

Examination of students' quantum physics knowledge construction with peers and metacognitive evaluations

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[This paper is part of the Focused Collection in Investigating and Improving Quantum Education through Research.] As one of the successful theories in physics, quantum physics, constructed out of physical ideas and expressed mathematically, provides the required information for the development of technology by describing atomic or quantum systems. For several reasons related to the characteristics of quantum physics knowledge, students struggle with understanding the concepts forcing them to change their classical thinking. However, previous research showed that discussing quantum physics phenomena with peers helped students in problem solving, improving understanding, and constructing quantum physics knowledge. In this study, we focused on students' knowledge construction in several different quantum physics topics with peer discussions depending on the kind of quantum physics problems and group structure. Examination of students' tests with 45 quantum physics problems and reflection reports for four sessions during the semester indicated that the occurrence of knowledge construction is associated with group size and gender composition but not with the kind of problems. The knowledge construction condition is also related to group size, and satisfaction with knowledge and disagreements in group discussions are important for the construction of quantum physics knowledge.

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I. INTRODUCTION

Quantum technologies provide advantages to existing technologies [1], so scientific understanding of quantum physics is essential for scientists, engineers, and teachers for the future development of science and technology. However, because of the abstract, counterintuitive, and mathematical nature, teaching and learning quantum physics is challenging. Previous pedagogical studies on quantum physics learning indicated students' difficulties in various quantum physics concepts such as Schrödinger equation, measurement, wave function, Hilbert space [2–7], angular momentum and spin [8–10], atomic spectra [11–12], quantization [13], superposition [14–15], probability [16–20], energy levels and transition, wave-particle duality, uncertainty [21–25], Zeeman effect [26], operators, observables, eigenvalues [27–29], and potential wells and tunneling [30–34]. The research conducted in different physics classrooms indicated that students' quantum physics performance was diverse, and they could not fully

understand the fundamental concepts [35]. In addition, it is pointed out that students' reasoning difficulties were similar to the challenges in introductory physics courses, such as overgeneralizing concepts in a context, not distinguishing closely related concepts, and not making sense of formalism [36]. The similarity of students' difficulties in quantum physics compared to those in classical physics could result from inadequate prior preparation, unclear goals, insufficient motivation, and the paradigm shift [35]. On the other hand, it is explained that students' difficulties in quantum physics might result from unproductive framing of quantum physics problems [37]. Effective quantum physics-specific research-based instructional methods and materials were developed to overcome students' difficulties, improve conceptual understanding, or facilitate productive problem solving by an appropriate epistemological framework in quantum physics learning. Visual quantum mechanics [38], quantum mechanics visualization (QuVis) [39], and PhET simulations [40] are the developed computer software that aims to help students visualize concepts and reduce conceptual difficulties in matter waves, potential energy diagrams, probability, wave functions, uncertainty, spin, photon, wave packets, photoelectricity, blackbody radiation, hydrogen atom, and quantum tunneling. Tutorials developed for fundamental physics [41] were also developed for quantum physics, such as Tutorials for Quantum Physics [42], Quantum Interactive Learning

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Tutorials (QuILTs) [43–44], Tutorials on Thinking about Quantum Entities [45], and Tutorials in Physics: Quantum Mechanics [46]. These materials were designed for students working in a group environment to develop their reasoning skills, understanding deeper topics, problem solving, and metacognitive awareness. The peer instruction method developed for the fundamental physics courses [47] was adapted to quantum physics courses [48], providing students an environment to discuss quantum phenomena with peers. The common characteristic of these methods and materials is the increased interaction among students in quantum physics classes, considering that learning is a social process from the sociocultural theory perspective [49]. In addition, discussion about quantum phenomena might help students find their formulations for the freshly learned concepts of quantum physics [50].

II. THEORETICAL FRAMEWORK AND LITERATURE

A. Knowledge construction and metacognition

This research, focusing on students' construction of quantum physics knowledge in peer groups and their reflection on learning, considers the role of the social environment on individuals' cognition. From the sociocultural and cognitive perspective, it examines knowledge construction and relates it to knowledge about knowledge or thinking.

Vygotsky's sociocultural learning theory [49] explains learning as a social process in which individuals interact with each other. Based on the social nature of knowledge construction, common knowledge of individuals collaborating increases [51] because collaborating pairs provide the zone of proximal development [49] that individuals cannot reach alone. In this way, in peer problem-solving environments, students in collaboration perform better than students working alone, and one possible explanation is that students' success in collaborative physics problem solving depends also on how they interact instead of their ability [52].

On the other hand, human communication is also crucial for constructing coherent knowledge structures called mental models [53]. To explain phenomena, individuals use their already constructed knowledge. From the cognitive perspective, for the individuals' reorganization of knowledge, for either weak or complete change (conceptual change), individuals' "dissatisfaction" with their previous conceptions is needed together with the intelligible, plausible, and fruitful nature of the new conceptions [54–56]. At that point, by conflicting ideas, peers might provide each other dissatisfaction with their previous conceptions when they get stuck in solving physics problems needed for knowledge organization.

Like more knowledgeable peers could provide a zone of proximal development for the less knowledgeable ones,

peers are also important for each other's metacognitive processes through reflective feedback. Metacognition (or cognition of cognition) corresponds to individuals' knowledge and experiences of their knowledge and cognition [57]. The knowledge dimension of metacognition involves awareness of one's cognition, and the experience dimension is about regulative activities about cognitive processing. It is considered the most powerful predictor of learning [58], so some physics education research focused on improving metacognitive knowledge and skills through classroom activities [59,60] to understand physics better. So, metacognition is highly important for the cognitive process of knowledge construction. Recent research indicated the influence of metacognition on the construction and revision of mental models of quantization [61] and reviewed metacognitive skills in learning upper-level quantum mechanics [36].

B. Peers in quantum physics learning

In two-round physics problem-solving environments (an individual and subsequent group round) with peer groups, students solve the problems first individually and then work together in the group round. Previous research about two-round problem-solving environments in various physics, chemistry, and biology topics showed an improvement in the percentages of correctly answered questions in the group round [5,48,62–74]. In some group cases, all members might be wrong simultaneously in the individual round; similarly, some group members might be correct. Considering the correctness of the answers in the individual round, two terms about knowledge construction emerged in the previous research [66,71,72]: "Co-construction," which means none of the group members answered the question correctly in the individual round, but the group arrived at the correct answer in the group round, and "construction of knowledge" which is at least one group member (but not all) answered the question correctly in the individual round and the group arrived at the correct answer in the group round. It was indicated that construction occurred in most cases (e.g., questions in an implemented test), and coconstruction was observed less often and at various rates [66,70–72,75]. Some physics education research has also examined what kind of peer discussions students performed during problem-solving activities. It was identified that students tried out, applied, and discussed new concepts with their peers when answering clicker questions in physics classes implementing Peer Instruction [76]. While working on tutorials, they discussed the content by asking questions and taught others by answering questions, discussed practical issues, created a base for discussions, and gave reflection, disagreement, confirmation, explanation, and elaboration about the discussed topics [77]. It was found that only 38% of peer discussions were standard conversations in which students discussed the aspects of at least one multiple-choice alternative and

their responses when answering clicker questions. The remaining 62% were categorized as nonstandard conversations, such as unanticipated student ideas, interpretation of the statistical feedback, and conversation pitfalls leading to unproductive interactions [78]. Research also showed that students' peer discussions could be on social, discursive, and disciplinary content [79].

The research examining peer discussions and the research-based instructional designs facilitating peer discussion are important for better learning of quantum physics. Deslauriers and Wieman [69] examined mastery and retention of conceptual understanding of quantum mechanics at the university level. The study included two equivalent modern physics classrooms with engineering physics students. One of the cohorts was taught through traditional lectures, and the other was taught with interactive engagement methods, including peer discussion and small group activities. The Quantum Mechanics Conceptual Survey (QMCS) [80] was implemented for testing and retesting after several months. The retention rate was high for both cohorts; however, it was identified that the experimental group taught by interactive engagement methods with peer discussion performed better than the group taught by traditional lecturing. As well as Deslauriers and Wieman's research, Singh and Zhu [48] attributed improvement in students' performances in quantum mechanics to peer discussion. The researchers further examined the effects of ConcepTests in peer-instructed, junior-senior level quantum mechanics classes. The researchers analyzed students' quiz scores in three experiment groups in different semesters, with one control group using traditional instruction. The ConcepTests hinted that discussing the questions with their peers in quantum mechanics classes improved students' understanding significantly. A similar result was found specifically on the quantum measurement subject [5]. In addition to previous research on performance gains, Sayer *et al.* [70] related students' knowledge construction to their performance in answering quantum physics questions. The researchers studied the effect of peer discussions through a modified approach to peer instruction in an upper-division quantum mechanics course. The data were collected with different instruments, including quantum physics questions developed over the years by the researchers. These involved prelecture reading quizzes, individual concept tests (ICTs), group concept tests (GCTs), and open-ended quizzes. As a modified approach, students in these classes were given ICTs to solve clicker questions about several different quantum mechanics topics (e.g., hydrogen atoms, identical particles, quantum statistical mechanics, perturbation theory, and the Schrödinger equation). After the individual round, students were asked to answer the same questions following peer discussion and to respond using GCTs. The researchers reported that all members of multiple groups improved their GCT after peer discussions. Few groups

were more productive in their collaborations. The researchers also observed that even when all group members answered the ICT questions wrong during the individual round, all group members could respond correctly on the GCT 31% of the time, which the researchers termed the "coconstruction of knowledge." Similarly, Brundage *et al.* [71] also focused on collaborative learning with peers at undergraduate level quantum physics course for physics students. The researchers implemented "Quantum Mechanics Formalism and Postulates Survey (QMFPS)" [81] in two rounds where students first solved the questions individually and then with their peers without any assistance from the instructor in the scheduled class. The researchers examined the group round performances of the groups who had "at least one correct answer" and "no correct answer" in the individual round by defining output as "construction" and "co-construction," respectively. The findings indicated that the construction of knowledge in quantum physics subjects occurred at higher rates, and co-construction was observed at various rates (0%–71%). The study of Didiş Körhasan *et al.* [82] indicating the role of peers in knowledge construction examined the instructional elements of students' mental models of quantization of physical observables in undergraduate modern physics classes for physics and physics teacher education students in Türkiye. One of the elements of instruction influencing students' mental models was identified as peers. Even in a lecture-based traditional teaching environment, peer interactions were critical to developing students' mental models of quantum physics topics. Analyses of their social interactions, especially involving the nearest person with whom students interacted, indicated that these peers shared similar mental models for the same cases of the phenomenon. The researchers emphasized peers' role in learning quantum physics with the construction of knowledge and links. Following a somewhat different track, Bungum *et al.* [22] employed the sociocultural perspective of learning to identify other effects of peer discussion by studying with preuniversity students to explore the potential of small group discussions in quantum physics learning. Data were collected from nine physics classes from seven upper secondary schools in Norway. Students were divided into groups of two or three, and 96 peer discussions, 55 on wave-particle duality and 41 on Schrödinger's cat were recorded on students' smartphones. After the researchers analyzed these recordings allowing epistemological and ontological questions, 70% of these peer discussions were found to be productive talk, with cumulative and exploratory explanations deepening the understanding of the concepts of quantum physics, leading the researchers to conclude that the use of small group discussions could enhance the understanding of abstract concepts. Besides the cognitive benefits, Shi [73] has also pointed out the metacognitive and affective benefits of peer discussions. The researcher explored the effect of peer interaction on

students' learning of quantum physics concepts using an experimental design studying with two electronics engineering student groups taking the introductory quantum physics course of a physics department at a university in China. Experiment group students were asked conceptual questions, and they answered the questions in two rounds when there was no clear consensus in the class related to a question. The two-point Chinese adaptation of Ireson's [83] scale was implemented as a pre- and post-test for both control and experiment groups. T-test analyses of the one-semester experiment results indicated that the experiment group students performed significantly higher than the control group on the post-test, although their pretest scores were similar. It was concluded that regular peer interactions in quantum physics classes provided opportunities for students to develop awareness of their current beliefs and motivation to consider alternative views if they were dissatisfied with them.

So, previous research about peer discussion in quantum physics courses has shown that students performed better in quantum physics problems, improved their understanding, and constructed and co-constructed their knowledge when they worked with their peers in quantum physics classes [5,22,48,69–71,73,82]. Nevertheless, there is still a need to examine peer discussions in quantum physics teaching and learning concerning the specific issues of quantum physics. For example, considering it is a physical theory constructed out of physical ideas and expressed mathematically [84], what kind of problems might be adapted to quantum physics classes for students' effective peer discussions? While previous research examined the role of gender composition, group formation, and group type on the group round performances in two-round physics problem-solving [62,85], the question remains how this might change for quantum physics problems. In addition, despite students reflecting on their knowledge through peer discussions, they might not be aware of what they know exactly. Discussing students' reflections and real performances in problem-solving groups helps understand group dynamics better in quantum physics education. For these reasons, we aimed to examine undergraduate physics students' construction of quantum physics knowledge with peers considering the nature of questions (conceptual, mathematical, visual) and the nature of groups (number or gender of group members). In addition, we also aimed to examine metacognitive evaluations, where students reflect on their knowledge and experiences [57] in peer learning. This results in the following research questions:

1. In what kind of quantum physics problems does knowledge construction occur more?
2. In what kind of groups does knowledge construction occur more?
3. How do students reflect on their learning experience during the process?

Considering these three questions, findings for constructing quantum physics knowledge (construction and co-construction) with peers might help create effective peer discussion environments.

III. METHODOLOGY

This research is a case study, defined as an “in-depth description and analysis of a bounded system” [86], focused on constructing quantum physics knowledge through peer discussions. Despite its exploratory and explanatory nature of “what” and “how” questions belonging to a qualitative perspective [87], it also used the quantification of qualitative data for the statistical examinations.

A. Sample and course

This study was conducted with third-year physics students registered in a Quantum Mechanics I course. The course includes the following topics: the postulates of quantum mechanics, wave packets, uncertainty, eigenfunctions, eigenvalues, operators, wave mechanics, simple problems in one dimension, harmonic oscillator, hydrogen atom, angular momentum, and spin. The courses covering engineering mathematics, advanced calculus, and modern physics were prerequisites.

Quantum Mechanics I is a 3-h course per week, with 2 h devoted to teaching topics and 1 h for problem solving. Students are given weekly homework, two midterm examinations, and a final examination, all of which are written and open-ended questions. The main instructional approach is traditional lecturing. Class participation is expected from the registered students ($N = 40$) during the 2022-23 fall semester. The ratio of female students is almost one over three.

B. Quantum physics problems and reports

In this research, the data were collected by tests with quantum physics problems and reflection reports, which are self and group reports. For the tests, quantum physics problems from previously developed research-based instruments such as Quantum Mechanics Survey (QMS) [7], Quantum Interactive Learning Tutorials (QuILTs) [43,44], Quantum Mechanics Conceptual Survey (QMCS) [80], Quantum Mechanics Formalism and Postulates Survey (QMFPs) [81], Quantum Mechanics Conceptual Test (QMCT) [88], Quantum Mechanics Visualization Instrument (QMVI) [89], and Quantum Mechanics Concept Assessment (QMCA) [90] in PhysPort [91] were obtained. Overall, 45 research-based multiple-choice problems were selected according to the course content, varying the number of problems in each test.

In the self-reflection reports, students stated how they benefited from the peer discussions by selecting up to three items from a given list [92] that best fit their experiences

with group discussions and explaining their reasons below the selected items, such as “I have learned the concepts that I never knew,” “I have learned the concepts better than I knew little,” etc. In the group reports, students were requested to reflect on the process of group discussions answering questions about the reasons for changing their answers from the individual round, in which problems they worked best or worst as a group, and how difficult it was to give a group answer after discussions. In addition, students were requested to select up to three statements from the given list [93] developed regarding conceptual change [54–56]. Students described their interactions with peers during discussion. This part aimed to identify to what extent students joined peer discussions solving quantum physics problems as a group, and so to what extent productive peer discussions occurred. In addition, students stated in self and group evaluation forms other issues, if any. Through self and group reports, students provided additional information on the specific cases they experienced and information about the nature of quantum physics problems and groups.

C. Research context

A total of 45 quantum physics problems were given to students in four sessions during a 15-week semester. Each group problem-solving session fit well with the topics taught and was implemented after two or three traditional instructor-centered lectures. Students worked with their self-constructed groups and solved the problems individually and then with their group members through peer discussion. Figure 1 presents a sample for the group problem-solving process describing the seating positions of students in the individual and group rounds together with the use of instruments.

As presented in Fig. 1, for each of the four group problem-solving sessions in the lecture hall, a printed test with quantum physics problems (test paper) and an answer form were distributed to each student. Students answered the problems in the test individually within 10–15 min. They solved the problems on the test papers and stated their answer choices both on the test papers and the answer forms. At the end of the individual round, only answer forms were collected considering students would use their

notes or calculations from the individual round for the group round. This was also used to control the threat to validity, such as the probability of changing individual round answers based on other members' answers or group decisions. Students were not informed about the time for the tests, and no feedback was given for the individual round.

After the individual round, students were requested to form groups for peer discussions in a subsequent group round in a few minutes. During this process, some students changed their initial seating positions to join a group. A printed group test was distributed to each peer group. In the group round, students solved each problem with peer discussion in 15–20 min. The groups with a small number of members (2 or 3) and the members of large groups (4–6) made some effort to increase interaction to discuss better. For example, the students of a peer group who sat on the front-line seats turned around to the group members in the back line. And even if four students were seated side by side, they discussed intensively by getting close to each other. During the whole process, individual and group rounds, students were not allowed to get information from textbooks, notebooks, or the Internet. In addition, students were not permitted to copy or note the problems to prevent students' unsupervised access to the problems as a precaution to keep the reliability and validity of research-based instruments. After the group round, self and group evaluation forms were distributed to each student to get reflections about their experience and ideas about the problems, groups, and themselves. Then, all tests and forms were collected.

Each problem-solving session took one class hour—50 min. The Appendix presents information about group problem-solving sessions. With the display of the number of problems and peer groups in the four group problem-solving sessions, a total of 389 peer discussion group instances existed. Students were asked to answer a minimum of 8 and a maximum of 14 multiple-choice quantum physics problems in each of the 4 tests. In the group problem-solving sessions, a minimum of 7 and a maximum of 10 peer groups were formed, the number of members varying from 2 to 6. The gender composition of the groups varied. The same-gender groups correspond to the groups

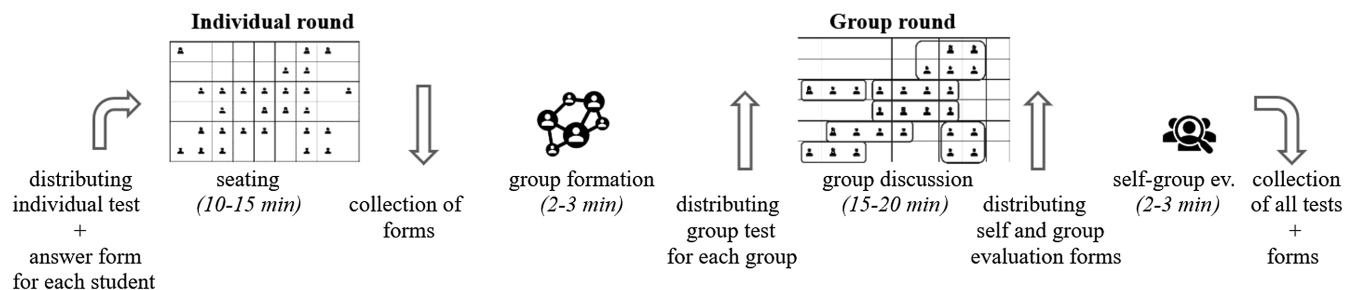


FIG. 1. Group problem-solving process.

with only female or only male students, and mixed groups include females and males together. Over four sessions, the number of students varied between 27 and 34, and 119 self and group reports were obtained (3 missing out of 122).

D. Data analysis

1. Coding of data

Students' tests (answer forms and problem solutions) and reflection reports (self and group reports) were prepared for the data analysis. All the data were coded qualitatively with the following coding scheme, and then qualitative and statistical analyses were performed on the coded data.

Codes for knowledge construction. All possible conditions about the change of an answer in the group round are considered: positive change from incorrect to correct answer, negative change from correct to incorrect answer, and no change from in/correct to in/correct answer. Table I presents the codes, letters, and sample symbolic forms for individual and group rounds to examine knowledge construction.

As presented in Table I, in this study, knowledge construction was examined for "positive change" corresponding to co-construction and construction of knowledge. "Co-construction" and "construction" were defined as follows:

- "Co-construction (CC)" means none of the group members answered the question correctly in the individual round, and the group arrived at the correct answer in the group round [66,70–72,75]. For this code, we have subcodes considering the type of incorrect as "all same incorrect," "all different incorrect," and "other" including some of the incorrect answers are the same, some of them are different, and/or no answers are given [62].

- "Construction (C)" of knowledge means at least one group member (but not all) answered the question correctly in the individual round, and the group arrived at the correct answer in the group round [66,71,72]. For this code, we also have subcodes considering the number of members having correct answers in the individual round as "less than half correct" and "equal or more than half correct" [62].

The groups, in which either all the group members answered the problem incorrectly in the individual round, and the group arrived at the incorrect answer in the group round (A), and the groups in which all the group members answered the question correctly in the individual round, and the group arrived at the correct answer in the group round (B) are the groups where no change was observed.

The groups in which all or some of the group members answered the question correctly in the individual round and the group arrived at the incorrect answer in the group round (D) are considered the negative change groups, where students changed from correct to wrong answer.

So, our focus on "knowledge construction" includes both construction (C) and co-construction (CC). Of the 389 instances, knowledge construction occurred in 185 (C = 172, CC = 13) and did not appear in 204 (A = 80, B = 74, D = 50).

Codes for the kind of quantum physics problems. Each of the 45 problems was coded as a "conceptual," "mathematical," or "visual" problem.

- A problem was coded as "conceptual" if the answer did not have to deal with mathematical calculations or formalism and reasoning was based on concepts. The second problem of QMCS [80] is an example of this coding. The problem asks what the result of measuring the distance of the electron from the nucleus in a hydrogen atom in the ground state would be.

TABLE I. The codes, letters, and sample symbolic forms for knowledge construction. The table presents a sample for the groups with three members; however, the coding rationale is the same for all groups with members different than three. Each sign represents a student's answer: "X" corresponds to the "same incorrect" answer, "x" corresponds to the "different incorrect" answer, "O" corresponds to the "correct" answer, "*" corresponds to any of "X" or "x" or "no answer (na)," "." corresponds to any of "X" or "x" or "no answer (na)," which is different from the following ".*".

Change in answer	Knowledge construction condition	Code letter	Special condition for individual round	Symbolic form for individual round	Symbolic form for group round				
Positive	Co-construction	CC	CC1	All same incorrect	<table><tr><td>X</td><td>X</td><td>X</td></tr></table>	X	X	X	O
			X	X	X				
			CC2	All different incorrect	<table><tr><td>x</td><td>x</td><td>x</td></tr></table>	x	x	x	O
	x	x	x						
	CC3	Other	<table><tr><td>.</td><td>*</td><td>*</td></tr></table>	.	*	*	O		
	.	*	*						
Construction	C	C1	Less than half correct	<table><tr><td>O</td><td>*</td><td>*</td></tr></table>	O	*	*	O	
		O	*	*					
		C2	Equal or more than half correct	<table><tr><td>O</td><td>O</td><td>*</td></tr></table>	O	O	*	O	
O	O	*							
No change	No-construction	A	All incorrect	<table><tr><td>*</td><td>*</td><td>*</td></tr></table>	*	*	*	X/x/na	
		*	*	*					
B	All correct	<table><tr><td>O</td><td>O</td><td>O</td></tr></table>	O	O	O	O			
O	O	O							
Negative		D	All/some correct	<table><tr><td>O</td><td>O</td><td>*</td></tr></table>	O	O	*	X/x/na	
O	O	*							

- If the answer to a problem deals with mathematical calculations or formalism and reasoning based on formulas, this type was coded as a “mathematical problem.” The third problem of QMS [7], asking the probability density (at time t) of a given wave function, is an example of a mathematical problem.
- If the information in a given visual element is used to get the answer and reasoning is based on the interpretation of the visual representation, this type of problem is coded as a visual problem. Conceptual or mathematical information could be presented as a visual representation. The 32nd problem of QMCA [90] is an example of a visual problem with conceptual information. The problem requests selecting the best energy eigenstate wave function sketch that shows the important qualitative features of the wave function for a given system. The 13th problem of QMVI [89] is an example of a visual problem with mathematical information. The problem asks for the energy of the lowest possible allowed energy state for a particle in an infinite well whose probability density is presented.

In this research, 18 of 45 quantum physics problems are conceptual, 14 are mathematical, and 13 are visual.

Codes about kind of groups. This code category has two separate and independent codes for the nature (kind) of groups: Group size and gender composition.

- First, for the group size, the groups were coded as “small groups” or “large groups” considering the number of group members. Groups of 2-member and 3-member were coded as a “small group”, whereas 4-member, 5-member, and 6-member groups were coded as a “large group”. Among 389 peer discussion instances, 185 of them ($N = 63$ for 2-member groups, $N = 122$ for 3-member groups) were small groups, and the rest 204 ($N = 159$ for 4-member groups, $N = 34$ for 5-member groups, $N = 11$ for 6-member groups) were large groups.
- For gender composition, if a group is composed of students of the same gender, either all members are girls or boys, we call this group a “same-gender group”. Another type of group, which is composed of girls and boys, no matter the dominance of gender in a group, we call a “mixed group.” Among 389 peer discussion instances, 154 ($N = 20$ for the groups with only girls, $N = 134$ for the groups with only boys) were same-gender groups, and the remaining 235 were mixed groups.

Coding of self and group reports. Students' explanations in the self and group evaluation forms were coded for “metacognitive reflections.” Metacognitive reflections of students were coded into three main categories: “awareness” of individuals' knowledge or thinking, “satisfaction” of individuals' knowledge, and “strategy selection,” which

is the selection of strategies for individuals' cognitive processes [61]. For the data analyses, 35 peer groups (see the Appendix) for four problem-solving sessions were categorized into three parts considering the positive change presented in Table I. That is, considering the number of occurrences of construction and co-construction in the groups in each of the four problem-solving sessions as (i) strong peer groups, whose members answered most of the problems correctly in a problem-solving session's individual and group rounds (B groups), (ii) co/construction peer groups, where construction and co-construction (C and CC groups) occurred in equal or more than half of the problems in a problem-solving session, (iii) novice peer groups, where construction and co-construction occurred in less than half of the problems in a problem-solving session, or negative change (D) and no-change (A) were observed.

The selection of three statements from the given list describing their interaction with peers during the discussion best was coded for “occurrence of productive peer discussion in the group round.” The statements were [93]:

1. “I answer the questions by myself without discussion.
2. We usually all had the same answer in the first round so there was no discussion.
3. We discussed what we had already known and arrived at the same conclusion.
4. We constructed new knowledge by combining our knowledge together.
5. We did not know how to approach the questions, so we usually just guessed.
6. We had different opinions and we kept our own ideas.
7. We were confused because we had different opinions.
8. We constructed new knowledge by revising our originally different opinions.”

The frequency of the selected items was used for the “occurrence of productive peer discussion in the group round.” The group members' chosen items in the group report after each test were organized first (see the Appendix). The numerical data for each item were separated according to group size and gender composition. Then, for the statistical analyses, the frequency of these categories was determined concerning group size and gender composition, and the percentage of each item was calculated. The third, fourth, and eighth items were coded for “productive peer discussion” considering weak or radical change [54–56].

2. Analysis

Both qualitative and quantitative analyses were used to answer the research questions. All the 389 peer discussion instances in test data were assigned to the codes. The qualitative codes were determined considering the research

aim, and then the data were assigned to the codes by consensus with the discussion of each. The resulting data were analyzed with descriptive statistics by transforming qualitative findings into numbers. Students' reflections on their learning experience during the group problem-solving process were analyzed using codes determined according to the literature and emerging data. Both parametric and nonparametric tests were used for statistical analysis of continuous and categorical data to investigate differences and relations among variables (e.g., knowledge construction, kind of quantum physics problems, kind of groups, etc.). Direct quotations and solutions from students' reflection reports and problem solutions in tests are presented in the Results section.

IV. RESULTS

The analysis of the 389 peer discussion instances for quantum physics problem solving with peers showed that in 172 groups (44.2%), some of the students had correct answers in the individual round, and the group could find the correct answer in the group round (construction, C). In 13 groups (3.3%), none of the members had a correct answer in the individual round; nevertheless, the group arrived at the correct answer in the group round (co-construction, CC). 204 groups (52.2%) were attributed to the “no-construction” condition (A, B, D) corresponding to no change or a negative change from the individual round to the group round. Table II presents frequencies and percentages for the knowledge construction condition.

As presented in Table II, we focus on the groups that have individual round correct answers and do not. From the 221 group instances in which at least one, but not all group members answered the question correctly in the individual round (C^a and D^a), 172 groups arrived at the correct answer in the group round (C, 77.8%). Similarly, from the 93 groups in which none of the group members answered the problem correctly in the individual round (CC^b and A^b), 13 groups arrived at the correct answer in the group round (CC, 14.0%). In addition, from the 75 groups in which all of the group members answered the problem correctly in

the individual round (B^c and D^c), 74 groups arrived at the correct answer in the group round (98.7%), indicating that students also discussed problems critically when all individual round answers were correct.

For each of the 45 problems, we had correct answer percentages, the ratio of correct answers to the total number of answers (correct and wrong answers and no answers) in both individual and group rounds. The individual and group rounds' correct answer percentages were examined by paired samples *t* test. As expected in the light of the literature, as an outcome of collaboration in the group round, the *t* test results indicated significant differences in the mean percentages of correct answers for the individual and group rounds [$t(44) = -8.414$, $p = 0.000$]. The difference (increase) in mean percentages between the rounds according to the kind of problems, group size, and gender composition are as follows: 15.0% for conceptual questions, 23.1% for mathematical questions, 19.4% for visual questions, 13.9% for small groups, 21.4% for large groups, 17.0% same-gender groups, and 20.1% mixed groups. The differences in the mean percentages of group rounds' correct answers were not significant for the kind of problems [$F(2, 42) = 0.059$, $p = 0.942$], group size [$t(88) = -0.064$, $p = 0.949$], and gender composition [$t(88) = 0.697$, $p = 0.488$]. Two-way between-groups analysis of variance was conducted to explore the impact of the kind of problems and group size on the group rounds' correct answers and the kind of problems and gender composition on the group rounds' correct answer percentages. There was not a statistically significant main effect for the kind of problems [$F(2, 84) = 0.353$, $p = 0.704$] and group size [$F(1, 84) = 0.066$, $p = 0.798$], and the interaction effect between the kind of problems and group size [$F(2, 84) = 1.003$, $p = 0.371$]. The second analysis also showed that there was not a statistically significant main effect for the kind of problems [$F(2, 84) = 0.422$, $p = 0.657$] and gender composition [$F(1, 84) = 0.345$, $p = 0.558$], and the interaction effect between the kind of problems and gender composition [$F(2, 84) = 0.325$, $p = 0.723$].

TABLE II. Frequencies and percentages for knowledge construction condition. D* ($N = 50$) corresponds to groups with all ($N = 1$) or some of the students ($N = 49$) being “correct” in the individual round. Each ^{a,b, and c} represents a specific individual round condition before the group round.

Knowledge construction condition	Frequency and percentage of all conditions	Frequency and percentage according to individual round condition (rate)
Construction	C ($N = 172$, 44.2%)	^a From some correct to correct ($N = 172$, 77.8%)
Co-construction	CC ($N = 13$, 3.3%)	^b From all incorrect to correct ($N = 13$, 14.0%)
No-construction	A ($N = 80$, 20.6%)	^b From all incorrect to incorrect ($N = 80$, 86.0%)
	B ($N = 74$, 19.0%)	^c From all correct to correct ($N = 74$, 98.7%)
	D* ($N = 50$, 12.9%)	^c From all correct to incorrect ($N = 1$, 1.3%) ^a From some correct to incorrect ($N = 49$, 22.2%)

TABLE III. Chi-square test results for knowledge construction depend on the kind of problems.

Examination	Compared groups, N	χ^2
Occurrence of knowledge construction depending on the kind of problems	C and CC vs A, B and D, 389	1.797, d.o.f. = 2, $p = 0.407$
Knowledge construction condition depending on the kind of problems	C vs CC, 185	3.489, d.o.f. = 2, $p = 0.165$
Construction with different rates of individual round correct answers depending on the kind of problems	C1 vs C2, 172	1.328, d.o.f. = 2, $p = 0.515$
Co-construction with different types of individual round incorrect answers depending on the kind of problems	CC1, CC2, and CC3, 13	3.993, d.o.f. = 4, $p = 0.486$

A. Construction of quantum physics knowledge with different kinds of quantum physics problems

To examine the role of the kind of problems with the first research question, “In what kind of quantum physics problems does knowledge construction occur more?”, Chi-square tests were performed. Table III presents the test results for knowledge construction depending on the kind of problems.

As presented in Table III, first, a Chi-square test was performed to examine the differences in the occurrence of knowledge construction (C and CC groups vs A, B, and D groups) according to the kinds of problems (conceptual, mathematical, and visual). No significant difference was identified in the occurrence of knowledge construction depending on the kind of problems [$\chi^2(2, 389) = 1.797, p = 0.407$]. Then, the Chi-square test was performed to examine the differences in the knowledge construction condition (C vs CC groups) regarding the kind of problems indicated no significant difference [$\chi^2(2, 185) = 3.489, p = 0.165$].

Table III presents also further analysis of the knowledge construction. A Chi-square test was performed to examine the differences in the construction with different rates of individual round correct answers (fewer and more; C1 vs C2 groups) regarding the kind of problems. No significant difference was identified in the construction with fewer and more correct answers in the individual round depending on the kind of problems [$\chi^2(2, 172) = 1.328, p = 0.515$]. The final Chi-square test performed to examine the differences in the co-construction with a different type of individual round incorrect answers (CC1, CC2, and CC3 groups) according to the kind of problems showed also no significant difference in the co-construction with different types of individual round incorrect answers depending on the kind of problems [$\chi^2(4, 13) = 3.993, p = 0.486$]. These results show that the construction with different rates of individual round correct answers and co-construction with varying types of individual round incorrect answers are not associated with the kind of problems.

Students' explanations in the group reports for the questions asking which problem their group worked on best or worst also indicated no statistical difference

regarding the kind of problems. Excluding “none of them,” “all of them,” and “no answer,” 91 and 70 explanations were attributed to the problems students worked on “best” and “worst,” respectively. Visual problems were the most stated problem type, both for worked best (46.2%) and worst (41.4%) as a group, by students. A Chi-square test was performed to examine the differences in the students' attributions for problems that worked best or worst according to the kind of problems (conceptual, mathematical, and visual). No significant difference was also identified in the attributions depending on the kind of problems [$\chi^2(2, 161) = 1.185, p = 0.553$]. No matter the kind of problem, qualitative findings also showed that students' reasons for the best and worst discussions were mainly about students' own or group members' limited knowledge of quantum physics topics and mistakes made during problem solving. A few students' reasons for good or bad discussion experiences pointed out group communication issues about agreements and convincing each other, as well as result orientation instead of the discussion process, which is how easy or difficult they agreed on.

B. Construction of quantum physics knowledge in different kinds of groups

To answer the second research question, “In what kind of groups does knowledge construction occur more?”, we focused on both group size (small and large groups) and gender composition (same-gender and mixed groups). Table IV presents the results of Chi-square tests performed for knowledge construction depending on group size and gender composition.

In the examination of the role of the kind of groups presented in Table IV, first, Chi-square tests were performed to examine the differences in the occurrence of knowledge construction (C and CC groups vs A, B, and D groups) according to group size (small groups and large groups), and then gender composition (same-gender groups and mixed groups). There was a significant difference in the occurrence of knowledge construction depending on the group size [$\chi^2(1, 389) = 19.970, p = 0.000$] and the gender composition of groups [$\chi^2(1, 389) = 6.456, p = 0.011$], where the effect strength phi was small

TABLE IV. Chi-square test results for knowledge construction depend on the group size and gender composition.

Examination	Compared groups, N	χ^2
Occurrence of knowledge construction depending on the group size	C and CC vs A, B and D, 389	19.970, d.o.f. = 1, $p = 0.000$
Occurrence of knowledge construction depending on the gender composition	C and CC vs A, B and D, 389	6.456, d.o.f. = 1, $p = 0.011$
Knowledge construction condition depending on the group size	C vs CC, 185	4.076, d.o.f. = 1, $p = 0.045$
Knowledge construction condition depending on the gender composition	C vs CC, 185	0.191, d.o.f. = 1, $p = 0.761$
Construction with different rates of individual round correct answers depending on the group size	C1 vs C2, 172	3.661, d.o.f. = 1, $p = 0.056$
Construction with different rates of individual round correct answers depending on the gender composition	C1 vs C2, 172	0.003, d.o.f. = 1, $p = 0.954$
Co-construction with different types of individual round incorrect answers depending on the group size	CC1, CC2, and CC3, 13	2.438, d.o.f. = 2, $p = 0.487$
Co-construction with different types of individual round incorrect answers depending on the gender composition	CC1, CC2, and CC3, 13	3.611, d.o.f. = 2, $p = 0.315$

for both variables with -0.227 and 0.129 , respectively. These results indicated that the occurrence of knowledge construction (C and CC) was observed in large groups statistically different than in small groups, and mixed groups statistically different than in same-gender groups. So, the occurrence of knowledge construction seems associated with group size and gender composition.

Then, Chi-square tests were performed to examine the differences in the knowledge construction condition (C vs CC) according to group size and gender composition. A significant difference was identified in the knowledge construction condition depending on the group size [$\chi^2(1, 185) = 4.076, p = 0.045$], where phi for the strength of association was small (0.148). However, no significant difference was identified in the knowledge construction condition depending on the gender composition [$\chi^2(1, 185) = 0.191, p = 0.761$]. These results indicated that knowledge construction (C) was observed in large groups statistically different than in small groups, and co-construction (CC) was observed in small groups statistically different than in large groups. However, an association between the knowledge construction condition and gender composition was not identified.

For the detailed analysis of the knowledge construction, Chi-square tests were performed to examine the differences in the construction with different rates of individual round correct answers in the individual round (fewer and more; C1 vs C2 groups) according to group size and gender composition. No significant difference was identified in the construction with fewer and more individual round correct answers depending on the group size [$\chi^2(1, 172) = 3.661, p = 0.056$] and the gender composition of groups [$\chi^2(1, 172) = 0.003, p = 0.954$]. Final Chi-square tests were performed to examine the differences in the co-construction with a different type of individual round incorrect answers (CC1, CC2, and CC3 groups) according

to group size and gender composition indicated no significant differences in the co-construction with a different type of individual round incorrect answers depending on the group size [$\chi^2(2, 13) = 2.438, p = 0.487$] and the gender composition of groups [$\chi^2(2, 13) = 3.611, p = 0.315$].

C. Cases for quantum physics problems about (co-)construction of quantum physics knowledge

For closer analysis, some cases were selected and dealt with students' individual and group round problem solutions. With selecting two cases from each conceptual, mathematical, and visual problem type, co-construction (CC) and construction (C) cases are discussed for the construction of quantum physics knowledge concerning the kind of groups regarding group size and gender composition. Selected cases had also different properties in terms of the number and rates of knowledge construction, percentage of individual and group round correct answers, and quantum physics topics. Table V presents the cases for the occurrence of knowledge construction with the frequency and rate of co-construction and construction, together with the problem specifications.

The 16th problem [QMVI, Ref. [89]] was a visual problem that examined students' understanding of probability density. Six knowledge construction instances and one co-construction were observed, and the rates of construction (C) and co-construction (CC) in the groups in this problem were 100% and 100%, respectively. That is, all students who were incorrect in the individual round found the correct solution in the group round. The problem asking the probability of position measurement of a particle in a specified region ($+2a, +3a$) required students' conceptual understanding of the normalization of the wave function and mathematical tasks for calculating probability where the relationship with the wave function is

TABLE V. Cases for occurrence of construction and co-construction of knowledge. C, Conceptual; M, Mathematical; V, Visual. QMS [7], QuILTs [43,44], QMCT [88], QMVI [89], QMCA [90]. *QuILTs: Spin 1/2, concept test 5.1. Rate means the comparison of C^a and D^a and CC^b and A^b in each problem (see Table II).

Problem no.	Problem ID	Kind	Subject	Occurrence of C and CC (number)	Individual and group round correct percentage (%)	Rate of C (%)	Rate of CC (%)
4	QMS 9	M	Expectation value	4, 1	25.8–67.7	100	20
6	QMS 20	C	Harmonic oscillator	3, 2	25.8–58.1	75	40
11	QMCA 13	C	Wave function	2, 0	32.3–35.5	50	0
15	QMCT 11	V	Potential step	5, 1	44.1–91.2	83.3	100
16	QMVI 6	V	Probability density	6, 1	58.8–100.0	100	100
40	*QuILTs	M	Spin	3, 3	16.7–70.0	100	60

constructed. The increase in correct answer percentages after the group round is 41.2% (58.8%–100.0%) overall since 14 (out of 34) students who selected the wrong answer in the individual round changed their answer in the group round. Five of the six construction (C) instances were groups with equal or more than half correct answers in the individual round (C2). A total of 11 comments about this problem were given by students. A member of a group with less than half correct answers (C1) in the individual round, which was also a mixed and small group with three members, explained that “because the idea of one of us was logical and the other two of us learned something.” Two students from the same C2 group, a mixed and large

group with six members, reflected on their learning as “I learned I should take square” and “it requires a mathematical manipulation based on physical realities.” Two members of another C2 group, a mixed and large group with four members, stated that this was the problem they worked best on with reasoning “we all contributed” and “we calculated together.”

The observed CC group was a mixed and large group with four members. Two of the individual round wrong answers were the same, and the other two were different (CC3), where the group discussion started with three different wrong answers. Figure 2 presents students' individual and group round solutions and drawings.

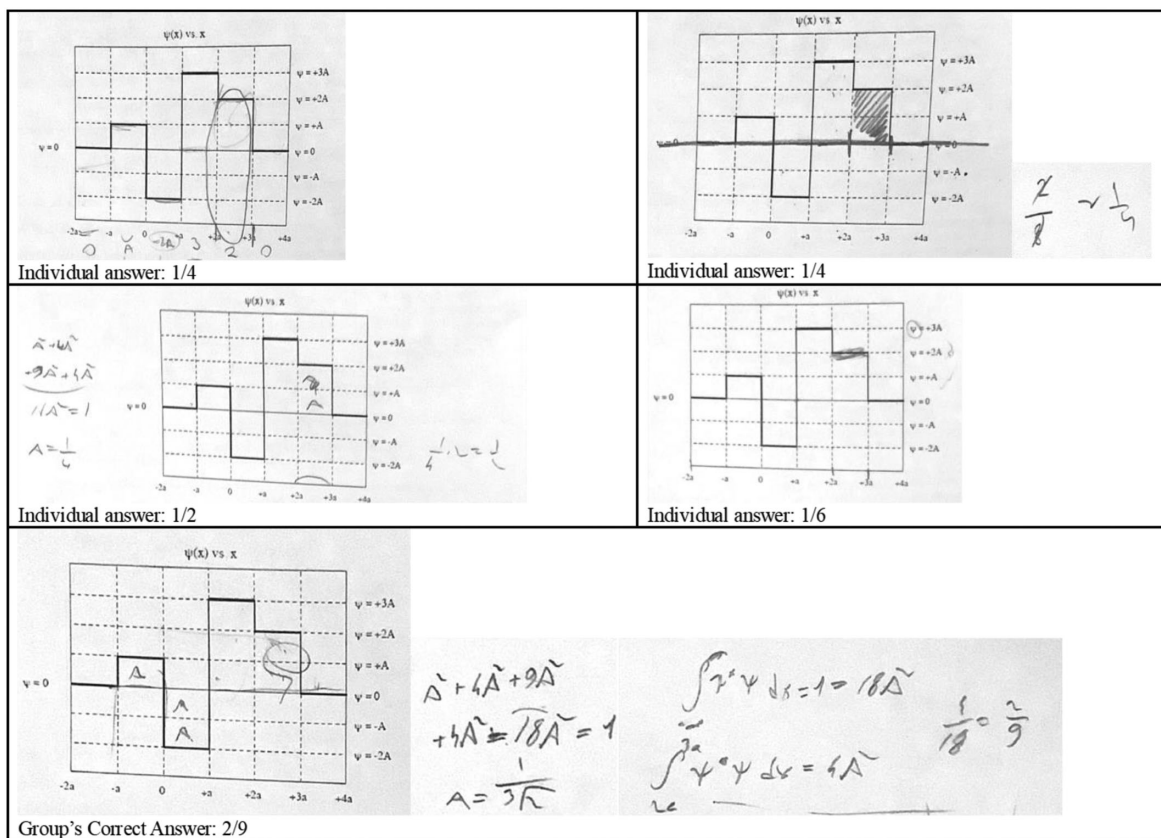


FIG. 2. Students' individual and group round solutions and drawings for the 16th problem.

As presented in Fig. 2, two students' answers (1/4 is a wrong answer selected by a total of 10 students in the individual round) indicated students' addition of the values without considering the square of the wave function for probability measurement, as the other eight students from different peer groups selected this choice in the individual round. The group round solution showed that normalization was discussed during the group round, and probability was calculated correctly, arriving at the correct answer (2/9).

The 40th problem [QuILTs, Ref. [43]] was a mathematical problem that examined conceptions of spin. Three construction and three co-construction instances were identified, and the rate of construction and co-construction in the groups in this problem were 100% and 60%, respectively. The problem asked about the orthogonality of pairs of spin states for spin 1/2 particles with the requirement of students' understanding of vectors and Hilbert Space. The problem preliminaries gave the eigenstates of the same and different spin components and choices were presented according to these conditions. Among all problems, this showed the greatest increase in correct answer percentages after the group round with 53.3% (16.7%–70.0%) overall. Of 30, 16 students who selected the wrong answer in the individual round changed their answer in the group round. One comment was given by a student of a C1 group, a same-gender and large group with four members, indicating that the problem was the problem discussed best in their peer group.

Three observed CC groups were as follows: (i) a mixed and small group with three members in which all members gave all different incorrect answers in the individual round (CC2), and (ii) a mixed and (iii) same-gender and small group with three members who gave partial different incorrect answers in the individual round (CC3). Students' notes on the tests indicated difficulty in

understanding that the eigenstates of different components of the spin operator are not orthogonal to each other, and states could be written as a linear superposition of the eigenstates. Three out of five groups, who answered this question wrong with all members in the individual round arrived at the correct answer in the group round, where the knowledge of spin states co-constructed.

The fourth problem [QMS, Ref. [7]] was a mathematical problem about expectation value. Four construction instances and one co-construction instance were observed, and the rate of construction and co-construction in the groups in this problem was 100% and 20%, respectively. The problem asked the expectation value of the energy for a given system ($t = 0$) by presenting the wave function. It required students' understanding of expectation value with correct representation of mathematics, correct use of the energy operator (Hamiltonian), and accurate understanding of complex numbers for the required calculations. The increase in the correct answer percentages after the group round is large (25.8%–67.7%). Two comments about this problem stated that group discussions were the worst in this problem. A member of a C2 group, which was also a mixed and large group with four members, explained that the “group polarized but we worked through it.”

All groups with less, equal, or more than half correct answers in the individual round arrived at the correct answer in the group round of the problem by construction, where the knowledge of expectation value was co-constructed by the correct mathematical process. However, only one out of five groups that answered this problem wrong with all members in the individual round arrived at the correct answer in the group round. This observed CC3 group was a mixed and small group with three members with two different wrong answers. Figure 3,

Co-construction (a mixed and small group with 3 members)

Construction (a same-gender and small group with 3 members)

Construction (a mixed and large group with 4 members)

FIG. 3. Samples from students' problem solutions are presented for two construction groups and one co-construction group in the group round.

from students' problem solutions, presents some samples of how students fixed problems with complex numbers in the calculations in the group round by construction or co-construction.

Two of the answer options contained the correct general mathematical representation of the expectation. Most students recognized this by selecting only these two wrong options (c and d) in the individual round. However, they had difficulty seeing the concrete expectation value for the given wave function. Accordingly, as presented in Fig. 3, in the group round, students focused on calculations to fix the errors and difficulty in the calculation.

The 15th problem [QMCT, Ref. [88]] was a visual problem that examined students' understanding of potential steps. Five construction instances and one co-construction were observed, and the rate of knowledge construction and co-construction in the groups in this problem were 83.3% and 100%, respectively. With a visual presentation of a potential step, the problem asked for the correct statement(s) about the wavelength of a particle ($E > V_0$). The problem required mainly the proper understanding of the conservation of energy. The increase in correct answer percentages after the group round is 47.1% (44.1%–91.2%) overall. Four comments about the problem belong to the C and CC groups. A member of a CC group (a same-gender and small group with three members) stated how their group discussed best this problem together “because I made a silly mistake and improved with discussion.” Five of the C groups (three C1 and two C2) in the individual round arrived at the correct answer in the group round by knowledge construction. However, there was also one D group. This small and mixed group with three members selected the wrong choice in the group round as “wavelength is decreased,” which had been chosen by two members, whereas one member had answered correctly in the individual round.

The sixth problem [QMS, Ref. [7]] was a conceptual problem that examined students' understanding of harmonic oscillator. Three construction (C) and two co-construction (CC) instances were observed, and the rate of construction and co-construction in the groups in this problem was 75% and 40%, respectively. The problem asked for the correct statements given for the system with answer options about the classical and quantum probabilities, and probability in different states and energies. The increase in correct answer percentages after the group round is medium (25.8%–58.1%) overall. Three of the five comments about this problem were about how group discussions were best for this problem. One member of a C1 group, a mixed and small group with three members, explained “we reminded each other some concepts about wavefunctions.”

Three groups with less than half and equal or more than half correct answers (one C1 and two C2) in the individual round arrived at the correct answer in the group round by construction, but one group did not. Similarly, two out of

five groups that answered this problem incorrectly with all members in the individual round arrived at the answer correctly in the group round. One of the observed CC groups was a mixed and small group with three members and the other was a same-gender and large group with four members.

The 11th problem [QMCA, Ref. [90]] was a conceptual problem that examined conceptions of the wave function. Two construction instances were identified, but no co-construction was observed. The rate of construction in the groups in this problem was 50%. The problem required students to make sense of the wave function and ask what part(s) of the time-dependent wave function is needed to predict the probability of a particular outcome of an arbitrary position measurement. The choices were presented as “only,” “either,” or “both” real and imaginary parts, together with the “not containing information” of a wave function. The increase in correct answer percentages after the group round was small (32.3%–35.5%) overall. One-third of students kept their ideas about only the real part of the wave function needed in the group round. Three members of a C2 group, a same gender and a large group with four members, commented that their discussion was the worst in this problem.

Two groups with less than half and equal or more than half correct answers (one C1 and one C2) in the individual round arrived at the correct answer in the group round of the problem by knowledge construction, but the two groups did not. These groups with a member(s) with individual round correct answers decided the answer after group discussion as “only the real part.” No co-construction was observed since four groups whose all members selected the same incorrect answer, “only the real part,” in the individual round. All members kept their answers the same in the group round and could not arrive at the correct answer with co-construction. These results of a discussion of conceptual problems indicated students' difficulties with the Born interpretation. Together with the conceptual limitations, such as making sense of “wave function” used for describing the wave behavior of particles, which has no physical interpretation, this could be associated with the mathematical limitations, such as making sense of complex numbers.

D. Occurrence of productive peer discussion in the group round and students' metacognitive evaluations

The third research question, “How do students reflect on their learning experience during the process?” was investigated in three categories considering the number of occurrences of construction and co-construction in the groups in each of four problem-solving sessions as (i) strong peer groups with most of the problems correct in the individual and group rounds, (ii) co/construction peer groups with knowledge construction and co-construction in equal or more than half of the problems, (iii) novice peer groups with knowledge construction and co-construction in

less than half of the problems, mostly negative change and no-change are observed.

Students' metacognitive evaluations covered three aspects: awareness of knowledge, which implies metacognitive knowledge or thinking; satisfaction of knowledge, comprising both feelings of being satisfied or feelings of being insufficient or not sure about knowledge; and the selection of appropriate strategies for individuals' cognitive processes during peer discussions reflected by students in reflective reports. Students mostly reflected on how they became aware of their knowledge or thinking through discussions (46.9%). They also commented on their satisfaction with knowledge as explained in peer discussions (29.6%). Some students explained they had some strategies in peer discussions (23.5%). When this was examined according to peer group categories, in strong groups, awareness and satisfaction of knowledge or thinking were dominantly identified with the same percentage (37.5%). In the co/construction and novice groups, awareness was dominant at 45.7% and 48.9%, respectively. In both cases, students stated how they recognized their mistakes and learned the correct answers, remembered the concepts studied before, connected the concepts (e.g., "wave nature of the particle with wave functions of the particle"), learned some concepts previously not known and improved their ideas, or recalled the concepts (e.g., "eigenvalues, momentum eigenstates, motion of a free particle"). In both co/construction and novice groups, nearly 29% of reflections of satisfaction were about strengthening the known (e.g., "probability of a wave," "wave function," "collapse," "bound state," "Hamiltonian," "time-dependent Schrödinger equation," "operators," "commutation relations"). One of the members of a same-gender and small group with three members in the novice peer groups indicated his satisfaction with conceptual aspects but was less satisfied with the mathematics as "I had understood the concepts very well but was a little rusty on some of the formulas. It helped me to further strengthen my knowledge on them. The questions that I changed answers for, I am bound to never forget again their contents." Regardless of all peer groups, 54.7% of students' reflections about dissatisfaction, which implied a feeling of "having insufficient" knowledge about quantum physics concepts. With similar rates, the co/construction and novice peer groups explained they had some strategies. Some of the direct statements of students are as follows: "Working on the problems without detailed calculations," "giving feedback to each other," "brainstorming," "reminding previously learned concepts," "mathematical derivations," "critical thinking," "discussing the point of view of on each member" (24.7% and 22.2%, respectively).

For "Explain your reason to change your answer in the individual round," 100 reasons were presented by students in the 119 self and group reports. Nine categories emerged from the data: (i) conceptual reasons, (ii) mathematical

reasons, (iii) attention, (iv) misreading or understanding about the problem, (v) guess or gut feeling, (vi) peers' logical or reasonable explanations, (vii) more knowledgeable peers, (viii) satisfaction of knowledge or answer, and (ix) group issues, together with a limited number of students stating they did not change their answers.

"Conceptual reasons" covered the students' reflections about misunderstanding concepts, lacking or wrong conceptions, confusing concepts, difficulty in recalling concepts, remembering wrongly, etc. In addition, it included the reasons based on structural mathematics [90]. Some examples from students' reflections in all peer groups were "I learnt that I should take square," "I realized that I did not know some concepts when it comes to imagining actual wave functions," "I didn't consider writing the momentum eigenstates in terms of position vector." This kind of reasons for students to change their answers was dominant (31.9%) in the novice peer groups. Strong groups also indicated only conceptual reasons based on structural mathematics as "I didn't take into account that $\Psi(x, t)$ can also contain imaginary parts." Reasonings about "mathematics," "guess or gut feeling," and "more knowledgeable peers" were identified as very limited, some of them were "I missed some parts in questions or could not foresee the solutions to integrals," "there wasn't reason, just gut feeling," "I just guessed and learned that I guessed wrong," "others know the question well," and "my friend knows better than me." "Attention," which causes students to select an item accidentally and carelessly, "misreading/understanding the problem", "peers' logical/reasonable explanations," and "group issues" such as the decision by considering the majority in the group answer or having consensus were also some reasons mentioned in both construction and novice peer groups. Some examples are as follows: "I accidentally choose the answers other than what is on my mind," "there were some parts that I did not read properly," "my friends' explanation was more logical," "they were the majority so I did it in that way," "we reached a common ground after discussions," "I changed some questions because my peers explained the correct versions." The last category, "satisfaction of knowledge/answer," was dominantly (36.7%) identified in co/construction peer groups. Students' reasons emerged from not being sure about the correctness of the answer, being sure about its wrongness, or not knowing what to do for a particular problem, indicated satisfaction of knowledge or answer, as the most dominant reasoning about a change of individual answer (30%) in overall. Some of the explanations were as follows: "I was not sure of my answer," "I have learned my mistakes," "I realized that my answer for the questions about Hermitian operator and eigenstates are wrong."

Students' evaluations about the difficulty of giving a group answer after discussions were examined with the same categories. The means of assessment for the strong, co/construction, and novice peer groups were 2, 4, and 3

points (out of 7), respectively. Strong peer groups indicated deciding on the answer was easy by saying “since our individual answers were same.” The co/construction peer groups explained their difficulty with different understandings, opinions, and disagreements in the group round. A member of a mixed and large group with four members reflected “we used to have a difference of opinion which confused some turmoil, but we got together to get to the finish line, which was satisfying.” This indicated the role of disagreements in the discussion round since this peer group had construction and co-construction in 9 problems (out of 11). Some expressions stated the importance of satisfaction of knowledge in group decisions “If you have strong reasons to choose your answer and believe yourself it becomes hard,” “we have some disagreements and were not confident in all questions. So, it is hard to decide an answer.” In the novice peer groups, students indicated the easiness of giving group answers by “we have discussed mathematically so it was easier,” and “in most concepts, we agreed with each other but there were questions that required a discussion. Overall, it was moderate, relatively on easy side.” Similar to the co/construction groups, reasons of novice group students also indicated the role of dissatisfaction of knowledge in decision making “Mostly we had the same answers and on those we differed one of us was more sure about it than the others and when that person explained their reasoning the others generally agreed”, “It is quite easy accept for the case no one have any idea”, “None of us were good enough on concepts.”

For the “occurrence of productive peer discussion in the group round,” the frequency of the items was examined. Figure 4 presents the percentages (%) of the items selected

by students according to group size, gender composition, and overall.

Figure 4 presented each item’s percentages to determine to what extent students joined peer discussions solving quantum physics problems as a group and to what extent productive peer discussions occurred. The underlined percentages for third, fourth, and eighth are “we discussed what we had already known and arrived at the same conclusion,” “we constructed new knowledge by combining our knowledge together,” and “we constructed new knowledge by revising our originally different opinions,” respectively, indicated the most selected items by students indicating the nature of peer discussions. According to group size, gender composition, and overall, the total percentages for these items, corresponding to productive peer discussions, indicated that more than half of the peer discussions were productive according to students’ reflections (for large groups 64.1%, small groups 57.6%, mixed groups 59.7%, same-gender groups 66.0%, and overall 61.8%). In other words, with a great proportion (61.8%), peer discussions were described as “productive” for the construction of quantum physics knowledge with either weak or radical change (e.g., revising originally different opinions). In some cases, 38.2% of the peer discussions indicated students’ confusion, guessing, or lack of discussion in the group round, which prevented productive talks. Chi-square tests with the frequency of the items for the occurrence of productive peer discussions in the group round (3, 4, and 8) and not (1, 2, 5, 6, and 7) according to group size and gender composition of groups showed no significant differences depending on the group size [$\chi^2(1, 262) = 1.072, p = 0.351$] and the gender

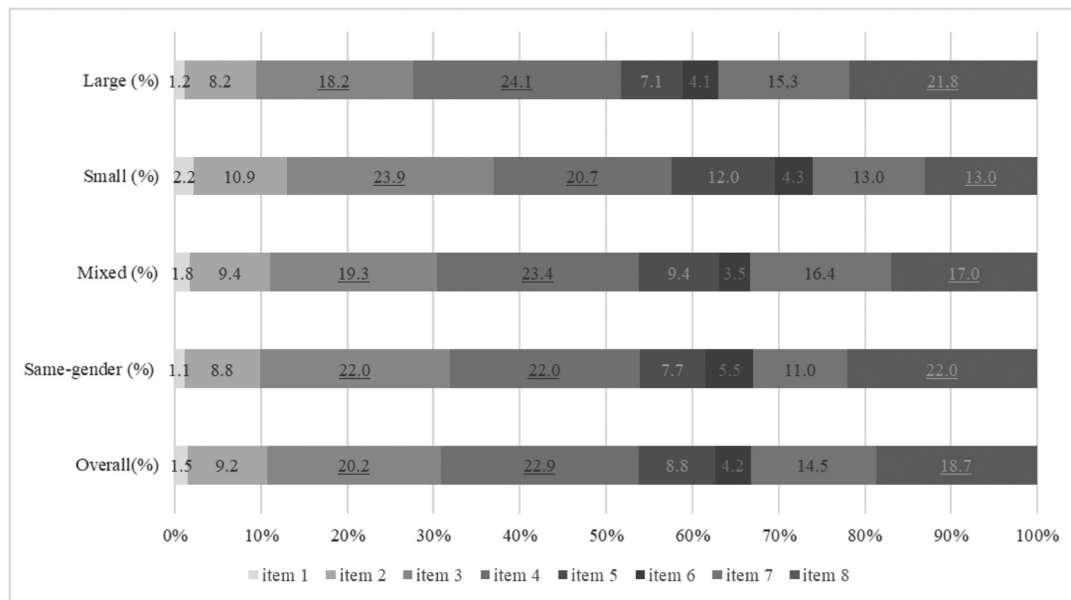


FIG. 4. Percentages (%) of the items according to group size, gender composition, and overall.

composition [$\chi^2(1, 262) = 0.994, p = 0.351$]. Therefore, according to students' reflections, the occurrence of productive peer discussions in the group round did not change for mixed or same-gender groups and large or small groups.

V. CONCLUSION AND DISCUSSION

Our research examining students' construction of quantum physics knowledge in self-constructed peer discussion groups indicated the following conclusions:

Conclusion 1. The occurrence of knowledge construction is associated with group size and gender composition but not with the kind of problems.

The results of this study indicated statistically higher correct answer percentages in the group round than in the individual round similar to previous research [5,48,62–74]. This increase emerges from students' knowledge construction, when at least one of the group members gave a correct answer in the individual round, and co-construction when none of the group members gave a correct answer in the individual round. Among all conditions of 389 group instances, 47.5% co/constructed knowledge. This effect might be the result of coaching by peers during peer discussion, which establishes the scaffolding needed for the learning of complex concepts [70] and reduces students' cognitive load [66]. In this way, students might activate their knowledge by triggering the relevant concepts [66], exchanging knowledge to deepen their understanding [22], or constructing their knowledge during social interaction [51,70].

When the occurrence of knowledge construction was compared with nonoccurrence according to the group size and gender composition, the results indicated construction and co-construction were observed in large groups (4 to 6 members) statistically more often than in small groups (2 and 3 members), and mixed groups statistically more often than in same-gender groups. In these groups, an increased number of feedback-providing peers or an extension of the zone of proximal development by interaction of peers might be advantageous in the knowledge construction and improving the percentages of co-construction (CC) or construction (C) of knowledge in comparison to no-construction groups (A, B, D). Students might feel more comfortable completing quantum physics tasks successfully in larger or mixed groups, and they could solve more complex problems more efficiently than they individually do. Since the students formed their own groups, they might feel also “more effective and productive” and “more receptive to their ideas” [85]. Besides, an increased number of conflicting or new ideas might cause students' dissatisfaction with their own conceptions quickly and promote reorganizing knowledge easier with the increased probability of intelligible, plausible, and fruitful conceptions for the intended conceptual change [54–56]. In addition, in large groups, an increased amount of metacognitive feedback might facilitate students' regulation of

their cognitive processes by searching for the best ways to improve their learning.

Previous examination of students' physics problem-solving group performance for each problem instance indicated the increased rates of arriving at a correct answer depending on the number of students with individual round correct answers in the group. In a calculus-based introductory physics course, the percentage of finding the correct answer was about 97.6% for the groups with all correct individual round answers, 85.3% for the groups with at least one member answered the problem correctly in the individual round, and 30.8% for the groups with all incorrect individual round answers [62]. For the quantum physics course case, the rates of this study were 98.7%, 77.8%, and 14.0%, respectively. These rates indicated that co-construction (CC) and construction (C) cases in quantum physics were smaller than in the introductory physics topics. Besides, previous research also indicated while construction (C) occurred in most of the problems of a test, and co-construction (CC) was observed less often (19%–31%) and with various rates for electromagnetism and quantum physics subjects [66,70–72,75]. In this study, in 32 of 45 problems, the condition for co-construction was fulfilled (in the groups were all incorrect in the individual round), 8 of the problems students co-constructed knowledge with various rates like previous research [71–72].

On the other hand, the occurrence of knowledge construction is not associated with these kinds of problems. In parallel, students' attributions for problems that worked best or worst according to the kind of problems do not differ statistically. The findings about the kind of problems show that knowledge construction could occur regardless of the nature of the problems.

Conclusion 2. The knowledge construction condition is associated with group size but not with the kind of problems and gender composition.

The data showed that both small and large groups had co-construction (CC) and construction (C) cases. However, the results indicated that construction (C) was observed at higher rates in large groups with four to six members. Conversely, co-construction (CC) was observed mostly in small groups with two or three members at higher rates. However, a relationship between knowledge construction and gender composition and the kind of problems was not identified. This means that the observation of construction or co-construction is not statistically different in mixed and same-gender groups and is also not different for conceptual, mathematical, and visual problems. A more precise analysis gave no hints that construction with varying rates of individual round correct answers (C1, C2) and co-construction with different types of individual round incorrect answers (CC1, CC2, CC3) would be associated with the kind of problems, group size, and gender composition. This could be interpreted that construction (C) with different rates of correct individual round answers (either less than

half or equal or more than half correct answers) occurs independent of the group size (either small or large), gender composition (same-gender or mixed groups), and kind of problems (conceptual, mathematical, or visual questions). Similarly, co-construction (CC), depending on a variety of incorrect individual round answers, occurs similarly in small and large groups, same-gender and mixed groups, and conceptual, mathematical, and visual questions.

Conclusion 3. Satisfaction with knowledge and disagreements in group discussions are important for knowledge construction.

Students presented numerous and various metacognitive reflections about the discussions in their peer groups. They mainly explained their awareness of knowledge or thinking (46.9%), satisfaction with their knowledge (29.6%), and the selected strategies during peer discussions (23.5%). In a social environment, peers can be sources and facilitators of each other's metacognitive knowledge and experience. This could be important for their knowledge construction since if students are aware of their knowledge, they could have affective evaluations about it [57], which we call satisfaction with knowledge. Many students in this study stated they were dissatisfied with their quantum physics knowledge when discussing the problems. This also explained the difficulty of arriving at a group decision. The dissatisfaction with knowledge is also important for conceptual change, so this facilitated how they approached the problems to get the correct answer in the group round. Previous research showed the importance of students' satisfaction with their knowledge and conflict with their peers for constructing scientific knowledge structures about quantum physics subjects [61,93]. So regular peer discussions could also give feedback to students about their knowledge condition and improve metacognitive skills throughout a semester. Students' reflections on two-round quantum physics problem-solving experiences could also be used to promote metacognitive skills in a similar way used in general physics subjects [60].

VI. IMPLICATIONS

In our case, many variables such as the occurrence of knowledge construction, knowledge construction condition, construction with varying rates of individual round correct answers, co-construction with different types of individual round incorrect answers, students' reflections about best and worst discussed problems, and the mean percentage of group round correct answers did not depend on kind of problems indicated it is not a significant variable for quantum physics knowledge construction in self-constructed peer groups. In the cases of knowledge construction observed, the solution process for any kind of problem is facilitated by different contributions such as calculation, making sense of new phenomena, or thinking through the concepts of quantum physics. As these conclusions could be applicable to other physics topics,

controlled experiments could be designed for the examination of changing media in students' peer problem-solving environment (e.g., computers or whiteboards).

When there is at least one (but not all) correct answer in the individual round, 22.2% did not arrive at the correct answer in the group round (D), but the construction rate (C, 77.8%) is large for quantum physics knowledge construction. On the other hand, peer groups with no correct individual round answers mainly could not arrive at the correct solution in the group round (86.0%). However, the rest of them (CC, 14.0%) found the correct answer by co-construction. Construction and co-construction of quantum physics knowledge showed that peer discussions helped handle mathematical difficulties and enhance conceptual understanding of various concepts such as wave function, measurement, Hilbert space, probability, uncertainty, operators, and spin, as identified in previous research [2–34]. Knowledge construction by mathematics was based on both the structural and the technical role of mathematics [94,95] such as interpretation of a mathematical form (e.g., the square of the absolute value of wave function) and successfully completing the mathematical calculations (e.g., calculation of probability and expectation value) with complex numbers, integration, or basic algebraic processes. Making sense of tunneling for a real particle, which is entirely quantum physical behavior, can be seen as the conceptual contribution of peer discussions on knowledge construction. Another example of co-construction concerns a group of two same-gender peers having the same incorrect answer for a scattering problem in the individual round but being able to solve it correctly in the group round and finding the correct waveform for a given potential step through peer discussion.

The observation that the occurrence of knowledge construction in quantum physics is only related to group size and gender composition suggests that the students' forming of large or mixed groups for quantum physics problem solving could increase the group round correct percentages and the appearance of construction or co-construction. This kind of assessment would show instructors how much knowledge construction could occur with the increased group round scores. The conclusion of the association of knowledge construction condition with group size might be informative for different uses of two-round problem-solving sessions in physics lessons. For a newly taught physics subject, when students do not have coherent knowledge structures (mental models), and in the individual round wrong answers are more probable, small groups might be more efficient for peer discussions than large groups for co-construction. Alternatively, small groups would be constructed for the students with individual round incorrect answers after an instructor considers the individual round answers. On the contrary, for the practice of subjects, for the cases with correct answers are more probable, students were allowed to construct their

peer groups with more than three members so that construction could be done in large groups. The conclusions of this study would also suggest controlled experiments for the examination of the role of the “instructor” in the construction of quantum physics knowledge by comparing “instructor-constructed” and “self-constructed” peer groups.

In contrast to the findings about the nature of peer discussion of astronomy concepts pointing out that almost 38% of conversations were classified as standard that students discussed on the clicker questions [78], the finding of this study showing 61.8% are productive is similar with the finding of 70.0% of peer discussions were found to be productive talk, with cumulative and exploratory explanations deepening the understanding of the concepts of quantum physics [22]. Although the student groups are different, this could be interpreted as the nature of peer discussion of microscopic and mathematically expressed subjects like quantum physics might be similar since students indicated more productive discussions than the macroscopic subjects. While students may discuss physics content, teach others, and perform metacognitive reflections during peer problem-solving environments, they

might also have conversations that lead to responses based on guessing strategies or incorrect information not represented in any answer option [76–79]. In this study, we may conclude that emerge of productive discussions (61.8%) allowed successful construction and co-construction of knowledge (C and CC groups) by weak and radical conceptual change; however, unproductive ones, including confusion, guessing, or lack of discussion, prevented this (A and D groups). Peer discussions might be critical in cultivating students’ reasoning skills, practicing the interpretation of physical formalism, and making them intellectually active which are necessary to develop a functional understanding of physics [96]. Instructors should integrate peer discussions into quantum physics classes and motivate and enhance students for productive talk of phenomena to construct coherent physics knowledge and provide them awareness and regulation of their own knowledge.

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APPENDIX: INFORMATION ABOUT GROUP PROBLEM-SOLVING SESSIONS

Session no.	Number of questions	Number of groups	Group instances	Research based-problems
1	12	10	120	QMS (six questions), QMCS (one question), QMVI (one question), QMCA (four questions)
2	11	9	99	QMCT (two questions), QuILTs * (four questions), QMVI (two questions), QMCS (two questions), QMS (one question)
3	14	7	98	QuILTs ** (five questions), QMCA (three questions), QMFPS (three questions), QMCS (three questions)
4	8	9	72	QuILTs (two questions), QMFPS (five questions), QMVI (one question)
Total	45	35	389	

QMS [7], QuILTs [43,44], QMCS [80], QMFPS [81], QMCT [88], QMVI [89], QMCA [90] *QuILTs bound and scattering states and possible wave function **QuILTs uncertainty principle and operators ***QuILTs spin 1/2 and Stern-Gerlach experiment.

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