

Investigating the effect of two-state approaches on students' understanding of quantum measurement: A quasiexperimental field study

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With the rise of quantum computing, interest has grown in using two-state quantum systems (qubits) at the secondary level to foster students' conceptual understanding. Quantum measurement, in particular, is central to quantum theory and its accurate conceptualization by students is crucial for grasping fundamental quantum principles. However, instructional methods typically make use of different contexts (i.e., different two-state systems), significantly affecting students' conceptual development in quantum physics. In this paper, we report findings from a cluster-randomized field trial involving 181 students taught through three inquiry-based, two-state approaches: the which-path-encoded single-photon, the polarization, and the double-well potential approach. All three approaches supported students' conceptual development, yet students taught using photon polarization and the double-well potential significantly outperformed those participating in a course following the which-path-encoded single-photon approach. Our findings indicate that students participating in the which-path-encoded single-photon approach often retain mixed-thinking frameworks, whereas those taught with photon polarization or the double-well potential approaches were more likely to develop toward quantum thinking. Thus, our findings underpin how influential the choice of (experimental) context is on students' conceptual development (also) in quantum physics.

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I. INTRODUCTION

Quantum physics was introduced into secondary school curricula in the final decades of the 20th century, primarily due to its cultural significance and foundational role in

atomic physics. During this period, instruction in quantum physics predominantly followed historical, atomic physics-based, or wave-oriented approaches. The historical approach focuses on recounting key milestones in the development of quantum theory (for example, through the evolution of atomic models) with the aim of fostering students' knowledge of the history of science and the nature of scientific inquiry. However, because this approach typically employs semiclassical representations (e.g., depicting quantum entities as particles moving continuously through space and time), it has been shown to contribute to persistent student

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misconceptions [1–7]. In contrast, the atomic physics and wave approaches tend to emphasize conceptual understanding relevant to chemical bonding and material structure, offering a more systematic foundation for explaining quantum phenomena [8].

In recent years, quantum technologies have emerged as a rapidly advancing and strategically important field, giving rise to a growing demand for so-called “quantum engineers.” As a result, the teaching of quantum physics has garnered renewed attention, which has significantly transformed the discourse surrounding quantum physics education. In particular, instructional approaches centered on two-state quantum systems (commonly referred to as two-state approaches) have gained prominence, largely due to the foundational role of the quantum bit (qubit), which itself is a two-state system.

The underlying rationale is that secondary school students should be introduced to the principles of quantum technologies early on if they are to pursue careers in this emerging domain. Accordingly, a coordinated effort has been undertaken to identify the core conceptual knowledge required by future quantum engineers [9], to delineate the quantum concepts appropriate for secondary education [10], to examine existing instructional methodologies at the secondary level [11], and to develop reliable instruments for assessing student understanding of these concepts [12]. These initiatives have been carried out by large-scale collaborations among scientists and educators, with the aim of establishing a broadly supported consensus within the field.

Quantum measurement is widely regarded by experts as one of the most fundamental and conceptually central topics in quantum physics [10,13]. Its significance extends beyond quantum technology education, underscoring the need for students to develop a robust understanding of quantum measurement even at the secondary school level. Moreover, the comprehension of quantum measurement represents a considerable challenge for students [14–16]. In this study, we examine and compare students’ conceptual understanding of quantum measurement across three distinct instructional approaches based on two-state quantum systems: the which-path-encoded single-photon approach (WP), the polarization approach (POL), and the double-well approach (DW).

II. RESEARCH BACKGROUND

A. Two-state approaches to quantum physics

A quantum system is a two-state system if it can be described by two quantum states [17–21]. One of the key pedagogical advantages of using two-state systems is their mathematical simplicity, which makes them especially accessible for educational purposes. Furthermore, they provide a natural entry point into a central concept of quantum information science: the qubit. Two-state systems

serve as effective platforms for introducing a wide range of foundational quantum concepts, including the quantum state, the superposition principle, the inherently stochastic nature of measurement outcomes, the incompatibility of certain observables, and the effect of measurement. Some instructional approaches also incorporate additional phenomena such as time evolution [22] and entanglement [23]. Given the diversity of physical systems that exhibit two-state behavior, a variety of experimental contexts are available for instructional use. Notable examples include the electron spin approach [18,20,24]; the which-path-encoded single-photon (WP) approach, which employs interferometric setups [25–28]; the polarization approach (POL) [6,14,17,29]; and the double-well potential approach (DW) [22,30].

Although nearly all two-state approaches cover the same fundamental quantum concepts, they do so within distinct physical contexts, each introducing unique characteristics to the instructional experience. The existence of multiple two-state models naturally raises the question of the potential strengths and limitations associated with each. While experts readily navigate across these contexts by identifying structural analogies and conceptual mappings, students often struggle to make such connections [31,32]. It is well established that students’ conceptual development is highly sensitive to the specific instructional context in which learning occurs [33–35]. Consequently, it is difficult to predict what students will internalize from each setting or how their understanding will transfer across contexts. For instance, conceptualizing quantum measurement through photon polarization (e.g., photons at a polarizer) differs significantly from interpreting it as the collapse of the position wave function (e.g., detecting photons at a specific position). These contextual nuances may introduce distinctive cognitive patterns in students’ learning that remain underexplored in the literature. Since the three approaches examined in this study (WP, POL, and DW) are grounded in fundamentally different physical settings, it is reasonable to hypothesize that students’ conceptions of quantum measurement will vary accordingly and that meaningful differences in their conceptual understanding may emerge.

B. Quantum measurement

Only recently have suitable assessment instruments been developed to evaluate the effectiveness of various instructional approaches in quantum physics education. For example, studies have shown that students taught via an interferometer-based two-state approach (commonly referred to as the which-path-encoded single-photon approach) demonstrated significantly greater gains in conceptual understanding compared to those taught using traditional methods at the secondary level [36]. In a separate effort, a questionnaire was designed to assess students’ understanding of a specific quantum concept, quantum measurement, rather than general quantum

reasoning, with a primary focus on university-level learners [37].

In 2024, a questionnaire was developed to specifically assess secondary school students' conceptual understanding of quantum measurement [12]. The study found that a tripartite classification of students' reasoning types in quantum physics provides a suitable framework for characterizing and evaluating their conceptual understanding across different instructional approaches. This categorization is consistent with existing frameworks in the literature on quantum physics education [6,38].

The response options for each item in the questionnaire (cf. Ref. [12] and Table VII in the Appendix) are designed to reflect distinct patterns of thinking, as follows:

- *The classical way of thinking:* When students interpret quantum phenomena using concepts and frameworks derived from classical physics. Quantum objects are conceived as possessing well-defined properties, meaning that all physical quantities have precise, determinate values, and that measurements merely uncover these preexisting attributes. Within this framework, students often interpret the statistical nature of quantum mechanics as stemming from incomplete knowledge or limitations of the measuring instruments, rather than as an inherent feature of the theory itself.
- *The mixed thinking:* Within this intermediate, or classical-quantum hybrid, reasoning framework, students' thoughts exhibit components that are accurate from a quantum physics standpoint, yet concurrently employ classical and mechanistic justifications. They continue to attribute classical properties to quantum objects but emphasize that experimentally verifying these well-defined properties is very difficult or even impossible. They distinguish between the empirically accessible world and the theoretical realm, postulating the existence of hidden properties intrinsic to quantum systems. According to this perspective, the inherent disturbance caused by measurement prevents exact determination of physical quantities, thereby necessitating a probabilistic description.
- *The quantum-mechanical way of thinking:* Students interpret quantum phenomena in terms of statistical ensembles, recognizing that quantum objects fundamentally lack certain properties (physical quantities are often inherently uncertain), even when ideal measurement devices are employed. Furthermore, an ideal measurement can alter the system's state, enabling the precise determination of some previously uncertain properties, while rendering other formerly certain properties uncertain.

It has been shown that two-state approaches outperform traditional methods in enhancing students' conceptual understanding of quantum measurement [12]. This finding suggests that two-state approaches are more effective in fostering a quantum-mechanical way of thinking. However,

the different two-state approaches, with their varying structures and emphases, likely lead to different effects on student learning. The development of a standardized assessment instrument [12] facilitates a direct comparison of students' conceptual understanding of quantum measurement across these different instructional approaches.

III. RESEARCH QUESTION

The research question guiding this study is the following: *What are the differences in students' learning gains regarding the conceptual understanding of quantum measurement when comparing the which-path-encoded single-photon approach, the polarization approach, and the double-well approach?*

This research does not attempt to rank the different approaches. Rather, our goal is to identify the fundamental differences and similarities in students' understanding supported through these instructional approaches and to determine which approach is more effective in conveying a specific, yet crucial concept: quantum measurement.

IV. METHODOLOGY

A. Study design and sample

We conducted a cluster-randomized field study to investigate the effect of different two-state approaches on upper-level secondary school students' learning of quantum physics, with a particular focus on quantum measurement: which-path-encoded single-photon approach (WP approach), polarization approach (POL approach), and double-well approach (DW approach). A detailed description and comparison of the interventions in the three comparison groups is provided in Sec. IV B. The instrument used to assess learners' conceptual understanding of quantum measurement at pretest and post-test points in time is described in Sec. IV C. The sample comprised a total of $N = 181$ secondary school students from Germany, Hungary, Italy, and Slovenia. A detailed overview of the study sample in terms of age and its distribution among different approaches is given in Table I. The participating students had not previously studied quantum physics, so the quantum physics curricula of different countries did not influence the students' thinking. As upper secondary school students participated, they had basic knowledge of mathematics and physics. Student participation in our

TABLE I. Overview of the study sample.

	WP approach	POL approach	DW approach
Number of students	75	66	40
Average age (years)	17.0	17.8	17.0
Number of classes	4	4	2
Number of instructors	1	3	1

study was voluntary and without financial compensation. They were also informed about the anonymity of their data and how it would be processed and were asked for their consent to participate.

B. Interventions

The three approaches investigated in this study introduce the same quantum concepts with a focus on quantum states, superposition principle, effect of measurement, quantum randomness and quantum interference, and an inquiry-based learning is taken. The approaches under comparison are presented below, and the comparison is summarized in Tables II and III.

1. Which-path-encoded single-photon approach (WP)

Students investigate the behavior of a single photon in a single-photon beam splitter setup through repeated measurements. A photodetector was placed at each of the two output ports of the 50/50 beam splitter: one for transmission and one for reflection. The students explore the probabilistic nature of quantum measurement: the outcomes (transmission or reflection) of individual measurements are unpredictable in principle, although the distribution of results over a large number of repeated measurements can be given by probabilities. The probabilistic nature of measurement arises from the existence of superposition states, which differ from the eigenstates associated with measurable outcomes. However, the measurement collapses the superposition state into one of the eigenstates.

Through a single-photon Michelson interferometer, the students find that a single photon cannot follow a certain path: even a single photon can produce an interference pattern in a sequence of repeated measurements, which

implies an intrinsic uncertainty in their position. If a certain position is assigned to the photons, the interference would not occur, demonstrating the effect of measurement. Students also come to recognize that, instead of describing photons in terms of spatial and temporal properties, the system is characterized by a quantum state. They describe the experiments using the quantum formalism, which is represented by the Dirac notation. Details on this approach can be found in Refs. [27,28].

2. Polarization approach (POL)

The students investigate the photon-polarizer interaction, where the students explore that the transmission or absorption of individual measurements are unpredictable by principle, although the distribution of results over a large number of repeated measurements can be described by Malus's law since it receives a new, probabilistic interpretation: $\cos^2 \theta$ is the probability of transmission of photons. As photons pass through a polarizer, their polarization changes, illustrating the effect of measurement.

Students learn that photons can be associated with either certain or uncertain polarization properties, where properties are understood as qualitative representations of eigenvalues. During the examination of polarization properties, the following concepts are introduced: (1) mutually exclusive properties (the qualitative form of the relationship between the eigenvalues of a given physical quantity); (2) incompatible properties (the relationship between the eigenvalues of noncommuting quantities); and (3) the uncertainty principle expressing the uncertainties of quantities.

Quantum states are introduced in a vector representation corresponding to the polarization directions. The probability

TABLE II. Comparison of the investigated approaches.

Approach	Context	Quantity under investigation	Remark	References
WP	Single-photon interaction with 50/50 beam splitter.	Number operator: eigenstates $ T\rangle$, $ R\rangle$ (assigned to the certain transmission and reflection, respectively).	The single-photon Michelson interferometer experiment is treated as two photon-beam splitter interactions.	[27,28]
POL	Single-photon interaction with polarizers and birefringent calcite crystals.	Polarization: eigenstate vectors $\mathbf{v}$, $\mathbf{h}$ (vertical and horizontal polarization states) or $\mathbf{d}$, $\mathbf{a}$ (diagonal and antidiagonal).	The uncertainty of spatial location is also demonstrated with birefringent calcite crystals, where polarization and spatial location are entangled.	[6,14,29,39]
DW	A single electron in a double-well potential.	Spatial location: left or right well with eigenvalue $x = L$ or $x = R$ and eigenstate $ L\rangle$ or $ R\rangle$. Energy: ground or first excited state energy with eigenvalue $E1$ or $E2$ and eigenstate $ E1\rangle$ or $ E2\rangle$.	Pseudo-two-state system: an idealization of an infinite double well with infinite barrier is used, where the approximation $ L\rangle = (1/\sqrt{2})(E1\rangle + E2\rangle)$ holds.	[22,30]

TABLE III. Comparison of the investigated approaches with respect to the features of quantum measurement.

Aspects of quantum measurement	WP approach	POL approach	DW approach
Statistical predictability	Photons at beam splitters	Photons at polarizers and birefringent calcites	A single electron in a double well
Superposition state (knowledge of state, measurement effect).	$\varphi_{0^\circ} T\rangle + \varphi_{90^\circ} R\rangle$, where $ T\rangle$ and $ R\rangle$ are the states of certain transmission and reflection, respectively, with the corresponding quantum phases φ_{0° , φ_{90° .	$\mathbf{u} = \psi_1\mathbf{v} + \psi_2\mathbf{h}$, where $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{h}$ are polarization state vectors and ψ_1 and ψ_2 are the corresponding probability amplitudes.	$ L\rangle = (1/\sqrt{2})(E1\rangle + E2\rangle)$, where $ L\rangle$, $ E1\rangle$, and $ E2\rangle$ are the states of being in the left well, possessing ground state or first excited state energy with certainty, respectively.
Measurement outcomes can be predicted with certainty in eigenstates.	In a Michelson interferometer, for certain mirror positions, the measurement outcome can be predicted with certainty.	Photons in certain polarization states will pass through or be absorbed by a polarizer.	Either the position or energy of electrons can always be measured precisely, but not both at the same time.
The measurement outcomes are mutually exclusive.	The certain reflection excludes the certain transmission.	Photons cannot pass through two perpendicular, consecutive polarizers.	Certain presence in the left well excludes certain presence in the right one. The certain possession of the ground state energy excludes the certain possession of the first excited state energy.

interpretation of state transitions in a measurement is then assigned to Malus's law, where the transition probability equals the square of the scalar product of the two state vectors associated with the state change. State vectors are also expressed as linear combinations (i.e., superpositions) of basis vectors (eigenstates). In a measurement, the superposition state changes to one of the eigenstates, with the corresponding probabilities.

Single-photon at a birefringent calcite crystal experiment is considered, in which the polarization state and the spatial location of photons are entangled. This entanglement reveals the uncertainty of the photons' spatial location. Observing either the spatial position or the polarization of photons demonstrates the effect of measurement. Details on this approach can be found in Refs. [6,14,29,39].

3. Double-well approach (DW)

The context of the double well is treated as a pseudo-two-state system, where the bases are spatial location or energy eigenstates and the possible spatial location properties are left or right well and possible energy properties are ground state energy and first excited state energy. The position and energy properties are incompatible, so the uncertainty principle holds between them.

Students use an idealized simulation to investigate the behavior of a single electron in a double well. They measure the spatial location and energy of electrons in a sequence of repeated measurements. The simulation allows setting the initial state, and the time and base for three

measurements. The simulation displays the results in terms of histograms with a few visual effects to indicate a successful measurement. Students can use the simulation on their mobile phones. Details on this approach can be found in Refs. [22,30].

C. Instrument

We used an instrument comprising a total of eight single-choice items to assess students' conceptual understanding of quantum measurement. The instrument used in this study has been developed within the framework of the DQC2step project [40]. It is the result of a comprehensive development process—including expert interviews and discussions as well as quantitative piloting—and has proven to be sufficiently instruction sensitive [12]. All of the eight items come with three to four answer options which have been designed

- based on students' answers in a qualitative prestudy, hence, reflecting students' thinking about quantum measurement in an authentic manner, and
- in a context-independent manner allowing (1) to translate the items into a specific educational context, (2) for comparisons between teaching approaches focusing on different physical realizations of qubits with regards to students' conceptual development.

The three to four answer options of each item have been designed to be indicators of either quantum thinking (correct answer option, score: 2 points), mixed thinking (score: 1 point), or classical thinking (score: 0 points) based

TABLE IV. An example item including answer options and scores. The scoring of the response options 0-1-2 reflects indicators for classical, mixed, and quantum-mechanical ways of thinking, respectively. The item reads as follows: “Indicate which statement is correct about the result of a measurement of a quantity P on a single quantum object?”

Answer option	Score
(a) We could predict the outcome of the measurement if we could fully obtain the necessary information about the quantum object.	0
(b) We cannot predict anything about a single measurement because the possible values of the result are not precisely defined.	1
(c) We can predict the probability of obtaining a certain value if the state of the quantum object is known.	2

on findings from a qualitative prestudy described in Ref. [12]. Consequently, students could score a maximum of 16 points on the eight single-choice items of the instrument [12]. The questionnaire allows us to examine the development of the students’ total scores. On the other hand, each student presents eight ways of thinking through eight items, so the frequency of the three ways of thinking can be examined from data relating to $N \times 8$ individuals. The employment of a three-point scale is further substantiated by its capacity to differentiate more precisely between students who exclusively conceptualize in classical terms and those who have partially acquired an understanding of quantum physics. This is due to the fact that we do not merely ascertain whether an answer is correct or incorrect; we also consider the possibility that there may be varying degrees of understanding. The term “mixed thinking” is employed in a broad sense. A more refined categorization may be possible, however, such an endeavor would necessitate the development of a significantly more intricate evaluation instrument. This approach would be ill-suited to a concise assessment intended for utilization within a classroom context.

An example item is given in Table IV and all the items of the instrument, including scoring descriptions, are provided in the Appendix. As illustrated in Table IV, each of the three answer options reflects a distinct way of thinking (classical, mixed, and quantum). The 0-point answer option (classical) suggests that students imagine quantum objects as classical particles, and therefore believe that accurate measurement requires information about the particle. The 1-point answer option (mixed) suggests that the student has already recognized the impossibility of making accurate predictions in certain scenarios. However, the reasoning is flawed because the student believes that there are external reasons for this. The 2-point answer option (quantum) employs quantum knowledge in a competent manner and accepts the necessity of providing a probabilistic description.

D. Data analysis

First, we provide an overview of the conceptual understanding of quantum measurement by students across all cohorts for both the pretest and post-test using descriptive statistics such as median Mdn , mean M , and standard deviation SD .

We used paired samples t tests to check the difference between pre- and post-test scores of students from the DW approach, the POL approach, and the WP approach, respectively, for statistical significance. We used Cohen’s d as a measure of effect size, alongside the established ranges of small ($d < 0.5$), medium ($0.5 \leq d < 0.8$), and large ($0.8 \leq d$) effect sizes [41]. While the difference between pre- and post-test scores does not provide a reliable measure of student learning at either end of the scale [42], Hake’s normalized gain g is not biased by student pretest scores [43]. The normalized gain g takes values less than 1, while values of $0.30 \leq g \leq 0.70$ may be associated with moderate learning gains (Ref. [44], p. 65).

In the next step, analysis of variance (ANOVA) was used to test whether there were statistically significant differences between the three cohorts with respect to students’ conceptual understanding of quantum measurement at the pretest and post-test points in time. Furthermore, post hoc Tukey-Kramer tests were used to check for statistically significant differences between the groups. As a measure of effect size for the overall comparison, we used partial eta squared (η_p^2), applying the commonly used categorization of small ($\eta_p^2 < 0.06$), medium ($0.06 \leq \eta_p^2 < 0.14$), and large ($0.14 \leq \eta_p^2$) effects [41]. Again, Cohen’s d was used as a measure of effect size for pairwise comparisons. To ensure the homogeneity assumption underlying ANOVA, we used Levene’s test [45]. In addition, the normal distribution of the data was assessed using the Shapiro-Wilk test [46].

To visualize how students’ understanding of quantum measurement develops, we use Sankey diagrams showing the proportion of responses indicating classical, mixed, or quantum thinking across all responses per cohort (i.e., number of students in cohort $\times$ items $\times$ answer options) in the pre- and post-tests.

Note that for all statistics applied to our data, we only used data from those students who completed all items of the instrument.

V. RESULTS

A. Overview of the data

Table V provides an overview of the proportion of correct responses for all items on the instrument, broken down by cohort, for both the pre- and post-test.

TABLE V. Percentage of correct responses (reflecting options worth 2 points and quantum mechanical reasoning) per item, comparing pretest and post-test results across the three instructional approaches (POL, polarization approach, DW, double-well approach, and WP, which-path-encoded single-photon approach).

Item	POL	DW	WP
Item 1—pre	24.2	45.0	42.7
Item 1—post	60.6	80.0	13.3
Item 2—pre	72.7	70.0	49.3
Item 2—post	86.4	75.0	61.3
Item 3—pre	22.7	35.0	57.3
Item 3—post	40.9	42.5	62.7
Item 4—pre	9.1	2.5	36.0
Item 4—post	33.3	50.0	6.7
Item 5—pre	16.7	27.5	12.0
Item 5—post	42.4	62.5	42.7
Item 6—pre	39.4	40.0	25.3
Item 6—post	77.3	52.5	37.3
Item 7—pre	24.2	5.0	25.3
Item 7—post	47.0	5.0	28.0
Item 8—pre	18.2	2.5	22.7
Item 8—post	57.6	35.0	9.3

Descriptive statistics on pretest and post-test scores for the three intervention groups are given in Table VI.

The difference between the students' pre- and post-test scores is statistically significant for either approach, as shown by paired samples t tests [47] for the polarization approach [$t(65) = 7.08$, $p < 0.001$; $d = 0.87$], for the double-well approach [$t(39) = 5.63$, $p < 0.001$; $d = 0.89$], and for the which-path-encoded single-photon approach [$t(74) = 2.71$, $p < 0.01$; $d = 0.31$], respectively. Accordingly, the median learning gains, expressed in terms of Hake's g , can be considered moderate for students following the polarization approach (median $g = 0.40$) and for students following the double-well approach (median $g = 0.37$), in contrast to participants participating in the which-path-coded single-photon approach (median $g = 0.11$).

B. Analysis of variance

The students' pretest scores (see Table VI) do not differ statistically significantly across groups according to a one-factor ANOVA [$F(2, 178) = 0.27$, $p = 0.761$]. However, at the time of the post-test, students' conceptual understanding of quantum measurement differed statistically significantly

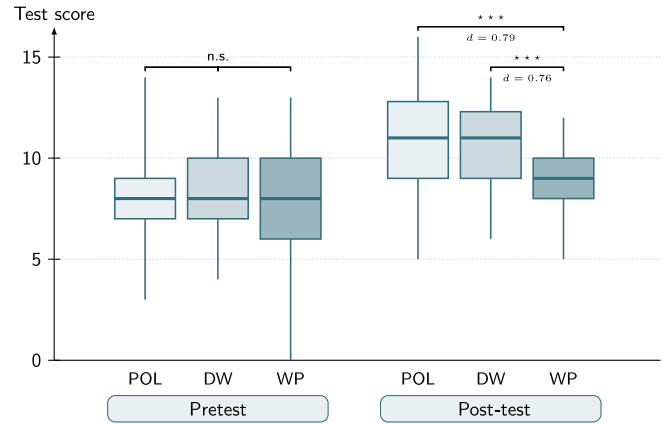


FIG. 1. Boxplots for the students' pre- and post-test scores for the polarization approach (POL), the double-well approach (DW), and the which-path-encoded single-photon approach (WP). The whiskers indicate minimum and maximum scores.

across the different cohorts [$F(2, 178) = 13.4$, $p < 0.001$; $\eta_p^2 = 0.13$]. Pairwise comparisons using Tukey-Kramer post hoc tests reveal no statistically significant difference in students' conceptual understanding of quantum measurement between the polarization approach cohort and the double-well cohort ($p = 0.986$). However, students who participated in the which-path-encoded single-photon approach on average achieve a highly statistically significant lower conceptual understanding of quantum measurement than students from the polarization approach cohort ($p < 0.001$) and students from the double-well approach cohort ($p < 0.001$), with medium effect sizes of $d = 0.79$ and $d = 0.76$, respectively. Figure 1 provides a graphical overview of the distribution of students' pre- and post-test scores by cohort.

C. Students' conceptual development

In the final analysis step, we analyzed the number of answer options chosen that serve as indicators for quantum-like thinking, for mixed thinking, and for classical thinking. In particular, we examined the percentage of transitions between responses indicating classical, mixed, and quantum thinking from pre- to post-test points in time [48]. By determining the relative frequencies of all such transitions, students' conceptual development for all three cohorts can be described by the Sankey diagrams shown in Figs. 2–4, which

TABLE VI. Descriptive statistics (M , mean, SD, standard deviation, Mdn, median, min, minimum, and max, maximum) for pre- and post-test scores across instructional approaches. The maximum test score was 16 points.

	M		SD		Mdn		Min		Max	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
POL	7.95	10.90	2.09	2.63	8.00	11.00	3	5	14	16
DW	8.30	10.80	1.99	2.14	8.00	11.00	4	6	13	14
WP	8.13	9.17	2.76	1.68	8.00	9.00	0	5	13	12

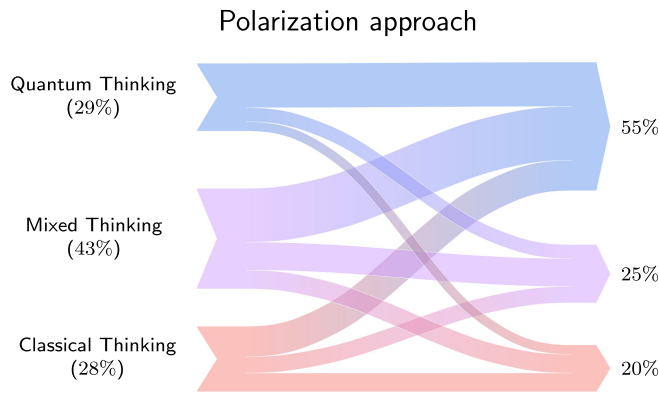


FIG. 2. Sankey diagram illustrating how students' responses indicating classical, mixed, or quantum thinking shifted from pretest to post-test under the polarization approach. Percentages represent the share of total responses (i.e., number of students $\times$ items $\times$ answer options) categorized into each thinking type.

illustrate how students' response patterns evolved from pretest to post-test in the three instructional approaches.

In the polarization approach (see Fig. 2), a clear trend emerges: a substantial proportion of students, initially distributed among responses indicative of quantum thinking (29%), mixed thinking (43%), and classical thinking (28%), shifted predominantly to responses indicative of quantum thinking at the post-test point in time (55%). Similarly, the double-well approach students show a strong progression toward responses indicative of quantum thinking, albeit slightly less pronounced (see Fig. 3). Here, 50% of the students' responses were consistent with quantum thinking at the post-test point in time; however, with responses indicative of mixed thinking still accounting for 35%. In contrast, the which-path-encoded single-photon approach (see Fig. 4) presents a markedly different picture. The final distribution shows that the majority of students (49%) retain responses indicative of mixed thinking, with only 33% of responses reflecting quantum thinking and

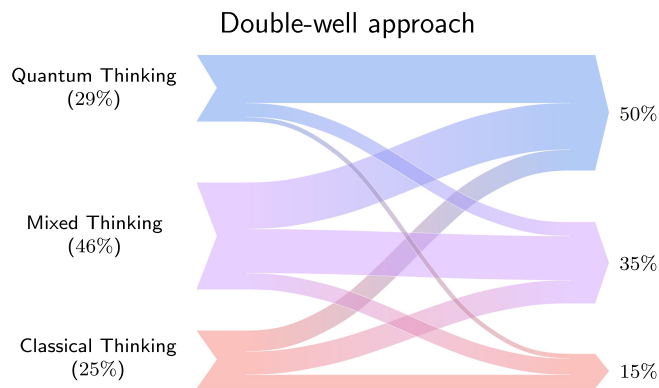


FIG. 3. Sankey diagram illustrating how students' responses indicating classical, mixed, or quantum thinking shifted from pretest to post-test under the double-well approach. Percentages represent the share of total responses (i.e., number of students $\times$ items $\times$ answer options) categorized into each thinking type.

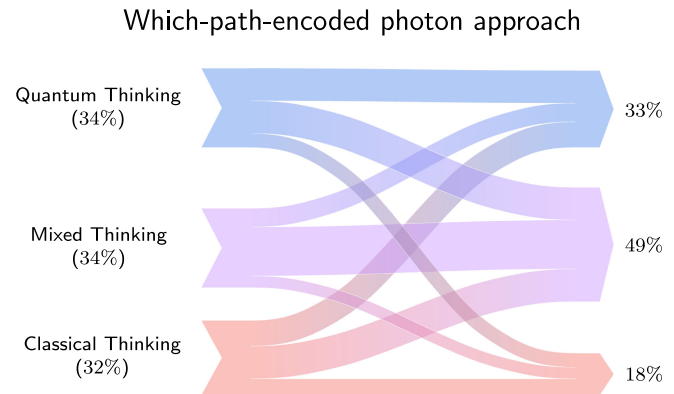


FIG. 4. Sankey diagram illustrating how students' responses indicating classical, mixed, or quantum thinking shifted from pretest to post-test under the which-path-encoded single-photon approach. Percentages represent the share of total responses (i.e., number of students $\times$ items $\times$ answer options) categorized into each thinking type.

18% remaining aligned with classical thinking. These patterns substantiate the quantitative results reported in Secs. VA and VB.

VI. DISCUSSION

A. Conceptual understanding of quantum measurement

We found no statistically significant difference in students' conceptual understanding of quantum measurement in two-state systems between those taught using the polarization approach (POL) and those using the double-well approach (DW) (cf. Table I). However, our results show that students exposed to the which-path-encoded single-photon approach (WP) performed significantly worse the conceptual understanding of this than students in the other two groups (cf. Table I). These findings suggest that the instructional approach (even if they are all based on two-state systems) plays a critical role in shaping students' conceptual understanding of quantum physics, with the WP proving less effective for learning about quantum measurement.

This may be because the central element of the WP approach is "motion" (or, more precisely, the lack of classical motion). Previous research on quantum physics education shows that overcoming classical motion is one of the most difficult tasks for students in every quantum approach at every level [2,3,5–7,14,38,49–55]. Rather than accepting the absence of classical motion, students often construct hybrid theories that blend classical and quantum concepts. For instance, they may assume that quantum objects follow well-defined trajectories, even though such paths are not supported by experimental evidence. Students instinctively strive to visualize quantum effects [2,3,5–7], but there is no way to describe or depict their continuous motion in space and time. According to the theory of conceptual shift [56], it is difficult for students to break with their worldview formed according to classical physics

and instead adopt quantum thinking to at least some extent [1,8]. Understanding quantum mechanics requires a reboot of knowledge [26]. Figures 2–4 clearly indicate that a significant proportion of students taught using the WP approach tend to adopt the so-called mixed thinking (regarding quantum measurement) and struggle at that level. Presenting quantum measurement through a less “loaded” concept [polarization (POL) or energy (DW)], students can more easily grasp certain aspects of quantum measurement.

It is worth noting that in a previous study [12], students who learned through the WP approach performed better in several items, likely due to the inclusion of a quantum eraser experiment involving polarizers, which extended the teaching-learning sequence compared to this study. Combining different instructional approaches, such as the WP and POL frameworks, may offer a more comprehensive learning experience by leveraging the pedagogical strengths of each.

B. Discussion of students’ development on the item level

Although the reliability of each question is low, we find it worthwhile to examine the distribution of student responses to each question.

The results of the first task (item 1) show a large difference between the cohorts, with a small proportion of students using the WP approach giving the correct answer, while a large proportion of students using the other two approaches (POL, DW) gave the correct answer. Perhaps, this is due to the fact that in the WP approach, students have strong memories of a single-photon beam splitter interaction, where they experience that the measurement result for a single photon is unpredictable. The fact that a quantum physical measurement is sometimes predictable (because the system is an eigenstate) appears in this approach only in exceptional cases: When the position of the mirrors in a single-photon Michelson interferometer gives a 0% or 100% probability of detection. Conversely, in the POL and DW approaches, students may develop a strong intuitive image associated with the task. For example, a horizontally polarized photon will certainly pass through a polarizer with horizontal permitted direction. Similarly, if an electron is located in the left-hand well and a position measurement is performed, it will be found there with certainty.

The WP approach also strongly underperformed in item 4. We think this may be because in the WP approach the concept of quantum measurement always comes up during the detection, so successive rapid measurements are difficult to imagine. However, in the POL approach and the DW approach successive measurements are very intuitive, for instance, when photons are emitted onto several successive polarizers (which is confirmed by a previous study [6]), or the energy of an electron is measured several times, one

after the other, in rapid succession. Similar arguments were made with respect to item 8.

The response pattern of item 6 shows a strong difference in students’ answers; the students from the POL approach performed best, while students from the WP approach received the lowest scores. This is perhaps because the WP approach to detection involves the “annihilation” of the photon, so the change of state is not so convincing. On the other hand, in the POL approach, the state of the photon has changed after passing through the polarizer, and this can be registered by a photodetector.

It is particularly interesting that almost no students taught with the double-well (DW) approach answered item 7 correctly (repeating the result of our previous study [12]), despite the fact that the mutual exclusivity of measurement outcomes is also present in this approach. In contrast, mutual exclusivity is more explicitly visible in the polarization (POL) approach, where orthogonal polarizers completely block light transmission, clearly illustrating this concept and visible with the naked eye.

VII. LIMITATIONS

It is important to highlight the limitations of our research. The presented study is a cluster-randomized design, which inherently lacks randomization at the individual level. This limitation may lead to systematic differences between cohorts, particularly concerning variables not accounted for in this research. To address this, future experimental studies should include additional control variables to enhance the robustness of the findings.

The instrument used to evaluate students has scoring limitations. For example, someone could score 8 points by choosing mixed-thinking answer options for every question. But it is also possible that four of their answers score 0 points (classical) and four score 2 points (quantum). The two cases are completely different in content, yet they result in the same total score. In other words, the total score does not distinguish between those who consistently demonstrate mixed thinking and those who think correctly in some cases and incorrectly in others.

Further element that vary across the approaches are the teaching methodology and the amount of instructional time required. The chosen approaches all use active learning methods, but not the same ones.

We think that a deeper insight into the students’ thinking, using qualitative methods, or an analysis of other fundamental quantum physics concepts would be a valuable future research task to complement our results.

VIII. CONCLUSION

In this paper, we investigated students’ conceptual understanding of quantum measurement in the context of three different two-state approaches to quantum physics: the which-path-encoded single-photon (WP), photon

polarization (POL), and double-well potential (DW) approaches, using a cluster-randomized field experiment.

Our findings show that all three instructional approaches supported the development of more elaborated student conceptions of quantum physics. However, students in the WP group demonstrated significantly weaker conceptual understanding of quantum measurement, primarily due to the high proportion of incorrect answers on items 1, 4, 6, and 8 (see Table V and the Appendix). No significant differences were observed between the POL and DW groups in this regard.

These results indicate that students’ understanding of quantum measurement is influenced by the physical context, even when the teaching methods and underlying mathematical formalism are similar.

Future research could extend this work by comparing students’ ideas about other features of quantum physics (e.g., quantum randomness or interference). It is

conceivable that different educational paths have different strengths and weaknesses, so that in the future an educational path could be created that uses several two-state approaches together, where the different approaches complement each other’s strengths. Moreover, follow-up and replication studies are necessary to confirm these findings and address current limitations.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available because of legal restrictions preventing unrestricted public distribution. The data are available from the authors upon reasonable request.

APPENDIX

TABLE VII. All items of the instrument used in this study. The correct answers are in bold font. The scores are given in brackets.

Item	Choices
1. Which of the following statements describes a method for predicting with certainty the result of a measurement of a quantity P for a single quantum object?	(a) There is no such method, because in quantum physics, it is not possible to predict the result of a single measurement. (1 point) (b) The quantum object must be prepared so that it attains a certain value of the quantity P. (2 points) (c) Almost infinitely accurate measuring equipment must be used. (0 points)
2. Indicate which statement is correct about the result of a measurement of a quantity P on a single quantum object?	(a) We could predict the outcome of the measurement if we could fully obtain the necessary information about the quantum object. (0 points) (b) We cannot predict anything about a single measurement because the value of the result is not precisely defined. (1 point) (c) We can predict the probability of obtaining a certain value if the state of the quantum object is known. (2 points)
3. Indicate the reason why the value of a given quantity may be indeterminate.	(a) A measurement in quantum physics disturbs the quantum object because quantum objects are microscopic. (0 points) (b) The laws of nature state that the results of individual measurements in quantum physics are not determined, except in a few special cases. (2 points) (c) In general, a quantum system cannot be brought into exactly the same state twice. (1 point)
4. A quantum object is first prepared in a well-defined state. Immediately afterward, a physical quantity is measured on it. Choose the statement that best describes the final state.	(a) The measurement process in quantum physics changes the state in an unpredictable way. (0 points) (b) The state in which the quantum object will be after the measurement is given by a distribution of states, each corresponding to one of the possible outcomes of the measurement. (1 point) (c) If preparation and measurement process concern the same physical quantity, the state of the quantum object remains the same. (2 points)

(Table continued)

TABLE VII. (Continued)

Item	Choices
5. A quantity A is first measured on a set of identical quantum objects. Then, a different quantity B is immediately measured on those quantum objects from this set which are in the state $ a\rangle$. Some of these quantum objects are then in a state $ b\rangle$. What can be said about their state?	(a) When B is measured, the objects in the state $a\rangle$ are transferred to another state $b\rangle$, which is completely different from the state $a\rangle$. (2 points) (b) The process of measuring B disturbs the state $a\rangle$, which is equal (or very close) to the state $b\rangle$ only if the disturbance caused by the measurement process is minimal. (0 points) (c) The process of measuring B will progressively evolve the state $a\rangle$ toward the state $b\rangle$ according to the laws of quantum physics. (1 point)
6. A quantum system that is in a certain state is subjected to a measurement. Indicate the statement that best describes the result.	(a) If the quantum system was in one of the possible result states at the beginning, its state becomes a superposition of possible result states after the measurement. (1 point) (b) If the quantum system was in a superposition of states corresponding to possible measurement results, it is in one of the possible result states. (2 points) (c) If the measurement is very accurate, then the measurement itself influences the possible results. (0 points)
7. A measurement apparatus consists of two devices arranged in series for the measurement of orthogonally polarized states $ H\rangle$ (horizontal) and $ V\rangle$ (vertical). A single photon in the equally weighted superposition state of $ H\rangle$ and $ V\rangle$ enters the apparatus. What can be predicted?	(a) When the single photon passes through the first device, it will certainly be absorbed by the second device. (2 points) (b) The probability that the photon passes through both devices is 25%. (0 points) (c) The probability that the photon passes through both devices is 50%. (1 point)
8. A given quantum object can be described using two states (two-state system). Imagine that this quantum object is in a superposition state and Alice performs a measurement on it. For this purpose, she chooses a suitable measuring device. Now Bob measures the same quantum object immediately afterward with an identical measuring device. Alice expects Bob to get the same result as she does.	(a) No, because the results of measurements in quantum physics are unpredictable. (1 point) (b) Yes, if she and Bob measure accurately enough. (0 points) (c) No, because the result of a second measurement cannot be predicted precisely. (1 point) (d) Yes, because if the same measuring device is used twice in a row for the same object, the result must be the same. (2 points)

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