it just collapsed, but seeing the wave spread out made it much clearer.

Third, the challenges in learning measurement concepts were attributed to difficulties in representation and mathematical expressions, as well as the lack of prior understanding of foundational concepts such as time evolution.

Preservice teacher G: I think I only understood about 30%–40% of the measurement concepts. I'm familiar with Dirac notation, but I don't fully understand it, especially when it comes to time evolution and measurement.

Preservice teacher I: Dirac notation appeared a lot, and I was familiar with it from previous quantum mechanics and linear algebra courses. But for those without that background, it might have been too little time to follow. Without prior review, it would have been hard to keep up with just the class alone.

Preservice Teacher J: I'm weak in mathematical expressions, so I struggled at first and still feel like I only understood about 60%. The integral forms and Dirac notation were especially challenging for me.

TABLE XVII. Analysis of QMS responses on uncertainty principle.

No.	Pretest (%)	Post-test (%)	Hake Gain
Q21	30	75	0.64

5. Uncertainty principle

Analysis of responses to individual QMS items. Among the QMS items, only one question addresses the concept of the uncertainty principle. The pretest, post-test results, and Hake gain values are presented in Table XVII.

According to Zhu and Singh [11], students often hold the misconception that the uncertainties in position or momentum are related to their expectation values. This misconception was similarly observed in the Q21 assessment, where 70% of preservice teachers demonstrated misunderstandings regarding the definition and meaning of uncertainty in the pretest. After the QuILTs instruction, 75% of preservice teachers exhibited a conceptual shift toward a scientific understanding, indicating they recognized uncertainty as a physical quantity related to standard deviation rather than expectation values. However, 20% of preservice teachers were still confused about the relationship between the expectation values of position and momentum with the uncertainty principle.

Analysis of responses to QuILTs test items. In this study, part 1 of the QuILTs on the concept of the uncertainty principle for wave functions, developed by Singh [31], was implemented in the instruction. Part 1 specifically addresses the uncertainty principle as applied to position and momentum. There are a total of four items related to the uncertainty principle of wave functions, and the pretest and post-test accuracy rates for each item are presented in Table XVIII.

In the pretest for Q1, 17% of preservice teachers correctly identified both the momentum and position for a free particle with a wave function e^{ikx} at $t = 0$. Additionally, 42% described that the momentum is well defined while the position is not, providing logical reasoning for their response. Another 5% selected the correct answer but failed to explain their reasoning, whereas 53%

TABLE XVIII. Analysis of QuILTs assessment results for uncertainty principle.

No.	Item contents	Pretest (%)	Post-test (%)
Q1	Definition of position and momentum	17	26
Q2	Momentum uncertainty through the wave function in position space	67	79
Q3	Position uncertainty through the wave function in momentum space	31	68
Q4	Measurement values and uncertainty	56	61

demonstrated an understanding of the uncertainty principle but were unable to apply it to the given problem. After the implementation of QuILTs, 75% of preservice teachers correctly identified that for a wave packet described by a delta function ($\delta(x)$), the position is well defined while the momentum is not, providing logical explanations. An additional 20% selected the correct answer but could not explain their reasoning, and 5% did not respond. These results suggest that most preservice teachers gained a clearer understanding of the position-momentum relationship and uncertainty in the given system, reflecting an effective conceptual shift. Q2 involves comparing momentum uncertainties based on a given wave function in position space. The pretest results revealed that 47% of preservice teachers selected the correct answer and provided scientifically valid reasoning. However, 21% chose the correct answer but gave inappropriate reasoning, while 32% selected incorrect answers. A detailed analysis of the pretest results shows that 11% of preservice teachers understood the shape of the position-space wave function but failed to connect it to the momentum uncertainty. Additionally, 5% attempted to explain the uncertainty using probability concepts, and 11% were unable to interpret the position-space wave function. Another 5% did not respond to the question. The post-test results showed an improvement, with 45% of preservice teachers selecting the correct answer. However, some misconceptions persisted. Among those who answered incorrectly, 15% demonstrated misconceptions about uncertainty, and 10% attempted to explain uncertainty using the expectation value. Moreover, 5% incorrectly stated that “greater position uncertainty leads to greater momentum uncertainty.” On the other hand, 35% of preservice teachers misunderstood the wave function’s shape but provided reasoning aligned with the uncertainty principle in terms of the relationship between position and momentum. The results for Q2 indicate that approximately 80% of preservice teachers had some level of understanding of the uncertainty principle. However, many struggled to analyze the wave function in position space and infer the corresponding momentum uncertainty.

As shown in Table XIX, the analysis of Q3 revealed that 58% of preservice teachers selected the correct answer and provided scientifically valid explanations. Following the implementation of QuILTs, the post-test results indicated that 50% of preservice teachers were able to correctly describe the solution using appropriate scientific concepts when given the momentum-space wave function. However, 25% responded that the position uncertainty could not be determined without knowing the Hamiltonian, but either omitted explanations or demonstrated misconceptions by attempting to use the expectation value in their reasoning. This suggests a lack of understanding of the Hamiltonian’s role and its relationship to temporal evolution. These results indicate that while 50% of preservice teachers could

TABLE XIX. Analysis of QuILTs assessment results for Q3.

Response type	Pretest		Post-test	
	Percent (%)	Student responses	Percent (%)	Student responses
Correct	58	• The uncertainty principle ($\sigma_x \sigma_p = \frac{\hbar}{2}$) can be used to determine the uncertainty. The position-space wavefunction can be obtained through Fourier transformation, allowing for the calculation of position uncertainty. • The momentum uncertainty can be determined from the momentum-space wavefunction, and the position uncertainty can then be calculated using the uncertainty principle.	50	• At $t = 0$, the position and momentum uncertainties can be determined through inverse Fourier transformation. However, for $t > 0$, the wavefunctions for position and momentum evolve over time, requiring knowledge of the Hamiltonian. • To determine the position uncertainty at $t > 0$, the time-dependent Schrödinger equation must be solved to obtain $\psi(x, t)$. • Without knowing H, the wave function cannot be determined, making it impossible to calculate the uncertainty
Correct with missing or incorrect explanation	11	No explanation provided.	25	• $(d\langle Q \rangle)/dt = i/\hbar \langle [H, Q] \rangle$ knowledge of H is necessary. • The time dependence of expectation values is governed by H, making it impossible to determine uncertainties for $t > 0$ without knowledge of H.
Incorrect with incorrect explanation	5	It cannot be determined because only the minimum value can be calculated.	20	• At $t = 0$, the position-space wave function can be determined. Knowing this, it can be expressed as a sum of eigenfunctions, allowing the position uncertainty to be calculated. • If the momentum-space wavefunction is known, the position-space wave function can be determined. Additionally, knowledge of k provides information to determine the energy eigenfunctions.

accurately describe the relationship between momentum and position uncertainty, the remaining half experienced confusion regarding the Hamiltonian's role in this context. Additionally, some preservice teachers continued to exhibit misconceptions by attempting to connect uncertainty with expectation values.

The pretest results for Q4 indicate that 27% of preservice teachers selected the correct answer. However, 11% provided insufficient explanations, and 16% either did not provide an explanation or gave inappropriate reasoning. Furthermore, 63% of preservice teachers failed to distinguish between measurement values and uncertainties. After the implementation of QuILTs, 40% of preservice teachers selected the correct answer in the post-test, but only 20% provided scientifically accurate reasoning. Common misconceptions included the belief that measuring a high velocity implies a determined velocity with low momentum uncertainty, or conversely, that high velocity corresponds to high momentum uncertainty. Additionally, some preservice teachers relied on experiential perspectives, reasoning that higher velocities make positions harder to observe, and incorrectly associated this with uncertainty. Another

misconception involved attempts to explain uncertainty using de Broglie wavelengths. These results indicate that a majority of preservice teachers still struggle with understanding the concept of uncertainty. While the QuILTs post-test results suggest that preservice teachers could describe the relationship between position and momentum given a wave function and explain the uncertainty principle to some extent, many experienced difficulties in reasoning about wave functions in relation to comparisons and interpretations. Although some preservice teachers demonstrated an understanding of the relationship between wave functions in position and momentum space, they still struggled with the role of the Hamiltonian in time-evolving systems. Moreover, many preservice teachers continued to exhibit misconceptions about uncertainty, with some interpreting uncertainty empirically or attempting to explain it using concepts like probability, expectation values, measurement outcomes, or de Broglie waves.

Analysis of interview data from students. Interviews with preservice teachers revealed that initial misconceptions about the uncertainty principle were corrected to align with scientific understanding.

Preservice teacher Q: I knew that a larger expectation value shifts the graph to the right, but I couldn't distinguish how it becomes narrower or broader (uncertainty). I learned that through the tutorial.

An interview with preservice teacher Q revealed that they understood uncertainty in terms of measurable values or expectation values rather than as the standard deviation of the wave function.

Preservice teacher E: I only knew the basics of Fourier transform, but seeing the graphs helped me understand the uncertainty between x and p .

Preservice teacher S: I thought I understood Fourier transform, but I realized through the post-test that knowing the Hamiltonian is key to describing time evolution.

Second, the concept was refined through direct observation of Fourier transforms. For instance, as seen in the interview with preservice teacher E, the use of simulations to study Fourier transforms allowed them to learn how changes in position-space and momentum-space wave functions relate to the uncertainty principle. Similarly, preservice teacher S demonstrated improved understanding of the relationship between position and momentum wave functions under time evolution. Preservice teachers recognized that momentum uncertainty could be inferred from the position-space wave function alone, while determining precise values under time evolution required knowledge of the system's Hamiltonian.

In addition to the conceptual changes observed in specific quantum mechanics topics, several participants reflected on how the instructional design of the QuILTs influenced their approach to learning. The QuILTs incorporated visualization tools and structured conceptual activities designed to facilitate student understanding and promote conceptual change, features that closely resemble effective classroom teaching practices. Recognizing these instructional features, some participants found the QuILTs particularly appealing and relevant to their future roles as teachers. They began to engage with the material not merely for personal understanding but with a teaching-oriented perspective, focusing on how to explain complex ideas clearly to future students. Initially, quantum mechanics was perceived as a difficult subject with limited relevance to secondary education. However, when working with the QuILTs, participants reported a shift in perspective, feeling that the activities mirrored the kind of teaching practices they aspired to implement. One participant noted, "Using the QuILTs felt like practicing how I would teach

this to my future students, and it made me study more seriously." Although this pattern was not consistent across all participants, these reflections suggest that the instructional design of the QuILTs, closely aligned with effective teaching strategies, made the program particularly attractive to preservice teachers and supported deeper conceptual engagement, ultimately facilitating their own conceptual change.

IV. DISCUSSION

This study analyzed the conceptual changes and patterns in preservice physics teachers' understanding of wave functions, bound and scattering states, time evolution of wave functions, quantum measurement, and the uncertainty principle through instruction utilizing QuILTs. The results revealed significant improvements in their understanding of quantum mechanics, particularly in the domains of possible wave functions and the uncertainty principle, while certain challenges persisted in other areas. In the domain of possible wave functions, preservice teachers demonstrated high accuracy rates on concepts such as "Possible Wave functions for a Given Potential" and "Plotting Wave functions in Infinite (Finite) Potential Wells." However, few preservice teachers provided scientifically accurate reasoning, and interviews revealed persistent misconceptions, such as the belief that only specific forms of wave functions are allowed. Questions like "Normalization of Free Particle Wave functions" showed lower accuracy rates, indicating a need for further emphasis on the application of the normalization condition. After QuILTs instruction, preservice teachers refined their understanding of the meaning of wave functions in quantum mechanics, but some still struggled with representing nonstationary states and correctly applying normalization conditions. In the domain of bound and scattering states, the Hake gain value for the concept "Determination of whether a given particle is in a bound or scattering state based on its position" showed moderate to high effectiveness. While preservice teachers became capable of qualitatively sketching wave functions for these states after instruction, initial confusion regarding the coexistence of bound and scattering states for a single energy eigenstate and directional preference persisted. This suggests that while QuILTs effectively corrected many misconceptions, additional targeted guidance is needed to fully address issues like directional preference and wave function interpretation. The time evolution of wave functions remained a challenging topic for many preservice teachers. Significant difficulties were observed in items related to the "Relationship between wave functions in a superposition of energy eigenstates and the Schrödinger equation" and the "Time evolution of the wave function following a sudden change in potential energy." While QuILTs instruction successfully corrected the misconception of representing the wave function using

a single phase, some preservice teachers continued to struggle with accurately expressing wave function coefficients. Additionally, variations in individual learning paces suggest that tailored preinstruction on foundational quantum mechanics concepts could enhance the effectiveness of QuILTs in this domain. In the domain of quantum measurement, preservice teachers demonstrated significant improvement, particularly in items like “Qualitative sketch of the wave function postmeasurement” and “Probability calculation for energy measurement.” However, misconceptions persisted regarding the distinction between eigenstates of observables and stationary states, with nearly half of the preservice teachers unable to differentiate between the two. This aligns with previous findings that the complexity of mathematical representations and a lack of foundational knowledge in time evolution are barriers to conceptual understanding. Finally, in the domain of the uncertainty principle, preservice teachers initially exhibited misconceptions about its definition and meaning. After QuILTs instruction, these misconceptions were largely corrected, and the process of graphing wave functions using Fourier transforms helped refine their understanding. This highlights the importance of visualization and active learning in addressing abstract quantum mechanics concepts. These findings suggest that while QuILTs instruction effectively fosters conceptual change in many areas, challenges remain in fully addressing misconceptions related to normalization conditions, nonstationary states, and the eigenstates of observables. Future studies could explore the development of supplemental instructional strategies to address these areas, such as individualized preinstruction or enhanced simulations for complex topics like time evolution and measurement. Moreover, additional focus on directional preference in bound and scattering states could further improve conceptual understanding in these domains. Beyond the observed conceptual changes, qualitative reflections from participants suggest that the structured and visualization-driven design of the QuILTs contributed to fostering deeper conceptual engagement among preservice physics teachers. Some participants reported engaging with the material not only for their own understanding but also from a teaching-oriented perspective, considering how to explain complex quantum mechanics concepts to future students. This alignment between the instructional design and the participants’ emerging professional roles as educators appears to have supported more meaningful conceptual change. These findings underscore the importance of instructional strategies that not only facilitate active learning but also align with learners’ future professional responsibilities, thereby enhancing conceptual understanding. Future research could further investigate how learner identity interacts with instructional design to influence conceptual change, potentially through comparative studies involving different student populations.

V. CONCLUSION AND IMPLICATIONS

This study identified various types of misconceptions held by preservice physics teachers regarding key quantum mechanics concepts. Participants demonstrated relatively high accuracy on quantitative tasks such as wave function normalization, time evolution, and energy measurement—skills typically emphasized in conventional instruction and supported by algorithmic problem-solving strategies. After QuILTs-based instruction, these scores improved further, suggesting that preservice teachers had developed procedural fluency in these areas.

However, even within this group, participants showed persistent difficulties with tasks requiring qualitative reasoning or visual interpretation of wave functions. This contrast indicates that frequent exposure and explicit practice influence familiarity and performance, and that symbolic proficiency does not necessarily entail conceptual understanding. The findings also highlight the importance of balanced instruction that integrates both quantitative and qualitative dimensions of quantum mechanics learning.

The QuILTs-based interventions proved effective in reducing common misconceptions and supporting participants in developing representational and conceptual fluency. Improvements in interpreting uncertainty, time evolution, and wave function superposition indicate that well-structured, research-informed scaffolding can support conceptual change even in abstract domains such as quantum mechanics. For preservice teachers, the need to understand and eventually explain these complex concepts to future students may have encouraged deeper engagement with the material. The visualization tools and structured activities in the QuILTs likely supported this process by providing concrete representations of abstract ideas.

This study contributes to the literature by focusing on preservice physics teachers within a non-Western context, specifically South Korea. In this system, most secondary physics teachers are trained through colleges of education, where their disciplinary coursework is similar to that of science majors, but often accompanied by limited explicit instruction in conceptual reasoning. The persistence of misconceptions in this cohort underscores the need to go beyond procedural training.

The observed learning gains through QuILTs indicate the importance of integrating research-based, concept-focused resources into teacher preparation curricula. Such resources can promote deeper conceptual understanding and help preservice teachers better internalize and communicate abstract quantum ideas. These results may inform reforms in teacher education programs in Korea and other countries with similar institutional and curricular structures.

Because this study focused specifically on preservice physics teachers, further research is needed to determine whether similar patterns are observed among undergraduate or general learners of quantum mechanics.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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