# Toward thinking quantumly: Student reasoning about the enigma of QM

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The physics education research (PER) literature on quantum mechanics (QM) often acknowledges and minimizes what we refer to as the enigma of QM, viewing these aspects of QM as a challenge to overcome, rather than an inherent feature to be leveraged for learning. Using the overlapping theories of knowledge-in-pieces and resource theory as connected to an epistemological framing perspective, we focus on students' descriptions of the enigma of QM and the epistemological frames that students view as relevant when approaching QM. Data comes from interviews with 10 undergraduate physics students who had learned quantum physics topics across different courses in the curriculum, ranging from sophomore-level modern physics to an advanced undergraduate-level QM course. To elicit students' ideas on QM, the interview protocol emphasized their perspectives and empowerment when possible. Iterative rounds of qualitative analysis coded the data for how students described enigmatic aspects of QM, along with three epistemological frames students identified as their reasoning strategies: reasoning about scale, reasoning about the connections between QM and mathematics, and reasoning about the connections between QM and classical mechanics. The results document that students are aware that something about QM is unexplainable, an enigma to acknowledge. We argue that acknowledgment of the enigma of QM is a key part of what we refer to as thinking quantumly. Additionally, the epistemological frame of reasoning about scale has not been widely discussed in the QM-PER literature, which students use to distinguish QM from classical mechanics and to explain why mathematical models are needed. These results are important for amplifying students' voices in how they approach enigmatic aspects of QM. Thus, our results begin to move the QM-PER literature beyond viewing enigmatic aspects of QM as obstacles to be corrected through refined instruction, toward an asset-based orientation as a source of meaningful insights for the learning process.

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## I. INTRODUCTION AND BACKGROUND

Quantum mechanics (QM) is widely considered abstract and technically difficult; furthermore, reasoning in QM is challenging, given the classical-to-quantum paradigm shift [1]. In various QM textbook introductions, there is a commonly acknowledged lack of consensus about fundamental principles, philosophical interpretations, and how anything should be taught, pointing to a broader example of disagreement within the field (e.g., Griffiths, McIntyre, and Townsend [2–4]). On the other hand, some consensus exists about the depth of learning challenges associated with quantum instruction. Sakurai and Napolitano prepare the reader for conceptual difficulty by writing: "We hope that by exposing the reader to a 'shock treatment' at the onset, he or she may be attuned to what we might call the 'quantum-mechanical way of thinking' at a very early stage." [5] Feynman offers details about the difficulties of learning QM in his lectures on physics [6] when he describes it as peculiar and mysterious:

> Because atomic behavior is so unlike ordinary experience, it is very difficult to get used to, and it appears peculiar and mysterious to everyone–both to the novice and to the experienced physicist. Even the experts do not understand it the way they would like to, and it is perfectly reasonable that they should not, because all of direct, human experience and of human intuition applies to large objects. We know how large objects will act, but things on a small scale just do not act that way. So we have to learn about them in sort of abstract or imaginative fashion and not by connection with our direct experience.

This quote by Feynman describes learning QM as surprising and difficult, noting that the field is "peculiar

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and mysterious" for both students and experts. The quote suggests that fundamental peculiarities arise when reasoning about atomic behavior without ordinary experience at the small scale. The quote also raises several questions about how to describe and learn QM as well as how to reason about scales in a "sort of abstract or imaginative fashion." While reasoning about subatomic particles is regarded as perplexing, peculiar, or mysterious, others describe QM as a counterintuitive phenomenon challenging our classical intuitions [7]. Some interpretations emphasize that QM produces strange experimental results [8], while others describe QM as "wonky," "bizarre," or "weird" [9]. While recognizing that there is a range of perspectives on capturing the challenges associated with QM, we refer to the field and these related phenomena throughout as the enigma of QM, highlighting that QM baffles learners and presents itself as mysterious, a scientific puzzle or riddle to be pondered. Importantly, QM challenges students with new mathematics and conceptual ideas that present it as remarkable.

With a distinct conceptual and enigmatic basis, QM has fundamentally changed how physicists view the physical world and has raised questions about quantum instruction and curricular sequencing [10]. From an instructor angle, Pollock et al. [11] compiled comprehensive teaching materials for student-centric QM courses that can be flexibly adapted for institutions. Knowing that many of the challenges are rooted in mathematics, one study implemented an instructional sequence for secondary science students that reduced reliance on Dirac notation and documented that the new teaching-learning sequence was generally well accepted, but there were also learning difficulties encountered [12]. Another approach to the mathematical challenge is to emphasize philosophical understandings rather than mathematical formalizations [13], with an underlying assumption of the importance of procedural reasoning and ideas [14]. Similarly, with a focus on instruction, but with more emphasis on students' interpretations, Bailey and Finkelstein [15] emphasize the importance for instructors to recognize students' varied interpretations, noting nuances within physics learners' perspectives, ideas, and beliefs, including their attention to various philosophical approaches to QM (e.g., the Copenhagen interpretation commonly taught in introductory modern physics courses).

Moving beyond instruction, a large body of QM-physics education research (PER) scholarship documents students' challenges across various topics using a difficulties perspective. As an example, Emigh et al. [16] examine core features of QM (e.g., time evolution) with a focus on categorizing difficulties according to various student errors, rather than emphasizing the range of ways students might conceptualize QM. This body of work has examined challenges across a range of QM topics, for instance, atomic spectra [17], the double-slit experiment [18], wave functions [19], including difficulties with the time-dependent wave function [20], distinguishing between three-dimensional Euclidian space and Hilbert space [21], the Zeeman effect for the hydrogen atom [22], quantum measurement [23], angular momentum [24], the uncertainty principle [25], and Dirac notion and formalism [26]. A review article argues that students lack an understanding of key QM concepts and that students overgeneralize ideas across unrelated contexts [27].

Some empirical PER articles examine students' misconceptions about specific quantum-focused topics to determine the extent to which they persist after instruction [28]. An assumption is that misconceptions interfere with the instructional target (scientific understanding) and are resistant to change. For change to occur, students need to experience dissatisfaction to replace the old idea with a new, more tangible concept [29–31]. The notion of dissatisfaction with existing conceptions before accommodation can occur is rooted in an older but widespread view of conceptual change [31] and is still prevalent within PER [28]. The emphasis on dissatisfaction and replacement can inadvertently feed into the notion that students lack relevant knowledge to bring to the QM classroom. But on the other hand, given that QM is widely acknowledged to be surprising and abstract in ways that go beyond our direct experiences, we wonder how one can successfully learn QM if these challenges are an inherent dimension. One possibility is that moving forward requires a shift toward embracing or acknowledging these surprising and abstract dimensions of QM.

Other scholarship has examined students' epistemological framing [32]. One approach views students' difficulties in QM as an unproductive frame rather than an inability to solve the problem [33]. Other framing approaches focus on mismatches in frames, shifts in frames, and insufficient resources within a frame [34]. Some work documents whether students check for global consistencies within their answers [35]. These articles documenting the above-mentioned challenges through the framing perspective have tended to emphasize different questions than the enigmatic aspects of QM. Furthermore, the QM-PER landscape is becoming increasingly vocal about students overgeneralizing concepts (e.g., Refs. [27,36]), and we wonder about the underlying causes for this overgeneralization.

Some articles have recognized the enigmatic aspects of QM and viewed them as obstacles to be corrected through refined instruction (e.g., Refs. [27,37,38]). Other perspectives have begun to embrace the enigmatic aspects of QM from a student perspective. Hoehn and Finkelstein [39] shift the focus away from correct answers by documenting students' views of the QM weirdness. More closely related to the current paper, a body of work has begun to recognize students' varied interpretations and complex reasoning to approach the various learning challenges associated with QM (e.g., Refs. [40–44]). Corsiglia et al. [45] examine complex systems by focusing on the precise moments

when students describe QM as nonintuitive, highlighting the value of students' interpretations in the educational curriculum. Moving forward requires a more precise focus on the enigmatic aspects of QM and on how students frame this foundational dimension of QM. Thus, our research questions are: (RQ1) How do undergraduate physics students describe enigmatic aspects of quantum mechanics? (RQ2) What frames do undergraduate physics students identify for how they reason about QM?

## II. ENIGMATIC ASPECTS OF QM

To better unpack what we mean by the inherent enigma of QM, we consider a combined example of conceptual reasoning from both QM and classical mechanics to stress the point and lay the groundwork for our data analysis section. Classical mechanics is built on determinism; objects are expected to be in a single, definite location at any given time. Since all the fundamental laws of classical mechanics are deterministic, if you know all there is to know about a system at a particular time, and you know how the system evolves in time (i.e., the law of motion), then you can predict the future state of the system with certainty. Thus, if you know the initial state of a classical system, future (and past) behavior is observed to be deterministic and thereby reversible. In QM, a system can be in a superposition of (energy) eigenstates, and the complex-valued coefficients comprising the superposition state evolve in time, each according to the corresponding eigen-energy. These time-dependent coefficients are related to the *probabilities* of measuring particular values for measurable quantities (i.e., observables), such as position, momentum, energy, spin component, etc. Thus, while the time evolution of the quantum state is deterministic, observable quantities are probabilistic, a distinguishing enigma of QM as compared to classical mechanics. The Heisenberg uncertainty principle describes this inherent, enigmatic aspect of QM by defining the uncertainty in a given pair of observables, e.g., $\Delta x \Delta p_x \geq \hbar/2$.

The Heisenberg uncertainty principle is one of many enigmatic aspects of QM. Students might also consider the inherent uncertainties and probabilistic aspects of QM as resulting from measurement error, assuming that if one had more precise equipment, then the issue of nondeterminism might be resolved. Others might focus on mathematical calculations as a means to avoid conceptualizing the ever-present enigma of QM. Another approach might be to emphasize how phenomena appear differently across various physical scales or use analogies to help decipher where a boundary for classical mechanics might exist. Given this range of puzzling phenomena and the various ways students might discount or engage in various rationalizations about the enigma of QM, we find it essential to draw on a theoretical perspective that acknowledges the inherent complexity in this sensemaking about QM. Importantly, we view reasoning about these collective issues as key to *thinking quantumly.*

## III. THEORETICAL APPROACH

Within PER and the larger science education research field, a body of scholarship has emphasized the learning process through an asset-based orientation [46,47]. Rooted in a complex systems approach to knowledge, the asset-based orientation recognizes the ontological problems associated with focusing solely on students' misconceptions and lack of knowledge, and instead foregrounds students' productive reasoning as a learning asset (e.g., [43,47–52]). Drawing on this asset-based orientation, the work we present uses the knowledge-in-pieces (KIP) lens [53,54] within the context of QM to foreground students' complex knowledge systems. A knowledge system may contain many elements (e.g., resources, intuitions, *p*-prims, mental models, and schemes) that operate at various grain sizes and in different ways across numerous contexts and learning settings. KIP emphasizes students' capabilities and potential for success, which some have dubbed the "good stuff" [55]; see also Ref. [56]. This perspective seeks ways to foreground students' existing ideas and experiences, their incoming resources, as a foundation from which new knowledge can be built [57].

Most prior work within the KIP perspective has documented students' knowledge about physics content within the context of classical mechanics (e.g., Refs. [49,53]). A few other articles have applied this perspective to students' reasoning about scientific practices, such as encountering anomalous data during data analysis [58] and examining students' reasoning to understand the role of prior knowledge as it relates to instructional analogy [59]. The key hypothesis undergirding the research presented here is that students have experiences with the everyday classical world (knowledge resources) along with other related topics, including mathematics and especially probability, which are considered an asset from which a deeper understanding of QM can be constructed. Another hypothesis is that students' good stuff [55] includes students' knowledge related to the enigmatic aspects of QM. Although we do not identify explicit elements or resources within the broad domain of QM learning, we apply overlapping theoretical frameworks of KIP [53,54] and resource theory [57] to our research design, data collection, and analysis. We are interested in the fine-grained details of how students reason about QM, particularly how students describe enigmatic aspects of QM, and their self-identified approaches for moving past many of the previously discussed learning challenges.

Foregrounding students' approaches to the enigmatic aspects of QM within KIP aligns with scholarship that has examined learners' resources about the nature of knowledge in physics, known as epistemological resources [60]. Related work also addresses epistemological framing, that

is, a set of expectations about an event or situation, including a locally coherent set of resources [61]. For example, engaging in a problem-solving activity in classical mechanics can be framed in various ways, ranging from manipulating formulas or equations to developing a qualitative or quantitative understanding of what is happening [62,63]. This scholarship on epistemological framing is rooted in linguistics and sociology [64,65]. Within PER, framing has been applied to a variety of topics, from work on physics teaching assistants [66] to pedagogical techniques [67]. Framing has also been applied to topics within QM using a difficulties perspective. As one example, Modir et al. [34] identified four frames (algorithmic physics, algorithmic math, conceptual physics, and conceptual math) to argue that student difficulties in QM stem from unproductive framing. Their analysis identified three categories of framing errors: mismatches between a frame and problem statement, random shifting between frames, and insufficient resource activation within a frame. Closer to this current work, Dini and Hammer [68] documented a graduate student grappling with classical-to-quantum thinking frame shifts to emphasize students' dynamic resource activation and highlight the importance of interpreting mathematical formalism in terms of physical meaning. Using a similar theoretical approach within the context of mathematical sensemaking, Dreyfus [41] demonstrated how mathematics can be used productively for QM learning when students look for meaning behind the numbers. These studies on framing have also documented the relevance of a more math-focused and classical-focused epistemology in QM (e.g., Refs. [34,41,68]). While these studies tend to focus on empirical analysis of students' solving QM problems, less is known about students' self-reflections of their own learning in QM.

Here, we leverage these perspectives to examine framing within QM from an asset-based orientation, with a focus on how learners find a coherent explanation when reasoning about the enigmatic aspects of QM. When students are engaged in this reasoning, one can view it as a process of sensemaking as a means for generating an explanation or figuring something out [69]. Importantly, within the context of QM, the process of figuring something out or achieving a resolution about the enigma of QM might appear different from that in classical mechanics. We refer to this process of sensemaking about the enigma of QM, even if it doesn't achieve resolution, as thinking quantumly. Although sensemaking is not the focus of our empirical analysis, this perspective provides background context for our study with a focus on (1) how undergraduate physics students describe enigmatic aspects of quantum mechanics and (2) what frames undergraduate physics students identify for how they reason about QM.

## IV. RESEARCH METHODS

### A. Study setting

Undergraduate students were recruited to participate in this study at a large research-intensive institution in the Western United States from the Department of Physics & Astronomy. The department has a robust undergraduate program with approximately 250 majors and awards approximately 50 bachelor's degrees annually. Presently, the department offers six core physics courses required for all undergraduate physics majors, spanning topics from introductory mechanics and electromagnetism through modern physics, thermodynamics, analytical mechanics, intermediate quantum mechanics, and electrodynamics. Students can also declare one of several emphases that include additional required and elective courses. Computation is integrated into the curriculum, including two required computational labs: one is taken concurrently with introductory physics and the other with analytical mechanics. Physics courses in the department offer a mix of teaching styles, ranging from traditional lecture-based and chalkboard approaches to more active learning strategies. In recent years, the department has focused on curriculum reform and scaffolding techniques, resulting in a more intentional and supportive curriculum that offers greater flexibility for students.

Students in this program are exposed to QM in at least two, and often three or more, distinct courses that build upon each other. The first exposure is in a fairly typical "Modern Physics" course, the third core physics course required for all majors. The second exposure typically occurs in a half-semester intermediate-level course, usually taken during the junior year. In one section of the intermediate course, research-based instruction is the predominant approach. For example, embodied and metaphorical representations have been added to the curriculum design to enhance conceptual understanding of complex subjects, such as spin states and time evolution [70]. This section has also adopted a spins-first approach (utilizing McIntyre's textbook [2]) rather than a position-first or wave-function-first perspective (e.g., Griffiths' textbook [3]). A position-first approach typically begins by solving the Schrödinger equation in a one-dimensional potential. In contrast, spins-first predominantly utilizes Dirac notation and emphasizes the postulates of QM and the matrix representation of quantum state vectors and operators [11]. The third exposure to QM is in a full-semester advanced undergraduate QM course, which is required for students in several, but not all, different emphasis areas. While there has been much debate throughout the PER literature (spins-first vs wave-function-first), students within this department are exposed to both approaches at different stages of their academic program.

### B. Recruitment and participants

Participants in this study were recruited from a pool of students spanning those with limited exposure to QM (typically sophomores) to senior undergraduate students, which allowed us to capture a broader range of ideas and approaches to reasoning about QM. Students were offered the opportunity to participate in a solo or paired interview, lasting approximately 1 hour. A QR code that linked to a recruitment survey was posted on bulletin boards and distributed throughout the main physics buildings to facilitate ease of communication. Importantly, there was variety in the interviewer's prior role with the population, as he had been a peer with some students and a teaching assistant for other students. Accordingly, we had a series of different recruitment emails tailored based on prior engagement. We then collected data on the participants' interview preferences via email. Those who selected a pair interview also chose a peer to interview with who also had a similar amount of coursework in QM. To amplify underrepresented voices, the study design promoted equitable representation: of the 10 participants, 5 self-identify as women and 5 self-identify as men.

At the time of the interviews, four students were enrolled in a sophomore-level modern physics course that covered wave particle duality, potential wells, and the basics of the Schrödinger equation from a more simplified approach to quantum notation. Two students had completed the sophomore-level modern physics course and were not enrolled in further quantum-related courses at the time of the interview. One student was enrolled in the junior-/senior-level advanced undergraduate QM course, which provides a more rigorous and formal exploration of quantum phenomena, including Dirac notation, harmonic oscillators, and a sophisticated mathematical formalism, using the spins-first McIntyre textbook [2]. Finally, three students had completed the junior-/senior-level advanced undergraduate QM course from the department's previous traditional instruction path, which preceded the more recent reforms of the undergraduate curriculum. Around the time of the interviews, the three senior undergraduate students were involved in various research-based projects, spanning astrophysics, geology, and semiconductors.

### C. Interview method and protocol

The study was designed to capture undergraduate students' reasoning by prompting them to think aloud during conversations about various topics in QM. Prioritizing students' perspectives, we utilized resource theory both to elicit and document individuals' knowledge resources and to understand how students collectively influenced each other's sensemaking activities [71]. Sensemaking in educational literature emphasizes students' articulation of various concepts as they work to figure it out for themselves [69]. Our goal was to use a method that would create a space to gather rich insights into how students activate ideas when grappling with the enigma of QM. Guiding our overall approach is an informal, open-ended interview or unstructured interview style, where the goal is to dig deep into experiences, as guided by key questions [72,73]. We therefore prepared a comfortable environment for students to discuss their self-internalized views, ideas, and experiences with QM. The lead author collected field notes and conducted the interview sessions.

First, the lead author engaged with the students through an ethnographic approach, learning alongside them throughout the semester, during which the interview protocol was developed and iterated. Due to the ethnographic efforts and the first author's prior role as an undergraduate physics student, the interviewer was familiar with half of the students and was a recognizable face to all of the participants. This familiarity facilitated the building of a rapport between the interviewer and student, helping to draw out relevant experiences and perspectives from students [74].

Next, we incorporated ideas from Vossoughi and Escudé [75] by redefining the roles of the researcher and the recording equipment. For example, participants were empowered to determine camera placement and told they could turn off the recording devices at any time for any reason. Each interview began with the lead author self-identifying as a doctoral student in the Department of Educational Psychology and outlining their interest in the educational aspects of QM. The interviewer shared anecdotes about recent challenges in undergraduate physics learning and prioritized addressing students' questions to build rapport with the students [74].

Finally, the interviewer reminded each student that the purpose of collecting video recordings was not to elicit correct responses but to explore ideas about QM concepts. Throughout the process, it was emphasized that there is no particular right or wrong way to answer. Depending on the progression of the conversation, through an open-ended interview protocol, various prompts were designed to capture students' views on the role of math in physics, the potential well, and the Heisenberg uncertainty principle. For instance, we probed for their conceptual explanations rather than asking students if they could solve questions about the potential well. Think-aloud-type questions prompted participants to reflect on their unique understanding and, in the case of pair interviews, allowed them to discuss ideas freely among themselves. The flexibility and comfortable environment of the interview created a space to capture individual reflections and explore students' thoughts, feelings, and ideas. At the end of the interview, each student completed an online demographic survey and had the opportunity to choose their pseudonym. Students were provided a $40 gift card for their participation. For a subset of interview prompts, see the Appendix.

TABLE I. Summary of codebook used to guide identification of students' descriptions of enigmatic aspects of QM, along with three epistemological frames students identified as their reasoning strategies.

| Code name | Definition | Example(s) |
|---|---|---|
| Quantum, it's just different | The interviewee discusses QM and mentions something about it being "unexplainable." They might use any number of synonyms, such as different, weird, strange, mystery, or other language that implies nuances in QM that are distinct | "What in the world is quantum? It's so hard to understand because it's so different." "I think quantum mechanics has like such this like, aura of like, mystery around it" |
| Quantum, at tinier scales | The interviewee discusses scale (e.g., small or large) and mentions an implication of that scale. This includes talking about the range or scope of the scale (small-scale objects to larger classical objects) or discussing whether the scale is or is not applicable | "Really small subatomic particles behave very differently than, like, a normal thing" |
| Quantum, it's just math | The interviewee discusses the role of mathematics and its implications for QM. Examples include math being necessary, helpful, or foundationally important to understanding QM | "You really have to have a mathematical view to get a good idea of what quantum is even about, because just trying to think about it without the math is like, what are these weird particles?" |
| Quantum, in terms of classical | The interviewee discusses classical mechanics, ideas, or knowledge (including teaching and learning experiences) and its implications for QM. This includes discussing how terminology does or does not align with QM | "I'm thinking of it classically again with that relaxation time, thermodynamics, and our galaxy, galaxies and classical dynamics, all that in terms of classical, I keep trying to connect it to classical because that's what I know really well" |

Note: This table presents a summary of the thematic codes developed through qualitative analysis. The thematic codes are the descriptions of the evidence observed that informed the classification of how students' described enigmatic aspects of QM (row 1) and the classification of the three epistemological frames that students identified as their reasoning strategies: Reasoning about scale (row 2), reasoning about the connections between QM and mathematics (row 3), and reasoning about the connections between QM and classical mechanics (row 4).

### D. Analysis

We begin our analysis by examining ideas and themes from the data, following the guidelines of the observe, schematize, and systematize (OSS) cycle (Refs. [56,76,77]). Entire interview transcripts were prepared using *AssemblyAI* [78] and further refined during the coding process. While epistemological framing analysis can incorporate multiple verbal and nonverbal features, including tone, pitch, gesture, and body language [65,79], the data presented prioritizes conceptual clarity by focusing on the verbal content of the students' discourse [80] as they describe enigmatic aspects of QM in their voice. After four solo interviews and three paired interviews, saturation was achieved as the key ideas that surfaced had been heard previously throughout the video and audio data [81]. Focusing on nuanced explanations and themes, we began exploring the various frames identified by participants. The first, second, and third authors examined the data together, iterating codes through collaborative viewing sessions and analytic conversations [82,83]. Following code generation, the first and third authors conducted interrater reliability, yielding a Cohen's kappa value of $\kappa = (.84)$, indicating near-perfect agreement [84]. Working as a group, we generated excerpts using in vivo strategies to present the code names as grounded in participants' natural flow and language [53,76,85], thereby amplifying these students' voices further. For a more extensive discussion of the qualitative data analysis process employed, known as *knowledge analysis*, including the associated assumptions, goals, process, and strategies for increasing validity and reliability of the coding, refer to Ref. [56].

To address RQ1, the following section focuses on how students describe enigmatic aspects of QM, identified by the *in vivo* code *Quantum, it's just different* (see row 1 of Table I). RQ1 emphasizes the students' varied descriptions of the enigmatic aspects of QM, based on the data showing that they described QM as enigmatic. Subsequently, RQ2 focuses on the perceived implications by the students as they reason about QM as documented through using the *in vivo* codes that focus on the epistemological frames: *Quantum, at tinier scales; Quantum, it's just math; and Quantum, in terms of classical* (see rows 2, 3, and 4 of Table I). Thus, RQ1 emphasizes the enigmatic aspects of QM while RQ2 emphasizes the reasoning processes that emerged, although there is a relationship between the results since the frames identified in RQ2 might be impacted by their descriptions of the enigmatic aspects of QM. The analysis concludes with an extended example from the pair interview with Wayne and Daul to demonstrate how their overlapping epistemological frames shape

their thinking quantumly about the enigmatic aspects of QM. The case of Wayne and Daul was chosen as a representative example of the interplay of codes observed in the data, presented in an evident and illustrative way.

## V. RESULTS: ANALYSIS OF STUDENTS' EPISTEMOLOGICAL IDEAS

*RQ1: How do undergraduate physics students describe enigmatic aspects of quantum mechanics?*

All 10 students include statements about QM being unexplainable, different, weird, or strange, including ideas that situate QM as distinct from everyday expectations. While there is nuance in the exact words and contexts students use to describe enigmatic aspects of QM, the notion of an enigma was common, particularly when responding to the question, "What is QM?" As students described enigmatic aspects of QM, three themes emerged: The first theme occurred when students recognized that something about QM is unexplainable, an enigma to acknowledge. The second theme occurred when students recognized that QM allows remarkable results. The final theme occurred when students considered why QM is perceived as difficult. To demonstrate these three themes and address RQ1, we now offer representative example cases of students describing enigmatic aspects of QM, identified by the *in vivo* code *Quantum, it's just different* (see row 1 of Table I). Key phrases are bolded to highlight moments particularly relevant to the discussion.

### A. Something about QM is unexplainable, an enigma to acknowledge

To address RQ1, we present an example of the first theme: that something about QM is unexplainable, an enigma to acknowledge. The example comes from a discussion with Antares, enrolled in the advanced undergraduate QM course, toward the end of the interview. While discussing the infinite square well, the conversation focused on why the potential is defined as zero at some point inside the square well and infinity outside. Antares described QM as weird and considered the role of mathematical equations. The following quote captures her view that something about QM is unexplainable, an enigma to acknowledge:

Antares: That makes sense in its own way like **sure it does that and it always does that it's never going to change and that's like the basics of quantum mechanics, right? Like why does it do that? We don't know it just does it.** Um, but like, time evolution's pretty weird too. I don't know really, you know. Just things that I don't really, [whispers] understand.

The above quote captures the epistemological idea that *Quantum, it's just different* when Antares said, "Like why does it do that? We don't know it just does it." We interpreted this statement to reveal that there is something about QM that is unexplainable, an enigma to acknowledge. Furthermore, we recognize that her use of the term "we" is ambiguous; she might be referring to herself or the broader physics community, or she might be saying that humans are not able to figure this out. Additionally, Antares also acknowledged the inevitability of the unexplainable behavior when saying, "Sure it does that and it always does that." In this example, Antares is engaging in sensemaking about QM with a focus on acknowledgment of the enigma, which is a key part of what we refer to as thinking quantumly.

### B. QM allows remarkable results, and QM is perceived as difficult

The following quote addresses the remaining two themes from RQ1: QM allows remarkable results, and why QM is perceived as difficult. During a discussion with two students who had completed the junior-/senior-level advanced undergraduate QM course, Asgeir offered his epistemological idea that QM allows cool results because QM is different from classical mechanics. Zack considered the relative difficulty of various science topics and wondered if, because we cannot see a single atom, that is why QM is perceived as difficult. The moment occurred toward the end of the interview, while they were discussing chemistry. Asgeir began the dialogue about electron orbitals, the SPDF notation, and the uniqueness of QM and was immediately followed by Zack contemplating why QM is perceived as difficult:

Asgeir: This is important because it applies to, chemistry and like, it affects how molecules will like, work with each other in like the atmosphere or in your body or in a beaker kind of a thing. Where it's like, **these are the cool things that quantum allows us to do because it's so different from classical mechanics.**

Zack: Yeah, I think that connection to chemistry is also very important, as much as I hate it, like I keep saying [laughs], um, because, because people can understand, [pauses] grasp chemistry much easier than, [pauses] quantum, which actually brings up something, a very interesting question which is, like, why? **Why can people understand chemistry much better than, quantum?** Is it because, is it because you're dealing with more particles, more than one particle and you're taking averages, kind of like thermo?

The above quote captures the epistemological idea that *Quantum, it's just different* when Asgeir said, "these are

the cool things that quantum allows us to do because it's so different from classical mechanics." We interpreted this statement to reveal that QM allows remarkable results. Next, Zack discussed the idea that QM is perceived as difficult, "Why can people understand chemistry much better than quantum?" and added a detailed explanation about leveraging the number of particles ("is it because you're dealing with more particles"). Following the above quote, Zack then mentions, "you can't really just see one atom, but you can see a bunch of atoms together, like a couple moles of an atom interact with each other," and suggested that through fun chemistry applications, such as reactions, that is what "gets people's attention." In these quotes, these two students recognized that QM allows remarkable results and explained their ideas about why QM is perceived as difficult. Supporting their views was a focus on the consideration of the scale of individual atoms within the context of chemistry.

To address RQ1, the above quotes show how these students describe enigmatic aspects of QM, which are representative of similar epistemological ideas and themes among other students in the data. We observe that these students' epistemological ideas are a source of meaningful insights for learning, in line with other asset-based orientation studies (e.g., Refs. [46,60]), and we expect that they could be incorporated into the knowledge-building and refinement process. Antares described that something about QM is unexplainable, an enigma to acknowledge. Asgeir described the cool things QM allows because QM distinguishes itself from classical mechanics. Zack questioned why one science can be perceived as more conceptually accessible, offering ideas for why QM is considered difficult. Based on these results, the next section will examine epistemological frames that the students identify as their reasoning strategies.

## VI. RESULTS: ANALYSIS OF STUDENTS' EPISTEMOLOGICAL FRAMES

*RQ2: What frames do undergraduate physics students identify for how they reason about QM?*

To address RQ2, the following subsections focus on the epistemological frames students identified for how they reason about QM. Those are reasoning about scale, reasoning about the connections between QM and mathematics, and reasoning about the connections between QM and classical mechanics. The following three cases presented below are these students' epistemological frames for navigating enigmatic aspects of QM identified by the *in vivo* codes: *Quantum, at tinier scales; Quantum, it's just math;* and *Quantum, in terms of classical* (see rows 2, 3, and 4 of Table I). The analysis concludes with an example demonstrating how the overlapping epistemological frames shape students' thinking quantumly about the enigmatic aspects of QM.

### A. Reasoning about scale

All 10 students commonly reasoned about scale; while there was nuanced variation in the exact context of the scale, two themes emerged: The first occurred when students reasoned about scale to distinguish QM from classical mechanics. The second theme occurred when students reasoned about scale to explain why mathematical models are needed. The following example of reasoning about scale demonstrates both themes; key phrases are bolded to highlight moments particularly relevant to the discussion. The moment occurred while Richard, enrolled in a sophomore-level modern physics course, was reasoning about scale to understand how really small things act. Richard reasoned that small-scale objects behave differently from classical mechanics and explained why mathematical models such as the Heisenberg uncertainty principle and the Schrödinger equation are needed:

Richard: From what I understand, because quantum mechanics is, like, specifically talking about subatomic particles, like, how they interact, how they act. Just like things really small is, like, what I always think of. And, I think, from what I get, like they just, **things really small, subatomic particles behave very differently than, like, a normal thing, like the normal like kinematics, where you have a ball, you roll the ball.** Like, we have Newton's laws for everyday life and things we do. Um, but I feel like and I just don't know perfectly but I feel like from what I've heard or read or just like, when people talk about quantum it's just like these, other things like **when you get inside of an atom or a molecule and get really small, they don't act, like, in the same way. And so having the uncertainty principle or, like, Schrödinger's equation is and the basics that we've learned in this class is like so that we can, start to understand how they do act.**

The above quote captures the epistemological frame *Quantum*, at tinier scales when Richard said, "subatomic particles behave very differently than, like, a normal thing," and discussed familiar rules from classical mechanics, "we have Newton's laws for everyday life." Using a familiar classical mechanics frame, balls rolling and Newton's laws, and reasoning that because small-scale objects behave differently from classical expectations, the uncertainty principle and the Schrödinger equation are necessary to "start to understand how they do act." We interpreted his comments to suggest that identifying really

small things was helpful for reasoning about QM and distinguishing classical mechanics, and he goes on to add details about why mathematics (uncertainty principle and Schrödinger equation) are needed to understand the behavior. In this way, he is highlighting the importance and relevance of two mathematical models. Richard is engaging in sensemaking about QM with a focus on reasoning about scales, what we refer to as thinking quantumly.

Across the interviews, students commonly reasoned about small-scale objects behaving differently to distinguish QM from classical mechanics and to explain why mathematical models are needed. In some instances, students also reasoned about both the large scale ("black holes colliding") and the small scale, to describe that scale-extreme objects deviate from expectations. Other students reasoned that QM operates continuously beneath the classical scale, suggesting that QM is the current best probabilistic mathematical model for describing small-scale objects. The remaining two epistemological frames we discuss next, mathematical and classical, are consistent with other well-documented approaches in the literature (e.g., Refs. [27,33–35]), which were observed frequently throughout our data. These documented cases, from an asset-based orientation, add to the literature by demonstrating that students are aware of the enigmatic aspects of QM and identify the mathematical and classical frames as their reasoning strategies.

### B. Connections between QM and mathematics

To continue addressing RQ2, we present an example of the epistemological frame reasoning about the connections between QM and mathematics. For instance, one approach to unexplainable QM involved reasoning about the role of mathematics during the pair interview with Amelia and Donna, enrolled in a sophomore-level course in modern physics. Amelia described an enigmatic aspect of QM that she considers unexplainable without math. The following quote captures her view of the electron spin: that math is needed to describe this enigma:

Amelia: An electron, like, isn't spinning, but it has [enunciates] spin, to it, kind of thing that like, you wouldn't think it's not, like, intuitive, but like, **the math shows that that's the way it is kind of thing.** So, like, I guess it's the hard to explain or **unexplainable, things that, just shouldn't be but are.**

The above quote captures the epistemological frame *Quantum, it's just math* when Amelia said, "the math shows that that's the way it is." We interpreted this statement to suggest that there is something that cannot be fully explained without the math, an electron's spin that "isn't spinning." This quote aligns with the ideas discussed in RQ1 by framing the enigma of QM as something to be acknowledged rather than something that can be explained without reliance on mathematical interpretation. The connections between QM and mathematics address RQ2 by identifying math as the necessary tool for thinking quantumly about a concept that Amelia considers unexplainable, an electron's spin.

While there was some variation in how students discussed the connections between QM and mathematics, students commonly identified math as a resource for reasoning about QM. In some instances, as students described subatomic particles, they recognized that mathematical models are not necessarily direct representations of the physical systems. Others suggested that QM is the "current best model" to describe "particles and systems" to explain why QM is difficult to comprehend as being inherently probabilistic.

### C. Connections between QM and classical mechanics

To continue addressing RQ2, we present an example of the epistemological frame reasoning about the connections between QM and classical mechanics. For instance, one approach to the conceptual difficulty of QM involved reasoning about why classical mechanics emerges during a solo interview with Lily, an undergraduate student who had completed the junior-/senior-level advanced undergraduate QM course. The following quote captures her view that classical mechanics is the remarkable result of QM.

Lily: I don't remember who said the quote but, they said, it's not remarkable, that the universe behaves the way quantum mechanics says it does, **what's remarkable is that, on, our scale of existence, um things behave classically.** Like the fact that you get classical mechanics out of quantum mechanics compounded upon quantum mechanics, you know, thousands, millions, trillions of times. **That, you know is sort of the remarkable thing, but, of course, it's not our lived experience, which is why it's, I think difficult conceptually.**

The above quote captures the epistemological frame *Quantum, in terms of classical* when Lily said, "What's remarkable is that, on, our scale of existence, um things behave classically." She flipped the traditional model and described the emergent phenomenon, classical mechanics, that becomes visible, "compounded upon quantum mechanics, you know, thousands, millions, trillions of times." Addressing themes from RQ1, she described how QM allows remarkable results and considered why QM is perceived as difficult. The connections between QM and classical mechanics address RQ2, as Lily described why the universe behaves classically. Lily is engaged in

thinking quantumly with a focus on reasoning about the connections between QM and classical mechanics.

All 10 students identified three frames that are used to help them approach the enigmatic aspects of QM. Those are reasoning about scale, reasoning about the connections between QM and mathematics, and reasoning about the connections between QM and classical mechanics. We observe students' reflections on their cognition to capture their ideas and approaches, that is, students' thinking quantumly about the enigmatic aspects of QM. From the above quotes, Richard provided context for an enigmatic aspect of QM, the boundary where classical knowledge breaks down, and affirmed the utility of math in starting to understand how small-scale objects act. Amelia used the example of an electron not spinning to suggest that math is essential to explain the unexplainable. Lily described that QM allows the remarkable results of classical mechanics to emerge. Next, this final case example demonstrates that the various epistemological frames discussed above are not isolated but are instead woven into how students approach the enigmatic aspects of QM.

### D. Overlapping epistemological frames

Addressing both RQ1 and RQ2, the concluding example presents the overlapping frames of reasoning about scale and reasoning about the connections between QM, mathematics, and classical mechanics. The following quotes demonstrate how various ideas combine to shape how students reason and think quantumly about the enigmatic aspects of QM. The example comes from a discussion with Wayne and Daul discussing implications of the Heisenberg uncertainty principle, two students who had completed the sophomore-level modern physics course and were not enrolled in any further quantum-related courses at the time of the interview. Wayne and Daul identified all three epistemological frames to suggest that mathematical models are required for small-scale, abstract thinking. The dialogue below is broken into three sections for easy reading; however, they are all part of one long conversation with Wayne:

Wayne: **At tinier, um scales, like atomic scales, things, aren't always what they seem.** Like, if you try to map, the position of a particle, it's not gonna to be where it, you think it actually is at an atomic scale. And I think it's important for people to, um, use the Heisenberg, or to **learn the Heisenberg Uncertainty Principle, in order to know that, like at those tiny scales, you're not gonna be obviously certain, you're going to be uncertain.** You can't know that a particle is here, like exactly.

The above quote captures the epistemological idea that *Quantum, it's just different* and the overlapping epistemological frames: *Quantum, at tinier scales* and *Quantum, it's just math*. Wayne said, "At tinier, um scales, like atomic scales, things, aren't always what they seem" and described the importance of the Heisenberg uncertainty principle while reasoning about scale to conclude, "you're going to be uncertain." We interpreted this statement to reveal that there is something about QM that is uncertain, an enigma to acknowledge, and he identified the epistemological frames of scales and math for thinking quantumly. Next, we observed reasoning about the connections between QM and classical mechanics by relating concepts with everyday experiences and reasoning about scale:

Wayne: And then I think it's good also to um, take that principle and **relate it to like everyday life with big things, because, it still applies, although it's very tiny.** And like, I can say you're like right here because we can't really see that, you're like a wave and you can be here or here [laughs] and yeah, I think that's important, because the Heisenberg uncertainty adds up to like bigger life scales. **It still applies even though it's very small and then at tiny scales, it applies more,** because then you can say, yeah, it's actually gonna be in this range, like, at very, very tiny scales.

The above quote captures reasoning about scale, "it still applies, although it's very tiny," that positions QM as continuously operating beneath classical mechanics, "because the Heisenberg uncertainty adds up to like bigger life scales." Wayne evoked the overlapping epistemological frame *Quantum, in terms of classical* simultaneously with the mathematical and scale frames by saying, "relate it to like everyday life with big things." We interpreted his epistemological ideas about the enigma of QM to suggest that, depending on the scale of one's observation, different features of either classical mechanics or QM become apparent. Wayne demonstrated a method for thinking quantumly in an "abstract and imaginative fashion" by reasoning about scale. The dialogue concludes as he said:

Wayne: So I think, yeah that's important and then it like, it gets you into more of the abstractness of quantum mechanics, where, things, I guess with the, quantum tunneling and you go through a potential barrier and things can end up on the other side. If you throw a ball at a wall and somehow it ends up on the other side, I think **the Heisenberg Uncertainty Principle sets up your mind for that type of abstract thinking** [laughs]. Like it's really, really weird. (Interviewer: "what does that even mean?") Yeah, what does that mean [laughs]? It's really weird, like **the only way we can describe it is with probability.** There's a

tiny probability that it could be over here, somehow, it has enough energy to get over there and keep the same energy.

The above quote addresses RQ1 that something about QM is "really weird," an enigma to acknowledge; in reference to quantum tunneling and uncertainty, QM allows remarkable results. Addressing RQ2, Wayne reasoned about "very tiny scales" and the connections between QM and classical mechanics: "If you throw a ball at a wall and somehow it ends up on the other side," to frame mathematics as necessary, "the only way we can describe it is with probability." Note that while Wayne may have had some lingering uncertainty about tunneling mechanisms (e.g., he mentioned energy increasing but also staying the same), his focus on the probabilistic aspect of QM is clearly a solid foothold for exploring the enigmatic aspects of QM more deeply and highlights the value of adopting an assets-based orientation in this work.

Next, Daul continued the dialogue by asking, "At what scale does quantum actually kick in?" The concluding quote below demonstrates that something about QM is different, an enigma to acknowledge, and by reasoning about the connections between QM, classical mechanics, and mathematics, Daul recognized that the remarkable result is the discovery of QM:

Daul: We've evolved to interpret the classical world for, like survival reasons right. **We know the ball is gonna fall**, so we move out of the way or we know, we have a spear we could kind of gauge where, an animal is gonna be, so we can nail it [laughs]. But **quantum, it's just, it's just different,** it just breaks all that and we don't have any intuition for it. I think the power we've got through it is just **the mathematics and the abstractions** we put on, the, to figure it out. It's even remarkable I think we've even found it out in the first place.

The above quote captures the epistemological idea that "quantum, it's just, it's just different." We interpreted this statement to suggest that something about QM is different, an enigma to acknowledge: "we don't have any intuition for it." Thinking quantumly, Daul began with a familiar classical mechanics frame, "We know the ball is gonna fall," and shifted to focus on the mathematical interpretation.

Addressing ideas from both RQ1 and RQ2, Wayne and Daul demonstrated how various frames and ideas combine to shape how students reason about the enigmatic aspects of QM. The dialogue reflects the broader themes with a focus on reasoning about scales as a frame for making progress in explaining or approaching enigmatic aspects of QM. Furthermore, in this interview, the students' reasoning about the connections between QM, classical mechanics, mathematics, and scale might function as a foothold for exploring QM more deeply.

## VII. DISCUSSION

Our results show that students recognized that there is something about QM that is unexplainable, an enigma to acknowledge. Within this recognition, when students reflected on their perspectives, they identified three frames that are used to help them approach the enigmatic aspects of QM: reasoning about scale, reasoning about the connections between QM and mathematics, and reasoning about the connections between QM and classical mechanics. By positioning the enigma of QM as central to our analysis and highlighting the value of students' recognition of the enigmatic aspects, we identified a new frame for solving QM problems: reasoning about scale, which has not been widely discussed in the QM-PER literature. Collectively, these results are crucial for drawing attention to students' perspectives on the inherent challenges of learning QM as the field begins to move beyond a perspective that singularly or predominantly emphasizes the views of researchers or experts.

### A. Integrating findings with conceptual framework

Our results prompt us to raise the poignant question: what does it mean to learn something enigmatic or unexplainable? Further confounding this question is the widespread reality of the inherent learning challenges. From one perspective, the unexplainable aspects of QM are often minimized in the QM-PER literature and in QM instruction (e.g., Refs. [12,16,27,34,38,86]). Asking students to overlook this dimension—while they are consciously aware of it—creates a paradox with ramifications. How can students balance their awareness of the enigma with the realities of QM often being presented as a complete model because the math works out nicely? In one well-cited article [87], Singh investigates student difficulties in upper-level QM. The study examines a written test administered to students from several universities, revealing widespread misconceptions regarding conceptual understandings, conflicting judgments, and an inability to recall important details. The results indicate that students struggle to distinguish between concepts and tend to overgeneralize their understanding. In a more recent article [35], which examined student sensemaking and reasoning primitives in the context of degenerate perturbation theory (DPT), the analysis included an emphasis on students being "stuck" in the math or physical models as associated with their struggles to engage in expertlike sensemaking. The student's struggles were conveyed as a challenge to be solved, rather than a productive resource to be harnessed. Although these articles are different for many reasons, we wonder if, beneath these challenges, there might be profound

uncertainties about learning something enigmatic. Perhaps assessments or other data collection tools might be accidentally overlooking certain aspects of students' perspectives. The students in our study were able to conceptualize unexplainable and enigmatic dimensions of QM while also acknowledging the need for various strategies or frames to approach learning and thinking quantumly.

Although resource theory has in the past successfully addressed epistemological resources [60,63], here we wonder about a new category of epistemological resources related to the limitations of what is explainable within QM. That is, do QM students develop resources about what they can and cannot know within QM? How might these resources develop and then be utilized in QM learning? An underlying challenge with a focus on QM resources is that there are likely similarities and differences between these QM resources and previously studied classical resources [53]. Furthermore, we have not explored the intersection between frames [68], alternative philosophical approaches [15], QM knowledge resources [57], or their connection to adjacent fields such as chemistry, nuclear engineering, or quantum computing. Future work might examine questions about the range of these resources, their similarities and differences in development, and the various ways they support learning and thinking quantumly. In this way, resource theory might provide a path forward as we work towards conceptualizing what it means to learn something enigmatic or unexplainable.

Finally, elevating the unexplainable aspects of QM raises questions about other topics within physics that are unexplainable in similar and different ways. For instance, classical mechanics learning can be considered similarly complex and difficult from both conceptual and procedural perspectives. Rarely does a classical mechanics class delve into the larger epistemological questions about what is and is not explainable. Consider how students in this study are aware of the enigmatic aspects of QM and how they acknowledge the enigma. How might students apply similar and different approaches to understanding Newton's laws? How might students conceptually describe Newton's laws without mathematical interpretation? Such questions have broad relevance across the physics curriculum and could be explored in future scholarship.

### B. Implications for QM interviewing

Consider the entanglement of both students' emotions and their conceptual reasoning [88] within the context of QM. An underlying question is about the relationship between students' feelings about QM and emotions, as it may impact the discussion of their ideas about QM. Prioritizing students' perspectives, we consider a key aspect of capturing undergraduate students' reasoning about the enigmatic aspects of QM: creating a comfortable environment that allows deeper investigation of their ideas. Building rapport between the interviewer and the student [74] was necessary to draw out relevant experiences and perspectives from students. This study acknowledges that, although the students were situated within a comfortable interview environment, as much as possible, engaging in open-flow dialogue, students may express differing perspectives across the pair and solo interviews. Furthermore, we expect frames and knowledge resource activation to emerge distinctly across various settings, such as a discussion session, an exam, or a problem-solving context, and all of these settings might be impacted by the students' emotions about QM.

While these results were grounded in the perspectives of undergraduate students, future studies should consider interviewing graduate students and faculty, who may provide counterarguments and alternative ideas on the enigmatic aspects of QM. By interviewing individuals with different background experiences about QM, our results about epistemological approaches might be presented in a new light to build a more complete picture of how individuals reason about the enigmatic aspects of QM.

### C. Translating findings into instructional and curricular implications

Our focus on the unexplainable aspects of QM has implications for QM curriculum and teaching. Importantly, there are a handful of studies in this space, for instance, by emphasizing the philosophical aspects of QM learning [13] or by shifting the focus away from correct answers to recognize students' ideas of QM weirdness instead [39], including partnering with students to codesign curriculum [89]. We found that students have ideas about enigmatic aspects of QM and are actively thinking about the remarkable results. One implication of these results suggests that QM instruction further enhances students' understanding of the philosophical aspects of QM and their interpretations of this often-overlooked aspect. Currently, these topics are often reserved for alternative or boutique QM classes, rather than being mainstream topics within the required physics major coursework. Shifting these topics to the canon of required physics courses would necessitate additional support. Physics instructors who have previously skirted the philosophical aspects of QM would need new learning opportunities to acquire the pedagogical skills and teaching resources necessary to incorporate these topics into their classes.

Furthermore, this change may have ramifications for one's physics identity and construction of what it means to "do" physics. However, we argue that students are already thinking about these enigmatic aspects of QM, and it is our responsibility as physics instructors to value their ideas and elevate the topics that are important to them in our classes. In the long term, we hope that by implementing a curriculum shift to acknowledge and leverage the

enigmatic aspects of QM, we will make classes more responsive to our students, supporting them in developing a "quantum-mechanical way of thinking" [5], and supporting their overall professional growth as physicists.

Finally, our results highlight that scale is an important and often underappreciated topic within QM that is part of students' collection of ideas associated with thinking quantumly. Although notions of scale are pervasive in the physics curriculum, they are not often foregrounded in the QM-PER literature; stellar scale, however, has received some attention in the Astronomy education literature [90]. More careful consideration of the role of scale in QM learning, as connected with recognition of the enigma of QM, might continue to support students in thinking quantumly. QM continues to be an understudied topic in PER, and it is both challenging for students and one that sparks their curiosity. The field has made strides in building various curricular sequences and exploring new ways to support students. As the field moves forward, a careful understanding of the nuances associated with thinking quantumly and the range of ways it is both similar to and different from other areas of physics will support learners and QM instruction that are responsive to students' ideas while embracing the unique, abstract ideas inherent to QM.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they contain sensitive personal information. The data are available from the authors upon reasonable request.

## APPENDIX: INTERVIEW PROTOCOL QUESTIONS

This appendix presents a subset of interview prompts for a semistructured protocol, written in the voice of the interviewer. The open-ended interview was designed to allow for flexibility in asking questions based on the natural flow of participant responses. Therefore, not all questions were asked in each interview; instead, the questions served as a flexible guide, chosen based on how the conversation evolved. Sometimes, after one of the questions, the interviewer asked a more open-ended prompt to pull out additional information before moving on to the next question (e.g., could you tell me more about that?). A subset of representative interview prompts is provided below:

### Situating the interview

1. How are you doing this semester?
2. What challenges have you faced while learning QM? Are those challenges unique to QM?
3. What topics in QM do you find interesting or want to explore more?

### Mathematics and QM

1. What is the role of math in QM? How is that similar or different from your experience with math in other classes?
2. How is learning QM similar or different from learning about classical mechanics?
3. What recommendations, strategies, and resources do you have for learning QM? What aspects of learning QM have you found difficult?

### Potential well

1. Consider a topic that gets brought up frequently when learning QM, the potential square well. What does the square well mean to you?
2. Why do we care about wave functions and particles in a box?

### Heisenberg

1. What does the Heisenberg Uncertainty Principle mean to you?
2. How has this impacted our understanding of QM? What are the implications?

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