Test of understanding of electric field, force, and flux: A reliable multiple-choice assessment tool

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(Received 12 November 2024; accepted 15 May 2025; published 3 June 2025)

This study presents the development and validation of a novel multiple-choice test designed to assess university students' conceptual understanding of electric field, force, and flux. The test of understanding of electric field, force, and flux was constructed based on the results of previous studies using a phenomenographic approach to classify common errors in these concepts. The final 10-item test, designed with targeted distractors, was administered to 116 students enrolled in an introductory electricity and magnetism course. Statistical analyses, including item difficulty and discrimination indices, confirmed the reliability and discriminatory power of the test. Students' performance revealed significant misconceptions in applying the superposition principle, Gauss's law, and electric force interactions. The results of this study provide valuable insights into students' learning challenges and offer physics educators and researchers a useful tool for assessing conceptual understanding and guiding instructional improvements.

DOI: 10.1103/98gy-2m6g

I. INTRODUCTION

Conceptual understanding of physics at the university level is a central focus of physics education research (PER), particularly concerning the challenges students face in grasping fundamental physics concepts [1]. In particular, in electricity and magnetism, three core concepts—electric field, force, and flux—have been the subject of numerous studies. Researchers have explored students' difficulties with the concept of electric field [2–9], electric force [4,5,7,10–13], and electric flux [7,14–20].

Another critical area of PER involves assessment, particularly the development and validation of multiple-choice concept inventories. Docktor and Mester [1] highlighted several strengths of these inventories: (i) They facilitate large-scale quantitative studies; (ii) their widespread availability allows for use across institutions nationally and internationally; and (iii) they enable instructors to assess the effectiveness of their teaching.

Multiple-choice tests have been developed to assess students' understanding of electricity-related concepts [4,5,7,15,21]. However, reviewing existing tests, we identified a gap: the need for a concise assessment tool that comprehensively evaluates students' understanding of electric field, force, and flux. This article addresses this gap by presenting a new test designed for this purpose.

The objectives of this article are threefold: (i) To introduce a multiple-choice test that assesses students' understanding of electric field, force, and flux and to describe its design process; (ii) To demonstrate the test's reliability and satisfactory discriminatory power; and (iii) To provide an in-depth analysis of students' conceptual understanding based on the test results.

II. LITERATURE REVIEW

Several researchers have developed multiple-choice tests to assess students' understanding of electricity and magnetism concepts. Madsen *et al.* [22] conducted a comprehensive review of six widely used tests in physics education research: (i) the Brief Electricity and Magnetism Assessment (BEMA; 30 items) [5], (ii) the Conceptual Survey of Electricity and Magnetism (CSEM; 32 items) [4], (iii) the Diagnostic Exam for Introductory Undergraduate Electricity and Magnetism (DEEM; 66 items) [21], (iv) the Electricity and Magnetism Conceptual Assessment (EMCA; 30 items) [7], (v) the Symmetry and Gauss's Law Conceptual Evaluation (SGCE; 25 items) [15], and (vi) the Magnetism Conceptual Survey (MCS; 30 items) [23].

When examining these tests collectively, several key points emerge: (A) The majority (4 out of 6) assess both

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electricity and magnetism concepts together. (B) The tests are relatively long, ranging from 25 to 66 items. (C) None of the tests specifically target electric field, force, and flux concepts simultaneously.

Given these observations, there is a clear gap in the literature for a concise test that evaluates these three fundamental concepts together. The test introduced in this article addresses this gap. While some of the items in our test are inspired by or related to those in previous assessments, our focus on the electric field, force, and flux as a unified set of concepts is novel. Table I outlines the key differences between our ten-item test and relevant items from the prior tests, where applicable.

From Table I, we can identify three key differences between our test items and those from previous assessments: (i) the inclusion of newly developed items, (ii) the use of more targeted distractors to address specific misconceptions, and (iii) the introduction of entirely new distractors. Additionally, an important distinction is that our test items were designed based on insights from previous studies in which we analyzed student responses to open-ended questions, as reported in several of our earlier publications [9,12,13,18–20]. These foundational studies informed the construction of our items to better capture students' conceptual difficulties.

III. METHODS AND TEST DEVELOPMENT

In this section, we address objective 1: the presentation of a multiple-choice test that evaluates students' understanding of electric field, force, and flux, along with its design process. This research builds upon our prior work, where we comprehensively classified the most common errors made by university students when learning these concepts. The earlier study employed a qualitative methodology, utilizing an open-ended questionnaire, and analyzed students' responses through a phenomenographic approach, as recommended in the literature [24,25]. The research results have been published in various articles [9,12,13,18–20], and we provide a brief overview of these findings as they informed the development of the multiple-choice test.

One of the main findings in Ref. [9] was the identification of a "hybrid" representation of vectors and field lines, and how this representation impacts students' application of the superposition principle. This hybrid representation was a distractor in item 1 of the multiple-choice test. In Ref. [13], we found that students sometimes answered questions about electric interactions as though they were magnetic interactions. Based on this, we added a distractor in item 2 that relates to magnetic force. Similarly, in Ref. [12], we observed that students often conflated the concept of electric force with the idea of impetus,

TABLE I. Differences between our ten items and the items from the previous tests.

Item	Related items from previous tests	Difference with items from previous tests
1	Question 5 of BEMA Question 8 of EMCA	We include new distractors, such as option D, which has a high percentage of selection. (More details are in the results section.)
2	Questions 3 and 4 CSEM Questions 1 and 2 BEMA	We include more specific distractors. In the CSEM and BEMA questions, two charges are presented (+Q and +4Q), and two separate questions are asked about the magnitude of the forces exerted on each charge. In our test, we present two charges ($+q$ and $+3q$) and ask a single question about the magnitude and direction of the two forces exerted on the two charges. We ask for the vector representation of the two forces on a grid.
3	Question 2 of DEEM	We include new distractors, such as option C, with a considerable percentage of selection.
4	Question 4 of DEEM	We include new distractors, such as option B, which has a high percentage of selection. (More details are in the results section.)
5 and 6	Question 9 of SGCE	We include more specific distractors. Our items present the same situation as question 9 of the SGCE. The SGCE asks to order the magnitudes of the three fluxes through the three surfaces. Instead, we divide the problem into two items. In the first item, we compare the fluxes through surfaces A and B, and in the second item, we ask about the comparison between the fluxes through surfaces B and C. The most important point is that we include reasoning justifications in the options.
7	No related item	Does not apply. New item.
8	No related item	Does not apply. New item.
9 and 10	Question 7 of the SGCE	We include more specific distractors. Our items present a situation like question 7 of the SGCE. The SGCE question asks simultaneously about the electric field and the electric flux. Instead, we divide the problem into two items. The first item asks about the electric flux, and the second is about the electric field. The most important point is that we include reasoning justifications in the options.

where movement is incorrectly assumed to imply force, which we incorporated into the distractors of item 4. Further insights from [19,20] informed the creation of distractors in items 7 and 8 based on students' varied explanations.

Building on these prior findings, we converted the open-ended questions into the multiple-choice items presented in the following section. To test the reliability of the multiple-choice test (TU-E3F), we conducted a quantitative study with 116 students. All participants were enrolled in a calculus-based, undergraduate introductory electricity and magnetism course at a private university in northeastern Mexico. The course included 3 h of weekly lecture-based sessions, which followed a widely used textbook [26], and 1.5 h of laboratory sessions based on well-established instructional tutorials [27]. The TU-E3F was administered to eight different groups of students and taught by five different instructors. To introduce additional variability, half of the students in these groups, randomly assigned, completed the TU-E3F, while the other half took a magnetism concepts test. Finally, regarding the sample size, it is important to note that physics education researchers, such as Engelhardt [28], following established guidelines in test design [29], recommend having 5 to 10 times as many students as test items when conducting item analysis. As our test consists of 10 items and was administered to 116 students, our sample size meets this recommendation.

A. Design of the test of understanding of electric field, force, and flux

To design a multiple-choice test that effectively evaluates students' understanding of electric field, force, and flux, we built upon the insights gained from the open-ended questionnaire, as recommended by Beichner [30]. We initially developed a draft version of the test and administered it to students after their electricity and magnetism course. Based on the analysis of this pilot administration, we refined the test and produced the final version—a 10-item multiple-choice assessment, provided in the Appendix. It is important to note that while additional items were initially considered, they were excluded from the final version as they did not meet the criteria recommended by Ding *et al.* [5] for reliability and discriminatory power. The finalized test was administered to 116 students at the end of their electricity and magnetism course. Table II outlines the key concepts assessed by the test (electric field, force, and flux) and describes each item.

B. Analysis of the test's reliability

To evaluate the test's reliability and discriminatory power (objective 2), we performed the five statistical tests suggested by Ding *et al.* [5]. Three measures focus on individual test items: the item difficulty index, the item discrimination index, and the item point-biserial coefficient. Table III shows these values for each item on the test.

TABLE II. The concepts evaluated in the test (electric field, force, and flux) and the description of the items.

Item	Concept	Description
1	Field	Two fixed-point charges ($+q$ and $-q$) are presented. The question asks students to choose the representation of the electric field in a position.
2	Force	Two fixed-point charges ($+q$ and $+3q$) are presented. The question asks students to choose the representation of the force exerted on each charge.
3	Force	A point charge $+q$, at rest, within an electric field is presented. The question asks students to choose the representation of the direction of the force on the charge.
4	Force	A point charge $+q$, in motion, within an electric field is presented. The question asks students to choose the representation of the direction of the force on the charge.
5	Flux	Three concentric spherical Gaussian surfaces of different sizes are presented, along with a charge $+q$ in the center and a charge $-q$ to the right of the positive charge, outside the first two surfaces but inside the third. The question asks students to compare the electric flux through the smallest surface with the electric flux through the medium surface.
6	Flux	It is the same situation as item 5. The question asks students to compare the electric flux through the medium surface with the electric flux through the largest surface.
7	Flux	Two fixed-point charges $+q$ and $-q$ are presented, aligned horizontally, and within a spherical Gaussian surface with a center in the middle of both charges. The question asks students to determine the electric flux through the surface.
8	Field	A point charge $+q$ is present, located within a spherical Gaussian surface but not in its center. The question asks students to determine if using Gauss's law to calculate the electric field at any position on the surface is possible.
9	Flux	In the first instant, a point charge $+q$ is shown in the center of a spherical Gaussian surface on which a position, P, has been marked. Then, in the second instant, another point charge $+q$ is added to the right of the first charge outside the Gaussian surface. The question asks students if there is any change in the electric flux through the Gaussian surface between the first and second instants.
10	Field	It is the same situation as item 9. The question asks students whether there is any change in the electric field at position P between the first and second instants.

TABLE III. Item difficulty index (P), item discriminatory index (D), and point-biserial coefficient (r_{pbs}) for each item of the test.

	1	2	3	4	5	6	7	8	9	10
P	0.43	0.18	0.41	0.22	0.42	0.34	0.60	0.18	0.70	0.24
D	0.72	0.41	0.79	0.52	0.86	0.83	0.69	0.24	0.55	0.34
r_{pbs}	0.56	0.49	0.59	0.49	0.65	0.69	0.58	0.19	0.45	0.42

Moreover, two measures focus on the test as a whole: the Kuder-Richardson reliability test and Ferguson's delta test.

The item difficulty index (P) assesses the difficulty level of each test item [5]. According to the criteria outlined by Refs. [31–34], this index should ideally fall between 0.2 and 0.8. As shown in Table III, most items fall within the recommended difficulty range, except for items 2 and 8, which have slightly lower difficulty values (0.18). However, these items were retained for two key reasons: First, they are highly consistent with findings from previous studies on the same concepts and align with the results of our analysis of the open-ended versions from which they were adapted. Second, despite their marginally lower difficulty indices, they provide valuable insights into students' misconceptions, particularly regarding Coulomb's law, Newton's third law, and Gauss's law in nonconventional charge distributions. Given their diagnostic potential and their difficulty indices near the acceptable threshold, we determined that their inclusion would enhance the overall assessment. Moreover, the test's average difficulty index (0.37, as seen in Table IV) remains well within the recommended range.

The item discriminatory index (D) evaluates each item's ability to distinguish between high- and low-performing students. This index ranges from $[-1, 1]$ and is considered acceptable if it is beyond 0.2 [5,32–34]. Literature has pointed out that this combination of high-discrimination behavior and acceptable difficulty index averages in a multiple-choice test is an effective way to improve test validity [35]. As displayed in Table III, all items on this test meet this criterion, calculated using the 25%–25% method, segmenting the sample into quartiles, and using only the uppermost and lowermost quartiles to produce the calculation [5,32]. Additionally, the average discriminatory index (0.60, as shown in Table IV) surpasses the recommended threshold, confirming that the test items effectively differentiate between student performance levels.

TABLE IV. Summary of the results of the five statistical tests.

Test statistic	Desired values	Test
Difficulty index	[0.2, 0.8]	Average: 0.37
Discriminatory index	≥ 0.3	Average: 0.60
Point-biserial coefficient	≥ 0.2	Average: 0.51
Kuder-Richardson reliability index	≥ 0.7 for group measures	0.70
Ferguson's delta test	> 0.9	0.96

The point-biserial coefficient (r_{pbs}) measures the consistency of each item in relation to the overall test, with a recommended threshold of 0.2 or higher [5]. Table III provides the coefficient values for each item, all of which meet the criterion. The test's average point-biserial coefficient (0.51, as shown in Table IV) also exceeds the recommended value, indicating good internal consistency across the test items.

The Kuder-Richardson reliability index evaluates the internal consistency of the entire test, with a recommended threshold of 0.7 or higher for group measures. The reliability index for our test is 0.7 (Table IV), meeting this criterion. Ferguson's delta, which assesses the discriminatory power of the test as a whole, should exceed 0.9 [5]. The test's Ferguson's delta value is 0.96 (Table IV), indicating discriminatory solid power. A summary of the five statistical measures is provided in Table IV.

We also emphasize that all items in this test were adapted from open-ended questions previously administered to a similar student population. The responses to these questions were analyzed using phenomenography, which allowed us to identify descriptive categories [24,25] that informed the construction of the multiple-choice options. These open-ended items underwent a validation process, as detailed in our previous studies [9,12,13,18–20]. This process ensured content validity by grounding the items in documented conceptual difficulties. Additionally, given the alignment of our test items with established instruments and widely recognized physics concepts, we consider the TU-E3F to possess strong face validity. Based on the statistical indices presented in Table IV and the rigorous item development process, we conclude that the test demonstrates reliability, internal consistency, and satisfactory discriminatory power.

IV. STUDENTS' UNDERSTANDING OF ELECTRIC CONCEPTS, AS INDICATED BY THE TEST RESULTS

In this section, we present a detailed analysis of students' understanding of the concepts assessed by the test, addressing objective 3. Our analysis follows the methodology used in our previous studies [36,37], allowing for a consistent approach to evaluating student performance and conceptual understanding.

A. Students' overall performance on the test

The average test score was 3.74 out of a possible 10 points, with each item worth 1 point. Expressed as a percentage, this corresponds to 37%, aligning with the average difficulty index value of 0.37 (Table IV). The score distribution is positively skewed, indicating that most students scored on the lower end of the scale, which

TABLE V. Division of items by difficulty level.

Item difficulty level	Range of correct answer percentages	Items
High	≤30%	2, 4, 8, 10
Medium	(30%, 60%)	1, 3, 5, 6
Low	≥60%	7, 9

suggests the test was challenging for them. Given this skewness, quartiles provide a more meaningful measure of spread. The median score was 4, meaning that students scoring at the median failed to answer six of the ten test items correctly.

B. Clusters of items according to level of difficulty

As presented in Table V, we categorized the test items based on their difficulty levels, as some previous works have done [36,37]. Items were classified as "high" difficulty if 30% or fewer students answered correctly, "medium" difficulty if 30% to 60% of students answered well, and "low" difficulty if 60% or more of the students answered correctly. This provides an initial glimpse into the balance of difficulty of the full test, which consists of items at different difficulty levels [35].

When analyzing the item classification, three key points stand out. First, the four high-difficulty items primarily assess the concepts of force (items 2 and 4) and field (items 8 and 10). As outlined in Table II, the force-related items evaluate interactions between two point charges (item 2) and the force on a moving charge in an electric field (item 4). The two field-related items (items 8 and 10) are focused on applying Gauss's law. Notably, none of the high-difficulty items assess the concept of flux.

Second, the medium-difficulty items cover all three concepts: force (item 3), field (item 1), and flux (items 5 and 6). Finally, it is worth noting that the two low-difficulty items (items 7 and 9) both evaluate the concept of flux. Items 7 and 9 refer to a direct application of Gauss's law that only requires students to identify the enclosed charge, which may account for their lower difficulty level.

C. Students' understanding of each electricity concept

We consider students' understanding of each electricity concept evaluated in the test, examining their performance on each item separately (Table VI). The analysis focuses mainly on errors with the highest selection percentage.

1. Electric field

Items 1, 8, and 10 assess students' understanding of the electric field concept. Item 1 presents two-point fixed charges ($+q$ and $-q$) and asks students to choose the correct representation of the electric field at a given position. This item evaluates the application of the superposition principle. Only 43% of the students answered correctly (option E). The most common error, selected by 21% of students, was option D, which displays two arrows representing the individual contributions of each charge. This suggests that students did not graphically add the vectors. This error aligns with a previous finding from Ref. [9], where students were observed to use a hybrid representation combining vectors and field lines. According to the rules for field lines, they originate from positive charges and terminate at negative charges. Some students drew vectors that started at positive charges or ended at negative charges, instead of drawing them at the position where they needed to represent the field. Even though vectors can be translated to different locations, these students did not transfer them correctly to the point of interest, which prevented them from performing the vector sum. When using this hybrid representation, students may focus on the individual contributions of each charge without correctly adding the vectors graphically [9].

Items 8 and 10 evaluate students' understanding of the electric field when applying Gauss's law. Item 8 presents a scenario where a point charge $+q$ is located inside a spherical Gaussian surface but not at its center and asks

TABLE VI. The three electricity concepts in the test (field, force, and flux), the items, and the percentages of selection of a particular choice for each item. The correct answer is in boldface, and N is for students that did not answer.

Concept	Item	A (%)	B (%)	C (%)	D (%)	E (%)	F (%)	G (%)	H (%)	N (%)
1. Field	**1**	11	1	7	21	**43**	16			2
	8	10	**18**	51	9	10				1
	10	14	16	23	**24**	22				0
2. Force	**2**	**18**	22	52	3	4	0	0	1	1
	3	**41**	13	11	7	9	3	16		1
	4	10	30	4	3	**22**	5	24	1	0
3. Flux	**5**	22	15	**42**	11	10				0
	6	**34**	12	27	5	20				2
	7	5	**60**	13	19	3				0
	9	**70**	6	11	7	6				0

whether Gauss's law can be used to calculate the electric field at any point on the surface. Only 18% of students answered correctly (option B), classifying this as a high-difficulty item. The correct answer indicates, "No, because the electric field is not uniform over the entire surface." The most common error, chosen by 51% of students (option C), was: "Yes, because it is a charge enclosed in a Gaussian surface." These students incorrectly assume that Gauss's law can be used to calculate the electric field at any point on the surface solely based on surface characteristics, previously noted in Refs. [18,19]. In this case, the charge is enclosed within the surface.

Item 10 builds on this by presenting a situation where a point charge $+q$ is initially placed at the center of a spherical Gaussian surface, with a point P marked on the surface. In the second scenario, an additional point charge $+q$ is introduced to the right of the first charge outside the Gaussian surface. Item 9 asks whether the electric flux through the surface changes between the first and second instances, while item 10 asks if the electric field at point P changes between these instances. Only 24% of students answered item 10 correctly (option D), another high-difficulty item. The correct answer states: "It decreases because the field generated by the external charge superposes that of the internal charge." The most frequent error, chosen by 23% of students (option C), was: "It remains the same because the second charge is not enclosed by the surface." This indicates confusion between electric flux and electric field, as students failed to apply the superposition principle and understand how the electric field changes when the charge distribution is altered. This item is right after an item about whether the additional charge affects the electric flux; the students might continue thinking that only the enclosed charge makes a nonzero contribution to the flux and therefore to the field. Another common misconception, selected by 22% of students (option E), was: "It decreases because both charges repel position P." These students erroneously believe there is a direct interaction between the charges and point P.

2. Electric force

Items 2, 3, and 4 assess students' understanding of electric force. Item 2 presents two fixed point charges ($+q$ and $+3q$) and asks students to choose the correct representation of the force exerted on each charge. Only 18% of students answered correctly (option A), marking this as a high-difficulty item. The most frequent incorrect response, chosen by 52% of students, was option C, which depicted forces with correct directions but incorrect magnitudes—specifically, it showed a greater force acting on the smaller charge ($+q$). This error implies a misunderstanding of Newton's third law, linked to diSessa's "Ohm's p-prim," where students incorrectly assume that greater effort yields greater results [38]; students believe that a larger charge ($+3q$) will exert a larger force on the smaller charge ($+q$). This error is related to two possible associations: (i) the magnitude of the source charge, where students believe that the larger charge exerts the larger force, and (ii) the magnitude of the electric field, where students assume the stronger field generated by the larger charge will produce a greater force on the smaller charge. Option B was the second most selected by 22% of students, which also presented forces with correct directions but incorrect magnitudes, showing a greater force on the larger charge ($+3q$). This suggests an inverse misunderstanding of diSessa's p-prim, where students mistakenly think the smaller charge exerts a stronger force on the larger one [13].

Item 3 presents a point charge $+q$ released from rest in an electric field and asks students to select the correct representation of the electric force's direction. Only 43% of students answered correctly (option A). The most frequent incorrect response, chosen by 16% of students, was option D, which stated that "the force exerted on the charge is zero." Students who selected this option may be experiencing two possible misconceptions, as noted in Ref. [12]. First, they might incorrectly apply ideas from magnetism, mistakenly believing that if a charge is at rest, the electric force is zero—confusing electric forces with magnetic forces, where motion is required to produce a force. Second, students may hold an impetus-based misconception, derived from diSessa's "Ohm's p-prim" [38], where they believe that if there is no motion, there can be no force, equating force directly with movement.

Item 4 presents a point charge $+q$ in motion within an electric field and asks students to determine the direction of the force acting on the charge. Only 22% of students answered correctly (option E), making this another high-difficulty item. The most frequent error, selected by 30% of students, was option B, which depicted the force as being perpendicular to the electric field [13]. This confusion likely arises from students misapplying Lorentz's law from magnetism, where the force on a moving charge is perpendicular to both the magnetic field and velocity [12]. The second most common error, chosen by 24% of students, option G, which presented a vector sum of the charge's velocity and the electric field. This reflects a common misconception rooted in pre-Galilean mechanics, where students believe that a force is required to keep an object in motion [13].

3. Electric flux

Items 5, 6, 7, and 9 evaluate students' understanding of electric flux. The scenarios for items 5 and 6 are the same: three concentric spherical Gaussian surfaces (A, B, and C) surround a positive charge $+q$ at the center, while a negative charge $-q$ is placed outside surfaces A and B but inside surface C. In this setup, item 5 asks students to compare the electric flux through the smallest surface (A) with that through the medium surface (B). Only 42% of students answered option C correctly: "The electric flux through A is the same as through B, because the

enclosed charge is the same." The most frequent misconception (22%, option A) was selecting the answer: "The electric flux through A is greater, because A is closer to the charge $+q$." This indicates that these students incorrectly believe electric flux depends on the surface's proximity to the electric charges in the system.

Item 6 asks students to compare the electric flux through the medium surface (B) with that through the largest surface (C). Only 34% of students answered correctly (option A): "The electric flux through B is greater, because the charge enclosed in B is greater than in C." However, 27% of students selected the most common incorrect answer (option C): "The electric flux through B is less, because C encloses more charges than B." These students incorrectly assume that electric flux depends solely on the number of enclosed charges, without considering the charge's sign.

Item 7 presents two point charges, $+q$ and $-q$, aligned horizontally within a spherical Gaussian surface, with its center positioned between the two charges. Students are asked to determine the electric flux through the surface. Only 63% of students correctly selected option B: "The electric flux is zero because the net charge enclosed by the surface is zero." The most frequent incorrect answer (19%, option D) stated: "The electric flux is $\oint \vec{E} \cdot d\vec{S}$ by Gauss's law, but it cannot be calculated in this case." This suggests that some students believe Gauss's law is applicable but find the solution too mathematically complex or think the problem cannot be solved with available tools.

As mentioned earlier, item 9 presents a situation where, in the first instance, a point charge $+q$ is located at the center of a spherical Gaussian surface with a marked point P. In the second instance, an additional point charge $+q$ is placed outside the surface. Item 9 asks whether the electric flux through the Gaussian surface changes between the first and second instances. Only 70% of students correctly answered (option A): "No, because the additional charge is not enclosed by the Gaussian surface." The most frequent error (11%, option C) was the belief that "Yes, because the additional charge does affect the flux through the Gaussian surface." This misconception suggests that some students incorrectly think an external charge can influence the flux through a Gaussian surface, even if it is not enclosed.

D. Identification of incorrect answers with higher frequency percentages

For instructional purposes, we identify the incorrect test answers with a frequency rate greater than 20%. Table VII shows the ten incorrect answers that fulfill this criterion.

TABLE VII. Incorrect answers on the test with frequency rates greater than 20%. (Answers are in order of decreasing percentage.).

Error	Item	Option	%	Concept	Description
1	2	C	52%	Force	In the interaction of two fixed point charges ($+q$ and $+3q$), the bigger the charge, the greater the force it will exert. This error is associated with two inappropriate conceptions: (1) "P-prim magnitude of the source": thinking that the charge $+3q$, because it is of greater magnitude, exerts a greater force on the charge $+q$. (2) "P-prim magnitude of the field," thinking that the charge $+3q$, by producing a greater electric field, will exert a greater force on the charge $+q$.
2	8	C	51%	Field	It is possible to use Gauss's law to calculate the electric field at any point on a Gaussian surface, in a situation where it is not possible, based solely on surface characteristics, specifically that the surface encloses the charge.
3	4	B	30%	Force	In the interaction of a point charge $+q$, in motion, within an electric field, the direction of the force exerted on the charge is perpendicular to the electric field (confusion with Lorentz's law of magnetism).
4	6	C	27%	Flux	The electric flux depends on the number of enclosed charges, regardless of their sign.
5	4	G	24%	Force	In the interaction of a point charge $+q$, in motion, within an electric field, the direction of the force exerted on the charge is between the velocity of the charge and the electric field (misconception of the pre-Galilean "force" needed to keep objects in motion).
6	10	C	23%	Field	Flux-field confusion when applying Gauss's law in analyzing the superposition principle and the electric field change when modifying a system's charge distribution.
7	10	E	22%	Field	Interaction between charges and positions, when applying Gauss's law, in the analysis of the principle of superposition and the change of the electric field when modifying the charge distribution of a system.
8	2	B	22%	Force	In the interaction of two fixed point charges ($+q$ and $+3q$), the smaller the charge, the bigger the force it will exert ("inverse p-prim").
9	5	A	22%	Flux	The electric flux through surfaces depends on their proximity to the electric charges in the system.
10	1	D	21%	Field	In representing the electric field in a position (in a system with two fixed point charges), choose two independent vectors as answers. This error is associated with two possible procedures: (1) Confusion between field and force: students think that there is interaction between charges and position, either attraction or repulsion; (2) Focusing on the contribution by each charge without adding the vectors graphically.

Upon analyzing Table VII, we identified that four errors are related to the concept of electric force, four to the electric field, and two to the electric flux. In the four errors concerning electric force (errors 1, 3, 5, and 8), two notable trends emerge. The first is that two errors involve the interaction between charges (errors 1 and 8) and are linked to diSessa's "Ohm's p-prim": directly in error 1 (52%), and inversely in error 8 (22%). The second trend is the occurrence of two errors related to the interaction of moving charges within an electric field (errors 3 and 5), which involve confusion with Lorentz's law (error 3; 30%) and a misconception of pre-Galilean mechanics (error 5; 24%).

In the case of the four errors involving the concept of electric field (errors 2, 6, 7, and 10), two additional patterns can be observed. The first is that three errors pertain to the application of Gauss's law (errors 2, 6, and 7): reasoning based purely on surface characteristics (error 2; 51%), confusion between flux and field (error 6; 23%), and the mistaken belief that charges interact with specific positions (error 7; 22%). The second trend is the misunderstanding of the superposition principle, evident in error 10 (21%), where students incorrectly selected two independent vectors to represent the electric field at a specific position. Table VII shows that some students who made this error also exhibited the same confusion seen in error 7, thinking that charges and positions interact directly.

Finally, we observe two errors related to electric flux (errors 4 and 9). Both are connected to misunderstandings about the dependencies of electric flux: students incorrectly believed that flux depends on the number of enclosed charges, regardless of their sign (error 4; 27%), and that flux varies with the proximity to charges in the system (error 9; 22%).

V. CONCLUSIONS

A. Findings

The first objective of this study was to develop and present a novel multiple-choice test designed to evaluate students' understanding of electric field, force, and flux. A key element of the design process involved classifying common student errors through open-ended problems and building on findings from previous studies using a phenomenographic approach [9,12,13,18–20]. This process informed the creation of a reliable ten-item test that is distinct from existing tools by targeting these three critical concepts simultaneously. The finalized version of the test is provided in the Appendix.

The second objective was to demonstrate that the test is a reliable evaluation tool with strong discriminatory power. By performing five statistical tests recommended by Ding *et al.* [5], we confirmed that the test meets all the necessary criteria for reliability. The outcome was a multiple-choice test with good difficulty indices, excellent discrimination capability, and effective distractors, as suggested in the literature [35]. These results validate the test as a useful tool for evaluating student understanding in different educational contexts, tracking learning gains, and assessing the impact of instructional materials specifically designed to address conceptual challenges.

The third objective was to perform a detailed analysis of students' understanding of the concepts evaluated by the test. Our analysis revealed that students at the median level had difficulty correctly answering six out of the ten items. Additionally, the most significant challenges were observed in four items (items 2, 4, 8, and 10), which assess the concepts of force and field. Interestingly, none of the items assessing the concept of flux fell into the high-difficulty category. The items related to force focused on the interaction of two-point charges (item 2) and the force on a moving charge in an electric field (item 4), while the field-related items (items 8 and 10) involved the application of Gauss's law. Some of the students' difficulties stem from common misconceptions in mechanics, such as misapplying the superposition principle and Newton's laws. These findings indicate that students face persistent challenges when applying fundamental mechanics concepts in the context of electromagnetism. This difficulty may arise from the challenge of transferring previously learned mechanics concepts and the possibility that some students may not have fully grasped these foundational ideas in their earlier courses. Consequently, their misconceptions are repeated in the new context of electromagnetism, potentially compounding their learning difficulties. This diagnostic test provides a valuable tool for identifying such difficulties and assessing students' ability to apply Newtonian reasoning in an electricity and magnetism context. To address these challenges, instructional approaches should emphasize explicit connections between mechanics and electromagnetism, reinforcing fundamental principles and reasoning strategies that aid in problem solving across both domains.

In addition to the overall performance analysis, we closely examined each item, identifying the percentages of correct and incorrect responses and uncovering the incorrect reasoning and misconceptions behind students' errors. Notably, ten incorrect answers were selected by more than 20% of students. Most of these frequent incorrect answers are consistent with previous studies in other tests and in our own research based on the open-ended versions of these same items. We recommend that physics instructors specifically target these misconceptions when planning their lessons. Ultimately, the analyses presented in this study provide physics educators and researchers with a comprehensive overview of the common difficulties students encounter when applying these electric concepts. This insight could also inform the development of new instructional materials to improve students' conceptual understanding.

B. Limitations of the study

Despite the contributions of this study, there are two main limitations to acknowledge. First, the Test of Understanding of Electric Field, Force, and Flux (TU-E3F) was designed specifically for students in calculus-based physics courses,

as the test includes items that require an understanding of integral calculus, particularly in the context of electric flux. While it is possible to adapt the test for algebra-based physics courses by rewording these items and modifying their representations, such an adaptation has not yet been undertaken. Therefore, the current version of the test is not suited for algebra-based students. Future research could explore modifications that make the test accessible to a broader range of physics students.

Second, as shown in Table VI, some distractors had very low selection rates (≤3%). While this might suggest a limited impact on the test's discriminatory power, these distractors were intentionally retained to ensure comprehensive coverage of documented misconceptions. Although certain misconceptions may be less prevalent in our current study population, they could be more common in different instructional settings or future studies. Moreover, these distractors help preserve the conceptual integrity of the test by ensuring that incorrect responses stem from well-documented misunderstandings rather than test-taking strategies. Future research may focus on refining item effectiveness using broader student data to enhance the test's diagnostic capability.

C. Future work and implications

Beyond addressing the limitations, this study opens multiple avenues for further research. One direction is the adaptation of the *TU-E3F* for algebra-based physics courses, particularly by modifying items involving electric flux and integrals. This adaptation could allow for the test's wider application across diverse student populations. Additionally, longitudinal studies could assess the effectiveness of instructional interventions targeting the misconceptions identified in this study. The test could also be used in different institutional contexts to analyze how various teaching strategies influence conceptual understanding. Finally, exploring alternative assessment formats, such as two-tier multiple-choice questions or open-ended justifications, could provide deeper insights into students' reasoning processes.

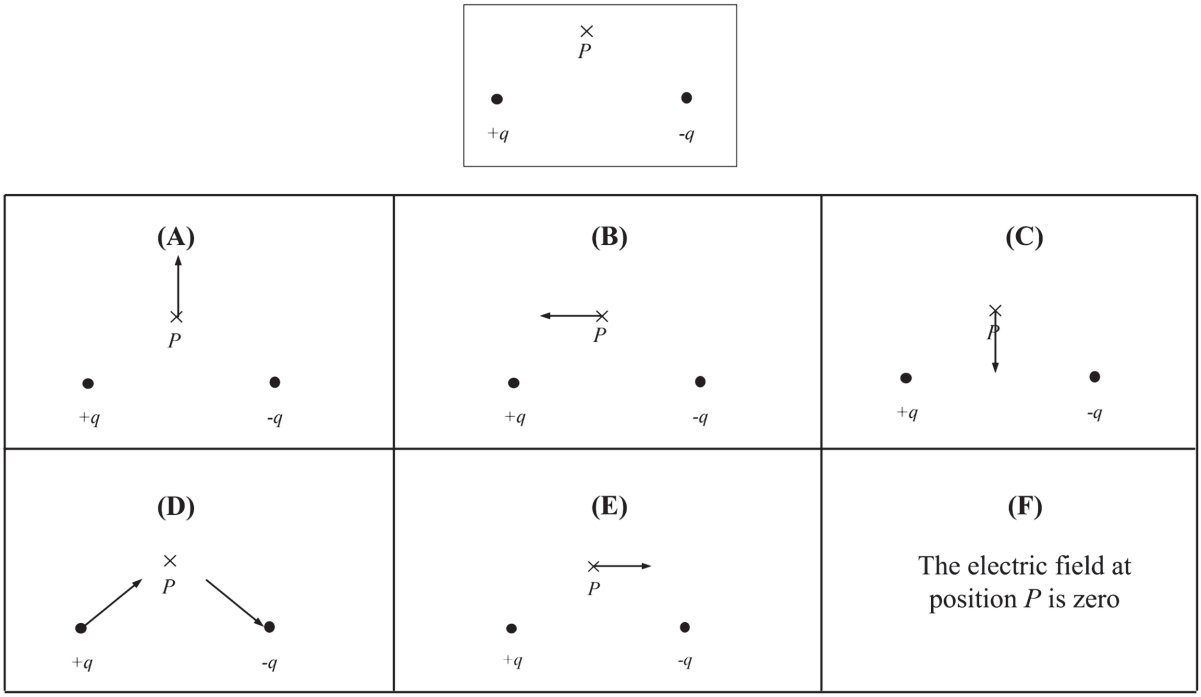
Overall, the *TU-E3F* represents a reliable and conceptually targeted tool for assessing students' understanding of electric field, force, and flux. By identifying key misconceptions and areas of difficulty, this test offers valuable insights for both physics educators and researchers, facilitating the development of more effective instructional strategies in electricity and magnetism courses.

ACKNOWLEDGMENTS

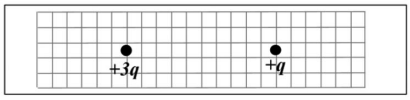
The authors acknowledge the technical and financial support of Writing Lab, Institute for the Future of Education, Tecnologico de Monterrey, Mexico, in the production of this work.

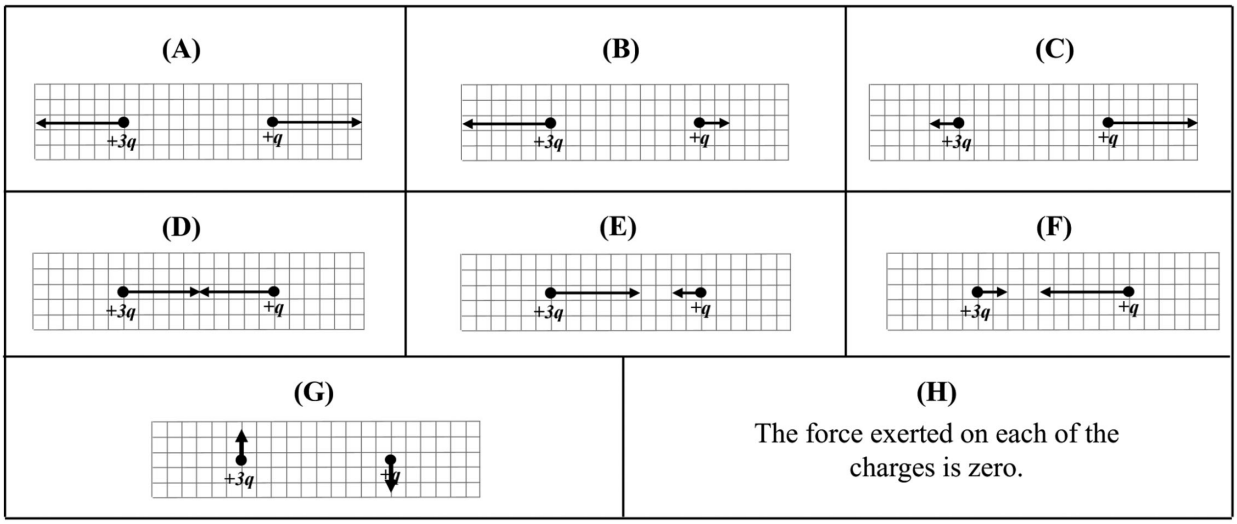
APPENDIX: TEST OF UNDERSTANDING OF ELECTRIC FIELD, FORCE, AND FLUX (TU-E3F)

1. The figure shows two fixed point charges $+q$ and $-q$ and a position P located at the same distance from both charges. Which of the options shown below is the best way to represent the electric field at position P?

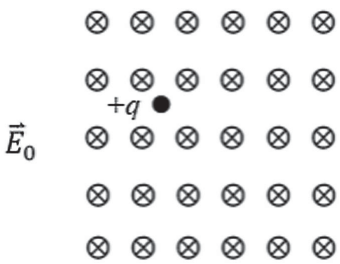


2. The figure shows two fixed electric charges $+3q$ and $+q$. Which of the following options shows the force exerted on each charge due to its interaction with the other?



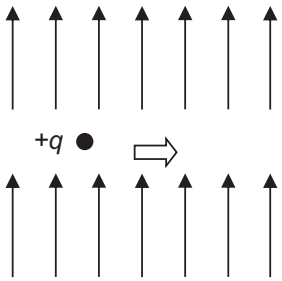


3. A charge $+q$ is released from rest in a region with a uniform electric field $\vec{E}_0$ directed into the page, as shown in the figure. Which of the following options represents the direction of the force exerted on the charge $+q$ due to its interaction with the electric field?



(A) Into the page (⊗)
(B) Out of the page (⊙)
(C) Left (←)
(D) Right (→)
(E) Up (↑)
(F) Down (↓)
(G) The force exerted on the charge is zero

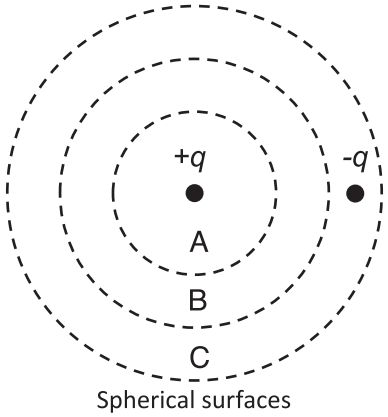
4. The figure shows a point charge $+q$ with velocity $\vec{v}$ located in a region of a uniform electric field $\vec{E}_0$. Which of the following options represents the direction of the force exerted on the charge $+q$ due to its interaction with the electric field?



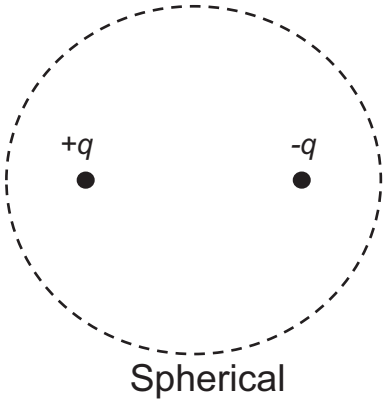
(A) Into the page (⊗)
(B) Out of the page (⊙)
(C) Left (←)
(D) Right (→)
(E) Up (↑)
(F) Down (↓)
(G) Diagonally up and to the right (↗)
(H) The force exerted on the charge is zero

To answer questions 5 and 6, consider the following situation:

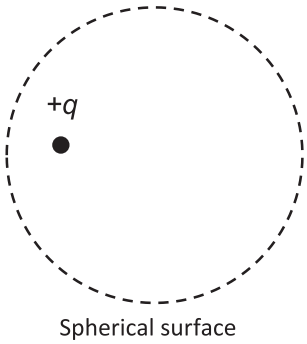
There are three imaginary spherical surfaces A, B and C (represented by dotted lines in the figure). There is also a charge $+q$ located in the center of the three surfaces, and a charge $-q$ located between surface B and surface C.



5. How does the electric flux (surface integral of the electric field $\oint \vec{E} \cdot d\vec{S}$) through surface A compare to the electric flux through surface B?
   (A) The electric flux through A is greater, because A is closer to the charge $+q$.
   (B) The electric flux through A is greater, because A contains a charge but B does not.
   (C) The electric flux through A is the same as through B, because the enclosed charge is the same.
   (D) The electric flux through A is less, because B is larger.
   (E) The electric flux through A is less, because B is affected by both charges.
6. How is the electric flux (surface integral of the electric field $\oint \vec{E} \cdot d\vec{S}$) through surface B compared to the electric flux through surface C?
   (A) The electric flux through B is greater, because the charge enclosed in B is greater than in C.
   (B) The electric flux through B is greater, because B is closer to both charges.
   (C) The electric flux through B is less, because C encloses more charges than B.
   (D) The electric flux through B is less, because C is larger.
   (E) The electric flux through B is less, because C contains a charge, but B does not.
7. There are two point charges $+q$ and $-q$. Additionally, an imaginary spherical surface enclosing both charges is shown (represented by dotted lines in the figure). What is the amount of electric flux through the spherical surface shown?



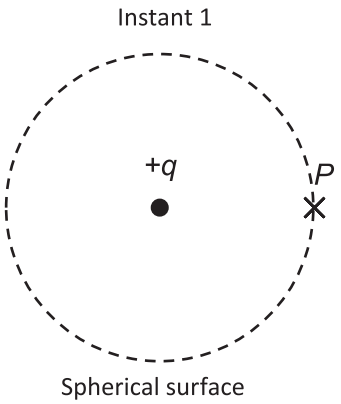
   (A) The electric flux is zero because the electric field is zero across the surface.
   (B) The electric flux is zero because the net charge enclosed by the surface is zero.
   (C) The electric flux is $\frac{kq_1q_2}{r^2}$ from the positive charge to the negative charge.
   (D) The electric flux is $\oint \vec{E} \cdot d\vec{S}$ by Gauss's law, but it cannot be calculated in this case.
   (E) The electric flux is $\oint \vec{B} \cdot d\vec{S}$ by Ampere's law, but it cannot be calculated in this case.
8. There is a point charge $+q$ enclosed in an imaginary spherical surface (represented by dotted lines in the figure). Using only this surface, is it possible to apply Gauss's law ($\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$) to calculate the electric field at any position on the surface?

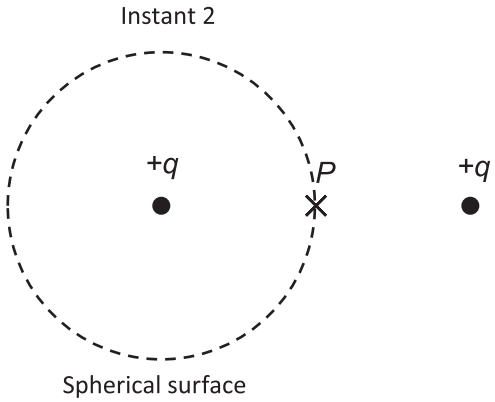


   (A) No, because Gauss's Law only works for charges centered on a surface.
   (B) No, because the electric field is not uniform over the entire surface.
   (C) Yes, because it is a charge enclosed in a Gaussian surface.
   (D) Yes, because Gauss's Law is used to calculate the electric field in this type of case.
   (E) Yes, using $\oint \vec{E} \cdot d\vec{S}$ although the mathematical process would be more complex.

To answer questions 9 and 10, consider the following situation:

At a certain instant 1, there is a point charge $+q$ at the center of an imaginary spherical surface (which in the figure is represented by dotted lines). Later, at instant 2, a charge $+q$ is added outside the imaginary spherical surface. A position P is located on the imaginary surface, halfway between the two charges, as marked.





9. Does the electric flux (surface integral of the electric field $\oint \vec{E} \cdot d\vec{S}$) through the surface change when adding the second charge at instant 2?
   (A) No, because the additional charge is not enclosed by the Gaussian surface.
   (B) No, because the additional charge does not affect the electric field through the surface.
   (C) Yes, because the additional charge does affect the flux through the Gaussian surface.
   (D) Yes, because the additional charge repels the Gaussian surface.
   (E) Yes, because there are more field lines through the Gaussian surface.

10. Does the electric field at position P *increase, stay the same, or decrease* when the second charge is added at instant 2?
   (A) It increases because there is repulsion between the charges.
   (B) It increases because both charges are positive and affect position P.
   (C) It remains the same because the second charge is not enclosed by the surface.
   (D) It decreases because the field generated by the external charge superposes that of the internal charge.
   (E) It decreases because both charges repel position P.

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