

Effect of scale comprehension on student reasoning about astronomical phenomena.

II. Moon phases

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This paper is the second in a two-part series that investigates students' reasoning on astronomical phenomena. In this work, we examined how (if at all) students rely on spatial scale assumptions when reasoning about Moon phases. To that end, we conducted $N = 25$ semistructured interviews with last-year high school students. A personalized, physical scale model of the Sun-Earth-Moon system was constructed for every participant individually and placed on the interview table. Students were encouraged to clarify their thoughts by interacting with the scale model or by making a drawing. The scale estimates underpinning the scale model on the table were assessed through an interactive online survey, which was administered several days to weeks prior to the interviews. Only students with severely underestimated values for the relevant sizes and distances were selected for an interview. This selection procedure was motivated by both pragmatic and empirical considerations. We observed a wide variety of explanations for Moon phases, some of which were not found in the existing literature. Several students used their (inaccurate) scale ideas to reason about Moon phases. However, occasionally an interviewed student expressed arguments inconsistent with the scale model on the table. In these cases, scale assumptions seemed to be not taken into account when reasoning about Moon phases. The implications of these findings are discussed.

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I. INTRODUCTION

Over several decades of astronomy education research, Moon phases have proven to be a remarkably difficult topic for students to learn [1]. Many alternative ideas on the cause of Moon phases have been documented to persist into adulthood, despite targeted instruction at school. Yet, Moon phases are considered an important topic in education. As Trundle *et al.* [2] state, understanding Moon phases might not be essential for the purposes of everyday life, but “the cycle of lunar phases, so beautiful and so evocative to the human race, is a phenomenon of nature that begs

understanding.” Moreover, it is included in the primary school curriculum for most of the Organisation for Economic Co-operation and Development member countries [3]. For instance, in the Next Generation Science Standards of the United States [4], teaching the cause of lunar phases is prescribed for grades 6–8, while the mere observation and pattern recognition of the phases are suggested for first grade already.

Among others, one hurdle for a thorough understanding of Moon phases is an appreciation of the huge scales involved in the Sun-Earth-Moon system [5,6]. These scales are important because they constitute students' mental models of the Sun-Earth-Moon system [7]. It has been argued that the difficulties understanding Moon phases originate from students' struggles to develop accurate and robust mental models [8–10]. One can imagine several concrete implications of scale assumptions on Moon phase observations. For instance, they determine the probability of lunar eclipses, which are often confused with Moon phases [11–14]. Also, they determine the degree of

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similarity of the Moon phases as seen simultaneously by multiple observers from different positions on Earth.

Despite the consensus on the importance of scale [1,15], little effort has been devoted to examining explicitly the relation between scale comprehension and conceptual understanding of Moon phases. This work aims to fill this gap. We conducted semistructured interviews with $N = 25$ students in their last year of high school on the topic of Moon phases. The interviewed students were selected based on their ideas of the spatial scales in the Sun-Earth-Moon system, which were assessed by an online survey several days to weeks prior to the interview. The students' answers on this survey allowed us to construct a physical, personalized scale model of the Sun-Earth-Moon system for every student and place it on the interview table. Interaction with the scale model aided in clarifying students' reasoning during the discussion. As such, we searched for an answer to the following two research questions:

1. Which explanations do students in the last year of high school suggest for Moon phases?
2. How (if at all) do students rely on their ideas of spatial scales in the Sun-Earth-Moon system while reasoning about Moon phases?

While the current work covers the astronomical topic of Moon phases, the companion paper holds the same objectives for the topics of apparent Moon and Sun sizes and solar eclipses [16].

In Sec. II, we provide a thorough overview of the existing literature on students' understanding of both Moon phases (Sec. II A) and astronomical scales (Sec. II B). In Sec. III, we elaborate on the online survey used for the selection of participants based on their scale understanding (Sec. III A), as well as on the protocol followed during the interviews (Sec. III B). The results of this study are presented in Sec. IV, followed by a discussion in Sec. V and the conclusions in Sec. VI.

II. BACKGROUND

A. Moon phases

Acquiring a thorough understanding of Moon phases is a great challenge for students. This is apparent from the numerous alternative ideas on the cause of Moon phases held by students of all ages, as well as by pre- and in-service teachers. Roald and Mikalson even speak of a “plethora of different, more or less complicated conceptions” [17]. Plausibly the most well-known and most prevalent of these is the idea that the Moon phases are caused by the Earth's shadow covering (part of) the Moon [11–14,18–22], which Stahly names the “eclipse explanation” [14]. Other common conceptions include the belief that clouds cover part of the Moon [11,14,20], the Sun casts a shadow on the Moon [13,20], the Moon has a “dark side” and rotates [18,23], and a varying amount of sunlight

illuminates the Moon [20,24], although this list is far from exhaustive.

Many researchers have investigated the effectiveness of interventions to enhance students' understanding of Moon phases, with varying approaches and results. For instance, Trundle *et al.* [2] report an inquiry-based physics course intervention during one semester with 12 undergraduate students. They report that most participants shifted from an alternative conception of Moon phases before instruction to a scientific conception after instruction. Seven participants continued to hold the scientific conception six months after instruction, while five participants reverted to more alternative and/or fragmented conceptions. Wilhelm *et al.* [25] used a quasiexperimental design to explore how instructional method and gender influenced learning about Moon phases. They found an overall positive impact of the experimental design, with girls benefiting more than boys. Ogan-Bekiroglu [26] used a model-based teaching method to test its effect on preservice teachers' conceptions of Moon phases. She concluded that periodic observations of the Moon and sharing information with peers helped participants to gain scientific knowledge. However, some features, such as the daily time difference of Moonrise, were not successfully explained by the students' mental models. Taylor *et al.* [27] developed a four-phase, 12-lesson pedagogical strategy to enhance astronomical knowledge of grade 7–8 pupils by stimulating mental model building. While the students' scores on other astronomical topics (such as tides) improved impressively from pre- to post-test, for Moon phases, only 16% of the participants selected the correct response in the post-test (compared to 0% in the pretest). The authors conclude that “understanding about the Moon remained difficult for these pupils despite the intervention.” Similar conclusions are formulated by, e.g., Ucar [28], Callison and Wright [7] and Hermann and Lewis [29].

The question rises, then, why the topic of Moon phases is so challenging for students. People base their understanding of the world on direct observations and interactions [30,31]. However, in the case of Moon phases, daily experience is not an assistant but rather a hurdle for learning [9]. Thinking and reasoning about Moon phases requires mentally shifting from a geocentric to an allocentric perspective [10,32]. The exclusively geocentric human perspective on the sky is therefore one of the causes for many alternative conceptions listed above [9].

Mentally shifting to an allocentric perspective is only one of the cognitive abilities necessary for a thorough understanding of Moon phases. Others are logical reasoning and spatial abilities [8,33], specifically on the relative motions and vast scales involved in the Sun-Earth-Moon system [5,34] and the recognition of the intrinsic three-dimensionality of that system [10,35]. We will elaborate more on spatial scales in the next section. However, scales and multidimensionality are not independent. As Eriksson

and colleagues have shown, size and distance are essential aspects for the perception and extrapolation of three-dimensionality from two-dimensional representations [36]. When students fail to acknowledge the intrinsic 3D nature of the Sun-Earth-Moon system, in which the Moon's orbit is tilted with respect to that of the Earth, they can be expected to confuse a full Moon with a lunar eclipse, thus drifting towards an eclipse explanation for Moon phases.

In a classical classroom, most if not all representations of the Sun-Earth-Moon system are two-dimensional. When learning about Moon phases, an essential competence is thus to interpret these discipline-specific representations and conceptually extrapolate a 3D model from them [36]. However, research has shown that students are often much less capable of doing so than generally assumed [35,37,38]. This poses considerable limits on the effectiveness of classical teaching materials. Multiple authors have investigated the effectiveness of astronomy textbook representations in explaining Moon phases. Dove [39] concluded that textbook diagrams can be confusing for students, either because the Moon's orbit is displayed aligned with the ecliptic or because multiple Moons are shown at once. Pena and Quilez [40] argue that illustrative images on Moon phases are not by themselves sufficiently explanatory. In addition, strong inducements to attract attention are required in form of written text, as well as specific instructions by the teacher. These results were confirmed in a later study by Testa *et al.* [41], who point out that students' difficulties in correctly interpreting textbook images may stem from, e.g., the compositional structure of the image and the presence of both real and symbolic elements. An example of the former is an exaggeration of the eccentricity of the Moon's orbit around the Earth, which is irrelevant and therefore hinders students' understanding of why we see different phases of the Moon. They also suggest that presenting eight Moons with different phases in one image around the Earth might suggest that different phases are visible from different locations on Earth. While this discussion suggests that inaccurate textbook representations may induce common alternative conceptions of Moon phases, there are certainly more factors playing a role. This was neatly shown by the research of Madura *et al.* [42], who documented common alternative conceptions of Moon phases with visually impaired students.

Two more suggestions can be made on why Moon phases prove so challenging for students. First, Trundle *et al.* [2] investigated representations of the Moon in children's books and found that these often misinterpret the Moon by displaying inaccurate shapes or sizes. While this may be regarded as rather innocent given the young age of the audience, the authors argue that these representations reinforce common misconceptions about Moon phases, specifically the erroneous idea that they are caused by the Earth's shadow. If so, it might substantiate why this belief is so deeply entrenched and difficult to overwrite. Second, it

has been shown that many teachers do not sufficiently understand astronomical topics covered in class themselves [7] and consequently experience a lack of confidence when teaching about it [43,44]. In fact, research has shown that the astronomical content knowledge of preservice physics and science teachers is no better than that of preservice teachers in other (nonscience) fields [45]. This plausibly leads to limited support for students when learning about these topics, while the previous discussion on textbook representations indicated the crucial role of teacher scaffolding.

B. Scale comprehension

While the main topic of interest in this work is Moon phases, student explanations are viewed in the context of scale comprehension. An extensive overview of the research literature on students' knowledge of astronomical scales, together with potential sources of difficulties, has been written in the companion paper [16]. Therefore, we only provide a summary here.

A solid understanding of spatial scales is generally considered to be an important asset in astronomy education [46], crucial to understand a wide range of astronomical phenomena to a deeper level [15]. Additionally, students' ability to conceptualize scale is significantly correlated with their visual-spatial skills [47], which in turn has been shown to strongly correlate with many science, technology, engineering, and mathematics (STEM) disciplines [48–52]. It could thus be argued that an improved understanding of astronomical scales both has a direct positive impact on students' astronomical literacy and plausibly contributes to their performance in other STEM fields.

Numerous studies have demonstrated poor understanding of astronomical sizes and distances by students in primary school [53], secondary school [54–57], university [12,58] and teacher education programs [13,59,60]. While the evidence for a lack of understanding of spatial scales is overwhelming, specific results of the cited literature are occasionally contradictory. One reason for this disagreement may be different assessment methods, which all have their respective limitations. For instance, asking participants for scale estimates in terms of kilometers or miles is considered problematic, since students generally have difficulties grasping the meaning of such large numbers [61–64]. Multiple choice queries, on the other hand, imply the risk of students choosing the largest option, whatever that may be [58]. Ranking questions are considered more solid, but the information they provide is rather limited.

The enormous dimensions of astronomical systems make visual representations of true scales often challenging, if not impossible. Therefore, textbooks and online web pages generally present schematic illustrations. This was, for instance, shown by Taylor and Grundstrom [65] for representations of the Earth-Moon system. They recommend that authors always mention explicitly when an

illustration is not to scale, which was found often not to be the case. Textbook illustrations generally exaggerate one specific feature of an astronomical system to clarify a certain concept, thereby losing scale accuracy [41]. While this is justifiable, it can potentially lead to erroneous interpretations. For instance, when the eccentricity of the Earth's orbit around the Sun is exaggerated to illustrate its ellipticity, students can easily be misled toward the interpretation that a varying distance to the Sun causes seasons [66]. This example is only one out of four erroneous conceptions about the significance of size and distance in the changing temperatures between seasons described by Ojala [66], all of which are claimed to stem from textbook illustrations.

Few studies have attempted to quantitatively establish a relation between students' ideas of spatial astronomical scales and their explanations for astronomical phenomena. Fanetti [34] did so for the phenomenon of Moon phases (hypothesizing that students who placed the Moon very close to the Earth would be more likely to hold an eclipse model for Moon phases), but did not find any such relation. Wilhelm *et al.* [67] showed how teachers in grades 6–8 held similar alternative conceptions as their students and additionally had a limited understanding of the spatial scales in the Sun-Earth-Moon system. They state that the teachers' difficulties with visualizing the Earth-Moon scale could explain some of their low scores on the Lunar Phases Concept Inventory. Studies by Rudmann [68] and Türk [69] reported significant correlations between students' spatial thinking and their achievements in astronomy. Scale comprehension is generally underrepresented in classic tests of spatial thinking, which tend to emphasize mental rotation and perspective taking [70,71]. Nonetheless, scale comprehension is recognized in the literature as part of spatial magnitude or spatial scaling skills and thus falls within the broader umbrella of spatial ability [72,73]. We can therefore also expect scale comprehension specifically to correlate with astronomy performance. This hypothesis seems to be supported by results of, e.g., Madura *et al.* [42], who identified “a misunderstanding of distance, scale, and relative location of astronomical objects” as a common theme that runs through a wide range of alternative conceptions of Moon phases.

III. METHODOLOGY

The methods used in this work are largely identical to those described in the companion paper [16]. We will briefly describe this methodology here and refer the interested reader to the previous paper for more details.

A. General outline

We conducted 25 interviews with last-year high school students (age 16–18) and documented their explanations for the phases of the Moon. Three students went to school

in Sweden, while all other students attended schools in Flanders, Belgium. These participants were the same as those described in companion paper [16]. The results from both parts of this series stem from one set of interviews, where first the astronomical topics of part I. were discussed (being the apparent sizes of the Moon and the Sun, and solar eclipses) and thereafter the topic of this paper, Moon phases. All Belgian students learned about Moon phases during a (limited) number of geography classes, most often in the year prior to the year of participation in this study. The Swedish students were introduced to this topic earlier and in a more distributed manner across primary and secondary education, although it is described in less detail in the national curricula.

The participants were selected and invited for an interview based on their answers to an interactive online survey, which was administered a few days to weeks prior to the interviews. This survey probed the students' ideas on the spatial scales in the Earth-Moon-Sun system and used customized visualizations as feedback after every question. Students were also asked to make a self-assessment of certainty for every answer, including a ‘pure guess’ option. This online survey was to a large extent similar to the one administered in our previous work [74], in which we elaborate more on the specific questions and include illustrations of the visualizations. Apart from scale-related questions, the online survey deployed in this study also included one multiple-choice question on the cause of seasons and one on the cause of Moon phases. These questions were based on the AMoSS test [75] and the Lunar Phases Concept Inventory [76,77], for the seasons and Moon phases question, respectively. These questions were not included to fully gauge students' reasoning on these topics, but rather to discern which students were susceptible to common misconceptions (such as the conception that seasons are caused by a varying distance to the Sun or Moon phases are caused by the Earth's shadow). The question formulation and set of provided answers of the multiple-choice question on Moon phases are discussed in Sec. IV A.

Using the students' answers to the scale-related questions in the survey, a personalized physical to-scale model of the Earth-Moon-Sun system was constructed for every student and placed on the interview table. This model was drawn on paper and therefore two-dimensional. It is clear that this procedure was not feasible for students whose answers to the scale questions in the survey were too large, as their scale model would not fit on the interview table. As such, we selected students for interviews that severely underestimated the sizes and distances in the Earth-Moon-Sun system. This selection procedure was not only driven by pragmatic motives but also by empirical ones. Our previous work [74] showed that the majority of students strongly underestimate these scales. Therefore, the selected participants represent a large group of students within the broader population. Moreover, students with poor spatial

skills can be expected to also underperform in astronomical conceptual understanding [47,68,69]. It would make intuitive sense that this also applies to scale comprehension specifically. For instance, to students who think that the Moon is very close to Earth, it would be more logical that Moon phases would be caused by the Earth's shadow. Since this work aims to uncover how incorrect spatial scale comprehension relates to alternative reasoning on Moon phases, this was an interesting group of students to focus on. The interviews with Belgian students were conducted in Dutch, so all transcripts in this report have been translated to English later on. The interviews with Swedish students were conducted in English, with a native Swedish speaker present in the room to whom the participants could turn to as an interpreter whenever necessary.

B. Interviews

The physical and personalized scale model of the Earth-Moon-Sun system was placed on the table between the interviewer and the student, already before the student entered the room. At the start of the interviews, students were pointed to the scale model and informed that the relative sizes and distances were retrieved from their survey answers. They were encouraged to make any adjustments to the model they deemed necessary, by replacing and/or resizing the celestial bodies. After the students were satisfied with the scale model, they were assured that additional adjustments could still be made at any time throughout the interview. During the subsequent discussion on Moon phases, students were repeatedly encouraged to use the scale model to illustrate their reasoning.

The discussion on Moon phases started by checking whether the students were familiar with the term “Moon phases”. If not, they were told that it refers to the Moon's shape seemingly changing over time, as seen from Earth. They were then asked to try and explain why that happens. As mentioned before, they were encouraged to make drawings or use the scale model on the table to illustrate their thoughts. Often, the interviewer asked follow-up questions like “When is it full Moon?”, “When is it half Moon?”, “Why is that?” to better gauge the students' level of understanding.

To that same aim, students were thereafter shown a picture of a gibbous Moon, displayed in Fig. 1. The interviewer told the students: “This is a picture of the Moon when it is more than half, but not quite yet full. When do we see such a Moon?”. Follow-up questions were again asked frequently, such as: “How come we don't see that last bit of the Moon?”.

There was also a second, more specific reason to include the photo of Fig. 1 into the interview protocol. It was anticipated that the photo would lead to confusion for students who assigned Moon phases solely to the Earth's shadow. Because, in that case, the nonvisible part of the Moon would have a “round” (convex) contour instead of



FIG. 1. Photograph of a gibbous Moon shown to the students during the interviews. Source: [78].

the “slicelike” (concave) contour observed in the gibbous Moon (a sketch of these contours is drawn in Fig. 7). The authors found it an interesting question whether the students would be aware of that incompatibility and if so, how they would react to it.

To finish the interview, two hypothetical questions were asked about Moon phases. They were: “What would happen to the Moon phases if the Moon were suddenly much further from the Earth than it is now [according to the scale model]?”, and “What would happen to the Moon phases if the Moon were suddenly much larger than it is now [according to the scale model]?”. These questions were added to investigate whether students were able to reason logically with their explanatory model for Moon phases, even though the explanation itself may be wrong. For example, it was anticipated that students who assigned Moon phases solely to the Earth's shadow would reason that when the Moon would be further away, a smaller part of its orbit would lie in the Earth's shadow cone. Similarly, we expected these students to argue that if the Moon were larger, it would less often (or never) fully lie in the Earth's shadow, thereby reducing the frequency of new Moons.

For the sake of completeness, we added the interview protocol in Appendix B.

C. Data analysis

All interviews were video- and audio recorded and verbatim transcribed. Gestures, drawings and interactions with the scale models were included in the transcriptions. The participants' names were replaced by pseudonyms in the transcripts—and thereby also in this report. A coding scheme was developed for the students' explanation of Moon phases and for their arguments on the hypothetical questions separately. The codes contained information about the different elements present in the suggested explanations and their scientific correctness. These schemes were then tested against newly conducted interviews and

refined when necessary. This process continued iteratively until it converged and all new data could logically be assigned a code in the constructed scheme. The coding schemes, together with an explanation on their structure and content, are shown in Appendix A. Transcript excerpts with identical codes were compared to assure that the provided explanations were based on similar arguments. As such, recurring themes and argumentations could be uncovered, as well as the students' mental models. In the coding procedure, special attention was paid to whether and how students used the scale model to guide their thoughts, as this was of primary importance to reach our research objectives.

The coding of all participants was performed by the first author of this work, while an additional coding of five participants (20% of the total dataset) was done by one of the coauthors to calculate interrater reliability scores. We obtained an overall Cohen's κ score of 0.77. Since the set of possible codes differed between questions (see Appendix A), this κ score was calculated by taking the mean of the question-specific κ values. However, these question-specific values were also evaluated carefully. In fact, in a first round of interrater reliability calculation, the overall κ score was acceptable, but the score for one question in particular (that on the cause of Moon phases, using the codebook of Table II) was too low. Therefore, adaptations and clarifications were added to the codebook and a second round of three cross-coded participants was performed. This time, all obtained values were sufficiently high. While the obtained overall κ score of 0.77 indicates a substantial agreement between the raters, it should be taken into account that the limited number of cross-coded participants can potentially cause it to be imprecise. To limit the error on κ , codes that were never assigned by either of the raters were excluded from the calculation. Additionally, we report an overall percentage agreement of roughly 95% and a mean Jaccard score of 0.75, which aligns well with the recovered κ score. Together, these values guarantee a high reliability of the coding procedure.

IV. RESULTS

A. Online survey

The results of the scale-related question in the online survey were already discussed in the companion paper [16] and will only briefly be reviewed here. Most of the 249 students who completed the survey underestimated the probed relative distances. On the two relative sizes questions, students made both over- and underestimations so that the correct values lied within the interquartile ranges of the estimation distributions. These interquartiles ranged from 4 to 10 for the ratio Earth/Moon (with the correct answer being 4) and from 15 to 200 for the ratio Sun/Earth (with the correct value being 109).

Just over one third of all answers were pure guesses, and the nonguesses roughly followed a uniform certainty

distribution. Approximately one quarter of all answers to scale-related questions were adjusted after interacting with the customized visualizations. Due to the general underestimation of the probed sizes and distances in the survey, at least two students could be selected for an interview from every class. This selection was exactly identical to the one applied in the companion paper [16], as the results of both parts stem from the same set of interviews. The selection was done by setting the following criteria: First, we wanted the Sun-Earth-Moon system to fit on the interview table. The average available space was roughly 2 m, and the minimal size of any circle (in practice, of the Moon) in the scale model was set to 2 mm. That implies that the ratio of the entire system's size compared to the Moon's size could be a factor of 1000 at most. This was the case for 101 students in our sample, or just over 40%.

As a second criterion, we required interviewees to have correctly answered the size-related ranking question of the survey. That is, they correctly ranked the Moon, Earth, and Sun in increasing size. For students who answered this ranking question incorrectly, the survey would later on ask how much *smaller* the Sun is compared to the Earth, for instance. We argue that these students lacked the most fundamental content knowledge in astronomy and therefore were not considered suitable for participation. This second criterion reduced the pool of potential interviewees to 80, or roughly 32%. Finally, for the same reason, we required interview participants to have ranked the distance "Earth—Moon" as smaller than "Sun—Earth." As a result, the final number of potential interview participants became 75, or approximately 30% of our total survey data pool.

We now turn to the results of the survey question on the cause of Moon phases. By combining the students' answers to this question with their scale estimates of the survey, we can reproduce some of the results of Fanetti [34]. She hypothesized that college students who believed the Moon to be too large and/or too close to the Earth would be more likely to explain Moon phases in terms of the Earth's shadow. However, she could not confirm this hypothesis with her data.

The survey question on Moon phases was formulated as follows: "Why does the Moon, as seen from Earth, appear to change shape over time? Multiple answers are possible." The following options were given:

- Sometimes the Moon is (partly) in the shadow of the Earth [hereafter referred to as "Earth shadow"]
- Sometimes the Moon is (partly) in the shadow of the Sun [hereafter referred to as "Sun shadow"]
- The position of the Moon relative to those of the Earth and the Sun changes over time [hereafter referred to as "Relative positions M-E-S"]
- Sometimes certain objects block our sight on (a part of) the Moon [hereafter referred to as "Blocking objects"]

TABLE I. Frequencies of every cause for Moon phases. Note that these numbers sum to more than the number of participants, since the options are not mutually exclusive.

Model	Number of occurrences
Earth shadow	166 (66.7%)
Relative positions M-E-S	106 (42.6%)
Sun shadow	68 (27.3%)
Blocking objects	32 (12.9%)
Rotating dark side	13 (5.2%)
Do not know	5 (2.0%)

- The Moon has a bright and a dark side and rotates around its axis [hereafter referred to as “Rotating dark side”]
- I really don’t know [hereafter referred to as “Do not know”]

The choice of these options was motivated by earlier inventories used in the literature, especially the Lunar Phases Concept Inventory [76,77]. Table I presents the frequencies of the various options. As mentioned in the question, students could tick multiple boxes, which is why the total number of occurrences adds to more than the number of participants. The most popular option is “Earth shadow,” with two thirds of the participants. This confirms the well-known alternative conception that Moon phases are caused by the Earth casting its shadow on (part of) the Moon [11–14,18–22]. The second most frequently chosen option is that of the relative positions M-E-S, the only scientifically correct option. Thereafter, with rapidly decreasing frequencies, follow the Sun’s shadow, blocking objects and the rotating dark side.

In Fig. 2, we investigate the relation between students’ responses to the Moon phases survey question and their answers to the open-ended question, “How many times does the Earth fit in the distance between the Earth and the

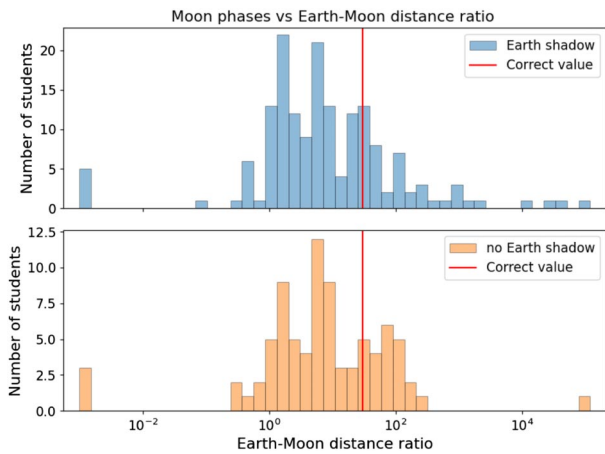


FIG. 2. Students’ estimates of the Earth-Moon scale factor, comparing students including and excluding the Earth’s shadow as a reason for Moon phases.

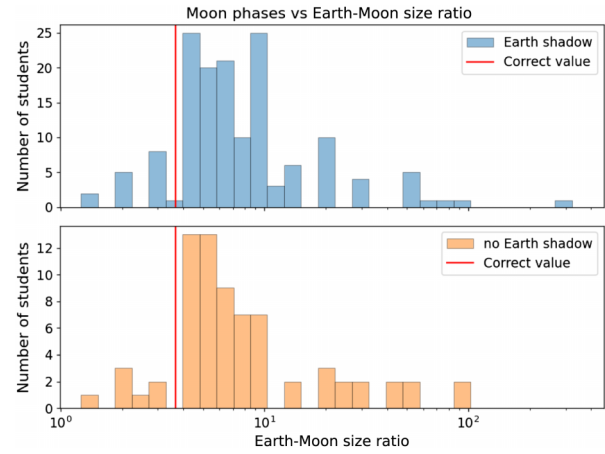


FIG. 3. Students’ estimates of Earth and Moon diameters, comparing students including and excluding the Earth’s shadow as a reason for Moon phases.

Moon?”. In line with previous studies (see, e.g., Ref. [27]), we refer to this number of Earth diameters as the “distance ratio” of the Earth-Moon system. A wide range of estimates was expressed for this question, which is why they are plotted on a logarithmic scale in Fig. 2. This resulted in practical issues for those students who made an estimate of zero, so these answers were manually changed to 10^{-3} . No statistically significant difference was found between the group of students who included Earth’s shadow in their responses to the Moon phases question and those who did not ($p > 0.05$). This result was obtained by performing a Mann-Whitney U test.

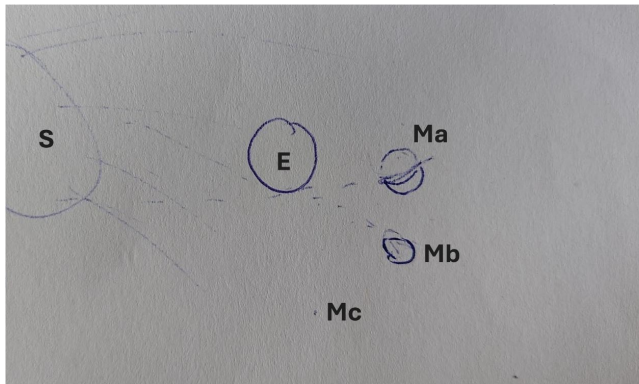
In Fig. 3, we display the students’ estimates for the Earth/Moon size ratio. We again compare the students who had Earth’s shadow included in their answers to the Moon phases question to those who did not. Again using a Mann-Whitney U test, we found no statistically significant difference in the scale estimates between the two groups of students ($p > 0.05$).

From the quantitative survey data, we thus found no relation between the students’ scale conceptions in the Earth-Moon system and their inclusion of the Earth’s shadow as a cause for Moon phases. This is very much in agreement with the results of Fanetti [34].

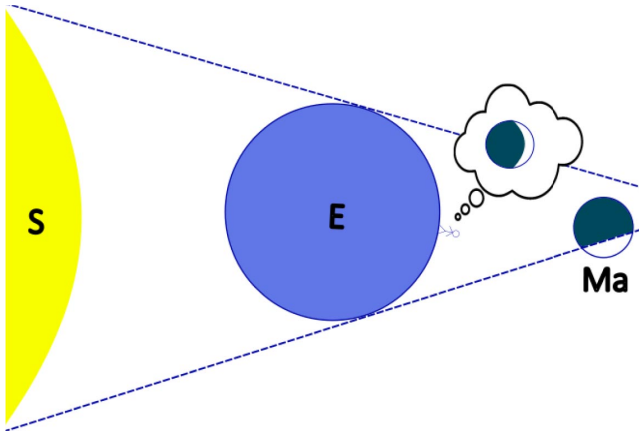
B. Interviews

1. Moon phases

Correct explanations. Only three students in our interview sample were able to provide a correct and complete explanation for Moon phases. However, this was not surprising, given the large number of students suggesting the Earth’s shadow as (part of) the explanation in Table I. Yet, it was lower than anticipated by the researchers, as all students had learned about Moon phases at some earlier time in school.



(a) Original drawing from Mila, with added letters S, E, Ma, Mb, Mc.



(b) Reproduced illustration

FIG. 4. Earth shadow model as an explanation for Moon phases, as (a) drawn by Mila and as (b) reproduced by the authors.

Earth shadow models. More surprising than the small number of correct explanations was the even smaller number of students assigning Moon phases solely to the Earth's shadow. There were only two students who indicated that every non-full Moon is due to the Earth's shadow covering some part of the Moon. In this model, we observe a full Moon whenever the Moon is outside of the Earth's shadow cone. This idea is illustrated in Fig. 4(b) and drawn by Mila in Fig. 4(a). Mila expresses her thoughts as follows:

- M: So... at first I thought that the Earth itself was in the way of the Moon, but I don't know... that if the Sun is here... [draws the Sun (S) at the left side of Fig. 4(a)]...
- I: Yeah?
- M: ...that it lights up this part of the Moon, which we see, but that this part, we're in the way of it ourselves [draws the Moon at position Ma in Fig. 4(a), together with the Sun's ray dividing the Moon in a dark and illuminated part].

- I: Ah right, whereas if the Moon is here [points at the Moon in position Mb of Fig. 4(a)]?
- M: Then we do see it fully.
- I: Then it's what? A full Moon?
- M: Umm, yeah or maybe just a tiny bit less, because I think... yeah, if you really draw rays like that, the Earth might still be blocking just a small part [draws a Sun's ray to Moon Mb], but okay, yeah.
- I: Yeah okay, but so for example here [indicates point Mc in Fig. 4(a), roughly first quarter position]—is that a full Moon?
- M: Yeah.

When Mila is shown the picture of the gibbous Moon in Fig. 1, she realizes immediately that this shape of the Moon does not fit in her explanation of Moon phases. She says that she had not thought about that yet, and that if the Earth were in front of the Moon, the gibbous Moon shape would not be observed. She clearly becomes confused and dissatisfied with her explanation and tries to come up with a solution that incorporates the gibbous Moon shape but does not really succeed. She concludes: “I just can't think for a minute and I'm stuck in, I think, wrong patterns, the whole time already.”

Mixed models. Mixed models were observed more often than either correct explanations or pure shadow models. Students holding these models explained Moon phases by a combination of correct elements (such as stressing the Earth's perspective on the Moon) and incorrect elements (such as the Earth's shadow playing some role). For example, multiple students started out with a correct explanation of half Moon phases, but got confused when asked to indicate the full Moon position. The idea that the Earth's shadow causes a full Moon to be invisible seemed to be a hurdle that students needed to overcome. Some students were able to do so, thereby correcting their explanation toward the scientific one. An example of such a participant was Hannah, who first talked about the Earth blocking the sunlight but then seemed to change her mind. This was deduced from the following interview fragment:

- H: So the Moon orbits the Earth, but the Sun always stays in the same place, and so, if you have the Earth here [gestures a ball in the air with her hand], and the Sun here, and the Moon here [makes the same gestures for Moon and Sun, with Moon in its first quarter phase], then the Moon is only lit on that side by the thing [gestures the direction of sunlight onto the Moon] so we only see that side [indicates the half of the Moon opposite to the Sun], no, that side is lit [corrects and indicates the Sun-directed half of the Moon], and so that [opposite] side is dark, because this [Sun-side] side is being lit by the Sun, so then we see this

- shape [draws a half Moon in the air], and that's the first quarter. And so there [indicates the position to the Earth opposite from where she just gestured the Sun to be], we don't see a Moon in the sky...
- I: Wait, where? Can you show that [with the scale model on the table], where we don't see the Moon?
- H: Like this [places the Moon post-it in the full Moon position].
- I: Like this we don't see the Moon?
- H: Because the Earth blocks all the sunlight, so the Moon can't be lit.
- I: Okay.
- H: And so here [puts the Moon post-it in last quarter position] it's only lit on that [Sun-directed] side, and here [places the Moon post-it in new Moon position] it's..., no wait... [hesitates] No, this is new Moon. Here only this [Sun-directed] side is lit, but we only see this [opposite] side, so that's new Moon, and that's [places the Moon post-it back at the full Moon position] full Moon.

However, other students did not seem to manage to reject the idea that the Earth's shadow obscures every full Moon. Instead, these students adjust the rest of their explanation to become consistent with the Earth shadow idea. An example of such a student is Gregory, who starts out with a correct explanation of half Moon phases but is led astray by the idea of the Earth's shadow. Initially, this "only" leads to an interchange between the full and new Moon positions. This can be seen from the following excerpt:

- I: Okay, can you show how that [Moon phases] works with this [scale] model?
- G: Yeah, that's... if it's here [the Moon post-it is currently between Earth and Sun], no wait, somewhere here [moves the Moon post-it to first quarter]—that's the first quarter.
- I: Uh-huh, why so?
- G: Because it's... positioned like this relative to the Earth and like this relative to the Sun [draws imaginary lines between the objects], so it's only partly lit [places his hand on the light-dark boundary on the Moon post-it], and then here... [places the Moon post-it at full Moon position] I think... Here you can't see the Moon because, with the Sun... the Earth blocks it, and then there's no light to see the Moon. And then at the last quarter [moves the Moon post-it 90 degrees further] you have that [Sun-directed] part that's lit, and then here [Moon between Earth and Sun] you can, yes.
- I: Yeah, what do we see here [with the Moon between Earth and Sun], at this position?
- G: A full Moon, I think.

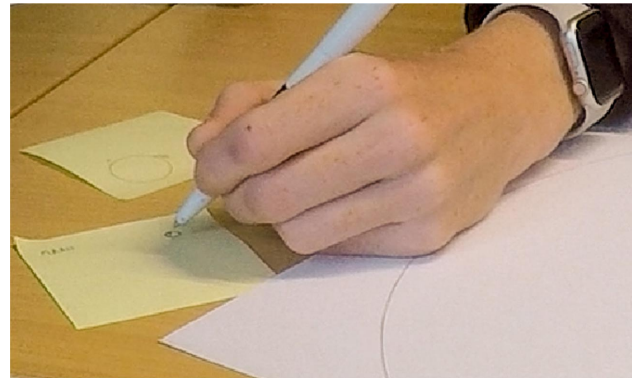


FIG. 5. Screenshot of the interview with Gregory, shortly after he has colored the half of the Moon that is illuminated by the Sun (not obscured). The post-it on which Gregory is drawing represents the Moon, the other post-it represents the Earth and the larger paper represents the Sun.

Up until this point, Gregory's explanation is very similar to Hannah's above (before she changed her mind about the new Moon position). However, when Gregory is shown the photograph of the gibbous Moon (Fig. 1), he tries to include this into his explanation, changing his previous statements. This causes his eventual model to misinterpret all Moon phases, including half Moon phases. Illogically, Gregory even states that the side of the Moon opposite to the Sun is illuminated (see Fig. 5). This can be induced from the following fragment, together with Fig. 5:

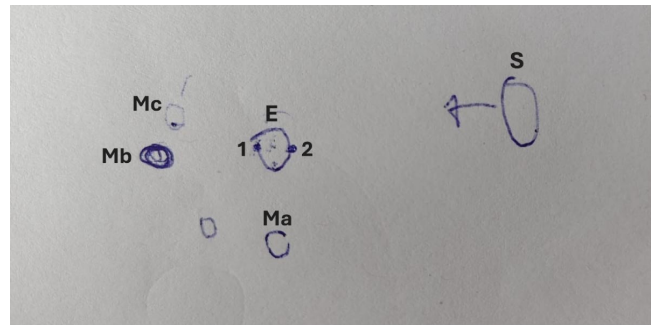
- G: If I had to indicate that [gibbous Moon position] with the Moon [post-it] or something, I'd think it would be like, here or so [places the Moon post-it at a position between his full Moon (actually new Moon) and first quarter positions], like it's not completely...
- I: Yeah, close to the full phase [points to the position between Earth and Sun, which G. thinks is the full Moon position], but not quite yet?
- G: Yeah, not quite yet.
- I: Okay, and which part of the Moon is lit right now?
- G: Yeah, like... wait, here...
- I: Can you indicate it [hands over a pen]?
- G: Ah yeah, I think at first quarter it's this part [hovers the pen over the Sun-directed half of the Moon], I think. And then... no wait, it's this part [hovers the pen over the opposite half of the Moon], and then when it moves more over here [towards the position between Earth and Sun], more and more of it gets lit... [hovers over a larger part of the Moon with the pen]
- I: You can color it, go ahead, color the part that's lit.
- G: Oh okay, I think at last quarter this part is lit [colors half of the Moon on the side opposite the Sun, as shown in Fig. 5], and then it continues and

continues [gradually extends the colored part into the Sun-facing half of the Moon], until... that part, like we see over there [points to the gibbous Moon picture].

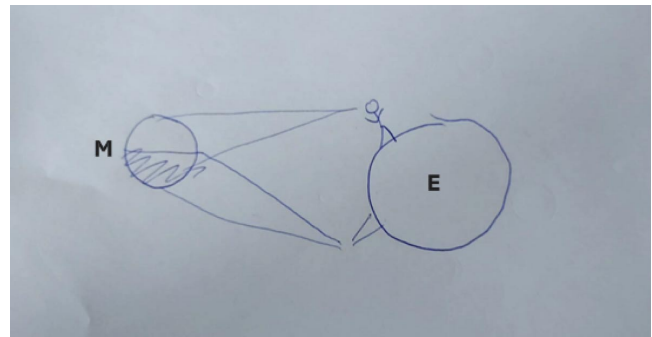
Other explanations. Some students' explanations contained other features than relative positions, reflection of sunlight, and the Earth's shadow. For instance, Alice suggested that the Moon itself emits light as well, thereby influencing the observed Moon phase. Christopher argued that "some bodies had like dark spots, and I think that might be what the Moon has. And I think it has like one where it's facing the Sun and one that's never facing the Sun." Christopher reasons that as the Moon rotates, more or less of these dark spots face the Earth, thus causing Moon phases. Both the idea of a self-luminous Moon and that of dark spots (or rather, a dark side of the Moon) are well-known alternative conceptions in the literature [5,18,23]. Later in the interview, Christopher adds another possible cause for Moon phases. He starts talking about stars behind the Moon and their impact on the observed Moon phase. He argues: "They the background stars are actually very similar to the Sun when it comes to light, and I think that might also, when I just think about it now, it might actually be a reason why we see different Moon phases. And when you have those two stars, for example, you have two different stars here; it might be that when they show more on the Moon, it might show different types of phases of the Moon"

Another interviewed student, Noah, suggested that the observed Moon phase depends on the geographic position of the observer. At the start of the next interview excerpt, Noah has only drawn the Sun (**S**) and the Earth (**E**) in the drawing of Fig. 6(a). He hovers his pen above the Earth and says that when the Moon is there, in the air, there is a full Moon. Then he seems to reconsider and says the following:

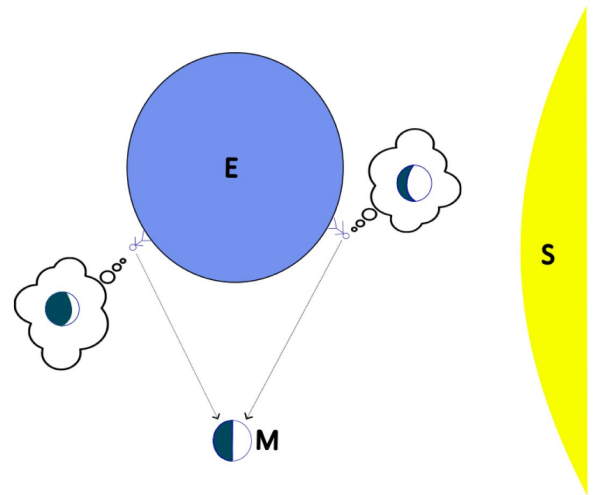
- N: Yeah, depends on perspective though.
 I: Because?
 N: Because, yeah, if you're all the way on top of the Earth, you'll see like half of the Moon.
 I: And if I'm standing where, I see the full Moon?
 N: Uhm, the full Moon will then be here somewhere [points at a position on the paper on the side of the Earth opposite to the Sun, roughly where later the Moon **Mc** will be drawn].
 I: Does that depend on perspective or on where the Moon is, or both?
 N: Depends on perspective, because if you're like on that side [points to the "bottom" part of the Earth], then you would still see a part of the black.
 I: Wait, so imagine the Moon is here [draws the Moon on position **Ma** in Fig. 6(a)]. Do I see different things when I'm standing there or there or there [points subsequently at positions **1**, the



(a) Original drawing from Noah, with added letters and numbers.



(b) Original drawing from Charlotte, with added letters **E** and **M** to indicate Earth and Moon, respectively.



(c) Reproduced illustration

FIG. 6. Influence of the geographic location of the observer on the observed Moon phase, as drawn by (a) Noah and (b) Charlotte, and as (c) reproduced by the authors.

center of the Earth and **2** in the drawing of Fig. 6(a)]? Or not?

- N: Yes, you will then see black on this side [points at position **1**] and more white on this side [points at position **2**].

I: Yeah, so here [points at position **1**] I see nothing of the Moon? Or just a bit?

N: I think just a bit.

- I: And here [position 2] I see everything? Or also just a bit?
- N: Also just a bit.
- I: But more than there [points at position 1]?
- N: Yes.

We provide a schematic representation of this idea in Fig. 6(c). Noah suggests that even though the Moon is in its first-quarter position, not every observer on Earth will see a half Moon. As one moves more toward the Sun-side of the Earth, more than half of the Moon will be visible, and less than half will be visible for observers on the opposite side of the Earth. It is interesting to note that this explanation would be accurate if the Moon were as close to the Earth as the representations of Fig. 6 suggest. We will return to this point more elaborately in the discussion. Noah was not the only student suggesting that geographic location was an important factor for the observed Moon phase. For instance, Charlotte, who made the drawing of Fig. 6(b), even stated that the upper person in Fig. 6(b) could see a full Moon while the lower observer sees a new Moon.

Interestingly, Noah later on also claims that when the Moon is at position **Mb** in the drawing of Fig. 6(a), the Earth blocks the sunlight from reaching the Moon. Then, no Moon is visible, regardless of the position of the observer. When shown the gibbous Moon picture of Fig. 1, Noah builds further on the Earth's shadow argument. He thereby neglects the possibility that an observer at position 2 in Fig. 6(a) would observe moon **Ma** as a gibbous Moon. When asked where the Moon should be in order to observe a gibbous Moon, Noah responds:

- N: Yeah, you can do it here... like here [draws the Moon at position **Mc** in Fig. 6(a)], and then the Earth is a little bit in the way of the Moon. And then you can... because here is then the shadow [points to the side of the drawn Moon that is behind the Earth], because the sunlight partly falls on the Earth and then doesn't fall on that side.
- I: Ah yeah, so the Moon actually has to stand almost in the Earth's shadow?
- N: Yeah.
- I: We don't see this part because here is the Earth's shadow?
- N: Yeah.
- I: Yeah, because the shadow of the Earth, that's round, so then I would actually think we don't see a part like this [draws shape **A** in Fig. 7], do you understand? And not a part like this [draws shape **B** in Fig. 7].
- N: Ah yeah, okay yeah.
- I: So how does that happen, do you think?
- N: Like, that position... I'm trying to find that orientation again, because it's a bit difficult.

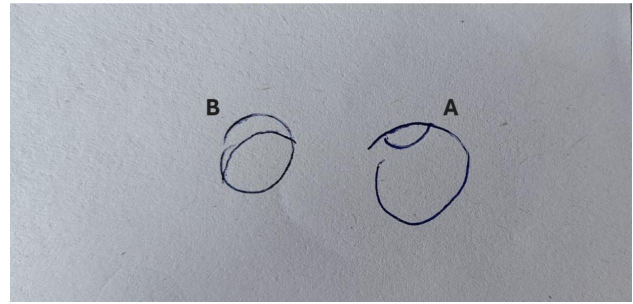


FIG. 7. Shapes of the “missing part” of a gibbous Moon, as drawn by the interviewer during the interview of Noah.

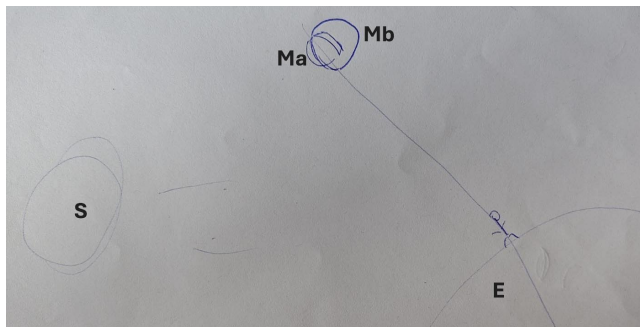
Like maybe it's standing here to... [makes a move to draw but does not really do so, falls silent]

- I: That's a hard question, maybe?
- N: Yeah, that orientation is like a thinking question, you kind of know it's going to be somewhere...

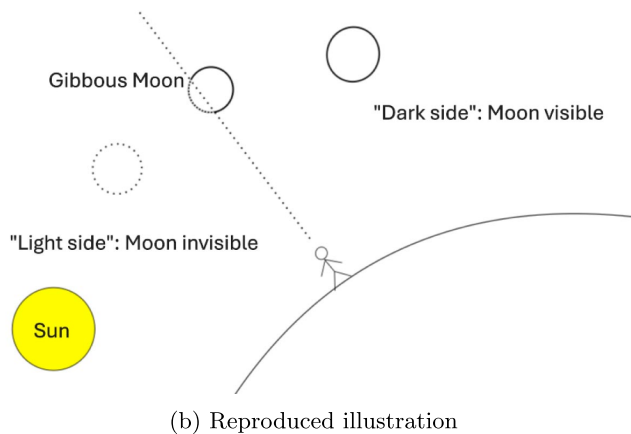
This is a neat counterexample to the reaction of Mila, described in the previous subsection. Like Mila, Noah also holds an Earth shadow conception for Moon phases. However, unlike Mila, Noah does not notice the inconsistency between his explanation and the shape of the gibbous Moon on his own. When it is pointed out to him, he tries to reconcile his explanation with this contradictory insight but fails to do so.

One last and highly peculiar explanation was given by Emily. She divides space into a “light” and “dark” zone, the border between the two being straight upward from the observer. In her model, we see (a part of) the Moon only when it is in the dark zone, because otherwise it is outshone by sunlight. She explains her thoughts in the following excerpt while drawing Fig. 8(a).

- E: Yeah so, here is your little guy [starts making the drawing of Fig. 8(a)], and here is the Sun, and so here is light [draws the line upwards from the drawn person and points to the Sun-side of that line], for example right, and on the other side there is shadow [points to the opposite side]. And the Moon, right, rises here [gestures how the Moon rises from the left of the drawing], uhm, and then for example for someone standing in the light, yeah will then... yeah I guess the Moon would actually not really be visible. And then from here onwards [points at the light-dark border] you'll have for example this... so then you have such a Moon, a half one [draws Moon **Ma** in Fig. 8(a), where the visible half is on the right side], and then that's because this other side [towards the Sun] is not visible because of the light.
- I: Oh okay, yeah yeah.



(a) Original drawing from Emily, with added letters **S**, **E**, **Ma**, **Mb**.



(b) Reproduced illustration

FIG. 8. Emily's model for Moon phases, as (a) drawn by Emily and as (b) reproduced by the authors.

E: And that's what I think because that's the shadow then, this side [opposite to the Sun], and yeah I think it was something like that.

[...]

I: I've brought one last picture, that's this one [shows the picture of the gibbous Moon]. So here you see that the Moon is more than half but also not quite yet full. If you see this Moon, what's that situation then [points to the drawing of Fig. 8(a)]?

E: Yeah, that the Moon... Oh, that's really stupid right, but that the Moon is like this then [draws Moon Mb in Fig. 8(a)], and that little part is here [on the left, "light" side of the border].

I: Ah yeah, so that we do see this larger part [on the right], but not this smaller one [on the left]?

E: Yes.

Textbook resemblances. When asked if they were familiar with the term "Moon phases," multiple students answered that they had learned about them in school, during geography class. Consequently, these students seemed to rely heavily on what they still remembered from those classes, rather than on logical reasoning. For instance, when

Fien was trying to point out the position of the full and new Moon in her drawing of Fig. 9(a), she said; "[...] I am basing this purely on what I still remember from geography. But yeah, it was either this one, or it was, uhm, these two [pointing at two Moons in the drawing] half, or, and then full and, and nothing, but I am not sure anymore what order it was." The drawings made by students like Fien to explain Moon phases showed a strong similarity to typical textbook images on this topic. Apart from Fien, this was especially the case for Alice and Pascale, whose drawings are shown in Figs. 9(b) and 9(c), respectively, accompanied by a typical textbook image in Fig. 9(d).

The students making these memory-based drawings rarely arrived at a correct explanation for Moon phases. In Fien's drawing of Fig. 9(a), she drew a line through two Moons. She believed these to be the correct positions for a half Moon, while actually they are in the positions of the full and new Moon. Alice, who drew Fig. 9(b), talked about the Moon emitting light itself, as discussed before. And Pascale indicated in her drawing of Fig. 9(c) that there would be a crescent Moon in the last quarter position. Therefore, students who copied illustrations they remembered seeing in textbooks did not perform better in explaining Moon phases. We revisit this finding in the discussion.

2. Hypothetical questions

At the end of the interview, we asked students what they thought would happen to Moon phases if (a) the Moon suddenly were much further away from Earth and (b) the Moon suddenly were much larger than it is now, according to the scale model on the table. These questions were found quite difficult by the students. More than half of the answers were that nothing would change to the Moon phases, except for the fact that the Moon would maybe look larger or smaller overall. The students who provided a hypothesis on what would happen to the Moon phases formulated a broad variety of arguments. Some of these were very logical given the explanatory model for Moon phases that the student started from, others less so.

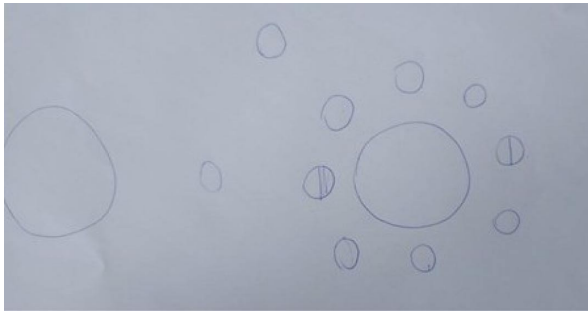
Celine displayed a pure Earth's shadow model for Moon phases. When asked what would happen in case of a more distant Moon, she gave the exact anticipated answer that we would observe a full Moon more often. She explains:

C: I think that if [the Moon] is further from the Earth, I, what I said, that theory, that it can be illuminated more by the Sun so that it would be full more often then, more visible.

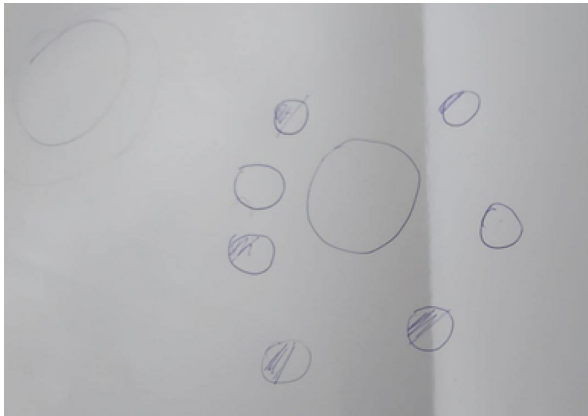
I: Uh-huh, so you would see this less often [points at the photo of the gibbous Moon] and less often a crescent and so on?

C: Yes.

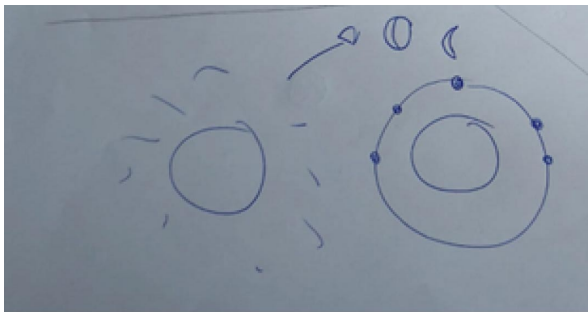
This is a good example of where a student can reason logically from an incorrect explanation as a starting point.



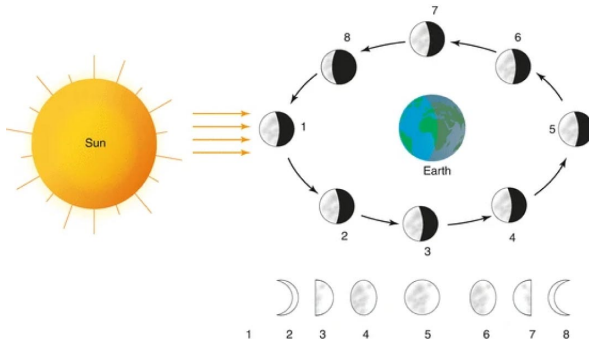
(a) Original drawing from Fien.



(b) Original drawing from Alice.



(c) Original drawing from Pascale.



(d) Typical textbook image. Source: [79]

FIG. 9. Drawings of the Moon phases by (a) Alice, (b) Fien, and (c) Pascale, together with (d) a typical textbook image on this topic [79].

However, Mila explained Moon phases in the same way as Celine but provided a very different hypothesis for the distant Moon question. Mila argues that when the Moon is further from the Earth, it takes longer for the Moon to travel across the shadow of the Earth than when it is close by. She explains her thoughts as follows:

M: If it [the Moon] is further, and the Sun stays at its position, then I think, it will take longer for like a whole... [makes a circular movement with her finger in the air] from new Moon to full Moon will take longer, because...

I: Okay?

M: ...yeah, it just stays longer in the shadow of the Earth, I think.

I: Now, if the orbit...?

M: ...because if it goes equally fast... Yeah then it will go less quickly from here to here [points at the edges of the Earth's shadow zone at the further distance to the Moon in the scale model on the table], then that it would go from here to here [points at the edges of the Earth's shadow zone at the original, closer distance. Her two index fingers pointing at the edges are much closer together now than before].

From her gestures, it is clear that Mila visualizes the shadow zone of the Earth to expand radially outward, instead of decreasing with distance in a cone shape. As such, the distance the Moon has to cover in the shadow zone increases with distance. To further illustrate this idea, Mila uses two miniature Moons to show the Moon's movement at the two distances simultaneously. She argues that because the speed of both Moons is the same, the Moon at the closer distance is already at the far edge of the shadow zone when the farther Moon still has a lot more distance to travel. This is an example where not only the starting explanatory model for Moon phases is wrong, but also the reasoning in this hypothetical question contains illogical elements (like the assumption that shadows extend radially with distance).

Mila also provided an interesting answer to the question of what would happen to Moon phases if the Moon were larger:

M: Yeah that we would... yeah if it's a lot bigger [...] then it will, uhm, never fully lie in the shadow of the Earth, so it will, yeah, never... then we will always be able to see it in a way, I think.

I: And so imagine it's this big [draws a larger Moon on the post-it in the scale model] and it's in the same place as where you'd normally have a new

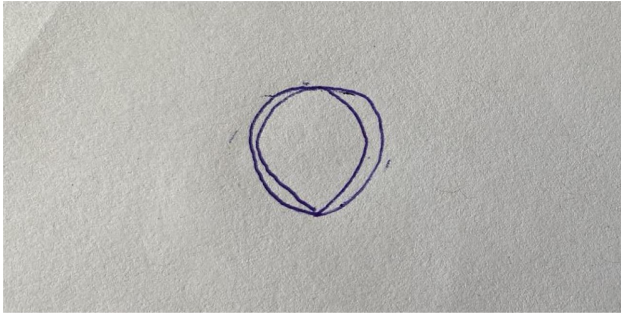


FIG. 10. Shape of the new Moon when the Moon would be larger, as drawn by Mila.

Moon right, so in this place [places the Moon post-it at full Moon position, as that is where Mila indicated new Moon position], can you draw what the Moon would then look like?

M: Then I think just, wait so this is, wait so this is the Moon [starts making the drawing of Fig. 10], then I would think... like this.

I: Ah yeah, so the middle part is what?

M: What we can't see.

I: So the shadow of the Earth?

M: Yes.

I: Okay, and the outer part we can see?

M: Yes.

Unlike her reasoning on the more distant Moon question, Mila's answer to the larger Moon question is very logical.

Students with nonshadow explanations for Moon phases were challenged by the hypothetical questions. For instance, Emma explained Moon phases without mentioning any shadow effects. She could link several positions of the Moon with the corresponding phase but struggled to clearly explain the mechanism of reflection and the role of the Earth's perspective. When asked what would happen if the Moon were further away, she responded as follows:

I: Okay, and imagine hypothetically, the Moon would not be at this distance, but a lot further, like here [moves the Moon post-it further from the Earth], and it would move at this distance and not and that [smaller] distance. Do you think that would change anything in the shapes of the Moon we see? Or in at what times we would see what shape?

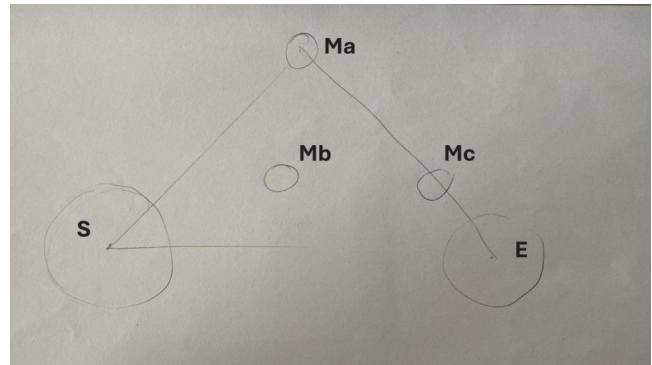
E: Uh, probably yeah.

I: What do you think would change?

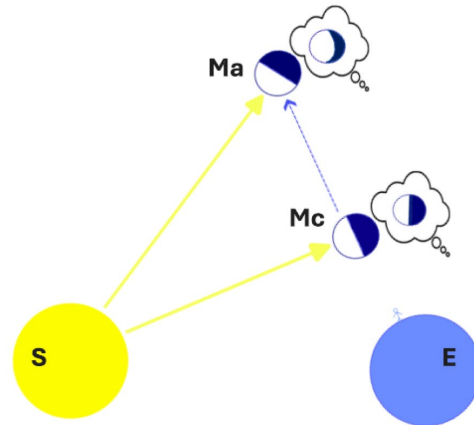
E: Yeah the angle between like, that the Sun forms with the Moon and the Earth [gestures], that would be different right.

I: Okay, can you try to draw that, or is that difficult?

E: Pff, uhm, so if it's here... [draws Moon **Ma** in Fig. 11(a)] Wait, uh... I don't know...



(a) Original drawing from Emma, with added letters.



(b) Reproduced illustration

FIG. 11. Emma's explanation for a differently observed Moon phase when the Moon were farther from the Earth, as (a) originally drawn and (b) reproduced by the authors.

v So we're trying to compare, here it's rather far [points at Moon **Ma** in Fig. 11(a)] and here it's closer [draws Moon **Mc** in Fig. 11(a)]. Would we see another shape when the Moon's here [at **Ma**] compared to when the Moon's here [at **Mc**], you think?

E: Uhm, I think that has to be different.

I: Right, but you can't really explain how different? Or can you?

E: Yeah, I think here [at **Ma**] it's illuminated more than there [at **Mc**].

I: Okay, why?

E: Uhm, yeah the angle here [at **Ma**] is also larger, but I'm not sure...

This argument is another example where scale assumptions are highly relevant. Interestingly, Emma was not the only student making this argument. At least one other student also referred to the changing angles between the three celestial bodies. A third student who could arguably

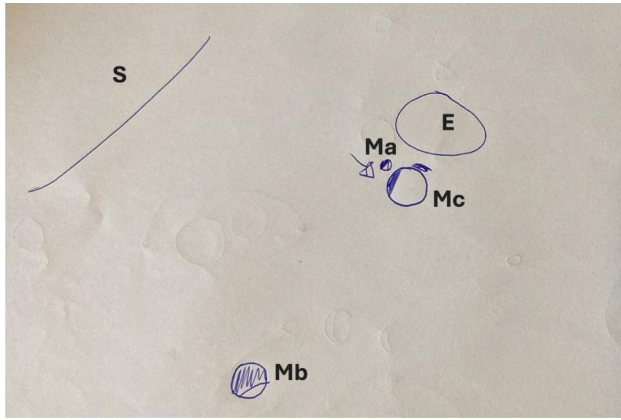


FIG. 12. Drawing made by Pascale while responding to the two hypothetical questions on Moon phases.

be put in the same category was Pascale. Her reasoning was expressed most clearly after the interviewer asked her to draw what she meant. In the following excerpt, she explains her thoughts while drawing Fig. 12:

- I: Wait, could you maybe draw that on a new paper [turns over the paper with previous drawings]?
- P: Yeah [starts making the drawing of Fig. 12 on the backside of the paper], so this was the Earth, and first I had put that here [draws the Moon at position **Ma**] and now the Moon is here [draws the Moon at position **Mb**] and the Sun comes from here. Uh, here [at **Ma**] it will only be able to illuminate the side, say, and here [at **Mb**] there will also be sunrays from here [points at the upper right part of the drawn Sun], so that the front, as we see it, like how we look at the Moon, will also be illuminated by the Sun.
- I: And how will we see it then? Because, what shape was it here [at **Ma**]?
- P: Uh, here it was like, half Moon, so then we'll see this piece [colors part of the Moon at **Ma**].
- I: Okay, and there [at **Mb**]?
- P: Uh, and here [at **Mb**] we'll then see that piece [colors part of the Moon at **Mb**].
- I: Right, so more?
- P: Yes, more.
- I: Right. And imagine that the Moon does not change in distance, but it suddenly becomes a lot bigger [puts the Moon post-it back at the original, smaller distance from the Earth in the scale model], let's say as large as the Earth actually.
- P: Yes.
- I: Do you think that changes anything to the shapes?
- P: Uh then it would get smaller, I think, and then, with the, like actually almost the same reasoning so say the Earth gets, uh the Moon gets bigger [draws the Moon at position **Mc** in Fig. 12]...
- I: Yes?

- P: ...then less sunrays will be able to go to this side, right, to the front [indicates the Earth-facing side of Moon **Mc**], then uh, only the side will be illuminated and then that'll be a smaller piece [colors part of the Moon at **Mc**].
- I: Okay, so instead of half Moon, it will be less here [at **Mc**]?
- P: Hmm, yeah like one quarter or something.

From this fragment and drawing only, it is not easy to follow Pascale's reasoning completely. When the Moon is farther or larger, different proportions of the Moon are illuminated by the Sun because sunrays hit the three Moons in Fig. 12 differently. However, it is unclear how Pascale imagines these sunrays to travel and what prevents the same rays from hitting all three Moons. From her quotes, it could be interpreted that Pascale thinks that for closer Moons, the Earth blocks the rays coming from a certain part of the Sun (the right upper part she pointed to), and that causes part of the Moon to be obscured. However, one cannot be entirely sure of this interpretation, as the interviewer did not ask further clarifying questions.

To conclude, we briefly discuss the interview of Mark. Mark gave an interesting answer to the hypothetical question of a more distant Moon. He responded as follows:

- M: Yeah, well, it might affect the rotation and the orbit around the Earth in such a way, because, euhm, there's always one visible side of the Moon that we always see, you know, the, the Moon doesn't really rotate around its axis [gestures]. We don't, we don't really see the backside of the Moon at all.
- I: Okay.
- M: So we might, might see more of like a backside, if the orbit was to change. So yeah, the relationship between the, the, the rotation of the Moon and its orbiting around the Earth would be different compared to at a small orbit.
- I: And would that imply that we see different shapes?
- M: Uhm, I think we would still see the same kind of like, half Moons and full Moons, but on different surfaces of the Moon, yeah.

We interpret from the above excerpt that Mark refers to the phenomenon of tidal locking, which causes the same side of the Moon to be directed toward Earth at all times. Mark argues that this depends on the "relationship" between the Moon's rotation and revolution, and that this relationship would change in case of a larger distance between the Moon and the Earth. This is a rather sophisticated argument, and although it was not at all anticipated by the authors, it was considered scientifically correct. However, Mark incorrectly states that tidal locking implies that the Moon does not rotate on its axis, which is a common alternative conception in the literature [12,13,22,39,80].

V. DISCUSSION

The results presented in this work strongly confirm that explaining Moon phases is exceptionally difficult for students. Despite the fact that all students learned about Moon phases in school—most of them one year prior to the interview—very few were able to give a correct explanation. Surprisingly, also very few students held a “classical” eclipse model, where all phases of the Moon are caused by the Earth’s shadow. (For the exact number of occurrences of every explanation for Moon phases, see Table II in Appendix A.) Mixed models recurred much more frequently, in which both scientific and alternative causal elements were implemented. For instance, many students were able to point out the positions of the first and third quarters and explain why we then only see half of the Moon, but they were confused when asked about the position of the full Moon. The Earth’s shadow appeared to be a hurdle that students needed to overcome in order to recognize the position of the full Moon as such. Some students were able to move from such a mixed model toward a more correct explanation during the interview by rejecting the idea that the Earth’s shadow (necessarily) obscures the full Moon position. However, other students built further on the erroneous eclipse explanation. They stated, for instance, that a new Moon stands in the shadow of the Earth, and therefore, a full Moon should then be on the opposite side of the Earth, in between the Earth and the Sun. Occasionally, the erroneous interchange of the new and full Moon was extrapolated to other phases as well. For instance, when the full Moon was thought to be in between the Sun and Earth, then the gibbous Moon should be next to that position (the actual crescent Moon position). An extreme example of such reasoning was observed in the interview with Gregory, who eventually switched all Moon phases around. Even though he started with a correct indication of a half Moon, he ended the discussion by indicating that it is always the side of the Moon opposite to the Sun that is visible from Earth.

One reason for the recurring feature of the Earth’s shadow might be the two-dimensional nature of both the scale model on the table and the drawings made by the students. As the literature shows [35,37,38,81], the discrepancy between the two-dimensional representations of Moon phases and the inherently three-dimensional nature of the Sun-Earth-Moon system plausibly enhances the eclipse explanation. Like with textbook images, participants of the interviews were implicitly asked to extract three-dimensional information from the two-dimensional scale model, which is something students are known to struggle with [36]. As such, using a three-dimensional scale model could potentially have alleviated the confusion on the full Moon position, thereby improving students’ explanations.

As an answer to our first research question “Which explanations do students in the last year of high school

suggest for Moon phases?,” we can conclude that there is a wide variety of explanations. We documented many different explanations for Moon phases, some of which were not found in the existing literature. The best example is potentially the explanation of Emily, shown in Fig. 8. It would be interesting to verify whether this explanation resurfaces in replication studies, as it was suggested by only one student in our sample. In Fig. 6, we illustrated the influence of the observer’s geographic position on the observed Moon phase. This idea was proposed by both Noah and Charlotte while drawing Figs. 6(a) and 6(b), respectively. Although several authors have already reported the incorrect idea that different Moon phases are simultaneously seen from different locations [20,21,24,26,33], none of these studies described the exact reasoning of Fig. 6. They either present the location dependence as a mere statement without a description of the students’ reasoning, or the argument behind the location dependence does not align with the argument of Fig. 6. We therefore consider this argument a second newly documented erroneous explanation of students for Moon phases.

The second research question, “How (if at all) do students rely on their ideas of spatial scales in the Sun-Earth-Moon system while reasoning about Moon phases?” requires a rather nuanced answer. On one hand, as the results in Sec. IV A showed, we found no relation between students’ ideas of the spatial scales in the Earth-Moon system and their inclusion of the Earth’s shadow as a cause for Moon phases. On the other hand, multiple student arguments relied on scale assumptions. We discuss a few of those arguments here in detail.

First, the idea that different observers on Earth see different Moon phases at the same time, as illustrated in Fig. 6, is highly scale-dependent. This argument starts from an underlying scale assumption, namely that the Moon is close enough to the Earth for the difference in perspective to be of any importance. However, in Noah’s scale model, the Moon is at ± 20 Earth diameters from the Earth. As such, the difference in observed Moon phase between two observers on opposite sides of the Earth would be minimal. In the scale model of Charlotte, who made the drawing in Fig. 6(b), the Moon was much closer to the Earth so that the drawn argument was much more relevant for her than for Noah.

Second, Emma argued that a more distant Moon has a different phase because the angle from the Moon to the Sun and Earth changes [as drawn in Fig. 11(a)]. This argument is highly scale-dependent. The angle that the Moon makes with the Sun and the Earth depends on the distances to both, and the degree to which that angle changes depends on what is meant exactly by the Moon being suddenly “a lot farther.” However, in Emma’s scale model, the Sun was sufficiently far away for this effect to be negligible. The displacement of the Moon post-it to the further position, as performed by the interviewer, would not have significantly changed the angle from the Moon to the Sun and the Earth.

Like Noah, Emma thus builds her argument on scale assumptions that are not in line with the scale model on the table. However, in the drawings made by Noah and Emma, the three celestial bodies are much closer together than in their scale models [see Figs. 6(a) and 11(a)]. It thus seems that both students base their arguments on their schematic drawings, rather than on their scale-accurate representations of the Sun-Earth-Moon system.

Third, Celine and Mila both held the eclipse model for Moon phases but made opposite conclusions about what would happen if the Moon were suddenly farther from Earth. While Celine argued that it would then be more often full Moon and less often all other phases, Mila reasoned that it would take the Moon longer to travel across the shadow of the Earth. As argued in Sec. IV, we consider the former reasoning much more logical than the latter, because the latter implies a radially extending shadow zone. However, Mila did provide a logical hypothesis on the new Moon's shape in the case of a much larger Moon, as illustrated in Fig. 10. From these observations, we infer that students with identical explanatory models for Moon phases can follow completely different reasonings when answering the hypothetical questions. Clearly, the degree of scientific accuracy and sophistication of these answers can differ greatly, both over different students for one question and over different questions for one student.

We can thus conclude that multiple students attempted to formulate an answer to the questions based on underlying scale assumptions. However, several remarks should be made. First, some of these students built their arguments on scale assumptions that were incoherent with the scale model on the table. It could be hypothesized that these students did not take their scale model into account at all when formulating these arguments. However, it could also be argued that they did take the scale model into account, but had difficulties estimating the magnitude of the considered effect (being the effect of different perspectives for Noah or the effect of a more distant Moon for Mila). Second, it could be argued that the hypothetical questions force students to take scales into account when reasoning about Moon phases, even when they would not be inclined to do so by themselves. While that may be a legitimate critique, it can still be informative to see how students respond to such questions. We observed that roughly half of the participants answered that nothing would change in Moon phases if the Moon were larger or further away. This can be illustrative of two things: the perceived independence of Moon phases on spatial scales and/or the perceived difficulty of these questions. Among the students who did engage in a potential change in Moon phases, we observed both logical and illogical arguments. This reflects how difficult it is for students to reason about Moon phases. Especially the question on the effect of a more distant Moon was challenging, as four students doubted whether it had an influence on the velocity and orbital period of the

Moon. The relation between semimajor axis and orbital period (Kepler's third law) was not something these students had learned yet, which presumably adds to the difficulty of the question.

One factor that appears to hinder logical reasoning on Moon phases is an insufficiently solid understanding of light propagation, reflection, and shadow formation. Many of the illogical arguments documented in this work can be linked to erroneous reasoning on those topics. For example, one recurring conception was that (a part of) the Moon is obscured when the sunlight coming from one particular part of the Sun cannot reach it. This was, for instance, argued by Mila when drawing Moon **Mb** in Fig. 4(a), as evidenced by the sunray hitting that Moon and the accompanying quote. The same erroneous argument was seemingly (but somewhat arguably, as noticed) made by Pascale when thinking about the larger (**Mc**) and more distant (**Mb**) Moons in Fig. 12. Other errors on shadow formation were made, for example, by Gregory (when he indicated the illuminated half of the Moon in Fig. 5) and by Mila (when arguing that the shadow cone of the Earth extends radially outward). This finding is much in line with previous research, which concluded that a necessary requirement to understand Moon phases is an acquaintance with optical concepts such as light reflection, angles, and visual fields [82,83].

The large number of students who suggested scale-independent answers (or no answer at all) should not be omitted in this discussion. Roughly half of the participants' answers to the hypothetical questions fell into these categories. Why is it that so many students seemingly regard Moon phases as independent from spatial scale assumptions, even when Moon phases are believed to be (partly) caused by the Earth's shadow? One hypothesis could be that the cause of Moon phases is stored in students' memory as declarative knowledge. They remember (erroneously) that "Moon phases have to do with the Earth's shadow," just like they often remember (erroneously) that "seasons have to do with the distance to the Sun." As long as they are not triggered to critically evaluate these ideas in light of the involved spatial scales, these conceptions can seamlessly coexist with seriously flawed scale ideas. Only when students are actively stimulated to think about Moon phases in the context of spatial scales (like they were when asked the two hypothetical questions), they move beyond the declarative knowledge level toward effective reasoning. When the declarative knowledge of students is too shallow, this may impede their ability to formulate a logical answer to the hypothetical questions. This hypothesis is supported by the work of Hansen *et al.* [37,84], who examined the difference between students' spatial and declarative astronomical knowledge. They showed how the effectiveness of two distinct learning environments varied between both knowledge domains: a project-based course using a computational model proved

more effective for spatial knowledge, and traditional lectures were better suited for declarative knowledge.

Finally, we found that mere memorization of textbook images did not seem helpful for students to explain Moon phases. This was deduced by comparing students' drawings with typical textbook representations (see Fig. 9) and evaluating these students' explanations. This agrees with findings in the literature that show how textbook representations are often incorrectly or incompletely understood and require sufficient scaffolding by teachers to fully transfer the incorporated knowledge [39–41].

VI. CONCLUSION

This work investigated how last-year high school students explain Moon phases. We paid special attention to the role of spatial scales in these explanations. To this end, we first administered an interactive online survey probing both students' understanding of astronomical scales and their ideas on the cause of Moon phases. Second, we conducted semistructured interviews, with a personalized scale model of the Sun-Earth-Moon system on the interview table. Students were encouraged to use this model to illustrate their reasoning. The survey results showed no relation between students' scale ideas and their inclusion or exclusion of the Earth's shadow as a cause for Moon phases. The interview data show a broad variety of student explanations for Moon phases. Correct and complete explanations were scarce, while the Earth's shadow was mentioned as (part of) the cause by roughly twice as many students. We also documented a number of other alternative explanations for Moon phases, some of which were not found in previous research literature. The role of spatial scale understanding in students' explanations for Moon phases did not emerge unambiguously from our data. On one hand, multiple students made arguments on both the cause of Moon phases and the hypothetical questions that were based on underlying spatial scale assumptions. However, these scale assumptions were occasionally contradictory to the students' scale model on the interview table. Many other students did not take scale assumptions into account in their reasoning, either because they did not deem them relevant or because our questions proved too challenging for them. We argued that besides scales, accurate reasoning on Moon phases can also be impeded by an insufficient understanding of other scientific concepts, such as light propagation and the extraction of 3D information from 2D representations. Future work should explicitly investigate the respective importance of these variables for students' understanding of not only Moon phases but also other (astronomical) topics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because of legal restrictions preventing unrestricted public distribution. The data are available from the authors upon reasonable request.

APPENDIX A: CODING SCHEMES

The coding of student answers was done on the level of the different topics, being (i) the suggested cause(s) for Moon phases and (ii) the responses to the hypothetical questions. No coding with smaller grain sizes (e.g., on the level of individual sentences) was performed. In this appendix, we will elaborate on the coding procedure for both topics. The coding of both topics started from the verbatim transcriptions of the interviews, supplemented with the original recordings when necessary. The codes were constructed using inductive category formation [85], based on the scientific correctness and completeness of the students' arguments. As such, for every newly considered datum, it was decided whether it could be assigned an existing code or required a restructuring or extension of the existing scheme. In the latter case, all previously considered data were reexamined while taking the revised coding scheme into account. This process continued iteratively until it reached saturation, and no adjustments were required to the coding schemes anymore. When students provided a certain explanation but later on changed their minds and adopted a different reasoning, both codes were assigned to their answers.

When all student answers were assigned one or several codes per question, the answers within one coding group were regarded collectively. By repeatedly reading the transcripts and watching the video recordings, it was checked whether these students really expressed the same ideas. Special attention was given to how students used the scale model to explain their ideas, to the gestures they made, and to the drawings they produced. If the explanations were seen as identical, no changes were made to the codes. The researcher then selected one or a few suitable students of this group (based on the unambiguous formulation used in the explanation and clear drawings) to describe the respective model in this work. If not all explanations of one code were deemed identical, some student explanations could potentially be reallocated.

1. Cause for Moon phases

The codes for this topic started with a Roman number, simply indicating whether the student was able to formulate a hypothesis on the cause for Moon phases (II.) or not (I.). In the latter case, this was the full code of that student's answer. To categorize the explanations students offered for Moon phases, we first constructed a list of elements that could potentially contribute to the argument. Examples of these elements were A. Reflection of sunlight by the

TABLE II. Coding scheme for the cause of Moon phases.

Elements	Combination	Specification	No.
II. A. Reflection of sunlight by the Moon	II. AB. Moon reflects sunlight; visible proportion of the Moon depends on relative positions S–E–M	II. AB. 1. Correct and complete explanation	3
		II. AB. 2. Correct but incomplete explanation (e.g. no specification which configuration leads to which phase)	7
II. B. Relative positions Sun–Earth–Moon	II. ABC. Explanation impacted by reflection, relative positions and Earth’s shadow	II. ABC. 1. First and third quarter explained correctly, new Moon occurs when Moon is in Earth’s shadow, confusion on full Moon position	3
		II. ABC. 2. First and third quarter explained correctly, new and full Moon positions interchanged	2
II. C. Shadow of the Earth		II. ABC. 3. Confusion on which Moon side is illuminated and/or shadowed, so that the positions of most to all phases are incorrect	2
		II. ABC. 4. All Moon phases (except full) are caused by Earth’s shadow covering (part of) the Moon	2
II. D. Shadow of the Sun	II. ABF. Explanation impacted by reflection, relative positions and observer location	II. ABF. 1. Different phases for observers at different locations, but only when Moon is near first or third quarter position	0
		II. ABF. 2. Different phases for observers at different locations, for all Moon phases	1
II. E. Moon has dark/light spots and rotates	II. ABCF. Explanation impacted by reflection, relative positions, Earth’s shadow and observer location	II. ABCF. 1. Different phases for observers at different locations, only near first/third quarter	1
		II. ABCF. 2. Different phases for observers at different locations, for all Moon phases	0
II. F. Observer location	II. ABCG. Explanation impacted by reflection, relative positions, Earth’s shadow and intrinsic luminosity of the Moon		1
		II. BD. All Moon phases (except full) caused by Sun’s shadow covering (part of) the Moon	0
II. G. Intrinsic luminosity of the Moon	II. BCD. All Moon phases (except full) are caused by Earth’s and/or Sun’s shadow covering (part of) the Moon		1
		II. E. Moon has (a) dark/bright side(s) and rotates; observed phase depends on which half faces Earth	1
II. H. Others	II. H. Alternative, not classifiable in other categories		1

Moon, B. Relative positions of the Earth-Moon-Sun, C. Shadow of the Earth. For every student answer, we then identified which of these elements were present. The letters of all the present elements were combined to identify that specific group of explanations. As some of these codes still represented a broad range of different explanations, they were further subdivided into more specific types of explanations. These specifications were indicated with Arabic numerals. The full and final coding scheme for this topic (for students providing an explanation, that is, whose code started with II.) is displayed in Table II. Of course, not all possible combinations of the elements in the first column are included in this table. With a list of elements as long as the one in this coding scheme, such an exhaustive set of combinations would be far too long. Therefore, we only included those codes that were either uncovered by previous research or naturally emerged from

the data. In the last column of Table II, we show the number of occurrences of every explanatory model for Moon phases in our dataset.

2. Hypothetical questions

As described in Sec. III, two hypothetical questions on Moon phases were asked to the students. Both considered a hypothetical change in the spatial scales of the Earth-Moon system and asked whether that would invoke a change in how we observe Moon phases. The students’ answers to both questions were coded using the coding scheme in Table III. The first level of each code indicated whether a student thought the new hypothetical scales would cause I. no change in how we observe Moon phases, II. some type of change, or III. the student simply did not know. For students in the second category, a further subdivision of the code was made based on the nature of their argument. In

TABLE III. Coding scheme for the hypothetical questions on Moon phases.

Change?	Reasoning	Specification
I. Nothing changes		
II. Something changes	II.A. Reasoning with shadow model of the Earth	II.A.1. Frequency or duration of (a) certain Moon phase(s) change(s) II.A.2. The configuration of a certain Moon phase alters or interchanges with another, e.g., gibbous Moon becomes full Moon II.A.3. New, currently nonobservable Moon phases would be visible II.A.4. Other type of argumentation
	II.B. Reasoning without shadow model of the Earth	II.B.1. Frequency or duration of (a) certain Moon phase(s) change(s) II.B.2. The configuration of a certain Moon phase alters or interchanges with another, e.g., gibbous Moon becomes full Moon II.B.3. New, currently nonobservable Moon phases would be visible II.B.4. Other types of argumentation
III. Do not know		

group II.A., arguments that involved the shadow of the Earth were categorized, while group II.B. contained arguments that did not. This distinction was made because the Earth's shadow was put forward by many interviewees as (part of) the cause for Moon phases, and we wanted to investigate whether these students reasoned similarly or differently about the hypothetical questions than students with a correct explanation for Moon phases. Moreover, as mentioned in Sec. III, we anticipated certain specific argumentations for a hypothetically larger or more distant Moon by students with an Earth's shadow explanation for Moon phases. As arguments in both groups II.A. and II.B. could still be of very different types, we formulated four specifications to complete the codes in either group. These specifications were indicated by Arabic numerals.

APPENDIX B: INTERVIEW PROTOCOL

In this appendix, we include the interview protocol used in all interviews in this study. The protocol is provided in English, but we note that the interviews with students in Belgium were conducted in Dutch. As the interviews were semistructured, additional follow-up questions could differ between participants. In the following, **I** refers to the interviewer, while **S** refers to the student.

1. Introduction

I: In the next twenty minutes or so, I will ask you some questions about how you think our Solar System and the surrounding space look like, as well as about a few astronomical phenomena. We are purely interested in what you think. You can take as much time as you need to answer, and there are no right or wrong answers as long as you share your own thoughts. I would like to ask you to think out loud as much as possible throughout the entire interview. That can be difficult, but I will remind

you if I notice that you are thinking without saying anything. Does that sound okay to you?

S: [...]

I: Great, thank you.

2. Reviewing the scale model

I: As you can see, I have placed a scale model of the space around Earth on the table. This circle represents the Earth, this one the Moon, and this one the Sun. The relative sizes of these three objects and the distances between them are based on the answers you provided in the online survey a while ago. In that survey, you saw a visualization for each question and had to indicate whether you agreed with it. Do you remember that? Here, we will do something similar: Now that you see everything together, do you agree with this scale model of the Earth-Moon-Sun system? Or would you like to change anything, such as making objects bigger/smaller or moving them?

S: [...]

I: Okay, if at any point during the interview you want to make additional changes, that is still possible.

The interview continues with the topics of apparent Moon and Sun sizes and solar eclipses, which are described in the companion paper [16].

3. Moon phases

I: The next thing I want to talk with you about are the Moon phases. Do you know what I mean by Moon phases?

S: Yes, Moon phases are.../No, I don't know.

I: (Indeed,) the fact that the Moon seems to change shape as seen from Earth; sometimes we see a full Moon, sometimes half Moon and so on. What would you say is the cause of that? You may use

- the scale model on the table or a piece of paper to draw if you like.
- I: Sometimes the Moon is not yet completely full but already more than half—it looks “three-quarters full.” Here is a photo as an example [shows the picture of Fig. 1]. What would the situation be when the Moon looks like this? You can use the scale model or make a drawing if you like.
- I: Suppose now for a moment that the Moon were suddenly twice as far from the Earth as it is now [moves the Moon post-it further from Earth in the scale model]. Would that change anything in how we see the Moon phases? If so, how?
- I: Suppose now for a moment that the Moon were in the same position as before [moves the Moon post-it back to original distance from Earth] but suddenly doubled in size? Would that impact how we see the Moon phases? If so, how?
- I: Okay, that was all. Thank you very much for participating.

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