Should quantum physics be approached with mathematical formalism in school? Effects of instructional approach on functional understanding under cognitive and affective controls

Fabian Hennig,[1*] Kristóf Tóth,[2] Joaquin Veith,[1]
Malte Ubben,[1] and Philipp Bitzenbauer[1]

[1]*Leipzig University, Institute of Physics Education, Vor dem Hospitaltore 1, 04103 Leipzig, Germany*
[2]*Department of Mathematics and Physics, Széchenyi István University, Egyetem sqr. 1, H-9026 Győr, Hungary*

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Evidence from physics education research suggests that the introduction of mathematical formalism in quantum physics instruction can support the development of students' conceptual understanding, particularly by promoting functional ways of reasoning about quantum phenomena. However, empirical findings on the effectiveness of such approaches at the secondary school level remain inconclusive, and little is known about the extent to which learning outcomes are shaped by instructional design as opposed to learners' affective and cognitive characteristics. This study addresses this gap by investigating the relative influence of instructional approach on students' understanding of single-photon states, while controlling for prior knowledge, cognitive load, situational interest, and physics self-concept. A quasiexperimental pre-post design was implemented with $N = 172$ upper secondary students, comparing a qualitative instructional condition with an approach supplemented by a reduced Dirac formalism. Learning outcomes were assessed using the framework of Functional Fidelity and Fidelity of Gestalt. A multivariate analysis of covariance, followed by univariate analyses of covariance, was conducted to examine the contributions of instructional and learner-related variables. The findings indicate that affective variables emerge as strong predictors of functional understanding and suggest that learning in quantum physics is driven less by instructional format than by learners' affective and cognitive prerequisites. In particular, fostering interest appears to be a key mechanism for supporting the development of functional understanding. The results highlight the importance of integrating affective dimensions into instructional design and provide a basis for future research on the interaction between instructional approaches and learner characteristics in quantum physics education.

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I. INTRODUCTION

Modern quantum technologies have advanced rapidly in recent years, leading to an increasing relevance of quantum physics (QP) beyond its purely theoretical significance [1]. Consequently, quantum physics has become an integral part of school curricula in various countries, where the photon is commonly introduced in the context of light-matter interaction [2]. However, physics education research indicates that learners develop a wide range of conceptions—so-called mental models [3]—of the photon [4,5]. This variability can be attributed, in part, to the nonintuitive nature of quantum phenomena, which often appear to contradict everyday experiences and thus hinder meaningful interpretation [6]. As a result, learners tend to transfer familiar concepts from classical mechanics, such as "position" or "trajectory," to QP, even though single quantum objects cannot be adequately described within a classical framework [7]. From a didactic perspective, this indicates that students' difficulties are not merely conceptual, but are rooted in an insufficiently elaborated understanding of the role and limitations of models, such that learning QP can be understood as a process of learning about models [8].

To systematically capture these mental models, Ubben and Heusler developed a framework in which learners' conceptions are described along two independent dimensions: Functional Fidelity and Fidelity of Gestalt [9]. In this context, Functional Fidelity characterizes "how far mental models in (quantum) physics are perceived as functionally accurate" [9, p. 1358]. Subsequent studies have shown that this framework is also applicable to

*Contact author: fabian.hennig@uni-leipzig.de

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mental models of the photon [10,11] and, importantly, that learners' conceptual understanding is positively correlated with the degree of Functional Fidelity in their thinking [12]. These findings suggest that fostering functional ways of thinking constitutes a central objective of QP instruction [4,13]. At the same time, they raise the question of how such functional reasoning can be effectively promoted in classroom settings [14,15]. Within previous work by Ubben and Bitzenbauer, abstraction processes have been proposed to support the development of functional thinking [12,16]. Complementary to this, studies by Hennig *et al*. [5,17] identified the introduction of a reduced Dirac notation as a promising instructional approach. However, empirical findings on the effectiveness of such approaches remain inconclusive [4,18]. In an experimental comparative field study, Hennig *et al.* [17] showed that students who received exclusively qualitative explanations about the photon exhibited a stronger tendency toward functional ways of thinking after instruction (see Sec. III A) than learners in the intervention group, who received the same qualitative explanations of single-photon experiments supplemented by a mathematical formalism of quantum physics based on a reduced version of Dirac notation. The authors interpret this result as evidence that different affective and cognitive factors may critically influence learning when formal representations such as the reduced Dirac notation are introduced [17].

Although the role of affective and cognitive learner characteristics in learning has been extensively investigated in educational research [19–22], it remains unclear which specific learner characteristics shape the learning of quantum physics when qualitative explanations are supplemented by a mathematical formalism. Addressing this gap, the present study investigates the following research question: How does learning quantum physics with a quantitative extension compare to learning at a purely qualitative level with regards to Fidelity of Gestalt and Functional Fidelity, controlled for

(a) self-concept in physics
(b) situational interest in quantum physics
(c) cognitive load
(d) prior knowledge?

II. RESEARCH BACKGROUND

To theoretically ground the analysis, learning is conceptualized in terms of representational quality, particularly through the constructs of Fidelity of Gestalt (FG) and Functional Fidelity (FF). These constructs capture complementary aspects of understanding, addressing both the structural and their functional component. The following sections elaborate this perspective in relation to the underlying Fidelities Model of Conceptual Development, mathematics in quantum physics education as well as affective and cognitive characteristics.

A. Fidelities Model of Conceptual Development

Even before formal instruction in quantum physics within a school context, learners already possess isolated ideas about central concepts of quantum physics [4]. These so-called mental models help them interpret phenomena but are not necessarily scientifically accurate. In an exploratory study, Ubben and Heusler showed that mental models of the atomic shell can be described using two independent dimensions, which they term "Functional Fidelity" and "Fidelity of Gestalt."

FF characterizes "how far mental models in (quantum) physics are perceived as functionally accurate" [9, p. 1358], whereas FG characterizes "how far the mental models in (quantum) physics are perceived as visually adequate depictions of something real" [9, p. 1357]. Depending on the manifestation of these two dimensions, four types of mental models can be distinguished (see Fig. 1).

The four types represent different combinations of the degrees of Fidelity of Gestalt and Functional Fidelity. A nondeveloped mental model (low FG, low FF) reflects an essentially absent conception. In an architectural mental model (high FG, low FF), the mental representation is assumed to resemble the photon's actual appearance but has no explanatory function. A dual mental model (high FG, high FF) combines this presumed resemblance with an awareness of the model's explanatory function. In a functional mental model (low FG, high FF), the photon's presumed appearance is largely irrelevant; instead, the

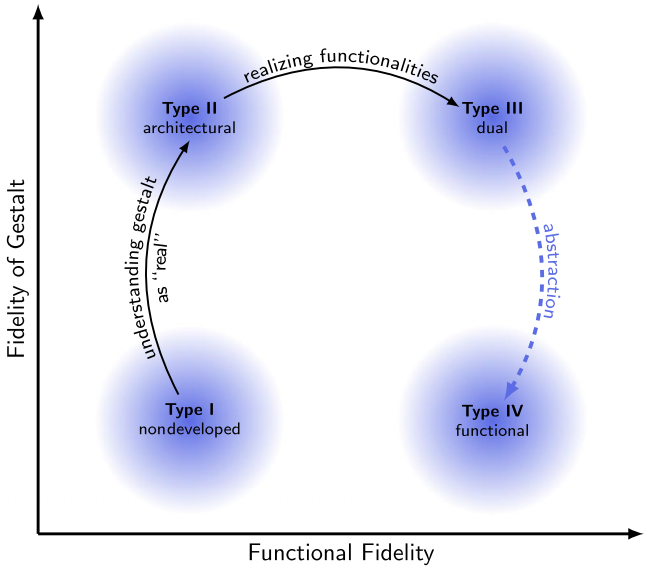


FIG. 1. Four types of mental models are shown: nondeveloped (low FF and FG), the architectural type (low FF, high FG), the dual understanding type (high FF and high FG), and the functional understanding type (high FF, low FG). It has been hypothesized that the degree of Functional Fidelity can be supported through the use of abstractions, which is why they have been highlighted in this figure.

mental representation is understood as a tool for explaining phenomena.

Bitzenbauer and Ubben demonstrated in a follow-up study that this model is also suitable for describing mental models of the photon [10], and that learners' conceptual understanding is significantly correlated with the degree of Functional Fidelity [12]. From this, it can be inferred that fostering functional conceptions represents a central objective of quantum physics instruction. Various approaches have been proposed to achieve this, such as abstraction processes [11,16], which aim to move structurally driven ways of thinking into the background. In this context, the mathematical description of quantum systems can play a crucial role, since within the formalism of quantum physics, central concepts from classical mechanics (such as "trajectory" and "position") are no longer applicable in the same way. Instead, concepts such as "state," "preparation," and "measurement" are used to describe quantum objects.

B. Mathematics in quantum physics education

Mathematics plays a foundational role in physics by enabling the description and analysis of physical systems [23]. This role is particularly pronounced in QP, where "the interplay between mathematics and physics plays a special role in quantum physics. While in classical physics, mathematical formalism is usually derived from other types of representations (diagrammatical, pictorial), in quantum physics, the mathematics takes on the conceptual role" [24, p. 88].

Given this distinctive status of mathematics in QP, a range of didactical approaches has been developed that address quantum phenomena while simultaneously providing access to the associated mathematical formalism [25–29]. A central commonality across these approaches is the use of Dirac notation [30], which affords a compact representation of quantum states, as "Kets provide an elemental representation of basic states" [31]. However, since school mathematics does not supply the formal theoretical underpinnings required for quantum mechanics, the implementation of Dirac notation in instruction necessitates substantial didactical reduction.

Core principles of QP can be effectively introduced using two-state systems, or qubits [32–34]. Owing to their conceptual and mathematical simplicity, such systems are widely employed in school-level instruction, for example, in contexts involving polarization phenomena [35–38], interferometric setups [28,39,40], the double-slit experiment [41,42], and double-well potentials [27].

Within these instructional contexts, kets are typically introduced as symbolic representations of quantum states [26,28,37,39,43]. Empirical findings indicate that students are able to recognize and accept the utility of a reduced form of Dirac notation [5,43]. In addition, real-valued probability amplitudes associated with basis states are often employed to support probabilistic reasoning [26].

Alternative approaches build on Feynman's phasor formalism [44], in which probability amplitudes are represented as phasors to account for interference phenomena [25,28]. In an experimental comparative study, Hennig *et al.* demonstrated that students' functional coherence in reasoning about photons, which is correlated with their conceptual understanding, can be systematically enhanced through an extension of the mathematical formalism. Follow-up results further suggest that both affective and cognitive factors play a significant role in this process [17].

C. Affective learning characteristics

Situational interest refers to a state of focused attention accompanied by an affective response that is triggered by environmental stimuli. It is typically transient and does not necessarily persist over time [45]. This state is characterized by heightened attention, curiosity, and fascination [46] and has been shown to support learning processes [46,47]. Accordingly, situational interest can be understood as an affective-cognitive activation state that influences both the depth of information processing and the willingness to invest cognitive effort.

In STEM contexts, situational interest is reciprocally related to domain-specific self-concept [48,49]. Self-concept can be conceptualized as a multidimensional memory structure comprising individuals' beliefs about their own attributes, preferences, competencies, and values [50]. It develops through social interactions as well as through intraindividual and interindividual comparison processes.

In the model proposed by Marsh and Shavelson, a distinction is drawn between academic and nonacademic self-concept [50]. Academic self-concept further differentiates into domain-specific components and refers to perceived abilities and competencies within a given subject domain, here specifically physics. A substantial body of empirical research demonstrates a positive association between ability self-concept and academic achievement [51,52]. Accordingly, students with higher domain-specific self-concept tend to exhibit more favorable performance trajectories, which can be explained both by motivational mechanisms and by differences in cognitive processing strategies [22]. In the present study, physics-related academic self-concept refers to students' perceptions and evaluations of their own competence, performance, and prospective achievement in physics.

Empirical research explicitly addressing affective learner characteristics in quantum physics education and outreach remains limited. In the closely related context of modern-physics outreach, Woithe investigated cognitive and affective outcomes of hands-on particle physics activities at CERN's S'Cool LAB. Her findings indicate

that situational interest and self-concept are shaped by an interplay of learner characteristics and perceived features of the outreach setting, including cognitive preparedness, cognitive load, and support by educators [53]. Anttila *et al*. found that university students' interest in learning quantum physics was associated with the perceived relevance of the subject, their quantum physics knowledge, and the frequency with which they encountered it; a one-day event also resulted in more positive attitudes and increased self-reported interest [54]. Similarly, Darienzo *et al*. reported improvements in high-school students' QIST-related attitudes, including self-efficacy, self-concept, perceived relevance, career aspirations, and quantitative fluency, following a one-week outreach program [55].

For the present study, situational interest and domain-specific self-concept were selected because they capture complementary affective-motivational aspects of learning. Situational interest reflects a transient and context-dependent response to the immediate learning environment, whereas domain-specific self-concept represents a more enduring perception of one's competence in physics [45,50]. Thus, situational interest captures learners' engagement with the specific quantum physics content, while self-concept captures their perceived competence in the broader subject domain.

D. Cognitive learning characteristics

We consider two central cognitive variables that are known to affect learning outcomes in complex scientific domains: prior knowledge and cognitive load. Prior knowledge is a key determinant of learning, as it shapes how new information is interpreted, organized, and integrated into existing schemas in long-term memory [56,57]. Cognitive load, in contrast, reflects the limitations of working memory during information processing and is central to Cognitive Load Theory [19,58].

We include these variables because they capture complementary constraints on learning: prior knowledge reflects the availability of cognitive resources in long-term memory, whereas cognitive load reflects the limitations of working memory during processing [20,59]. Their interaction is particularly relevant in complex scientific domains, where successful learning depends on both the accessibility of prior knowledge and the efficient management of cognitive resources. In particular, higher levels of prior knowledge can reduce intrinsic cognitive load by enabling the chunking of information into schemas, thereby facilitating more efficient processing [19,60].

1. Prior knowledge

Prior knowledge is one of the strongest predictors of learning success in physics education [61,62]. From a cognitive perspective, it reflects the availability of domain-specific schemas that enable learners to integrate new information into existing conceptual structures, consistent with theories of meaningful learning [63,64]. In highly abstract domains such as quantum physics, where instruction often requires ontological shifts and the restructuring or replacement of prior conceptions, differences in prior knowledge can substantially affect how learners interpret and assimilate new content [65,66]. To account for this variability, prior knowledge was measured before the intervention and included as a control variable in the analysis.

2. Cognitive load

According to Cognitive Load Theory (CLT; [19,58,59]), learning is constrained by the limited capacity of working memory. CLT distinguishes between intrinsic load (stemming from the inherent complexity of the material), extraneous load (caused by suboptimal instructional design), and germane load (associated with schema construction; [20,58,60]). Quantum physics is characterized by high element interactivity and abstract formalism, which can impose substantial intrinsic cognitive load [56,60]. Prior research on domain knowledge further suggests that learners' prior expertise strongly affects how such complexity is processed [57]. Measuring perceived cognitive load during the intervention allows us to estimate the processing demands imposed by the instructional design and to disentangle instructional effects from learner-dependent cognitive constraints.

By controlling for prior knowledge and cognitive load, we aim to isolate the specific contribution of the instructional intervention to students' learning outcomes.

III. METHODOLOGY

A. Interventions

In this section, a summary of the teaching-learning sequences employed in the study is provided. The teaching-learning sequences were implemented during regular school hours as part of the students' regularly scheduled physics lessons and were not offered as an extracurricular activity. Important parts of the learning process are real experiments, interactive screen experiments, GeoGebra animations, and worksheets created according to the principles of design-based research [67]. For a more detailed version, we refer the reader to the work by Hennig *et al*. [28].

The teaching-learning sequence was designed so that the experimental and control groups followed exactly the same path, using the same pedagogical approach. The only difference was that, after interpreting the phenomena qualitatively, the experimental group was introduced to a simplified formal description of quantum phenomena using reduced Dirac notation. During this time, the control group practiced using qualitative exercises. For example, the control group qualitatively reasoned about the possible

detection outcomes at a beam splitter and about how the detection rate changes when the movable mirror of the Michelson interferometer is displaced, without representing the single-photon states mathematically.

1. Lesson 1: Classical optics approach to Michelson interferometer

Before studying quantum physics, the students had covered wave optics. Both groups were familiarized with 50:50 beam splitters (where the incident light intensity is divided equally between the transmitted and reflected beams) and then with a Michelson interferometer.

The phenomenology of beam splitter and Michelson interferometer was investigated, while the real experiments were extended by GeoGebra animations (depicted in Fig. 2, which are available through the study [28]). We emphasized the relevance of the phases of electromagnetic waves.

2. Lessons 2–3: A single-photon state at a beam splitter

Students in both the control and experimental groups began their quantum optics studies with the same experiment. Using a single-photon source, in a sequence of a repeated measurement a single photon is emitted onto a 50:50 beam splitter, after which two detectors are placed at each of the corresponding spatial locations (transmission and reflection). Since real quantum optical experiments are expensive, these experiments were conducted in a computer environment using an interactive screen experiment, based on the materials by Bronner *et al.* [68]. The experimental setup can be seen in Fig. 3: the lamps on the black boxes are illuminated when the detector in the box detects a single photon. The red or blue lamp lights up when a photon is found in the spatial position corresponding to the transmission or reflection, respectively.

The students made the following discoveries:

(1) One photon is never detected twice.

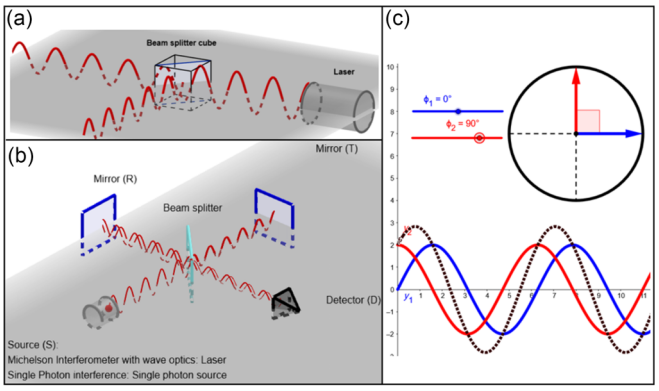


FIG. 2. This figure shows the GeoGebra animations used in the teaching-learning sequence, based on [28]: (a) beam-splitting cube, (b) Michelson interferometer, and (c) wave superposition and phase circle.

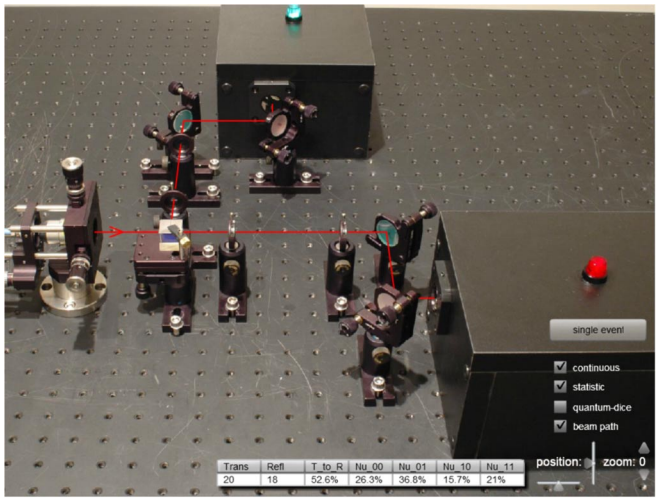


FIG. 3. The figure shows the interactive single-photon quantum optics screen experiment [68]. On the left a single-photon source is put, which emits a single photon onto the beam splitter, beyond which two detectors are positioned. The program measures the number of photons detected by each detector.

(2) The detectors exhibit perfect anticorrelation.

(3) In a large number of repeated measurements, the number of events can be predicted.

They provided the following explanations for these observations (discovering some laws of quantum physics):

(1) A single photon is indivisible (quantum nature).

(2) The behavior of one photon is usually unpredictable (indeterminism).

(3) The collective behavior of photons is statistically predictable (probabilistic description).

After the qualitative interpretation of the experiments, the instructional approaches for the two groups diverged. While the control group continued to analyze the experiments qualitatively, the experimental group was additionally introduced to a simplified quantum formalism using non-normalized, reduced Dirac notation.

In this study, the term "reduced Dirac notation" refers to a didactically simplified mathematical description of the single-photon experiments. The states associated with the transmitted and reflected paths are represented by kets with the quantities φ_T and φ_R as prefactors, for example, $\varphi_R|R\rangle$. These prefactors are not introduced as normalized probability amplitudes. Instead, their relation to the corresponding detection probabilities is established through

$$\frac{p_T}{p_R} = \frac{|\varphi_T|^2}{|\varphi_R|^2}.$$

Thus, the reduced notation relates the relative magnitudes of the prefactors to the relative probabilities of the corresponding detection outcomes without introducing the full mathematical formalism of quantum mechanics.

The students used this notation to describe the quantum state at different positions in the experimental setup. At the source, the state was represented as

$$|\psi\rangle_1 = \varphi_{0°}|S\rangle,$$

where $|S\rangle$ denotes the state emitted by the single-photon source and $\varphi_{0°}$ is a prefactor with a reference phase of 0°. Rather than treating such prefactors algebraically as complex quantities, the students represented them graphically as arrows, as illustrated in Fig. 2(c). The orientation of an arrow represents the phase, whereas its length is related to the detection probabilities through the ratio given above.

After the 50:50 beam splitter, the single-photon state was represented as the superposition

$$|\psi\rangle_2 = \varphi_{0°}|T\rangle + \varphi_{90°}|R\rangle,$$

where $|T\rangle$ and $|R\rangle$ denote the transmitted and reflected path states, respectively. The prefactors $\varphi_{0°}$ and $\varphi_{90°}$ are visualized by the blue and red arrows in Fig. 2(c), respectively.

The students are guided to make use of their experimental observations to read the superposition state $|\psi\rangle_2$ as follows:

- A single photon is neither in state $|T\rangle$ nor $|R\rangle$, but a different state, the superposition of them, which is expressed by the symbol "+."
- The phase coefficients $\varphi_{0°}$, $\varphi_{90°}$ express the phases of the corresponding states. The transmission is in phase with the source, while there is a phase shift of 90° in the case of reflection caused by the beam splitter.
- The probabilities are encoded into the phases. The ratio of the squares of their lengths (absolute squares) gives the ratio of the probabilities: $|\varphi_{0°}|^2/|\varphi_{90°}|^2 = p_T/p_R$. Since both arrows in this experiment are unit long, the ratio of the probabilities is 1∶ 1, meaning that each outcome occurs with the same probability of 1/2.

At this point, the students in the experimental group were introduced to mathematics as a tool for describing the experiments and their measurement outcomes without requiring a visualizable representation of the quantum-physical phenomenology.

3. Lessons 3–4: Single-photon states in a Michelson interferometer

Students spend two lessons studying a single-photon state in a Michelson interferometer. Initially, both groups are introduced to the experiment via the interactive screen experiment [68], as illustrated in Fig. 4. In experiments involving a large number of repeated measurements, students examine the detection rate (the number of photons measured per second) as a function of the position of the movable mirror of the Michelson interferometer. They discover that the detection rate depends on the mirror's position. The analysis of the experimental results focuses on the following observations:

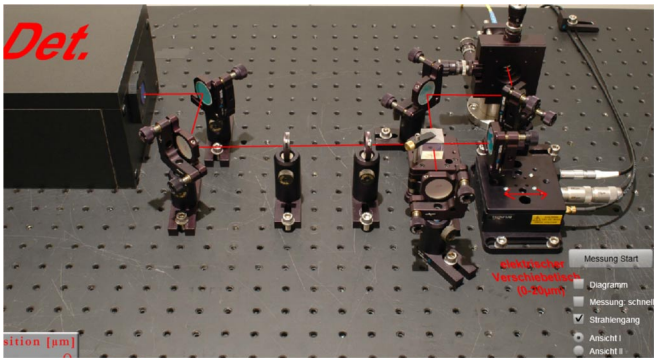


FIG. 4. The figure shows the interactive single-photon quantum optics screen experiment [68]. In the upper right a single-photon source is put, which emits single photons onto the beam splitter, behind which are two mirrors. One of the mirrors (right side in the picture) can be moved with great precision. The detector measures the detection rate.

1. Changing the position of the mirror affects the number of photons detected, even if only a single photon is emitted at a given time.
2. This observation is similar to what has been observed with electromagnetic waves. Therefore, the phenomenon is called quantum interference.

The students were then guided to the explanation. Photons cannot be considered as classical waves, since one photon cannot be detected twice. However, photons also cannot be classical particles that move along well-defined paths because then interference could not be observed. Photons are quantum objects that obey the laws of quantum physics. Quantum objects are delocalized and are capable of interference.

At this point, the two groups diverge once again. While the control group continues to examine further qualitative examples, the experimental group is introduced to a quantitative description of the phenomenon.

The construction of the quantum state proceeds step by step again. The single-photon states are emitted by the single-photon source (S) with a reference phase: $|\psi\rangle_1 = \varphi_{0°}|S\rangle$. This is followed by the beam splitter, which puts the state into a superposition, introducing a 90° phase difference in the term corresponding to reflection: $|\psi\rangle_2 = \varphi_{0°}|T\rangle + \varphi_{90°}|R\rangle$. Then, the learners examine different cases of the Michelson interferometer. For example, if the distances of the two mirrors from the beam splitter are equal, then no phase difference arises between the two arms of the interferometer, and thus the mirrors do not modify the state: $|\psi\rangle_3 = |\psi\rangle_2$. Next, the effect of the beam splitter must be taken into account again, since the mirrors reflect the light. At this point, the two components of the state are considered separately. The quantum state corresponding to transmission (T) changes under the action of

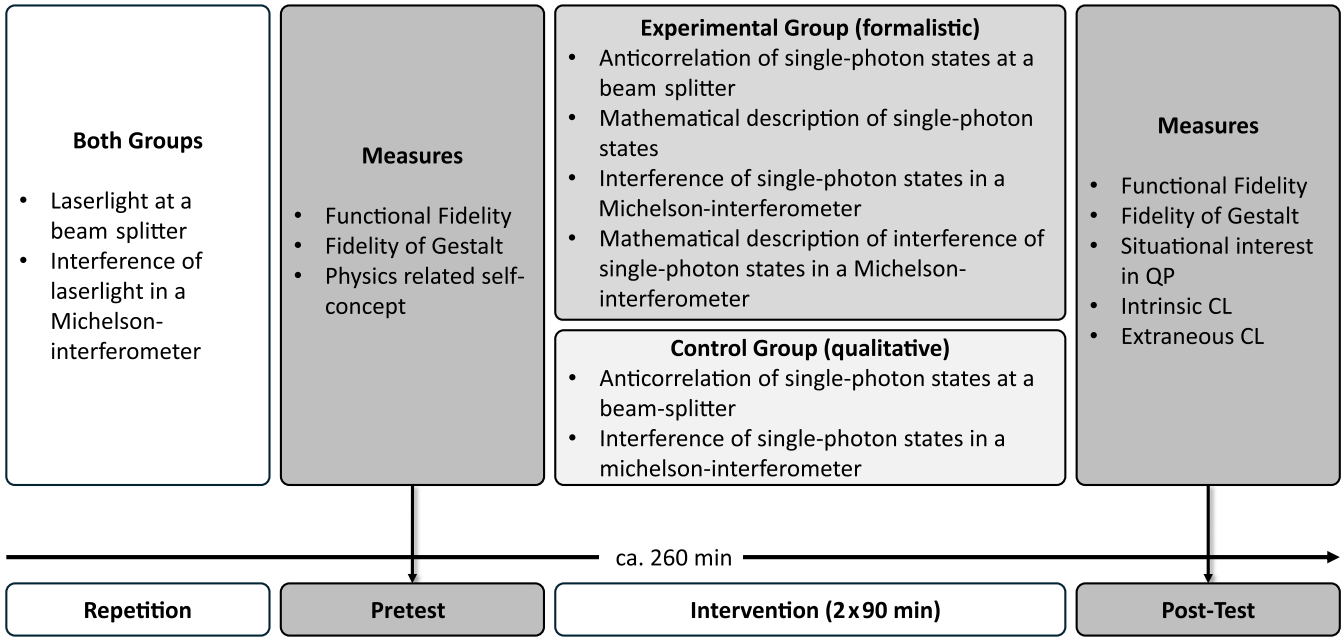


FIG. 5. Overview of the study design, including the different groups and the assessed variables as well as the required time.

the beam splitter as follows: $\varphi_{0°}|T\rangle \rightarrow \varphi_{0°}|S\rangle + \varphi_{90°}|D\rangle$, that is, the single-photon state carries the possibility of transmission (which corresponds to the source S) and reflection (which corresponds to the detector D). The reflection again increases the phase by 90°. The quantum state corresponding to reflection (R) changes under the action of the beam splitter as follows: $\varphi_{90°}|R\rangle \rightarrow \varphi_{90°}|D\rangle + \varphi_{180°}|S\rangle$.

Thus, from the state $|\psi\rangle_3$ we obtain the state $|\psi\rangle_4 = \varphi_{0°}|S\rangle + \varphi_{90°}|D\rangle + \varphi_{90°}|D\rangle + \varphi_{180°}|S\rangle$. Since opposite phases cancel each other, $|\psi\rangle_4 = 2\varphi_{90°}|D\rangle$. In this case, the single-photon state exclusively corresponds to finding the photon at the detector with state $|D\rangle$, and thus the probability of detection is 100%.

Using similar arguments, by examining specific (and later general) cases, students discover the detection probability of single-photon states, as presented in the article cited in Ref. [28] in more detail. The essence of the formal part is that students are able to statistically predict experimental results using mathematical tools. Various specific examples provide an opportunity to practice and master some part of the simplified formal language. Students simply discover the usefulness of the formalism in a topic that is particularly difficult to imagine, perhaps supporting their functional thinking.

B. Study design and sample

The study was designed as a quasiexperimental pre-post comparison with cluster-based assignment, in which intact classes were allocated to one of two instructional conditions. The study design and the process are illustrated in Fig. 5.

The sample comprised a total of $N = 172$ upper secondary school students (ages 16–18) who were encountering quantum physics in formal instruction for the first time. The control group ($n_C = 77$) was recruited from schools in Germany, whereas the experimental group ($n_E = 95$) was drawn from schools in Hungary. As a consequence of this allocation procedure, the quasirandomization introduces limitations that are addressed in more detail in Sec. V E.

The instructional conditions were designed to be structurally comparable. In the control condition, properties of single-photon states were discussed exclusively at a qualitative level. In the experimental condition, the same qualitative explanations were supplemented by a quantitative description using the reduced Dirac notation introduced in Sec. III A. While students in the experimental condition engaged with this formal extension, students in the control condition worked on additional qualitative tasks during the corresponding instructional time. Both conditions comprised four instructional sessions of 45 min each and were implemented by the research team using a standardized instructional script to ensure consistency in content and structure across conditions.

C. Instruments

To address the research question, students' levels of functional and representational coherence in reasoning about single-photon states, as well as their physics-related self-concept, were assessed prior to the intervention. Physics self-concept was deliberately measured in the pretest, as it is conceptualized as a relatively stable learner characteristic.

Following the four-session instructional phase, students' functional coherence in reasoning about photons was assessed again. In addition, situational interest in quantum

TABLE I. Summary of the scales that were utilized, along with the Cronbach's α values and anchor examples as well as the abbreviations used in Tables II and III.

Variable	Pretest	Posttest	Cronbach's α	Source	Anchor example
Functional Fidelity (FF)	✓	✓	$\alpha = 0.652$	Hennig *et al.* [17]	"The accurate position of a single photon at any given time between the beam splitter and the detectors cannot be determined in principle."
Fidelity of Gestalt (FG)	✓	✓	$\alpha = 0.706$	Hennig *et al.* [17]	"In reality, when scientists do experiments, they cannot measure both parts of a single photon at the same time."
Self-concept in physics (self-concept)	✓		$\alpha = 0.917$	Adopted from Häussler and Hoffmann [70]	"I am good at understanding physics."
Situational interest in quantum physics (interest)		✓	$\alpha = 0.898$	Adopted from Woithe [71]	"I want to spend more time on quantum physics."
Intrinsic cognitive load (intrinsic load)		✓	$\alpha = 0.775$	Adopted from Leppink *et al.* [69]	"The topics covered in the activity were very complex."
Extraneous cognitive load (extraneous load)		✓	$\alpha = 0.689$	Adopted from Leppink *et al.* [69]	"The instructions and/or explanations during the activity were very unclear."

physics and perceived intrinsic and extraneous cognitive load were measured at this time point.

Multiple established instruments were used to operationalize the constructs of interest. The items of the instruments used in this study are provided in Tables IV–IX in the Appendix. Most scales were based on five-point Likert items ranging from strongly disagree (1) to strongly agree (5). Exceptions were the situational interest scale, which employed a six-point response format, and the cognitive load scale, which followed the instrument developed by Leppink *et al.* and employed an 11-point response format ranging from strongly disagree (0) to strongly agree (10) [69]. The reliabilities, indicated by Cronbach's α, were reported in Table I. Moreover Scale scores were computed as the mean of the corresponding items.

D. Data analysis

To address the research question, a one-way MANCOVA was conducted, with instructional condition (qualitative vs. formalistic) as the between-subjects factor.

The degree of Functional Fidelity and the degree of Fidelity of Gestalt in the posttest were treated as dependent variables. The results of a correlation analysis revealed a moderate negative correlation between the two variables ($r = -0.622$, $p < 0.001$), indicating that a multivariate analysis was appropriate [72]. Based on this design, a multivariate analysis of covariance (MANCOVA) was conducted using R version 4.5.0.

To control for individual baseline differences, specifically, variations in affective and cognitive characteristics, all measured constructs were included as covariates in the model.

To further examine the multivariate effects, follow-up univariate analyses of covariance (ANCOVAs) were conducted to analyze the contributions of the individual dependent variables in detail.

The assumption of homogeneity of regression slopes was tested by including interaction terms between the instruction condition and all covariates in the model [73]. None of these interaction terms reached statistical significance ($p > 0.292$), indicating that the assumption was satisfied.

In addition, all model assumptions were carefully evaluated. Multivariate outliers were assessed using the Mahalanobis distance, and no critical outliers were identified [73]. The multivariate normality of the residuals was examined using Q-Q plots, which showed no substantial deviations. Homogeneity of covariance matrices across groups was tested using Box's M test, which was not statistically significant ($\chi^2(3) = 1.30$, $p = 0.73$), indicating that this assumption was also met.

IV. FINDINGS

A. Descriptive statistics

Table II provides an overview of the descriptive statistics of the measured variables. Prior to instruction, both groups exhibited comparable levels of functional and gestalt thinking, indicating similar baseline conditions in their conceptualizations of single-photon states. Following instruction, the degree of Functional Fidelity increased in both groups, whereas the prevalence of Fidelity of Gestalt decreased across both conditions.

B. Findings of the MANCOVA

In the MANCOVA, interest, self-concept, cognitive load, and pretest scores were included as covariates.

TABLE II. Overview of the descriptive statistics regarding the rating-scale items.

Group ↓	Variable →	FF_Pre	FF_Post	FG_Pre	FG_Post	Interest	Intrinsic load	Extraneous load	Self-concept
Formalistic	Mean	2.98	4.20	3.07	2.30	2.89	5.40	2.02	3.10
	SD	0.62	0.64	0.39	0.62	1.29	2.26	1.91	0.98
Qualitative	Mean	3.04	4.24	3.11	2.31	1.91	3.89	2.31	2.62
	SD	0.47	0.62	0.35	0.55	0.91	1.96	1.88	0.89

Due to slight deviations from a normal distribution, Pillai's trace was used as a robust test statistic [74].

1. Effects of the covariates

As shown in Table III, the MANCOVA revealed substantial effects for several covariates.

Functional Fidelity.—With respect to Functional Fidelity, pretest scores did not have a statistically significant effect on the combined dependent variables of Functional Fidelity and Fidelity of Gestalt in the context of photon understanding ($V = 0.021$, $F(2, 163) = 1.76$, $p = 0.175$).

Fidelity of Gestalt.—Regarding Fidelity of Gestalt, pretest scores showed a significant multivariate effect ($V = 0.076$, $F(2, 163) = 6.73$, $p = 0.002$).

Situational interest in quantum physics.—Interest in quantum physics exhibited a strong multivariate effect on the dependent variables ($V = 0.163$, $F(2, 163) = 15.83$, $p < 0.001$).

Self-concept in physics.—Similarly, self-concept in physics was significantly associated with the dependent variables ($V = 0.044$, $F(2, 163) = 3.75$, $p = 0.026$).

Intrinsic and extraneous cognitive load.—In contrast, neither intrinsic cognitive load ($V = 0.000$, $F(2, 163) = 0.04$, $p = 0.963$) nor extraneous cognitive load ($V = 0.009$, $F(2, 163) = 0.74$, $p = 0.479$) showed significant multivariate effects.

2. Effects of the instruction

Moreover, the multivariate analysis indicated that the type of instruction had no statistically significant effect on the degree of Functional Fidelity and Fidelity of Gestalt in the posttest ($V = 0.003$, $F(2, 163) = 0.224$, $p = 0.799$).

C. Follow-up univariate analyses

To provide more detailed insights into the significant multivariate effects, follow-up univariate ANCOVAs were conducted for each dependent variable.

In the posttest, situational interest in quantum physics emerged as a strong predictor of functional thinking ($F(1, 164) = 30.15$, $p < 0.001$, $\eta_p^2 = 0.16$). In addition, self-concept in physics showed a significant effect ($F(1, 164) = 7.54$, $p = 0.007$, $\eta_p^2 = 0.04$). No significant effects were found for intrinsic or extraneous cognitive load, nor for Fidelity of Gestalt in the pretest (all $p > 0.22$). Functional Fidelity in the pretest showed a marginal effect ($F(1, 164) = 3.00$, $p = 0.085$). The instruction condition did not have a statistically significant effect on posttest functional thinking ($F(1, 164) = 0.174$, $p = 0.677$).

For the degree of Fidelity of Gestalt, interest also showed a significant effect ($F(1, 164) = 18.18$, $p < 0.001$, $\eta_p^2 = 0.10$). In addition, Fidelity of Gestalt in the pretest was a significant predictor ($F(1, 164) = 4.16$, $p = 0.043$, $\eta_p^2 = 0.02$). No significant effects were observed for self-concept, cognitive load, Functional Fidelity in the pretest, or the instruction condition (all $p > 0.10$).

V. DISCUSSION

The MANCOVA results indicate that the instructional approach, operationalized as learning with or without

TABLE III. Summary of the MANCOVA (Pillai's trace). $^{*}p < 0.05$; $^{**}p < 0.01$; $^{***}p < 0.001$.

Predictor	df	Trace	F value	num df	den df	p
Group	1	0.0027	0.2242	2	163	0.7993
Interest	1	0.1626	15.8281	2	163	<0.001***
Self-concept	1	0.0439	3.7458	2	163	0.0257*
Intrinsic load	1	0.0005	0.0381	2	163	0.9626
Extraneous load	1	0.0090	0.7385	2	163	0.4794
FF_Pre	1	0.0212	1.7644	2	163	0.1746
FG_Pre	1	0.0762	6.7255	2	163	0.0016**

mathematical extension, does not have a statistically significant effect on the development of functionally conceptions of single-photon states.

At first glance, this finding appears to contradict the results reported by Hennig *et al*., who found that learners in a control group without mathematical extension showed a stronger tendency toward functional representations [17]. At the same time, the present results are consistent with those of Justice *et al*., who demonstrated that different instructional approaches do not uniformly lead to superior learning outcomes [75].

Taken together, these findings suggest that the inclusion of mathematical elements in instruction does not, in itself, enhance learning. Rather, its effectiveness appears to depend on contextual factors and learners' prior knowledge. This interpretation is in line with Michelini *et al*., who state that the "choice of mathematics is strongly dependent on the students, their interests and skills" [24]. This statement highlights the importance of carefully adapting the implementation of mathematical formalization to the specific characteristics of the learning group. A more detailed examination of the role of individual learning prerequisites, as addressed in the following section, is therefore warranted.

A. Discussion of RQ (a)

The findings indicate that self-concept in physics plays a central role in shaping learning outcomes, affecting not only their quality but also the mode of cognitive processing. While self-concept is positively associated with the degree of Functional Fidelity in learners' thinking, it is negatively associated with the degree of Fidelity of Gestalt. This pattern suggests that a higher self-concept is linked not only to improved learning outcomes but also to a qualitatively different way of engaging with instructional material.

These results are consistent with established theoretical assumptions and empirical findings, which posit a positive relationship between self-concept and conceptual understanding [22,51,52,76]. Learners with a strong physics-related self-concept tend to exhibit greater cognitive persistence and a higher willingness to engage with complex content, thereby enabling deeper engagement with underlying concepts and supporting the development of functionally accurate understanding.

At the same time, the negative association with Fidelity of Gestalt suggests a systematic shift in processing strategy. Learners with a high self-concept appear less reliant on superficial or formal task features and instead orient more strongly toward underlying conceptual relationships. In contrast, learners with a lower self-concept may depend more heavily on structural features, which can divert attention from the functional aspects of the photon model.

One plausible explanation for this pattern is that a higher self-concept reduces the tendency to avoid cognitively demanding processes, as proposed by Eccles and Wigfield [77]. Learners who perceive themselves as competent may be more inclined to engage with challenging material and complex physical concepts, thereby supporting deeper levels of cognitive processing [77]. In this sense, self-concept appears to be intertwined not only with motivational aspects of learning but also the depth and quality of cognitive engagement.

In this light, our findings suggest that self-concept functions as a mechanism that facilitates a shift from surface-oriented to conceptually grounded processing, thereby playing a critical role in the development of functional ideas about single-photon states.

B. Discussion of RQ (b)

The results of the MANCOVA, together with the subsequent ANCOVAs, indicate that interest is positively associated with Functional Fidelity and negatively associated with Gestalt fidelity. This pattern is consistent with theoretical assumptions.

The positive association between interest and Functional Fidelity can be attributed to a greater willingness of interested learners to engage with underlying concepts. Higher levels of situational interest are typically accompanied by increased attention, engagement, and cognitive persistence [21], which support deeper processing and the development of mental representations aligned with scientifically accepted interpretations. As a construct comprising both affective and cognitive components, interest can guide attentional processes and reduce reliance on intuitive or perceptually salient representations. In this way, higher levels of interest may facilitate the revision of initially plausible but scientifically inadequate conceptions and promote more conceptually grounded understanding [21].

The negative association with Fidelity of Gestalt suggests that interested learners rely less on superficial or formal task features [45,46] and instead focus more strongly on underlying functional relationships [78]. In contrast, learners with lower levels of interest may orient more toward easily accessible structural features, which can result in higher levels of Fidelity of Gestalt.

In addition, elevated situational interest may facilitate engagement with novel and counterintuitive concepts in quantum physics, such as the nonlocality of single-photon states [78]. Such content requires active cognitive engagement and a willingness to revise existing conceptions.

Situational interest can promote attention and cognitive activation and may also support persistence within a particular learning situation, while more developed phases of interest can contribute to sustained engagement over time [45]. Physics-related academic self-concept may likewise support persistence, particularly when engaging

with cognitively demanding material. The two constructs may therefore serve reciprocal roles. This interpretation is consistent with prior research suggesting that interest and self-concept function as positively interacting affective factors [79].

C. Discussion of RQ (c)

In contrast to the affective variables, cognitive load does not show a significant effect on learning outcomes in the present model. At first glance, this finding appears to contradict Cognitive Load Theory, which predicts that increased extraneous load should hinder learning, whereas intrinsic load reflects, to some extent, necessary cognitive processing (see Sec. II D).

One possible explanation for the absence of significant effects is that cognitive load cannot be considered independently of affective learner characteristics in this context. The results suggest that variables such as interest and self-concept are associated with differences in learning quantum physics and may be particularly relevant in explaining variance in outcomes, potentially in interaction with cognitive load. Learners with high interest and a strong self-concept may allocate cognitive resources more effectively to the learning material and may be less susceptible to distraction by features of the instructional environment. In contrast, learners with low interest may be more likely to shift attention away from the content toward peripheral aspects of the learning environment. This may manifest as increased extraneous load without directly affecting learning outcomes. Similarly, a low self-concept may lead to the avoidance of cognitively demanding content, thereby reducing intrinsic load without facilitating successful learning.

These considerations suggest that cognitive load operates less as an independent predictor in the present context and should instead be understood in relation to affective variables. In particular, interest can support persistence and learning during information processing, whereas higher academic self-concept is associated with stronger cognitive engagement in learning activities [47,80].

More generally, the findings point to a need for a more differentiated interpretation of cognitive load in physics instruction. While Cognitive Load Theory provides a framework for understanding cognitive processing [19], the present results indicate that affective factors may affect how cognitive load is experienced and managed.

A further explanation for the absence of significant effects relates to the distinction between cognitive processes and stable learning outcomes. Within Cognitive Load Theory, cognitive load is defined as a transient demand on working memory during learning [19]. It therefore primarily captures ongoing processing rather than the formation of long-term conceptual structures.

In contrast, the learning outcomes examined in this study, particularly Functional Fidelity, reflect conceptualizations that extend beyond the immediate learning situation. Their development requires sustained engagement, integration of knowledge, and the restructuring of existing mental models [10]. From this perspective, it appears unlikely that short-term variations in perceived cognitive load directly translate into differences in final learning outcomes.

By comparison, affective variables such as situational interest and self-concept mediate not only immediate processing but also the duration and intensity of engagement with the learning material. They therefore appear to play an important role in whether and how learning processes are sustained over time. Taken together, the findings suggest that these variables may be more strongly associated with learning outcomes than perceived cognitive load during individual learning episodes.

D. Discussion of RQ (d)

The findings provide a differentiated perspective on the role of prior knowledge in shaping learning outcomes. While a significant positive association was observed between Fidelity of Gestalt in the pretest and posttest, no corresponding effect was found for Functional Fidelity in the pretest within the multivariate model.

The association between pretest and posttest of Fidelity of Gestalt suggests that gestalt-based representations exhibit a degree of stability across instruction. Learners who initially ascribed more realistic Gestalt features to physical models tended to retain these representations following the instructional intervention. This pattern indicates that shape-based or structurally grounded representations are relatively persistent and resistant to change, consistent with prior findings in physics education research [6,81].

In contrast, Functional Fidelity in the pretest did not significantly predict posttest outcomes. This may indicate that functional modes of thinking are less reflective of stable prior conceptions and more strongly shaped by ongoing learning processes. The development of functional understanding appears to depend to a greater extent on factors such as interest and self-concept, which regulate the intensity and depth of engagement with the learning material.

From this perspective, Functional Fidelity in the posttest cannot be interpreted as a simple continuation of initial levels. Rather, it reflects processes of conceptual reorganization occurring during instruction. While Fidelity of Gestalt appears to capture the persistence of existing mental models, Functional Fidelity is more closely related to the construction of new conceptual structures and the transformation of prior conceptions.

Overall, the findings suggest that different components of prior knowledge play distinct roles in quantum physics learning. Gestalt-consistent representations exhibit relative stability, whereas functional understanding is more strongly shaped by current learning processes and affective conditions.

E. Limitations

The present study is subject to several limitations:

- The study is based on a quasiexperimental design in which intact classes were assigned to the respective conditions. As a result, randomization at the individual level was not possible, and systematic differences between groups cannot be ruled out.
- The groups differed in their baseline characteristics. The control group, which received qualitative instruction, consisted predominantly of students at a basic level, whereas the experimental group showed a more heterogeneous distribution. Thus, differences in variables such as interest and academic self-concept may have existed prior to the intervention and may have contributed to the observed outcomes.
- Key variables—including interest, cognitive load, and self-concept—were assessed using self-report measures. Such measures are inherently susceptible to biases arising from subjective perception, social desirability, and varying interpretations of the items.
- Data collection took place immediately after a relatively short two-week intervention. Consequently, the study does not capture long-term effects or more stable changes, particularly with respect to academic self-concept.
- The study focuses exclusively on the concept of the single-photon states within quantum physics. The generalizability of the findings to other areas of physics or to other domains is therefore limited.
- The two instructional conditions may not have differed to the same extent across all relevant dimensions. The absence of a significant instructional effect may thus be attributable, at least in part, to insufficient differentiation between conditions.
- The sample differs with respect to educational context, as the experimental and control groups were drawn from different countries. Consequently, cultural and systemic differences in educational practices may have influenced the results, thereby limiting internal comparability.
- Situational interest was assessed retrospectively after the teaching-learning sequence rather than during or immediately after individual learning activities. Given the situation-specific and transient nature of the construct, such retrospective assessments may not fully capture moment-to-moment variations in students' interest [82]. The results should therefore be interpreted as students' retrospective overall assessment of the situational interest associated with the intervention.

VI. CONCLUSION AND OUTLOOK

The findings indicate that the instructional approach, specifically the inclusion of mathematical extensions, does not exert a substantial influence on learning outcomes after controlling for relevant covariates. Learners' affective and cognitive characteristics were associated with variation in learning outcomes. Differences in learning outcomes therefore appear to be more closely related to individual prerequisites than to the instructional format itself. This result is consistent with prior research emphasizing the interplay between affective and cognitive factors in learning (see Sec. II C).

In particular, interest emerges as a key predictor of successful learning. The findings suggest that learning in quantum physics is less sensitive to variations in instructional design than to the extent to which learners engage actively and deeply with the content.

These results highlight the importance of fostering affective and cognitive factors within instructional settings. At the same time, cognitive load remains a necessary component of learning processes and should not be minimized indiscriminately. Rather, effective instruction requires an appropriate balance between cognitive demands and learners' capacity to engage with them.

Academic self-concept, in contrast, appears relatively stable and only limitedly susceptible to short-term instructional interventions. From this perspective, fostering interest represents a promising entry point. Increased interest has been shown to enhance attention, engagement, and persistence, thereby supporting sustained interaction with learning content.

For classroom practice, this implies the need to design learning environments that actively engage learners' interests. This may be achieved through contextualized instruction, the use of real-world problems, or the integration of experimental elements, all of which can establish personal relevance and promote active engagement.

Future research should therefore focus on identifying instructional approaches that effectively support the development of situational interest in physics, particularly in the domain of quantum physics. In addition, further investigation of the interaction between affective and cognitive variables and their differential impact on the degree of Functional Fidelity and Fidelity of Gestalt represents a promising direction. Overall, the findings underscore the importance of instructional approaches that account for both the cognitive and affective prerequisites of learners.

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DATA AVAILABILITY

Data supporting the findings and conclusions are available upon reasonable request from the corresponding author.

APPENDIX

TABLE IV. Rating scale items concerning the probabilistic behavior and indivisibility of single photons at a beam splitter. Items measuring Fidelity of Gestalt are marked with (FG), whereas items measuring Functional Fidelity are marked with (FF).

Statement	1 Strongly disagree	2 Disagree	3 Not sure	4 Rather agree	5 Strongly agree
For the next statements, consider an experiment in which a single photon is emitted onto a beam splitter and two detectors are placed behind it according to the transmission and the reflection.					
(a) Each photon is divided into two smaller parts, causing both detectors to register a photon simultaneously. (FG)	☐	☐	☐	☐	☐
(b) Each photon is split into two smaller parts; however, in real experiments, the detectors cannot measure both parts simultaneously. (FG)	☐	☐	☐	☐	☐
(c) The two detectors never sign simultaneously because a single photon cannot be divided. (FF)	☐	☐	☐	☐	☐
(d) It is not possible to predict with certainty which detector will find them. (FF)	☐	☐	☐	☐	☐
(e) If the information about each photon were fully accessible, we could predict exactly which detector would detect each individual photon. (FG)	☐	☐	☐	☐	☐
(f) Based on the accessible information about each photon, the calculations are too imprecise to predict exactly which detector will detect each individual photon. (FG)	☐	☐	☐	☐	☐

TABLE V. Rating-scale items concerning conceptions of single-photon trajectories at a beam splitter. Items measuring Fidelity of Gestalt are marked with (FG), whereas items measuring Functional Fidelity are marked with (FF).

Statement	1 Strongly disagree	2 Disagree	3 Not sure	4 Rather agree	5 Strongly agree
For the next statements, consider an experiment in which a single photon is emitted onto a beam splitter and two detectors are placed behind it according to the transmission and the reflection.					
(a) Each photon follows a specific path to a detector, even if this path cannot be observed. (FG)	☐	☐	☐	☐	☐
(b) Each photon follows a specific path, regardless of whether or not it is observed. (FG)	☐	☐	☐	☐	☐
(c) The accurate position of a single photon at any given time between the beam splitter and the detectors cannot be determined in principle. (FF)	☐	☐	☐	☐	☐
(d) It is feasible for a single photon to have no classically well-defined position. (FF)	☐	☐	☐	☐	☐
(e) At the beam splitter, each photon randomly selects a specific future trajectory. (FG)	☐	☐	☐	☐	☐
(f) Photons are too fast for their positions to be investigated experimentally. (FG)	☐	☐	☐	☐	☐
(g) Individual photons cannot be observed because the detector itself disturbs their position. (FG)	☐	☐	☐	☐	☐

TABLE VI. Rating-scale items concerning single-photon interference in a Michelson interferometer. Items measuring Fidelity of Gestalt are marked with (FG), whereas items measuring Functional Fidelity are marked with (FF).

Statement	1 Strongly disagree	2 Disagree	3 Not sure	4 Rather agree	5 Strongly agree
Consider individual photons in a Michelson interferometer. A single-photon source is used so that only one photon is emitted at a given time.					
(a) Changing the position of mirror M_T has no effect on the measurement outcome because single photons cannot interact with themselves. (FG)	☐	☐	☐	☐	☐
(b) The beam splitter divides each photon into two smaller parts that can interfere with each other. (FG)	☐	☐	☐	☐	☐
(c) The beam splitter does not make a single photon split; however, a single photon has no certain trajectory between the beam splitter and detector. (FF)	☐	☐	☐	☐	☐
(d) A single photon cannot be split by the beam splitter, but an interference pattern can be seen when the position of the mirror M_T is constantly changing. (FF)	☐	☐	☐	☐	☐
(e) When we change the position of mirror M_T, we obtain information about the positions of the photons. (FG)	☐	☐	☐	☐	☐
(f) When I move the mirror, I can use the measurement outcome to calculate the exact path of each photon to the detector. (FG)	☐	☐	☐	☐	☐
(g) Using the interference pattern, we could not calculate the certain trajectory of a single photon. (FF)	☐	☐	☐	☐	☐

TABLE VII. Rating-scale items measuring students' academic self-concept in physics.

Statement	1 Strongly disagree	2 Disagree	3 Not sure	4 Rather agree	5 Strongly agree
To what extent do you agree with the following statements?					
(a) I understand the material covered in physics very well.	☐	☐	☐	☐	☐
(b) I retain the material covered in physics very well.	☐	☐	☐	☐	☐
(c) I understand the material covered in physics lessons very quickly.	☐	☐	☐	☐	☐
(d) In my own assessment, my performance in physics is very good.	☐	☐	☐	☐	☐
(e) I believe that my classmates consider me very good at physics.	☐	☐	☐	☐	☐
(f) I believe that my physics teacher considers my performance in physics to be very good.	☐	☐	☐	☐	☐
(g) I expect my future performance in physics to be very good.	☐	☐	☐	☐	☐
(h) I think I am talented in physics.	☐	☐	☐	☐	☐

TABLE VIII. Rating-scale items measuring cognitive load during the quantum-physics teaching-learning sequence.

Statement	0	1	2	3	4	5	6	7	8	9	10
Please indicate on a scale from 0 to 10 how strongly you agree with each statement (0 = strongly disagree; 10 = strongly agree).											
(a) The topics covered in the activity were very complex.	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
(b) The activity covered formulas that I perceived as very complex.	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
(c) The activity covered concepts and definitions that I perceived as very complex.	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
(d) The instructions and explanations during the activity were very unclear.	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
(e) The instructions and explanations were, in terms of learning, very ineffective.	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
(f) The instructions and/or explanations were full of unclear language.	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐

TABLE IX. Rating-scale items measuring students' situational interest in quantum optics.

Statement	1 Strongly disagree	2	3	4	5	6 Completely agree
Please indicate on a scale from 1 to 6 how strongly you agree with each statement (1 = strongly disagree; 6 = completely agree).						
(a) I spend time on quantum optics in free time.	☐	☐	☐	☐	☐	☐
(b) I enjoy solving quantum optics problems.	☐	☐	☐	☐	☐	☐
(c) I discuss quantum optics with friends or family.	☐	☐	☐	☐	☐	☐
(d) I do not notice time passing when engaging with quantum optics.	☐	☐	☐	☐	☐	☐
(e) I want to spend more time on quantum optics.	☐	☐	☐	☐	☐	☐
(f) I liked quantum optics lessons.	☐	☐	☐	☐	☐	☐

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