

***Ab initio* theory of electron drag and wind forces on dislocations: Bridging quantum transport and electroplasticity**

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We develop a conserving Keldysh field theory for a moving dislocation represented by a discrete Kanzaki force and coupled to electronic and phononic baths. Within this framework the configurational electron wind force and the electronic drag emerge as the odd and even components of a single retarded stress correlation kernel constructed from the same deformation potential vertex, placing both mechanisms on the same many-body footing as standard transport coefficients. Symmetry of the slip geometry enforces a shear channel theorem in which electronic drag couples only to shear stress, while the electron wind is polar with respect to the Burgers vector, providing a microscopic mechanism for current-induced texture evolution. The formalism yields closed expressions for electronic drag and electron wind in terms of Fermi surface averages of deformation potentials, band velocities, and the screened Kanzaki projector, with no phenomenological relaxation times or adjustable masses. Evaluated from first principles for edge dislocations in face-centered cubic Al and basal dislocations in hexagonal close-packed Zn, these expressions give electronic drag coefficients that are several orders of magnitude smaller than phonon drag throughout the experimentally relevant temperature range, whereas the electron wind produces configurational stresses of order megapascals at current densities $J \sim 10^8\text{--}10^9\text{ A m}^{-2}$, consistent with reported electroplastic softening. Extension to finite magnetic field predicts a magnetoplastic analog of magnetoresistance, in which the drag is suppressed by cyclotron deflection while the wind remains essentially field independent. The theory therefore converts wind and drag from phenomenological fit parameters into parameter free functionals of the electronic structure and provides a direct route from electronic structure calculations to dislocation mobility laws in electroplastic and magnetoplastic flow.

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I. INTRODUCTION

The interaction of electric currents with dislocations remains one of the most debated problems in the physics of plastic deformation. Since the original experiments of Troitskii and Likhtman on Zn in 1963 [1], in which its flow stress was shown to decrease upon the passage of a high intensity current, the phenomenon now termed *electroplasticity* has been attributed variously to thermal and nonthermal origins [2,3]. The simplest interpretation has long been Joule heating [4–6]: The current increases the specimen temperature, thereby lowering the flow stress through its ordinary thermal dependence [2,7]. Indeed, several careful experiments have found no residual softening once the temperature rise due to the power density is properly accounted for, leading to the conclusion that no additional electroplastic mechanism operates under steady conditions (for instance, in Pb at 4.7 K [8] and in ferritic steels [9]).

However, alternative reports showcase flow stress decreases that far exceed those explainable by uniform heating [2]. Such findings have been grounded in a range of

proposed microscopic mechanisms, including momentum transfer from conduction electrons to dislocations (the so called *electron wind* effect [10]), localized Joule heating at dislocation cores and grain boundaries [11], field assisted diffusion and electromigration [12–14], or magnetoplastic depinning of dislocations from paramagnetic obstacles [15–17], to name but a few. Of these, the electron wind picture, developed in the 1950s and 1960s [10,18,19], remains conceptually the most immediate: the drift of the conduction electrons exerts a configurational force on the dislocation line capable of driving its glide and, consequently, the net plastic slip observed macroscopically as a sudden “softening” of the material. Indeed, under the application of a current, dislocations would be driven by this electronic momentum flux, and once the current subsides, the dislocation network would undergo an internal elastic reorganization.

It is clear, further, that the electron wind force is closely related to the *electronic drag* component of dislocation mobility, whereby a moving dislocation dissipates energy through electronic excitations in the crystal [18,20–22]. How both are related is less clearly articulated; although both effects originate in the same microscopic coupling between the dislocation’s strain field and the conduction electrons, they manifest in opposite limits: the electronic drag represents the dissipative back reaction in the absence of a driving current, while the electron wind corresponds to the configurational response generated by a steady electronic drift. Historically,

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the electronic drag has been regarded as negligible compared with its phononic counterpart [20,23], and although it has been suggested the electronic drag is affected by the applied current, this effect has typically been deemed negligible, and electroplasticity has therefore been interpreted not as a consequence of drag reduction but as the appearance of this additional electron wind configurational drive superposed upon the mechanical Peach-Koehler stress [24].

Kravchenko [10] first proposed that the drift of conduction electrons could exert a net mechanical pressure on dislocations, defining the *electron wind* force as a force $f_e \propto J$, where J is the current density. Subsequent phenomenological models, such as those of Lahiri *et al.* [25], retained the same linear scaling with current. This configuration force f_e translates into a resolved shear stress acting on the dislocation $\tau_b = f_e/b$ that estimates based on metallic conductivities and carrier densities [7,26] quantify in the range 10^2 – 10^3 Pa for $J \sim 10^9$ A m $^{-2}$, far below the values required to initiate dislocation glide [24,26]. Although experimental evidence for a directional wind component remained ambiguous for decades [2,27], recent real-time imaging has revealed electrically driven, polarity-dependent dislocation motion in single crystal ZnS, accompanied by charge sensitive core reconstructions and field-induced barrier reduction [28], providing the first direct observation of electronic wind driven dislocation motion.

To understand this configurational response, we must first understand the dissipative component from which it emerges. The first microscopic estimates of the electronic drag were obtained by Huffman and Louat [19,29,30] and by Brailsford [18], who modeled a dislocation moving through a free-electron gas and obtained an electronic drag force F given by $F = B_{el}v$, with $B_{el} \propto \rho_{el}^{-1}$ a viscous-like drag coefficient inversely proportional to the electrical resistivity ρ_{el} , and with only a weak temperature dependence at low T . Al'shits [31] extended the analysis to quantum and cryogenic regimes, demonstrating that the electronic term could dominate only when the phonon population is frozen out. By contrast, the dominant *phonon wind* contribution, treated extensively in later theoretical and numerical works [32–40], exhibits the well-known Bloch-Grüneisen scalings [41], $B_{ph} \propto T^5$ in the low-temperature limit [42–44] and $B_{ph} \propto T$ in the classical regime [44], exceeding the electronic drag by several orders of magnitude under ordinary conditions.

The implications of such microscopic effects for macroscopic electroplasticity, however, remain far from settled. It is essential to distinguish between the microscopic and collective manifestations of current-dislocation interactions. Macroscopic stress reductions or strain bursts observed under high intensity current pulses reflect the ensemble response of a correlated dislocation network, not necessarily the underlying microscopic cause. Upon applying a strong current, transient electron wind forces (or other localized mechanisms) can momentarily enhance dislocation mobility before thermal equilibration sets in. Once the pulse ceases, the network relaxes as internal stresses redistribute, often accompanied by bulk Joule heating due to the high current density. The local electronic effects acting directly on individual dislocations, such as electron wind, electromigration, or localized Joule heating, must therefore be distinguished from the collective

consequences that follow, including global Joule heating and the transient plastic reorganization of the dislocation microstructure.

At the theoretical level, existing frameworks treat electroplastic phenomena through disparate, often mutually incompatible hypotheses: the electron wind model interprets the current as a mechanical drive [7,24]; the localized heating picture attributes it to temperature elevation near the core [2,7]; magnetoplastic models invoke current-induced spin or Zeeman coupling effects [15,16]. None provides a unified microscopic account capable of disentangling configurational drive from dissipation, nor a single unified pathway with which to ground macroscopic observation on microscopic mechanisms. For instance, conventional kinetic formulations of the electron wind rely on semiclassical relaxation time approximations, in which the dislocation acts as a static scatterer and momentum loss is introduced phenomenologically [10]. Such approaches cannot enforce conservation laws nor separate reversible from irreversible components of the electronic response, leaving unresolved the fundamental question of whether a steady current can exert a genuine mechanical force on a dislocation independently of heating. A coherent resolution demands a conserving quantum transport framework in which the electronic stress response to the dislocation field is derived from first principles, allowing the configurational and dissipative contributions to emerge on equal footing.

The present work develops such a framework based on the Keldysh nonequilibrium field formalism [45,46]. The dislocation is treated as a dynamical source coupled to electronic and phononic quantum baths through its Kanzaki field [47–49], the discrete lattice force distribution which, when applied to the perfect harmonic crystal, reproduces the defect's elastic displacement field. In the long-wavelength limit, this coupling reduces to a homogeneous strain vertex acting on the electron gas. Integrating out the fast electronic degrees of freedom yields an effective real time action for the dislocation coordinate, from which both viscous drag and configurational (electron wind) forces emerge as distinct symmetry projections of the same microscopic stress-stress correlation function. No phenomenological relaxation times or ad hoc heating terms are required. Conservation laws follow automatically through Ward identities [50,51], and the measurable coefficients are expressed as Fermi surface averages of deformation potential vertices which can be computed from first principles electronic calculations.

This construction provides a unified, conserving, and parameter free description of current-dislocation coupling. In the weak field, steady regime appropriate to continuous currents, the Keldysh formulation reproduces the drag to known temperature scalings: $B_{el} \propto T^2$ for the electronic drag and $B_{ph} \propto T$ for phononic drag below and above the Debye temperature. The configurational response yields a linear electron wind stress $\tau_e = \eta_1 J/b$, whose magnitude follows directly from the same *ab initio* vertices. Extensions to finite magnetic field and localized core forces enter naturally through corresponding modifications of the Green functions and Kanzaki multipoles, likewise enabling systematic treatment of magnetoplastic and Joule heating effects within a single conserving framework.

By grounding the current-dislocation coupling in a microscopic quantum kinetic theory, the approach developed here clarifies which aspects of electroplasticity genuinely arise from electronic momentum transfer and which reflect secondary phononic or thermal processes. It provides a rigorous baseline for interpreting both steady and transient experiments, distinguishing genuine electron wind effects from indirect heating and establishing a direct route from *ab initio* electronic structure calculations to continuum dislocation mobility.

The paper is organized as follows. Section II introduces the Hamiltonian formalism and the Kanzaki force representation of the dislocation. Section III presents the nonequilibrium quantum Keldysh framework used to compute configurational (wind) and dissipative (drag) forces on a moving dislocation. Section IV applies this formalism to the case of an applied current and magnetic field, deriving unified expressions for the electron wind, electronic drag, Joule heating corrections, and magnetoplastic effects. Section V describes first principles evaluations of the deformation potential vertices using density functional theory (DFT) [and, for validation, the Korringa-Kohn-Rostoker (KKR) method], applied to an edge dislocation in face-centered cubic (fcc) Al and hexagonal close-packed (hcp) Zn, which we use to quantify from first principles the electronic drag and the electron wind force from first principles. Section VI summarizes the principal findings of this paper, and outlines future directions.

II. LATTICE DYNAMICS AND THE DISCRETE KANZAKI REPRESENTATION

Throughout, we work on a Bravais lattice with sites $\mathbf{R}$, ionic mass M , and force constant matrix $\Phi_{ij}(\mathbf{R})$, which obeys the usual point-group symmetries and translational invariance, $\Phi_{ij}(\mathbf{R}) = \Phi_{ji}(-\mathbf{R})$, for $i, j = 1, 2, 3$ the spatial directions; for ease of notation, we omit the position of atoms within the supercells, i.e., the $\Phi_{i\alpha, j\beta}$ notation familiar for example from Ref. [52]. The dislocation is modeled as a Kanzaki force field [47] acting on the perfect lattice. A Kanzaki field is a *configurational* body force that reproduces the elastic misfit of a defect on a discrete lattice: they are the forces that would have to be applied on the perfect lattice to induce the defect's topology. We therefore keep the microscopic harmonic Hamiltonian unmodified and introduce the Kanzaki force as an *external source* in the equations of motion.

A. Harmonic Hamiltonian and driven equations

The harmonic Hamiltonian contains canonical momenta $p_i(\mathbf{R}, t) \equiv M\dot{u}_i(\mathbf{R}, t)$ and reads

$$H_{\text{ph}} = \frac{1}{2M} \sum_{\mathbf{R}} p_i p_i + \frac{1}{2} \sum_{\mathbf{R}, \mathbf{R}'} u_i(\mathbf{R}) \Phi_{ij}(\mathbf{R}-\mathbf{R}') u_j(\mathbf{R}'). \quad (1)$$

The Kanzaki field $f^{\text{K}}(\mathbf{R}; X)$ represents the configurational misfit [47,48] or “disregistry” [49,53] of a straight Volterra cut with glide front $X(t)$ (e.g., $X(t) = \mp v_d t$ for a uniformly driven glide with steady v_d speed in the $\pm x_1$ direction) and

acts as a *driving force* in the Euler-Lagrange equations:

$$M \ddot{u}_i(\mathbf{R}, t) + \sum_{\mathbf{R}'} \Phi_{ij}(\mathbf{R}-\mathbf{R}') u_j(\mathbf{R}', t) = f_i^{\text{K}}(\mathbf{R}; X(t)) + f_i^{\text{ext}}(\mathbf{R}, t). \quad (2)$$

Here f^{ext} can represent an applied far field loading used below to recover the Peach-Koehler force acting on dislocations. Note that no Kanzaki term appears in H_{ph} ; aside from avoiding double-counting, this makes explicit that f^{K} is a deterministic background source fixed by the defect's configuration.

B. Discrete Kanzaki field from bond tractions

We now describe the form of the Kanzaki field following Refs. [54–56]. Let δ_α label a bond family and let b_j be the Burgers vector. Bonds straddling the slip plane $y = 0$ and overtaken by the front at X are selected by the *bond-crossing indicator*

$$c_\alpha(\mathbf{R}; X) = \frac{1}{2} \left[\text{sgn} \left(y_{\mathbf{R}} + \frac{\delta_{\alpha, y}}{2} \right) - \text{sgn} \left(y_{\mathbf{R}} - \frac{\delta_{\alpha, y}}{2} \right) \right] \Theta(X - x_{\mathbf{R}}). \quad (3)$$

The associated bond traction is $T_{\alpha i}(\mathbf{R}; X) = \Phi_{ij}(\delta_\alpha) b_j c_\alpha(\mathbf{R}; X)$ and the Kanzaki body force is its discrete divergence

$$f_i^{\text{K}}(\mathbf{R}; X) = \sum_{\alpha} [T_{\alpha i}(\mathbf{R}; X) - T_{\alpha i}(\mathbf{R} - \delta_\alpha; X)]. \quad (4)$$

This construction is analogous to the ones employed in harmonic lattice models by, e.g., Refs. [54,56,57]; the force distribution is purely discrete and symmetry-respecting; it requires no continuum limit, which would be given by the Burridge-Knopoff forces derived in linear elasticity [58] (for a discussion, q.v., Ref. [49]).

Fourier conventions and acoustic projector. We use the lattice Fourier transform $H(\mathbf{q}) = \sum_{\mathbf{R}} h(\mathbf{R}) e^{-i\mathbf{q} \cdot \mathbf{R}}$ and $h(\mathbf{R}) = \frac{1}{N} \sum_{\mathbf{q} \in \text{BZ}} H(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{R}}$. Let $\Phi_{ij}(\mathbf{q})$ be the dynamical matrix and $G_{ij}(\mathbf{q})$ the lattice Green function on the *acoustic* subspace, i.e.,

$$\sum_m \Phi_{im}(\mathbf{q}) G_{mj}(\mathbf{q}) = \Pi_{ij}^{\text{ac}}(\mathbf{q}), \quad \Phi_{ij}(\mathbf{0}) = 0, \quad (5)$$

with Π^{ac} the orthogonal projector onto the translational sector. Applying the transform to the Kanzaki problem gives

$$u_i^{\text{slip}}(\mathbf{q}; X) = G_{ij}(\mathbf{q}) \Pi_{jk}^{\text{ac}}(\mathbf{q}) F_k^{\text{K}}(\mathbf{q}; X). \quad (6)$$

Any net resultant or first moment carried by F^{K} is taken up by boundary reactions or by F^{ext} ; the projector removes the rigid translation mode, so solvability does not require $\sum_{\mathbf{R}} f^{\text{K}}(\mathbf{R}; X) = 0$, which indeed does not generally hold for straight dislocations [49,59].

Bond-crossing form factor and causal split. Fourier transforming the indicator produces the hallmark factor

$$\Psi_\alpha(\mathbf{q}; X) = e^{-iq_x X} S_\alpha(\mathbf{q}_{\parallel}) \frac{2i \sin \left(\frac{q_y \delta_{\alpha, y}}{2} \right)}{q_x + i0^+}, \quad (7)$$

with $\mathbf{q}_{\parallel} = (q_x, 0, q_z)$ and an in-plane structure factor S_{α} . In reciprocal space the Kanzaki drive is then (cf. Refs. [56,57,60])

$$F_i^K(\mathbf{q}, t) = \sum_{\alpha} \Phi_{ij}(\delta_{\alpha}) [1 - e^{-i\mathbf{q} \cdot \delta_{\alpha}}] b_j \Psi_{\alpha}(\mathbf{q}; X(t)) + F_i^{\text{core}}(\mathbf{q}), \quad (8)$$

where F^{core} parametrizes short-range core physics (e.g., multipoles) that one may add consistently as core-localized corrections of the Volterra dislocation that represent the core's true geometry [59].

The glide coordinate $X(t)$ may be viewed as the collective degree of freedom conjugate to the Kanzaki coupling. Integrating out lattice fluctuations then yields an effective nonlocal equation of motion with a retarded drag kernel, which will be derived microscopically in Sec. III.

Causal split of the slip form factor

The discrete Volterra cut produces the characteristic factor $(q_x + i0^+)^{-1}$ in the slip form factor, Eq. (7). It is convenient to record here its standard decomposition

$$\frac{1}{q_x + i0^+} = \mathcal{P}\left(\frac{1}{q_x}\right) - i\pi \delta(q_x), \quad (9)$$

so that, for any test function $\varphi(\mathbf{q})$ which is even in q_x ,

$$\int \frac{dq_x}{2\pi} \varphi(\mathbf{q}) \mathcal{P}\left(\frac{1}{q_x}\right) = 0, \quad (10)$$

$$\int \frac{dq_x}{2\pi} \varphi(\mathbf{q}) \delta(q_x) = \frac{1}{2\pi} \varphi(0, \mathbf{q}_{\perp}). \quad (11)$$

Since the structure factor $S_{\alpha}(\mathbf{q}_{\parallel})$ is even in q_x by inversion symmetry, the spatial envelope $B(\mathbf{q})$ in $\Psi(\mathbf{q}; X) = e^{-iq_x X} B(\mathbf{q})(q_x + i0^+)^{-1}$ inherits this evenness. Any quadratic quantity of the form $\sum_{\mathbf{q}} \Psi^*(\mathbf{q}) \mathcal{K}(\mathbf{q}, \dots) \Psi(\mathbf{q})$ with an even kernel $\mathcal{K}$ therefore probes the slip field only through the $q_x = 0$ plane of its spectrum in its imaginary part, while the principal value contributes to the real, reactive sector. The consequences of this projection for the retarded drag kernel and for the viscous drag coefficient are developed in Sec. III and Appendix A.

C. Electronic sector and strain response vertices

Having established the lattice sector, we now turn to the electronic degrees of freedom. The dislocation interacts with electrons through the strain field it generates, which is inhomogeneous in space, particularly about the dislocation core. To describe this interaction within a microscopic Hamiltonian, we need the operator that measures the electronic response to an imposed strain. The natural definition is the strain vertex $\Gamma_{ij}(\mathbf{k})$, obtained as the derivative of the Bloch Hamiltonian with respect to a uniform strain. Although defined in the homogeneous limit, this vertex provides the correct long-wavelength coupling that, when contracted with the discrete Kanzaki force field, yields the full inhomogeneous electron-dislocation interaction. This approximation is reasonable away from the core, because the far field has a $\approx 1/r$ decay, so at large distances, the strain appears quasi-homogeneous. However, near the core (within ~ 5 – $10b$), strain gradients are extremely sharp: $\nabla u \sim 1/b \sim 0.3$ – $0.5 \text{ \AA}^{-1}$, so

the strain gradient would peak at about $q \sim 1/b \sim k_F$ in most metals (e.g., in Al $k_F \approx 1.7 \text{ \AA}^{-1}$), and the homogeneous vertex alone will no longer suffice to capture the coupling with the Kanzaki forces. For such effect, higher order strain vertices are necessary, as we describe here.

Regardless, the starting point for all practical calculations is an effective one-electron Hamiltonian in the temporal gauge $\mathbf{A}(t) = -\mathbf{E}t$, $\phi = 0$, which preserves lattice periodicity,

$$H_{\text{el}} = \sum_{\sigma} \int d^3r \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[-\frac{\hbar^2}{2m_e} \nabla^2 + V_{\text{eff}}(\mathbf{r}) \right] \psi_{\sigma}(\mathbf{r}), \quad (12)$$

where V_{eff} is a mean-field potential representing the ionic cores and the electronic self consistent field. A uniform electric field can be introduced by minimal coupling in Eq. (12). Small ionic displacements modify V_{eff} , giving to first order

$$H_{\text{e-ph}} = - \sum_{\sigma} \int d^3r \psi_{\sigma}^{\dagger}(\mathbf{r}) [\nabla V_{\text{eff}}(\mathbf{r})] \cdot \mathbf{u}(\mathbf{r}, t) \psi_{\sigma}(\mathbf{r}), \quad (13)$$

which is the standard deformation potential form of the electron-lattice interaction.

For Bloch states of the undistorted lattice, H_{el} projects to a single particle matrix $h_{mn}(\mathbf{k}) = \langle u_{m\mathbf{k}} | H_{\text{el}} | u_{n\mathbf{k}} \rangle$ acting in the space of periodic functions $u_{n\mathbf{k}}$. Lattice distortions modify V_{eff} and therefore induce a strain-dependent variation $\delta h_{mn}(\mathbf{k})$ of this Bloch Hamiltonian. In a slowly varying displacement field $\mathbf{u}(\mathbf{r}, t)$, the perturbation may be expanded in spatial gradients introduced by Bardeen and Shockley [61], later generalized by Herring and Vogt [62] and formalized through the symmetry analysis of Bir and Pikus [63]:

$$\delta h(\mathbf{k}; \mathbf{q}, \omega) = \Gamma_{ij}(\mathbf{k}) \varepsilon_{ij}(\mathbf{q}, \omega) + i q_l \Xi_{ijl}(\mathbf{k}) u_j(\mathbf{q}, \omega) + O(q^2), \quad (14)$$

where $\varepsilon_{ij}(\mathbf{q}, \omega) = \frac{i}{2} [q_i u_j(\mathbf{q}, \omega) + q_j u_i(\mathbf{q}, \omega)]$ is the compatible (symmetric) strain, $\Gamma_{ij}(\mathbf{k})$ the homogeneous strain vertex, and $\Xi_{ijl}(\mathbf{k})$ the gradient vertex that encodes the leading sensitivity to strain gradients (elastic incompatibility enters via the curl of u [64], so it is retained through the Ξ term). Whereas the strain vertex Γ is commonly encountered in semiconductor theory (e.g., Refs. [65–68]), the gradient vertex Ξ_{ijl} extends the deformation potential concept to finite wave vectors, as implicit already Herring-Vogt transport theory [62] and later made explicit in first principles treatments of momentum resolved electron-phonon coupling [69].

Contraction of Eq. (38) with the discrete slip form factor $\Psi(\mathbf{q})$ yields the effective electron-dislocation coupling discussed in Sec. II. We may define the corresponding line vertex as (to first order)

$$V_n(\mathbf{k}) = b W_n(\mathbf{k}) = b \Gamma_{ij}(\mathbf{k}) \varepsilon_{ij}, \quad (15)$$

which carries units of energy times length and represents the microscopic work conjugate to the glide coordinate. The W_n is defined for convenience, as it provides the means to define dimensionless coefficients for the phonon wind and electronic drag in Sec. IV. We note that to second order, it no longer is proportional to the Burgers vector (see Sec. III C).

Equation (38) is the operator that we use to build the response functions; its contraction with the slip form factor $\Psi(\mathbf{q})$ produces the inhomogeneous electron-dislocation

coupling. Higher order terms may be needed in some cases. Here we describe the next two for completion. To do so, it is convenient to write it in two equivalent bases: (i) curvature of the displacement, $\kappa_{j,lm} \equiv \partial_l \partial_m u_j$, and (ii) gradients of the symmetric strain, $w_{ijm} \equiv \partial_m \varepsilon_{ij}$. In reciprocal space these lead to

$$\delta h^{(2)}(\mathbf{k}; \mathbf{q}, \omega) = q_l q_m \Upsilon_{ijlm}^{(u)}(\mathbf{k}) u_j(\mathbf{q}, \omega) + i q_m \Upsilon_{ijm}^{(\varepsilon)}(\mathbf{k}) \varepsilon_{ij}(\mathbf{q}, \omega), \quad (16)$$

with $\Upsilon_{ijlm}^{(u)} = \Upsilon_{ijml}^{(u)}$ symmetric in (l, m) and $\Upsilon_{ijm}^{(\varepsilon)} = \Upsilon_{jim}^{(\varepsilon)}$ symmetric in (i, j) . The two representations are linearly related by $w_{ijm} = \frac{i}{2}(q_i \kappa_{j,mm} + q_j \kappa_{i,mm})$ on the acoustic subspace, so one can trade $\Upsilon^{(u)}$ for $\Upsilon^{(\varepsilon)}$ (and vice versa) up to identities that vanish under rigid translations and rotations. A compact single tensor form that can be used in the kernels could then be

$$\delta h^{(2)}(\mathbf{k}; \mathbf{q}, \omega) = q_l q_m \Lambda_{ijlm}(\mathbf{k}) u_j(\mathbf{q}, \omega), \quad (17)$$

where Λ_{ijlm} is chosen to be symmetric in (l, m) and to satisfy the acoustic constraints $\Lambda_{ijlm} q_l q_m R_j(\hat{\mathbf{q}}) = 0$ for any rigid mode R (translations and rigid rotations), ensuring that $\delta h^{(2)}$ does not couple to nonelastic motions. In practice, Λ can be decomposed as $\Lambda_{ijlm} = \Lambda_{ijlm}^{\text{sym}} + \Lambda_{ijlm}^{\text{tr}}$ with $\Lambda_{ijlm}^{\text{sym}}$ built from derivatives of Γ and Ξ , and $\Lambda_{ijlm}^{\text{tr}}$ the residual curvature coupling that cannot be re-expressed as a divergence of lower order vertices.

These second-order Λ terms are expected to be parametrically small for ordinary metallic dislocations and will be neglected here. The relative size of the curvature contribution is controlled by the dimensionless ratio $\rho(q) = q \|\Lambda\| / \|\Xi\|$ evaluated on the wave-vector support of the screened Kanzaki projector. For compact cores, with characteristic radius $a_{\text{core}} \sim 1\text{--}5 |\mathbf{b}|$, the dominant spectral weight lies at $q \sim 1/a_{\text{core}} \lesssim 0.1\text{--}0.3 \text{ \AA}^{-1}$, well below typical metallic $k_F \sim 1 \text{ \AA}^{-1}$, so that the $O(q)$ gradient vertex governs the scattering and $O(q^2)$ curvature corrections remain subleading. Only in situations where the core develops substantial sub-atomic structure or extreme dissociation shifts significant weight to larger q would $\rho(q)$ approach unity and the Λ sector become comparable; physically, this corresponds to the regime in which the electronic response probes strain variations on length scales comparable to the mean free path. This same ordering will govern the discussion of core contributions to the wind force in Sec. II C.

In general, we will see that the homogeneous vertex Γ_{ij} governs elastic "renormalizations" in the compatible $q \rightarrow 0$ limit (meaning, the response to uniform strain which affects elastic properties like sound velocities or lattice stiffness), and hence it is responsible for T^m scalings in the electronic drag. However, dissipation proper arises from the finite q response on the Doppler shell ($\omega = \mathbf{q} \cdot \mathbf{v}_d$). Retaining the leading gradient coupling Ξ_{ijl} and the discrete Kanzaki slip form factor is thus essential to ensure a nonzero $T = 0$ drag and to capture the correct magnitude of the drag coefficients B_{el} and electron wind forces. A similar conclusion was reached by, e.g., Ref. [70], where the 0 K offset of the phonon drag contributions in Frenkel-Kontorova frameworks were ascribed to the Hessian of the system's Hamiltonian.

Formal definitions and conservation. The homogeneous strain vertex is defined abstractly as the derivative of the Bloch Hamiltonian with respect to uniform strain at fixed electron number,

$$\Gamma_{ij}(\mathbf{k}) = \left. \frac{\partial h_{\varepsilon}(\mathbf{k})}{\partial \varepsilon_{ij}} \right|_{\varepsilon=0}, \quad (18)$$

and the leading gradient vertex as the functional derivative with respect to the strain-gradient field,

$$\Xi_{ijl}(\mathbf{k}) = \left. \frac{\partial h_{\nabla u}(\mathbf{k})}{\partial \varepsilon_{lj}} \right|_{\nabla u=0}. \quad (19)$$

These definitions, independent of any particular electronic-structure method, suffice for the field theoretic treatment below. Specific details on how to compute Γ and Ξ within DFT and the KKR method are provided in Sec. V and Appendix C, respectively.

Ward identities and order of limits. The relevant conservation statement is a *stress* Ward-Takahashi type identity [50,51] that ties the strain vertex to the variation of the inverse Green's function under an infinitesimal coordinate deformation. Schematically, we may state that

$$\Gamma_{ij}(\mathbf{k}) = \left. \frac{\partial G^{-1}(\mathbf{k}, \varepsilon; \varepsilon_{ij})}{\partial \varepsilon_{ij}} \right|_{\varepsilon_{ij}=0}. \quad (20)$$

Here derivatives with respect to $\mathbf{k}$ must be taken in a Bloch representation that includes the $\mathbf{k}$ -dependence of the basis functions, so that Pulay terms are properly accounted for. In the homogeneous, compatible limit ($\mathbf{q} \rightarrow 0$ taken *before* $\omega \rightarrow 0$) this reduces to the familiar current like identity $q_i \Gamma_{ij} = \partial h / \partial k_j$, but only if the basis derivative (Pulay) corrections are included. We use this correspondence merely as a consistency check, not as a defining relation. Transport slopes at low frequency require the opposite order ($\omega \rightarrow 0$ first, then $\mathbf{q} \rightarrow 0$); this ordering is kept explicit in Sec. III so that sum rules are not silently violated.

III. NONEQUILIBRIUM FORMALISM: KELDYSH ACTION AND RESPONSE FUNCTIONS

The motion of a dislocation is a genuinely nonequilibrium process: as the glide front advances it performs mechanical work that must be absorbed irreversibly by phonons and electrons. Capturing this energy transfer requires a framework that keeps track of both causal response and fluctuation statistics, which equilibrium imaginary-time methods cannot do. The Keldysh closed time path formalism is therefore the natural choice. It allows us to construct an action that contains, on equal footing, the retarded and advanced kernels that govern dissipation and the Keldysh components that encode fluctuations. In this way the drag force can be derived as the causal response of the lattice-electron system to the slip-induced strain field.

Our strategy is as follows. We first set up the generating functional on the closed contour and define the Keldysh rotation that separates classical and quantum components of the phonon field. We then derive the quadratic phonon action and state clearly when the fluctuation-dissipation relation applies. This sets the stage for introducing the (deterministic) Kanzaki

drive and for coupling to the electronic stress vertex. Finally, we integrate out phonons and electrons to obtain a force-force influence functional for the glide coordinate and extract the small-velocity viscous coefficient. In subsequent sections we examine the role electromagnetic fields have on the drag coefficient and the dislocation's motion itself.

A. Closed contour, sources, and the Keldysh rotation

In the Keldysh formalism [71] we are not concerned with a pure ground state, but with the real time dynamics of a statistical ensemble. The ensemble is encoded with an initial density matrix $\hat{\rho}_0(u, \psi; u', \psi') = \rho_{\text{ph}}(u, u')\rho_{\text{el}}(\psi, \psi')$, which may represent a thermal phonon ensemble and an electronic Fermi sea, or any other specified statistical state. To compute the expectation values of operators at any later time, we must evolve this density forward with the system's Hamiltonian $H(t)$, and then evolve it back again, so that the trace over states is preserved. This double time evolution defines the closed time contour $\mathcal{C}$, and introduces two couples of each dynamical field, u^+ on the forward branch, and u^- on the backward branch.

The corresponding generating functional with external sources $\eta_{\pm}$ for the fields $\phi^{\pm}$ (bosonic or fermionic as appropriate) is

$$Z[\eta_{\pm}] = \text{Tr} \left[T_{\mathcal{C}} \exp \left(-\frac{i}{\hbar} \oint_{\mathcal{C}} dt (H(t) - \eta_+ \phi^+ + \eta_- \phi^-) \right) \hat{\rho}_0 \right], \quad (21)$$

where the classical source enters with opposite signs on the two branches so that functional differentiation produces physical response functions, and $T_{\mathcal{C}}$ denotes ordering of all operators along the Keldysh contour $\mathcal{C}$, ensuring that fields are arranged according to their position on the forward and backward time branches so that expectation values preserve the correct real time causality.

In effect, Z is the single object from which all real-time correlation and response functions are obtained by functional differentiation with respect to sources. The rotated sources are defined by

$$\eta^{c,q} = \frac{\eta_+ \pm \eta_-}{\sqrt{2}},$$

so that variation with respect to η^q generates response functions. For bosonic fields (phonons) with $\phi \equiv u$ and $\eta \equiv J$ one finds

$$\frac{\delta^2 \ln Z}{\delta \eta^q \delta \eta^q} \rightarrow D^R(t, t') = -\frac{i}{\hbar} \theta(t - t') [u(t), u(t')], \quad (22)$$

$$\frac{\delta^2 \ln Z}{\delta \eta^q \delta \eta^q} \rightarrow D^K(t, t') = -\frac{i}{\hbar} \{u(t), u(t')\}, \quad (23)$$

where D^R is the retarded Green's function and D^K the Keldysh (or symmetrized correlation) function. For fermionic fields (electrons) with $\phi \equiv c$ and $\eta \equiv \mathcal{J}$ the analogous definitions give the electronic propagators G^R and G^K . Other contractions yield the advanced function D^A and mixed correlators in the usual way.

The distinction between “c” and “q” fields arises because it is convenient to change variables from the branch fields to symmetric and antisymmetric combinations. For phonons this defines the orthonormal Keldysh rotation

$$u^c = \frac{u^+ + u^-}{\sqrt{2}}, \quad u^q = \frac{u^+ - u^-}{\sqrt{2}}, \quad (24)$$

which diagonalizes the source coupling: a classical source multiplies only u^q . Variation of the action with respect to u^q then yields an Euler-Lagrange equation for u^c that is automatically retarded, so causality is built into the formalism. No assumption is made about the statistical state of the bath beyond specifying $\hat{\rho}_0$. Whenever we later invoke fluctuation-dissipation for a given sector we will state explicitly that this sector is thermal.

B. Phonon action and fluctuation-dissipation

We now wish to pass from the contour generating functional Eq. (21) to an explicit action in the phonon sector. We start from the microscopic harmonic Hamiltonian given in Eq. (1),

$$H_{\text{ph}}[u, p] = \sum_{\mathbf{R}} \frac{p_i(\mathbf{R})^2}{2M} + \frac{1}{2} \sum_{\mathbf{R}\mathbf{R}'} u_i(\mathbf{R}) \Phi_{ij}(\mathbf{R}-\mathbf{R}') u_j(\mathbf{R}').$$

The contour trace Eq. (21) admits a path integral representation on the forward and backward branches. Specializing to the phonon Hamiltonian Eq. (1) and sources $J_{\pm}$ for the displacements, one may write

$$\begin{aligned} Z[J_{\pm}] &= \text{Tr} \left[T_{\mathcal{C}} \exp \left(-\frac{i}{\hbar} \oint_{\mathcal{C}} dt (H_{\text{ph}} - J u) \right) \hat{\rho}_0 \right] \\ &= \int \mathcal{D}u^+ \mathcal{D}p^+ \mathcal{D}u^- \mathcal{D}p^- \\ &\quad \times \exp \left\{ \frac{i}{\hbar} (S_+[u^+, p^+; J_+] - S_-[u^-, p^-; J_-]) \right\} \\ &\quad \times \mathcal{W}[u^+(t_0), u^-(t_0)], \end{aligned} \quad (25)$$

where $\mathcal{W}$ is the Wigner transform of the initial density matrix $\hat{\rho}_0$ and $S_{\pm}$ are the single-branch phase space actions. Because H_{ph} is quadratic, the momentum integral factorizes and can be carried out exactly, leaving a configuration-space path integral

$$\begin{aligned} Z[J_{\pm}] &= \int \mathcal{D}u^+ \mathcal{D}u^- \exp \left\{ \frac{i}{\hbar} (S_+[u^+; J_+] - S_-[u^-; J_-]) \right\} \\ &\quad \times \mathcal{W}[u^+(t_0), u^-(t_0)]. \end{aligned} \quad (26)$$

The resulting single-branch actions are

$$\begin{aligned} S_{\pm}[u^{\pm}; J_{\pm}] &= \int_{t_0}^{+\infty} dt \left[\frac{M}{2} \sum_{\mathbf{R}} \dot{u}_i^{\pm} \dot{u}_i^{\pm} \right. \\ &\quad \left. - \frac{1}{2} \sum_{\mathbf{R}\mathbf{R}'} u_i^{\pm}(\mathbf{R}) \Phi_{ij}(\mathbf{R}-\mathbf{R}') u_j^{\pm}(\mathbf{R}') \right. \\ &\quad \left. \pm \frac{1}{2} \sum_{\mathbf{R}} J_i^{\pm} u_i^{\pm} \right], \end{aligned} \quad (27)$$

so that the contour action is

$$S_C[u^\pm; J_\pm] = S_+[u^+; J_+] - S_-[u^-; J_-]. \quad (28)$$

Introducing the orthonormal Keldysh rotation Eq. (24) for both the fields $u^\pm$ and the sources $J^\pm$ yields the rotated variables u^c, u^q, J^c, J^q . In this basis the source term takes the form $J^c u^q + J^q u^c$. A physical external source (a force acting on the phonon coordinates, such as the Kanzaki term) couples with opposite signs on the two branches, so after the rotation one has $J^c = 0$ and only the “quantum” source $\eta \equiv J^q$ survives.

The initial ensemble enters through the Wigner weight $\mathcal{W}[u^+(t_0), u^-(t_0)]$. For a Gaussian state this weight is quadratic and may be written as

$$\mathcal{W}[u^+, u^-] \propto \exp \left\{ -\frac{1}{2} \int dt dt' u_i^q(t) C_{ij}(t-t') u_j^q(t') \right\}, \quad (29)$$

with $C_{ij}(t-t') = \frac{1}{2} \langle \{u_i(t), u_j(t')\} \rangle_{\rho_0}$ the symmetrized correlator in the initial state. Multiplying by $\mathcal{W}$ is therefore equivalent to adding a quadratic term in the (q, q) sector of the contour action,

$$\begin{aligned} & \exp \left\{ \frac{i}{\hbar} S_C[u^\pm; J^\pm] \right\} \mathcal{W}[u^+, u^-] \\ &= \exp \left\{ \frac{i}{\hbar} \left(S_C[u^\pm; J^\pm] \right. \right. \\ & \quad \left. \left. + \frac{1}{2} \int dt dt' u_i^q(t) \Sigma_{ij}^K(t-t') u_j^q(t') \right) \right\}, \quad (30) \end{aligned}$$

with

$$\Sigma_{ij}^K(t-t') = -\frac{i}{\hbar} C_{ij}(t-t'). \quad (31)$$

Thus, the lower right (q, q) block of the kernel is fixed entirely by the ensemble correlations of the initial state.

Putting these elements together, the quadratic contour action for the phonon sector in the Keldysh basis reads

$$\begin{aligned} S_{\text{ph}}[u^c, u^q] &= \frac{1}{2} \int dt dt' (u^c \quad u^q)_i(t) \\ & \quad \times \begin{pmatrix} 0 & [D^{-1,A}]_{ij}(t, t') \\ [D^{-1,R}]_{ij}(t, t') & \Sigma_{ij}^K(t, t') \end{pmatrix} \begin{pmatrix} u^c \\ u^q \end{pmatrix}_j(t') \\ & \quad + \int dt \eta_i(t) u_i^q(t), \quad (32) \end{aligned}$$

where the off-diagonal blocks are the inverse advanced and retarded propagators of the harmonic operator $M\partial_t^2 + \Phi$ with the appropriate contour boundary conditions. Fourier transformation in time and on the Bravais lattice then gives

$$D_{ij}^R(\mathbf{q}, \omega) = [(\omega + i0^+)^2 \delta_{ij} - \Phi_{ij}(\mathbf{q})/M]^{-1}, \quad D^A = (D^R)^\dagger, \quad (33)$$

and the Keldysh component is fixed by the choice of ensemble.

In thermal equilibrium, the Kubo-Martin-Schwinger relation yields

$$\begin{aligned} D_{ij}^K(\mathbf{q}, \omega) &= [D_{ij}^R(\mathbf{q}, \omega) - D_{ij}^A(\mathbf{q}, \omega)] \coth \left(\frac{\beta \hbar \omega}{2} \right) \\ &= (2n_B(\omega) + 1) [D_{ij}^R(\mathbf{q}, \omega) - D_{ij}^A(\mathbf{q}, \omega)], \quad (34) \end{aligned}$$

with $n_B(\omega) = (e^{\beta \hbar \omega} - 1)^{-1}$. This expression shows explicitly how temperature enters the bosonic sector, as each normal mode of frequency ω carries the Bose factor $\coth(\omega/2T)$. All expressions extracted from D^K inherit their T dependence from this factor. Out of equilibrium, no such relation holds, and D^K must then be determined from the actual phonon occupations or, in an interacting treatment, from the same conserving approximation that generates D^R and D^A . Only when the phonon sector is thermalized to a single temperature does the equilibrium identity Eq. (34) apply. In that case, the Keldysh component simply records that each normal mode of frequency ω carries the Bose population $n_B(\omega)$, so the phonon field is nothing more than a collection of independent harmonic oscillators, each assigned a thermal occupation at temperature T .

C. Electronic sector and Kanzaki coupling

The fermionic sector is formulated on the closed time contour in terms of Grassmann fields $\psi, \bar{\psi}$ associated with Bloch states. The bare electronic Hamiltonian is taken to be quadratic,

$$H_{\text{el}} = \sum_{\mathbf{k}} \bar{c}_{\mathbf{k}} h(\mathbf{k}) c_{\mathbf{k}}, \quad (35)$$

so that the Gaussian contour action becomes, after the standard Keldysh rotation $\psi^{c,q} = (\psi^+ \pm \psi^-)/\sqrt{2}$,

$$\begin{aligned} S_{\text{el}} &= - \sum_{\kappa} \int \frac{d\varepsilon}{2\pi} \left[\bar{\psi}_{\kappa}^q G_{0,\kappa}^{-1,A} \psi_{\kappa}^c + \bar{\psi}_{\kappa}^c G_{0,\kappa}^{-1,R} \psi_{\kappa}^q \right. \\ & \quad \left. + i \bar{\psi}_{\kappa}^q \Sigma_{0,\kappa}^K \psi_{\kappa}^q \right], \quad (36) \end{aligned}$$

with $\kappa = (\mathbf{k}, \varepsilon)$. The retarded and advanced propagators $G_0^{R,A}$ encode the chosen microscopic description, while the Keldysh self energy Σ_0^K fixes the distribution function.

Unless otherwise stated, we shall assume that the electronic subsystem to be in thermal equilibrium, so the Keldysh component of any Green function obeys the fluctuation-dissipation identity

$$G_0^K(\kappa, \varepsilon) = (1 - 2f_0(\varepsilon)) A_0(\kappa, \varepsilon), \quad A_0 = i(G_0^R - G_0^A). \quad (37)$$

where f_0 is the Fermi-Dirac distribution

Lattice distortions modify the Bloch Hamiltonian through the canonical variation derived in Sec. II,

$$\begin{aligned} \delta h(\mathbf{k}; \mathbf{q}, \omega) &= \Gamma_{ij}(\mathbf{k}) \varepsilon_{ij}(\mathbf{q}, \omega) + i q_\ell \Xi_{ij\ell}(\mathbf{k}) u_j(\mathbf{q}, \omega) \\ & \quad + O(q^2), \quad (38) \end{aligned}$$

which we use as the starting point for the Keldysh formulation.

Acoustic projection and scalar line vertex. Since the Kanzaki source drives the lattice through a line displacement, we first rewrite the vertex expansion Eq. (38) in a form where the perturbation is expressed through a scalar amplitude that captures the resolved acoustic displacement relevant for glide. For each direction $\hat{\mathbf{q}}$ one selects orthonormal acoustic eigenvectors $\{\mathbf{e}^{(\alpha)}(\hat{\mathbf{q}})\}_{\alpha=L,T_1,T_2}$ and the associated acoustic projector

of Eq. (5),

$$\Pi_{jm}^{ac}(\hat{\mathbf{q}}) = \sum_{\alpha} e_j^{(\alpha)}(\hat{\mathbf{q}}) e_m^{(\alpha)}(\hat{\mathbf{q}}), \quad (39)$$

which removes optical components. A long-wavelength acoustic displacement therefore satisfies

$$u_j(\mathbf{q}, \omega) = \Pi_{jm}^{ac}(\hat{\mathbf{q}}) u_m(\mathbf{q}, \omega), \quad (40)$$

so that Eq. (38) becomes

$$\begin{aligned} \delta h(\mathbf{k}; \mathbf{q}, \omega) &= \Gamma_{ij}(\mathbf{k}) \varepsilon_{ij}^{(ac)}(\mathbf{q}, \omega) \\ &+ i q_{\ell} \Xi_{ij\ell}(\mathbf{k}) \Pi_{jm}^{ac}(\hat{\mathbf{q}}) u_m(\mathbf{q}, \omega) + O(q^2), \end{aligned} \quad (41)$$

with $\varepsilon_{ij}^{(ac)}$ computed from the projected displacement.

For the line geometry relevant to dislocation glide, it is natural to factor out a scalar acoustic amplitude $u(\mathbf{q}, \omega)$ by writing $u_m(\mathbf{q}, \omega) = \hat{q}_m u(\mathbf{q}, \omega)$, since the dislocation couples only to modes whose displacement has a resolved component along the propagation direction. With $\mathbf{q} = q \hat{\mathbf{q}}$, the perturbation Eq. (41) may then be written as

$$\delta h(\mathbf{k}; \mathbf{q}, \omega) = i q W(\mathbf{k}; \hat{\mathbf{q}}, q) u(\mathbf{q}, \omega) + O(q^2), \quad (42)$$

where the coefficient W defines the scalar vertex

$$W(\mathbf{k}; \hat{\mathbf{q}}, q) = \hat{q}_i \hat{q}_j \Gamma_{ij}(\mathbf{k}) + i q_{\ell} \Xi_{ij\ell}^{(ac)}(\mathbf{k}; \hat{\mathbf{q}}) \hat{q}_m + O(q^2), \quad (43)$$

and the projected gradient vertex

$$\Xi_{j\ell} \equiv \Xi_{j\ell}^{(ac)}(\mathbf{k}; \hat{\mathbf{q}}) = \hat{q}_i \Xi_{ij\ell}(\mathbf{k}) \Pi_{jm}^{ac}(\hat{\mathbf{q}}). \quad (44)$$

Contour vertices and stress correlator. To first order, the homogeneous vertex Γ_{ij} defines the stress operator [72],

$$\hat{\sigma}_{ij}(\mathbf{q}) = \sum_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}/2}^{\dagger} \Gamma_{ij}(\mathbf{k}) c_{\mathbf{k}-\mathbf{q}/2}, \quad (45)$$

whose contour correlator is

$$\Pi_{ijkl}^{ab}(\mathbf{q}, t) = -\frac{i}{\hbar} \langle T_C \hat{\sigma}_{ij}^a(\mathbf{q}, t) \hat{\sigma}_{kl}^b(-\mathbf{q}, 0) \rangle, \quad (46)$$

with $a, b \in \{+, -\}$. After Keldysh rotation,

$$\Pi_{ijkl}^R(\mathbf{q}, t) = -\frac{i}{\hbar} \Theta(t) \langle [\hat{\sigma}_{ij}(\mathbf{q}, t), \hat{\sigma}_{kl}(-\mathbf{q}, 0)] \rangle, \quad (47)$$

$$\Pi_{ijkl}^A(\mathbf{q}, t) = +\frac{i}{\hbar} \Theta(-t) \langle [\hat{\sigma}_{ij}(\mathbf{q}, t), \hat{\sigma}_{kl}(-\mathbf{q}, 0)] \rangle, \quad (48)$$

$$\Pi_{ijkl}^K(\mathbf{q}, t) = -\frac{i}{\hbar} \langle \{\hat{\sigma}_{ij}(\mathbf{q}, t), \hat{\sigma}_{kl}(-\mathbf{q}, 0)\} \rangle. \quad (49)$$

In equilibrium,

$$\Pi_{ijkl}^K(\omega) = \coth(\beta \hbar \omega / 2) [\Pi_{ijkl}^R(\omega) - \Pi_{ijkl}^A(\omega)].$$

Keldysh electron-phonon coupling. The dislocation affects the electronic sector only indirectly, since the Kanzaki force first generates a lattice displacement and this displacement then enters the electronic action through the deformation vertices of Eq. (38). In the contour formalism the resulting electron-phonon coupling is obtained by inserting the acoustic displacement field into the canonical deformation expansion,

which yields

$$\begin{aligned} S_{\text{eph}} &= \sum_{\kappa, q} \int \frac{d\varepsilon d\omega}{(2\pi)^2} [\bar{\psi}_{\kappa+q}^q \Gamma_{ij}(\kappa) \psi_{\kappa}^c \varepsilon_{ij}^c(q) \\ &+ \bar{\psi}_{\kappa+q}^c \Gamma_{ij}(\kappa) \psi_{\kappa}^q \varepsilon_{ij}^q(q) \\ &+ \bar{\psi}_{\kappa+q}^q i q_{\ell} \Xi_{j\ell}(\kappa) \psi_{\kappa}^c u_j^c(q) \\ &+ \bar{\psi}_{\kappa+q}^c i q_{\ell} \Xi_{j\ell}(\kappa) \psi_{\kappa}^q u_j^q(q)], \end{aligned} \quad (50)$$

with $\kappa + q = (\mathbf{k} + \mathbf{q}, \varepsilon + \omega)$.

Kanzaki source and the line vertex Λ_i . The dislocation enters the contour theory through the Kanzaki force distribution,

$$S_{\text{slip}} = \int dt \sum_{\mathbf{R}, i} f_i^K(\mathbf{R}; X(t)) u_i^q(\mathbf{R}, t), \quad (51)$$

which in reciprocal space becomes

$$F_i^K(\mathbf{q}; X) = F_i^K(\mathbf{q}) e^{-iq_X X}. \quad (52)$$

This term acts only as a source in the phonon sector. The static lattice Green function $G_{ij}(\mathbf{q}, 0)$ determines the displacement generated by this source and defines the line vertex

$$\Lambda_i(\mathbf{q}) = G_{ij}(\mathbf{q}, 0) f_j^K(\mathbf{q}). \quad (53)$$

Thus, the Kanzaki-induced displacement and compatible strain may be written as

$$u_i(\mathbf{q}, \omega; X) = \Lambda_i(\mathbf{q}) S_X(\omega, \mathbf{q}),$$

$$\varepsilon_{ij}(\mathbf{q}, \omega; X) = \mathcal{U}_{ij}(\mathbf{q}) S_X(\omega, \mathbf{q}),$$

where

$$\mathcal{U}_{ij}(\mathbf{q}) = \frac{i}{2} (q_i \Lambda_j + q_j \Lambda_i). \quad (54)$$

The action S_{slip} does not couple directly to the electrons; it shifts the ionic field, and the electrons couple to that shifted field only through S_{eph} . Substituting the Kanzaki-induced displacement into the deformation expansion Eq. (38) therefore produces a direct glide-dependent electronic vertex. Since the scalar line vertex $W(\mathbf{k}; \hat{\mathbf{q}}, q)$ of Eq. (43) already contains both the homogeneous and projected gradient contributions, the resulting perturbation of the Bloch Hamiltonian reads

$$\delta h(\mathbf{k}; \mathbf{q}, \omega; X) = i q W(\mathbf{k}; \hat{\mathbf{q}}, q) \Lambda_{\parallel}(\mathbf{q}) S_X(\omega, \mathbf{q}) + O(q^2), \quad (55)$$

where $\Lambda_{\parallel} = \hat{q}_i \Lambda_i$. This expression is the one used throughout the remainder of the paper.

For later convenience, we introduce the band vertex

$$V(\mathbf{k}; \mathbf{q}) = i q W(\mathbf{k}; \hat{\mathbf{q}}, q) \Lambda_{\parallel}(\mathbf{q}), \quad (56)$$

which is the microscopic ingredient entering the electronic contribution to the influence kernel $\mathcal{D}_{el}^X(\omega)$ which we define below.

Indeed, the electronic back-reaction enters the glide problem only through those components of the stress tensor that couple to the slip-induced strain field. The fundamental object therefore becomes the four index correlator

$$\Pi_{ij,kl}^X(\mathbf{q}, \omega) = -\frac{i}{\hbar} \int_0^{\infty} dt e^{i\omega t} \langle [\hat{\sigma}_{ij}(\mathbf{q}, t), \hat{\sigma}_{kl}(-\mathbf{q}, 0)] \rangle_X, \quad (57)$$

with $X \in \{R, A, K\}$ and $\hat{\sigma}_{ij}$ defined in Eq. (45). The homogeneous strain vertex Γ_{ij} is already built into $\hat{\sigma}_{ij}$, so no further vertex contraction is needed at this stage. The Kanzaki construction enters through the screened slip vertex $\Lambda_i(\mathbf{q})$ and the associated strain pattern $\mathcal{U}_{ij}(\mathbf{q})$ [Eq. (54)], which is the strain field generated per unit slip spectrum S_X .

The slip geometry enables us to contract the rank 4 $\Pi_{ij,kl}^X$ kernel into an effective rank two kernel, and this is the form that we will employ in the sequel. To do so, we merely enforce the projection of Eq. (57) onto this pattern,

$$\Pi_{ij}^X(\mathbf{q}, \omega) = \mathcal{M}_{ij,kl}(\hat{\mathbf{q}}) \Pi_{kl,mn}^X(\mathbf{q}, \omega) \mathcal{N}_{mn}(\hat{\mathbf{q}}), \quad (58)$$

where the geometric tensors $\mathcal{M}$ and $\mathcal{N}$ encode the glide geometry and the Kanzaki strain; their explicit form is given in Appendix B. In practice, all electronic contributions to the influence kernel can then be written as

$$\mathcal{D}_{el}^X(\omega) = \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \Pi_{ij}^X(\mathbf{q}, \omega) \Lambda_j(\mathbf{q}), \quad (59)$$

so that the entire dependence on the dislocation geometry resides in $\Lambda_i(\mathbf{q})$, while Π_{ij}^X contains the intrinsic electronic dynamics.

Effective action for the glide coordinate. The full contour action is

$$S_{\text{tot}}[X, u, \psi] = S_{\text{ph}}[u] + S_{\text{el}}[\psi] + S_{\text{eph}}[u, \psi; X] + S_{\text{slip}}[u; X], \quad (60)$$

and integrating out u and ψ yields the effective action

$$e^{\frac{i}{\hbar} S_{\text{eff}}[X_+, X_-]} = \int \mathcal{D}[u_{\pm}] \mathcal{D}[\psi_{\pm}] \exp \left[\frac{i}{\hbar} (S_{\text{ph}}[u_+] - S_{\text{ph}}[u_-] + S_{\text{el}}[\psi_+] - S_{\text{el}}[\psi_-] + S_{\text{eph}}[u_{\pm}, \psi_{\pm}; X_{\pm}] + S_{\text{slip}}[u_{\pm}; X_{\pm}]) \right], \quad (61)$$

whose retarded and Keldysh kernels $\mathcal{D}^R$ and $\mathcal{N}$ carry the complete electronic and lattice back reaction. Their construction in terms of Γ_{ij} , $\Xi_{ij\ell}$, Λ_i , the scalar line vertex W of Eq. (43), the band vertex V of Eq. (56), and the projected correlators Eqs. (58) and (59) is given in the following sections.

D. General framework for the drag and configurational forces

Our goal is to obtain an explicit, effective theory for the slow collective coordinate $X(t)$, which measures the glide of the dislocation line. Phonons and electrons constitute an extended environment that reacts to any displacement of the dislocation core, producing both reversible and dissipative forces. Eliminating these fast degrees of freedom yields an influence functional for $X(t)$ that embodies the entire back-reaction of the medium through two point kernels. The procedure is formally identical to the Feynman-Vernon reduction [73] of a linear bath [74]: for fixed glide histories $X_{\pm}(t)$ on the forward and backward branches of the Keldysh contour, the phonon and electron fields enter quadratically, and their functional integrals can be carried out exactly. The result is an “effective action” for the glide coordinate alone.

After the Keldysh rotation $X_{c,q} = (X_+ \pm X_-)/\sqrt{2}$, the effective action takes the standard linear-bilinear form for a

slow classical coordinate coupled to a Gaussian environment (i.e., similar to a Caldeira–Leggett model [75]),

$$S_{\text{eff}}[X_c, X_q] = \int dt X_q(t) F_{\text{det}}[X_c](t) - \frac{1}{2} \int dt dt' X_q(t) \mathcal{D}^R(t-t') X_c(t') - \frac{i}{2} \int dt dt' X_q(t) \mathcal{N}(t-t') X_q(t'). \quad (62)$$

The term linear in X_q couples to the deterministic force F_{det} acting on the dislocation, which would include the applied Peach-Koehler force [76] and, under current flow, the electron wind. The mixed bilinear term contains the retarded kernel $\mathcal{D}^R$, which encodes all conservative and dissipative reactions of the surrounding medium. The quadratic term in X_q introduces the noise kernel $\mathcal{N}$ describing stochastic fluctuations of the configurational force.

This configurational force associated with the motion is the physical force conjugate to X , and is obtained by functional differentiation of the action,

$$F_X(t) = -\frac{\delta S_{\text{eff}}}{\delta X_q(t)} = F_{\text{det}}(t) - \int dt' \mathcal{D}^R(t-t') X_c(t') + \xi(t), \quad (63)$$

where $\xi(t)$ is a Gaussian random force with correlator

$$\langle \xi(t) \xi(t') \rangle = \mathcal{N}(t-t').$$

At this stage, $F_X(t)$ should be read as the coarse grained expectation value of the microscopic force operator, because the functional derivative of the influence action with respect to the force conjugate to $X(t)$ (the X_q once $X(t)$ has been rotated) returns the contour average of that operator after the electronic and phononic fields have been integrated out. The deterministic part F_{det} collects any externally imposed configurational forces, including the macroscopic Peach-Koehler force and, under current flow, the electron wind. The retarded kernel $\mathcal{D}^R$ contains the medium’s back-reaction: its imaginary part produces the viscous drag that opposes motion, while its real part renormalizes the potential curvature. Drag therefore emerges only dynamically, when $X(t)$ evolves in time under a finite driving force, and its magnitude is fixed by $\text{Im } \mathcal{D}^R(\omega \rightarrow 0)$. The electron wind, by contrast, is a non-equilibrium contribution to F_{det} arising from the odd part of the electronic distribution function, not from the equilibrium kernel itself.

The two kernels follow directly from the microscopic force operator that couples the dislocation to the surrounding medium. Both phonon and electronic sectors respond to the dislocation strain field, and thus may contribute to the configurational force. The phonon contribution remains purely equilibrium in character, producing reactive and dissipative terms but no directed configurational drive. A deterministic configurational force may nevertheless arise from externally imposed mechanical stresses, which enter F_{det} as a macroscopic Peach-Koehler force, rather than through the phonon kernel itself. In turn, the electronic subsystem can, under current flow, develop an additional nonequilibrium configurational component, as we will discuss in detail in Sec. IV.

For a given Kanzaki source $f_i^K(\mathbf{R}; X)$, the total operator acting on the medium is

$$F_X(t) = - \sum_{\mathbf{R}, i} [\partial_X f_i^K(\mathbf{R}; X)] u_i(\mathbf{R}; X),$$

where u_i denotes the total displacement field. The phonon and electronic sectors enter additively in the influence functional, and the retarded and noise kernels may be written in terms of the corresponding force-force correlators,

$$\mathcal{D}^R(\omega) = \frac{i}{\hbar} \int_0^\infty dt e^{i\omega t} \langle [F_X(t), F_X(0)] \rangle, \quad (64)$$

$$\mathcal{N}(\omega) = \frac{1}{\hbar} \int_{-\infty}^\infty dt e^{i\omega t} \frac{1}{2} \langle \{F_X(t), F_X(0)\} \rangle. \quad (65)$$

The real part of $\mathcal{D}^R$ gives the reversible renormalization of the potential energy landscape, while its imaginary part defines the dissipative response responsible for drag. These two components together constitute the full memory kernel linking the present force to the past history of motion.

Decomposing the microscopic correlations into phonon and electronic contributions yields

$$\mathcal{D}_{\text{ph}}^R(\omega) = \sum_{\mathbf{q}} (q_x)^2 F_i^K(-\mathbf{q}) D_{ij}^R(\mathbf{q}, \omega) F_j^K(\mathbf{q}), \quad (66)$$

$$\mathcal{D}_{\text{el}}^R(\omega) = \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \Pi_{ij}^R(\mathbf{q}, \omega) \Lambda_j(\mathbf{q}), \quad (67)$$

where the screened strain vertex Λ_i given in Eq. (53) mediates coupling of the dislocation field to the electronic subsystem through the lattice Green function $G_{im}(\mathbf{q})$. Note that the electronic part enters via $\hat{\sigma}$ in $\Pi^{R/K}$, not through a displacement field u_i .

The kernel $\Pi_{ij}^R(\mathbf{q}, \omega)$ is the electronic stress kernel obtained by projecting the four-index correlator Eq. (57) onto the slip-induced strain pattern, as defined in Eq. (58), and it encodes the dynamical screening of the strain field by the electron gas. The prefactor $(q_x)^2$ arises from expanding $e^{-iq_x X} \simeq 1 - iq_x X$ on each contour branch, while the causal split separates the conservative principal value from the dissipative contribution confined to $q_x = 0$, as discussed in Sec. II. The phonon and electronic baths therefore act in parallel, their responses superposing linearly in $\mathcal{D}^R$.

The corresponding noise kernels have the same structure,

$$\mathcal{N}_{\text{ph}}(\omega) = \sum_{\mathbf{q}} (q_x)^2 F_i^K(-\mathbf{q}) D_{ij}^K(\mathbf{q}, \omega) F_j^K(\mathbf{q}), \quad (68)$$

$$\mathcal{N}_{\text{el}}(\omega) = \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \Pi_{ij}^K(\mathbf{q}, \omega) \Lambda_j(\mathbf{q}), \quad (69)$$

where the Keldysh components D^K and Π^K encode the symmetrized correlations of phonons and electrons, respectively. In equilibrium they obey the Kubo-Martin-Schwinger relations

$$D^K = (D^R - D^A) \coth\left(\frac{\beta\hbar\omega}{2}\right),$$

$$\Pi^K = (\Pi^R - \Pi^A) \coth\left(\frac{\beta\hbar\omega}{2}\right),$$

so that the total noise kernel satisfies

$$\mathcal{N}(\omega) = \coth\left(\frac{\beta\hbar\omega}{2}\right) 2i \text{Im } \mathcal{D}^R(\omega).$$

The same elementary excitations that dissipate energy through the imaginary part of the retarded kernel also generate thermal fluctuations with amplitude fixed by the Bose and Fermi distributions. Out of equilibrium this relation ceases to hold because Π^K depends on the actual electronic occupations $f(\mathbf{k})$; however, the $\delta(q_x)$ structure persists, confining both phononic and electronic dissipation to the glide channel.

In this formulation the distinction between configurational and drag forces becomes transparent. The deterministic term F_{det} in the effective action represents external or nonequilibrium drives such as the electron wind; the real part of $\mathcal{D}^R$ renormalizes the conservative potential through static phonon and electronic screening; and the imaginary part, together with the noise kernel $\mathcal{N}$, describes viscous damping and the accompanying fluctuations. The glide coordinate thus behaves as a single classical degree of freedom coupled to two structured baths (phononic and electronic) whose reactive and dissipative responses are fully captured by the retarded kernel $\mathcal{D}^R$.

E. Low-frequency reduction and viscous drag at zero field

We first consider the intrinsic, field-free regime where the dislocation moves under a purely mechanical drive τb and the electronic system remains in thermal equilibrium. The glide coordinate $X(t)$ acts as a classical source that perturbs the lattice and electronic degrees of freedom. Within the Keldysh formalism, integrating out these baths is exact at Gaussian order and automatically separates the causal back-response from the fluctuation kernel. The former yields the deterministic restoring and viscous forces; the latter obeys the fluctuation-dissipation theorem (FDT), ensuring that both arise from the same spectral density.

On the contour, the Kanzaki coupling takes the Keldysh form S_{slip} given in Eq. (51). In Fourier space, this becomes

$$S_{\text{slip}} = \int dt \sum_{\mathbf{q}} F_i^K(\mathbf{q}) e^{-iq_x X(t)} u_i^q(\mathbf{q}, t).$$

A uniform glide therefore inserts the phase $e^{-iq_x X(t)}$, producing a Doppler coupling between the dislocation velocity and the bath's excitations. Expanding the exponential to first order in $X^q = (X_+ - X_-)/\sqrt{2}$ gives the effective quantum source

$$J_i^q(\mathbf{q}, t) = -\frac{iq_x}{\sqrt{2}} X^q(t) F_i^K(\mathbf{q}; X_c(t)),$$

whose contraction with the bath propagators generates the retarded kernel.

After integrating out the phonon or electron fields, the quadratic influence functional has the generic form

$$S_{\text{infl}}[X_c, X_q] = \frac{1}{2} \int dt dt' \left[x_q(t) \mathcal{D}^R(t - t') X_c(t') - \frac{i}{2} X_q(t) \mathcal{N}(t - t') X_q(t') \right], \quad (70)$$

with $\mathcal{D}^R$ and $\mathcal{N}$ the retarded and symmetrized force-force kernels introduced in the Sec. III D. In equilibrium, the FDT

relates them by

$$\mathcal{N}(\omega) = 2i \coth\left(\frac{\omega}{2T}\right) \text{Im } \mathcal{D}^R(\omega).$$

The drag force and the drag coefficient. From Eq. (63),

$$F_X(t) = - \int dt' \mathcal{D}^R(t-t') X(t') + \xi(t),$$

and in Fourier space,

$$F_X(\omega) = -\mathcal{D}^R(\omega) X(\omega) + \xi(\omega).$$

For sufficiently slow motion one expands $\mathcal{D}^R$ to first order,

$$\mathcal{D}^R(\omega) = K - i\omega B + \mathcal{O}(\omega^2), \quad (71)$$

with $K = \text{Re } \mathcal{D}^R(0)$ the conservative renormalization, and thus achieves

$$B = -[\partial_\omega \text{Im } \mathcal{D}^R(\omega)]_{\omega \rightarrow 0} \quad (72)$$

as the viscous drag coefficient. In the time domain,

$$F_X(t) = -K X(t) - B \dot{X}(t) + \xi(t), \quad (73)$$

so that $\text{Im } \mathcal{D}^R(\omega) \propto \omega$ produces a force linear in velocity.

Phonon and electronic drag. Thus, our Keldysh formalism enables us to directly compute the dislocation drag from the retarded force-force kernels $\mathcal{D}^R$. The phonon drag coefficient follows from $\mathcal{D}_{\text{ph}}^R$ in Eq. (67); for details on how to compute it, see Ref. [77]. Here we focus solely on the electronic drag.

For the electrons, we use the electronic part of the same kernel, Eq. (67), and expand its stress correlator Π_{ij}^R [Eq. (58)] for small frequency. Appendix B shows in detail that the imaginary part of Π_{ij}^R behaves as

$$\text{Im } \Pi_{ij}^R(\mathbf{q}, \omega) = -\omega \eta_e(T_e) \mathcal{P}_{ij} + \mathcal{O}(\omega^3),$$

with $\eta_e(T_e)$ an electronic viscosity and $\mathcal{P}_{ij}$ the appropriate projector (both defined in Appendix B). Inserting this into Eq. (67) gives the electronic drag coefficient

$$\begin{aligned} \mathcal{B}_{\text{el}}(T_e) &= \frac{1}{2} \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \mathcal{P}_{ij} \Lambda_j(\mathbf{q}) [-\partial_\omega \text{Im } \Pi_{ij}^R(\mathbf{q}, \omega)]_{\omega \rightarrow 0}. \end{aligned} \quad (74)$$

This recovers the viscous drag law

$$F_{\text{drag}}(t) = -\mathcal{B}_{\text{el}}(T_e) \dot{X}(t).$$

Equation (74) shows that dissipation appears only when $X(t)$ varies in time, since the phase factor $e^{-iq_x X(t)}$ mixes positive and negative frequencies of Π^R . In equilibrium the FDT relation between Π^K and $\text{Im } \Pi^R$ ensures that the same spectrum that produces viscous drag also sets the amplitude of the electronic noise.

Finally, the mobility law follows directly from the effective action with the low-frequency kernel,

$$B(T) \dot{X} = \tau b + \xi(t), \quad \langle \xi(t) \xi(t') \rangle = \mathcal{N}(t-t'), \quad (75)$$

which is the Langevin representation of the same influence functional. A Langevin mobility law of this form, which here is obtained from first principles, has been used previously to describe thermally activated motion [78], particularly in Frenkel-Kontorova type models [70,79]. It is important to

emphasize that in this work *drag* refers strictly to the viscous contribution proportional to the glide speed, arising from the $\omega \rightarrow 0$ limit of the imaginary part of the stress correlator $\mathcal{D}^R$. Other dissipative channels, such as radiative losses associated with higher time derivatives of the glide coordinate [77], are described by the same kernel but do not contribute to the viscous coefficient and are not to be regarded as drag. In particular, both phononic and electronic drag are properly understood as the zero frequency limit of $\text{Im } \mathcal{D}^R$, while frequency-dependent dissipative effects beyond this limit fall outside the notion of drag employed here.

Selection rule theorem, shear modes and viscous drag. In Sec. II the Kanzaki form factor was shown to enforce the $(q_x + i0^+)^{-1}$ dependence characteristic of a glide discontinuity. It is important to identify which part of the lattice form factor $\Psi(\mathbf{q}; X)$ actually contributes to dissipation. A formal proof of the ensuing result is given in Appendix A, where we establish that viscous dissipation in the low-frequency limit under discussion here samples the phonon/electron bath only through the $q_x = 0$ plane of its spectrum; here we summarize this result.

The discrete cut produces the characteristic factor $(q_x + i0^+)^{-1}$ [see Eq. (7)], the decomposition of which into real and imaginary parts has sharply distinct physical roles. What we wish to do is separate the "reactive" elastic back-response (i.e., the part thereof that generates the dislocation's own elastic field) from the genuinely dissipative channel. To that end, we invoke the following elementary distributional identities. Suppose $\Psi(\mathbf{q}; X) = e^{-iq_x X} B(\mathbf{q}) (q_x + i0^+)^{-1}$ with $B(\mathbf{q})$ smooth and even in q_x , since the structure factor S_α is even in q_x by spatial inversion symmetry of the lattice. For any test function $\varphi(\mathbf{q})$ even in q_x one has

$$\begin{aligned} \int \frac{dq_x}{2\pi} \varphi(\mathbf{q}) \mathcal{P}\left(\frac{1}{q_x}\right) &= 0, \\ \int \frac{dq_x}{2\pi} \varphi(\mathbf{q}) \delta(q_x) &= \frac{1}{2\pi} \varphi(0, \mathbf{q}_\perp). \end{aligned} \quad (76)$$

Hence, in quadratic functionals such as $\sum_{\mathbf{q}} \Psi^*(\mathbf{q}) K(\mathbf{q}, \omega) \Psi(\mathbf{q})$ with K even in q_x , only the $\delta(q_x)$ contribution survives in the imaginary part at $\omega \rightarrow 0$. The principal value part $\mathcal{P}(\frac{1}{q_x})$ contributes only to the reactive (real) part. From a physical point of view, the principal value represents reversible phase shifts of the lattice modes: it only stores and releases elastic energy, producing a back-force without dissipation. By contrast, the $\delta(q_x)$ factor singles out fluctuations with zero longitudinal wave vector along the glide axis. This is the main result proven in Appendix A.

Thus, for any admissible glide history and for the retarded kernel $\mathcal{D}^R(\omega)$ defined in Sec. IIIC, the imaginary part which enters the viscous drag coefficient Eq. (72) reduces exactly to its $q_x = 0$ component in the low-frequency limit. In practice, it is convenient to write the screened coupling to the medium as $\Lambda_i(\mathbf{q}) = G_{ij}(\mathbf{q}, 0) F_j^K(\mathbf{q})$, where G_{ij} is the static elastic Green function and $F_j^K(\mathbf{q})$ carries the slip form factor $\Psi(\mathbf{q})$ of Eq. (7). The latter contains the factor $(q_x + i0^+)^{-1}$ with an envelope that is even in q_x , so that the Sokhotski-Plemelj decomposition [80] singles out the $\delta(q_x)$ contribution in the imaginary part. The dissipative part of the drag kernel may then be expressed in terms of the effective two index stress

correlator defined in Eq. (B7) as

$$\text{Im } \mathcal{D}^R(\omega) = \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \text{Im } \Pi_{ij}^R(\mathbf{q}, \omega) \Lambda_j(\mathbf{q}), \quad (77)$$

with the understanding that the combination of the $(q_x + i0^+)^{-1}$ in the Kanzaki vertex and the even q_x envelope restricts the q_x integral to $q_x = 0$ in the imaginary part. The spatial projection therefore fixes $\mathbf{q} = \mathbf{q}_\perp$ perpendicular to the glide direction and removes any contribution from modes with finite longitudinal wave vector. Appendix A shows that this projection is exact for the low-frequency slope of $\text{Im } \mathcal{D}^R(\omega)$ that defines the viscous coefficient, irrespective of the detailed form of the glide history, and that the principal value part of $(q_x + i0^+)^{-1}$ contributes only to the reactive sector.

At such purely transverse momenta $\mathbf{q}_\perp$ and in the long-wavelength limit $|\mathbf{q}_\perp| \rightarrow 0$ the geometry of glide and the structure of the Kanzaki force remove any overlap with a homogeneous volume change of the surrounding medium. Appendix B makes this statement explicit for the electronic fluid by analyzing the four index stress kernel $\Pi_{ij,kl}^R$ in hydrodynamic form,

$$\text{Im } \Pi_{ij,kl}^R(\mathbf{q}_\perp, \omega) = \omega [\zeta \mathcal{P}_{ij,kl}^{(\text{bulk})} + \eta \mathcal{P}_{ij,kl}^{(\text{shear})}] + O(\omega^3),$$

with ζ and η the bulk and shear viscosities and $\mathcal{P}^{(\text{bulk})}$, $\mathcal{P}^{(\text{shear})}$ the corresponding projectors. The glide vertex is encoded in tensors $\mathcal{M}_{ij,kl}(\hat{\mathbf{q}}_\perp)$ and $\mathcal{N}_{mn}(\hat{\mathbf{q}}_\perp)$ constructed from the screened Kanzaki force and the slip geometry, and one finds that

$$\mathcal{M}_{ij,kl}(\hat{\mathbf{q}}_\perp) \mathcal{P}_{kl,mn}^{(\text{bulk})} \mathcal{N}_{mn}(\hat{\mathbf{q}}_\perp) = 0,$$

so that the bulk viscous channel is projected out of the contracted kernel at small $|\mathbf{q}_\perp|$. In other words, the topological slip discontinuity associated with glide has no mixed response with a homogeneous dilatation once the $q_x = 0$ projection has been imposed. The effective two index correlator that enters Eq. (77) therefore reduces to a purely shear object at small $\mathbf{q}_\perp$,

$$\text{Im } \Pi_{ij}^R(\mathbf{q}_\perp, \omega) = \omega \eta(T) \mathcal{P}_{ij}(\hat{\mathbf{q}}_\perp) + O(\omega^3),$$

where $\mathcal{P}_{ij}$ is a traceless, transverse projector onto the shear sector selected by glide. Substituting this form into Eq. (77) and using the $q_x = 0$ projection yields

$$\text{Im } \mathcal{D}^R(\omega) = \omega \eta(T) \sum_{\mathbf{q}_\perp} \mathcal{V}_i^*(\mathbf{q}_\perp) \mathcal{P}_{ij}(\hat{\mathbf{q}}_\perp) \mathcal{V}_j(\mathbf{q}_\perp) + O(\omega^3),$$

where $\mathcal{V}_i(\mathbf{q}_\perp)$ denotes the effective transverse vertex after the q_x integration, which includes the finite combination of $(q_x)^2$ and the $(q_x + i0^+)^{-1}$ factor evaluated at $q_x = 0$. This term alone controls the low-frequency slope of $\text{Im } \mathcal{D}^R(\omega)$ and hence the viscous drag coefficient defined by $B = -\lim_{\omega \rightarrow 0} \omega^{-1} \text{Im } \mathcal{D}^R(\omega)$.

It is important to make clear what this selection rule does and does not imply. The projection onto $q_x = 0$ concerns only the $\omega \rightarrow 0$ slope of the kernel and therefore only the viscous coefficient in the mobility law; by contrast, the finite-frequency part of $\text{Im } \mathcal{D}^R(\omega)$ retains its full Doppler structure and governs radiative losses, which contribute to the total

dissipated power but do not enter the definition of B . Dissipation in this context would be nonviscous but radiative in nature, not proportional to v_d , and it is important in the phonon sector, where a full account has been provided by the author in Ref. [77].

The present analysis concerns only the viscous drag coefficient, since the electronic route to dissipation is largely controlled by the low-frequency part of the stress response and is not subject to the radiative processes that influence the phonon channel; the coupling between ionic slip and electronic stresses is too weak to generate appreciable finite-frequency emission, so the electronic dissipation resides in the $q_x = 0$ sector alone. Now, the sequence of steps leading to the $q_x = 0$ projection is entirely kinematic and follows from the topology of the slip disregistry together with causality. Appendix A shows that, once the Kanzaki form factor and the $(q_x + i0^+)^{-1}$ structure are taken into account, the imaginary part of the retarded drag kernel that contributes to the low-frequency slope samples the bath only through its response at $q_x = 0$. In that sense the dissipative kernel relevant for viscous drag is effectively two dimensional and involves only fluctuations with wave vector perpendicular to the glide direction. Appendix B then shows that, at such transverse momenta and in the long-wavelength limit, the screened Kanzaki vertex becomes traceless, so that the contraction of the hydrodynamic stress correlator with the glide vertex removes the bulk sector and leaves only the shear projector. The electronic bulk viscosity drops out of the viscous drag coefficient, and an entirely analogous argument applies to the phonon stress field in the same low-frequency and long-wavelength regime.

The role of Doppler channels requires a separate remark. For a given glide history $X(t)$ the time-dependent vertex produces a Doppler factor $\delta(\omega - q_x v_d)$ in the Fourier representation, which characterizes phonon emission and absorption at finite frequency and finite q_x . In the definition of the viscous coefficient one first forms $\mathcal{D}^R(\omega)$ at general $(\mathbf{q}, \omega)$ and then takes the $\omega \rightarrow 0$ limit of $\omega^{-1} \text{Im } \mathcal{D}^R(\omega)$. In this order of limits, the Doppler δ collapses onto $q_x = 0$ and combines with the Sokhotski-Plemelj structure of the static form factor to produce the exact $q_x = 0$ projection in the viscous kernel. Radiative drag at finite frequency and high velocity remains governed by the full Doppler surface $\omega = \mathbf{q} \cdot \mathbf{v}_d$ and involves modes with $q_x \neq 0$, but these contributions do not enter the low-frequency slope that defines B . The selection rule formulated above is therefore a statement about the viscous drag coefficient alone: the dissipative part of the drag kernel that contributes to B arises exclusively from the shear channel of the medium at $q_x = 0$, whether carried by phonons or by electrons, while longitudinal or bulk distortions affect only the reactive part of the response or the finite-frequency radiative sector. In this way, the origin of dislocation drag in the viscous limit is fixed by the geometric character of glide itself. A moving dislocation transports a topological, volume preserving slip discontinuity, and it is this constraint, encoded in the $q_x = 0$ projection of the Kanzaki construction, that ensures only shear modes of the surrounding medium can absorb energy from its slow motion. The magnitude of the drag coefficient remains material specific through the values of the shear viscosities η_e and η_{ph} , but the fact that only shear viscosity enters the viscous drag law is a universal

consequence of the glide geometry rather than a detailed property of the microscopic spectrum.

Finally, it is worth emphasizing that all these statements concern the viscous, low-frequency limit that defines $\mathcal{B}$; finite-frequency radiative losses remain governed by the full Doppler structure of $\text{Im } \mathcal{D}^R(\omega)$ and are not encoded in the viscous drag coefficient. However, for later comparison with the mobility law it is useful to record the general relation between the drag kernel and the dissipated power. The instantaneous power transferred from the external drive to the dislocation is

$$P(t) = \dot{X}(t) F_{\text{drag}}(t),$$

$$F_{\text{drag}}(t) = - \int dt' \mathcal{D}^R(t-t') \dot{X}(t'),$$

so that, upon Fourier transforming and averaging over a long time window, one finds

$$P = \int \frac{d\omega}{2\pi} \omega \text{Im } \mathcal{D}^R(\omega) |\tilde{X}(\omega)|^2. \quad (78)$$

This formula describes dissipation at arbitrary frequency and contains both the viscous contribution and the finite-frequency radiative part. In the low velocity regime relevant for the mobility law the glide history is slow on microscopic scales, so that $\tilde{X}(\omega)$ has support only at small $|\omega|$. Approximating $\omega^{-1} \text{Im } \mathcal{D}^R(\omega) \simeq B$ then yields

$$P \simeq B \int \frac{d\omega}{2\pi} |\tilde{X}(\omega)|^2 = B \overline{\dot{X}^2},$$

which for steady glide at velocity v_d reduces to $P = B v_d^2$. Identifying the drag force density as $\tau_{\text{drag}} = -B v_d$ gives the usual relation $P = \tau_{\text{drag}} v_d$ in the low-frequency limit. Radiative phonon emission at higher velocities corresponds instead to the finite-frequency structure of $\omega \text{Im } \mathcal{D}^R(\omega)$ in Eq. (78); it contributes to the total dissipated power but is not encoded in the viscous coefficient B extracted from the $\omega \rightarrow 0$ slope. A full account of how to handle Eq. (78) in the phonon sector is given in Ref. [77], where the finite-frequency structure of $\omega \text{Im } \mathcal{D}^R(\omega)$ and the associated radiative processes are treated explicitly and are shown to showcase a characteristic separation of relaxation timescales. For the electronic sector, by contrast, Eq. (78) is largely unnecessary, because the glide motion probes frequencies that lie many orders of magnitude below the intrinsic electronic relaxation scales. The electronic stress response is therefore sampled only in its quasi static, long-wavelength regime and carries no accessible structure beyond the linear term in ω that defines the viscous coefficient. Any contribution from the higher frequency sector of $\text{Im } \mathcal{D}^R(\omega)$ would require excitations on electronic timescales which are not generated by the glide history, and which in practice are observed only in spectroscopies that inject energy directly into the electronic continuum, such as neutron or photon scattering. For the purpose of dislocation dynamics the low-frequency limit is therefore complete, and the electronic drag is exhausted by the shear viscosity that enters the $\omega \rightarrow 0$ kernel.

Temperature dependence of the electronic drag. The temperature dependence of $\mathcal{B}_{\text{el}}$ follows directly from the Keldysh structure of the electronic stress bubble. At Gaussian level the

Keldysh component of the stress correlator reads

$$\Pi_{ij}^K(\mathbf{q}, \omega) = i \int \frac{d^3k d\varepsilon}{(2\pi)^4} \text{Tr} [\Gamma_{ij}(\mathbf{k}) G_{\mathbf{k}}^K(\varepsilon) \Gamma_{kl}(\mathbf{k}+\mathbf{q}) G_{\mathbf{k}+\mathbf{q}}^A(\varepsilon+\omega) + \Gamma_{ij}(\mathbf{k}) G_{\mathbf{k}}^R(\varepsilon) \Gamma_{kl}(\mathbf{k}+\mathbf{q}) G_{\mathbf{k}+\mathbf{q}}^K(\varepsilon+\omega)], \quad (79)$$

with $G^K = (G^R - G^A)(1 - 2f)$ the usual Keldysh Green function. Using the equilibrium relation between Π^K and $\text{Im } \Pi^R$ quoted earlier, one can work directly with the spectral representation of the retarded bubble. In the long-wavelength limit and for small frequencies one finds the standard form

$$\text{Im } \Pi_{ij}^R(\mathbf{q}, \omega) = \omega \int \frac{d^3k}{(2\pi)^3} \left(-\frac{\partial f}{\partial \varepsilon_{\mathbf{k}}} \right) v_i(\mathbf{k}) v_j(\mathbf{k}) \eta_{\text{eff}}(\varepsilon_{\mathbf{k}}, T_e), \quad (80)$$

where $v_i(\mathbf{k}) = \partial \varepsilon_{\mathbf{k}} / \partial k_i$ is the band velocity and η_{eff} is an effective viscous coefficient that collects the imaginary parts of the electronic self energy entering the dressed propagators inside the bubble. More explicitly, if one writes

$$G_{\mathbf{k}}^R(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_{\mathbf{k}} - \Sigma^R(\mathbf{k}, \varepsilon)} \equiv \frac{1}{\varepsilon - \varepsilon_{\mathbf{k}} + i\gamma_{\mathbf{k}}(\varepsilon)/2},$$

with $\gamma_{\mathbf{k}}(\varepsilon) = -2 \text{Im } \Sigma^R(\mathbf{k}, \varepsilon)$ the damping rate, then a straightforward evaluation of the two spectral functions in the bubble shows that

$$\eta_{\text{eff}}(\varepsilon_{\mathbf{k}}, T_e) \propto \gamma_{\mathbf{k}}(\varepsilon_{\mathbf{k}}, T_e),$$

so the stress viscosity is set by the same imaginary self energy that determines the single particle scattering rate. The derivative $-\partial f / \partial \varepsilon_{\mathbf{k}}$ in Eq. (80) restricts the integral to an energy window of width $O(T_e)$ around the Fermi level, so the remaining temperature dependence is controlled by $\gamma_{\mathbf{k}_F}(T_e)$.

For electron-electron scattering in a Fermi liquid one has

$$\gamma_{\mathbf{k}_F}^{(ee)}(T_e) \equiv -2 \text{Im } \Sigma_{ee}^R(\varepsilon_{\mathbf{k}_F}, T_e) \propto T_e^2,$$

reflecting the usual phase space restriction of quasiparticle decay near the Fermi surface. Inserting this scaling into Eq. (80) gives

$$\text{Im } \Pi_{ij}^{R(ee)}(\mathbf{q}, \omega) \propto \omega T_e^2,$$

and therefore an electronic drag coefficient $\mathcal{B}_{\text{el}}^{(ee)} \propto T_e^2$ in Eq. (74). For electron-phonon scattering the imaginary part of the electron self energy contains the phonon Keldysh population factor $D_{\mathbf{q}}^K(\Omega) \propto (2n_{\mathbf{q}} + 1)$; in the classical regime $n_{\mathbf{q}} \simeq T_e / \Omega_{\mathbf{q}}$ and the dominant contribution at low frequencies is linear in temperature,

$$\gamma_{\mathbf{k}_F}^{(e-ph)}(T_e) \equiv -2 \text{Im } \Sigma_{e-ph}^R(\varepsilon_{\mathbf{k}_F}, T_e) \propto T_e,$$

which leads to

$$\text{Im } \Pi_{ij}^{R(e-ph)}(\mathbf{q}, \omega) \propto \omega T_e, \quad \mathcal{B}_{\text{el}}^{(e-ph)} \propto T_e.$$

Thus, the familiar Fermi liquid hierarchy $\gamma_{ee} \propto T_e^2$ and $\gamma_{e-ph} \propto T_e$ is inherited directly by the stress-stress kernel and, through Eq. (74), by the electronic contribution to the dislocation drag. These power laws are not phenomenological inputs but follow from the thermal occupation factors appearing in the fermionic and bosonic Keldysh propagators entering the stress bubble.

F. Current-free electron wind forces

Our purpose here is to show that the electronic environment cannot exert a wind force in equilibrium. In the absence of current, the electronic occupations remain even in momentum, so any configurational response must be conservative. Within the Keldysh reduction this fact is encoded in the real part of the retarded kernel, which captures the reversible polarization of the lattice and electron gas by the dislocation strain.

The conservative response enters through the zero-frequency limit

$$\Delta = \text{Re } \mathcal{D}^R(0) = \Delta_{\text{ph}} + \Delta_{\text{el}},$$

and appears as a renormalization of the curvature of the effective potential,

$$U_{\text{eff}}[X_c] = \frac{1}{2} \int dt dt' X_c(t) \Delta \delta(t-t') X_c(t') + \dots, \\ F_{\text{det}} = -\Delta X_c.$$

The fluctuation-dissipation relation

$$\Delta = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{\text{Im } \mathcal{D}^R(\omega')}{\omega'}$$

shows that this static renormalization derives from the same microscopic correlations that give rise to drag, although it involves their in phase component.

For the electronic sector one has

$$\Delta_{\text{el}} = \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \text{Re } \Pi_{ij}^R(\mathbf{q}, 0) \Lambda_j(\mathbf{q}),$$

where Λ_i is the screened strain vertex and Π_{ij}^R the stress correlator. This term represents the static screening of the dislocation strain by the electron gas. Because the equilibrium distribution $f(\varepsilon)$ is even, the screened stress produces no directed momentum transfer and hence no net configurational force. The expectation value $\langle F_X \rangle_{eq}$ vanishes identically. Only when a current is applied, displacing the occupations away from $f(\varepsilon)$ and breaking detailed balance, does a directional component appear.

Thus, the equilibrium electronic kernel modifies the stiffness but cannot generate a wind force. A nonconservative configurational drive requires a current, something which we address in Sec. IV.

IV. ELECTROMAGNETOPLASTIC RESPONSE

We now extend the zero field Keldysh treatment of Sec. III to include a uniform current and a transverse magnetic field. The influence functional for the glide coordinate remains our starting point. The electronic contribution to the retarded kernel is still obtained from the low-frequency slope of the stress-stress bubble, but here it must be evaluated at finite transverse wave vector selected by the discrete slip form factor. The Kanzaki construction fixes $q_x = 0$ while leaving $q_{\perp} = (q_y, q_z)$ to be integrated with the screened slip vertex.

Two conceptually distinct current effects must be separated. A uniform current produces a static electron wind stress, a deterministic force that adds to the resolved shear and is

independent of the glide velocity. The same current also modifies the viscous slope through the depletion of time-reversed phase space; in centrosymmetric metals this change is even in the current and therefore quadratic in J . A transverse magnetic field mixes velocity harmonics through the Lorentz operator and reduces the dissipative longitudinal response by the Drude factor at weak field; at stronger field Landau quantization of the *same* stress bubble produces quantum oscillations. Throughout this section we denote the magnetic field by B and the drag coefficient by $\mathcal{B}$ to avoid clashes.

Using the discrete Kanzaki source and lattice Green function introduced in Sec. II, the dislocation enters the electronic sector through the screened slip field $\Lambda_i(\mathbf{q}) = G_{ij}(\mathbf{q}) F_j^K(\mathbf{q})$. The causal structure of the slip form factor already shown in Eq. (7) enforces the separation $(q_x + i0^+)^{-1} = \mathcal{P}(1/q_x) - i\pi\delta(q_x)$, so that only the transverse $q_x = 0$ band contributes to dissipation (see Appendix A). Consequently, the relevant electronic response is the small-frequency, finite $q_{\perp}$ limit of the stress-stress correlator $\Pi_{ij}^R(\mathbf{q}, \omega)$ contracted with the long-wavelength strain vertex of Eq. (38). The retarded electronic kernel is therefore $\mathcal{D}_{\text{el}}^R \propto \Lambda^* \Pi^R \Lambda$, whose low-frequency slope defines the drag coefficient $\mathcal{B}_{\text{el}}$. Screening and near-core regularization follow the same prescriptions as in Sec. II and need not be repeated here.

A. Keldysh setup: Coupling, correlators, and the nonequilibrium distribution

The coupling between the glide coordinate and the electronic subsystem is already defined in Sec. III C: the Kanzaki source generates a displacement field u whose Fourier components enter the electronic action through the homogeneous and gradient vertices of the deformation expansion Eq. (38). The resulting microscopic coupling is encoded in the band vertex $V(\mathbf{k}; \mathbf{q})$ of Eq. (56), which incorporates both the scalar glide vertex $W(\mathbf{k}; \hat{\mathbf{q}}, q)$ and the screened Kanzaki displacement $\Lambda_i(\mathbf{q})$ defined in Eq. (53). The glide coordinate therefore couples to the electrons only through the operator

$$F_X(t) = \sum_{n\mathbf{k}} V_n(\mathbf{k}) c_{n\mathbf{k}}^{\dagger}(t) c_{n\mathbf{k}}(t),$$

which is the electronic back-force per unit line length associated with an infinitesimal glide.

Integrating out the electronic fields yields the standard Keldysh influence action

$$S_{\text{eff}}[X_c, X_q] = \int dt X_q(t) \langle F_X(t) \rangle \\ - \frac{1}{2} \int dt dt' (X_c \quad X_q)_t \begin{pmatrix} 0 & \chi_{XX}^A \\ \chi_{XX}^R & \chi_{XX}^K \end{pmatrix}_{t-t'} \begin{pmatrix} X_c \\ X_q \end{pmatrix}_{t'}, \quad (81)$$

where the kernels $\chi_{XX}^{R,A,K}$ are electronic response functions constructed from the same vertices V that enter the equilibrium reduction of Sec. III C. The first term, linear in X_q , gives the configurational force $\langle F_X \rangle$; the quadratic terms generate the electronic contribution to the drag and noise kernels.

In a stationary state the Keldysh propagator has the usual form

$$G_n^K(\varepsilon, \mathbf{k}) = (1 - 2f_n(\varepsilon, \mathbf{k}))(G_n^R - G_n^A),$$

so all nonequilibrium structures are encoded in the Fermi distribution f_n . To determine f_n in the presence of a weak uniform field, we recall that the equilibrium solution of the Keldysh kinetic equation is the Fermi function

$$f_0(\varepsilon) = \frac{1}{e^{\beta(\varepsilon - \mu)} + 1},$$

which satisfies $\Sigma^>G^< = \Sigma^<G^>$ and therefore yields $G^K = (1 - 2f_0)(G^R - G^A)$.

Subtracting the Dyson equations for $G^<$ and $G^>$ and performing a Wigner transformation gives the stationary quantum kinetic equation

$$\mathbf{v}_{n\mathbf{k}} \cdot \nabla_{\mathbf{r}} f_n + e \mathbf{E} \cdot \nabla_{\mathbf{k}} f_n = \mathcal{I}_{\text{coll}}[f], \quad (82)$$

where $\mathcal{I}_{\text{coll}}$ is the Keldysh collision integral.

In practice, the kernel $\mathcal{I}_{\text{coll}}[f]$ Eq. (82) contains phonon, electron-electron, and possibly impurity scattering, all of which act to relax the odd part of $f_n(\mathbf{k})$ in momentum space back toward the isotropic Fermi function f_0 . In the near equilibrium regime relevant here, the kinetic equation is linearized around f_0 and only the first angular harmonic of the deviation $\delta f = f - f_0$ survives, so that the detailed structure of the collision integral reduces to a single transport time τ_{tr} which controls momentum relaxation.

The relaxation time approximation (RTA) then takes the form

$$\mathcal{I}_{\text{coll}}[f] = -\frac{f_n(\mathbf{k}) - f_0(\varepsilon_{n\mathbf{k}})}{\tau_{\text{tr}}}, \quad (83)$$

which restores f_0 in the absence of the field and is accurate whenever momentum relaxing scattering dominates and varies smoothly on the Fermi surface.

This structure echoes the heuristic treatment of Brailsford [18], who imposed a phenomenological relaxation of the electronic distribution back to the Fermi function. However, our formulation does not rely on ad hoc assumptions about the dissipative channels but rather emerges from the linearized Keldysh kinetic equation itself, with the collision kernel and its angular structure fixed by the microscopic self energies; further, our formulation goes beyond Brailsford's by deriving the transport relaxation directly from the Keldysh stress bubble with fully dressed propagators. This replaces his model scattering rate by the exact microscopic object that governs the decay of the odd part of the electronic distribution, and embeds similar physical mechanism in a parameter free and internally consistent response framework. Indeed, our representation of $\mathcal{I}_{\text{coll}}$ Eq. (83) is not an arbitrary ansatz but the controlled low order limit of the Keldysh collision integral under weak driving. The use of the same conserving approximation in the conductivity ensures that τ_{tr} ultimately drops out in favour of the measured conductivity, so the microscopic structure of the scattering enters the wind force and the drag only through the macroscopic transport coefficients.

Solving Eq. (82) with Eq. (83) for a spatially uniform dc field yields the standard drifted distribution

$$f_n(\mathbf{k}) = f_0(\varepsilon_{n\mathbf{k}}) - (-\partial_{\varepsilon} f_0)_{\varepsilon_{n\mathbf{k}}} \hbar \mathbf{v}_n(\mathbf{k}) \cdot \delta \mathbf{k}, \quad \delta \mathbf{k} = \frac{e\tau_{\text{tr}}}{\hbar} \mathbf{E}. \quad (84)$$

This is the weak field stationary solution used in all nonequilibrium response functions below. Although the macroscopic current densities relevant for electromigration may be very large, the microscopic electric field experienced by the conduction electrons remains small in the only sense that matters for the Keldysh kinetic equation. The drift vector $\delta \mathbf{k} = e\tau_{\text{tr}}\mathbf{E}/\hbar$ is several orders of magnitude smaller than k_F in most metals, because the conductivity is high and the transport time short, so maintaining a large current requires only a modest field. Equation (84) is the stationary solution appropriate when the external drive varies slowly on electronic relaxation timescales, so that the electronic subsystem remains close to local equilibrium and acts as a fast bath for the dislocation coordinate. If one instead requires genuinely nonstationary or nonthermal electronic occupations (for example under ultrafast pulsing), the contour construction is unchanged: one solves the time-dependent kinetic equation for $f_n(\mathbf{k}, t)$ and substitutes the corresponding $G^K(t, t')$ in place of the stationary form. The retarded sector and the reactive versus dissipative split of the effective kernel remain the same; only the Keldysh occupations become explicitly time dependent. Generally however, the electronic sector will remain close to local equilibrium, its distribution retaining the Fermi-Dirac form with a slight momentum shift. This is the regime in which the Wigner transformed kinetic equation admits a controlled expansion around f_0 and in which the stationary drifted solution Eq. (84) follows uniquely from the contour dynamics. The use of the weak field form of f_n is thus not a phenomenological supposition but the physically correct limit in which both the wind force and the electronic drag must be evaluated.

The electronic response kernels $\chi_{XX}^{R,A,K}$ entering S_{eff} are evaluated using propagators built from this distribution. Their analytic structure remains that of equilibrium, but detailed balance is broken through the asymmetric G^K , which is the sole source of the electron wind force discussed next.

B. Keldysh reduction to the electron wind force

The configurational force conjugate to the glide coordinate follows directly from the contour operator defined in Sec. III C. Its expectation value is

$$\langle F_X(t) \rangle = -i \text{Tr}[\bar{V} G^<(t, t)], \quad (85)$$

with $G_n^<(\mathbf{k}; t, t') = i \langle c_{n\mathbf{k}}^\dagger(t') c_{n\mathbf{k}}(t) \rangle$ the lesser Green function and $V_n(\mathbf{k})$ the band vertex defined in Eq. (56). All dependence on glide, on the Kanzaki construction, and on the electronic structure is already contained in this vertex. Equation (85) is therefore the unique microscopic starting point for both the electron wind force and the electronic contribution to drag.

For a homogeneous steady state, all electronic correlators depend only on time differences, so the equal time lesser Green function can be expressed through its mixed

frequency-momentum representation as

$$G_n^<(\mathbf{k}; t, t) = \int \frac{d\varepsilon}{2\pi} G_n^<(\varepsilon, \mathbf{k}).$$

Inserting this into the contour expression for the force Eq. (85) gives

$$\langle F_X \rangle = -i \sum_n \int \frac{d^3k}{(2\pi)^3} \int \frac{d\varepsilon}{2\pi} V_n(\mathbf{k}) G_n^<(\varepsilon, \mathbf{k}). \quad (86)$$

In the Keldysh representation, the lesser function in a stationary state has the spectral form

$$G_n^<(\varepsilon, \mathbf{k}) = i f_n(\varepsilon, \mathbf{k}) A_n(\varepsilon, \mathbf{k}),$$

with f_n the (possibly nonequilibrium) distribution and A_n the single particle spectral density. In the sharp quasiparticle limit we approximate

$$A_n(\varepsilon, \mathbf{k}) = 2\pi \delta(\varepsilon - \varepsilon_{n\mathbf{k}}),$$

so that the energy integral in Eq. (86) collapses onto the band shell:

$$\begin{aligned} \langle F_X \rangle &= \sum_n \int \frac{d^3k}{(2\pi)^3} \int \frac{d\varepsilon}{2\pi} V_n(\mathbf{k}) f_n(\varepsilon, \mathbf{k}) 2\pi \delta(\varepsilon - \varepsilon_{n\mathbf{k}}) \\ &= \sum_n \int \frac{d^3k}{(2\pi)^3} V_n(\mathbf{k}) f_n(\mathbf{k}), \end{aligned} \quad (87)$$

where $f_n(\mathbf{k}) \equiv f_n(\varepsilon_{n\mathbf{k}}, \mathbf{k})$ is the distribution evaluated on the quasiparticle shell. This yields the compact expression

$$\langle F_X \rangle = \sum_n \int \frac{d^3k}{(2\pi)^3} V_n(\mathbf{k}) f_n(\mathbf{k}), \quad (88)$$

which we shall use as the starting point for the wind force in the quasi static regime.

The equilibrium part of f gives no net configurational force; the linear correction in the electric field produces the electron wind discussed in Sec. IV B 1.

The same contour functional also fixes the dissipative back-reaction of the electronic subsystem. Expanding the influence action to second order in the glide velocity produces a term linear in $\dot{X}(t)$, which defines the electronic drag force $F_{\text{drag}} = -B_{\text{el}} \dot{X}$. As we discussed in Sec. III E, its coefficient is given by the low-frequency limit of the retarded kernel in Eq. (72), which we can adapt specifically to the effective action as written in Eq. (81), as $B_{\text{el}} = \frac{1}{b^2} \lim_{\omega \rightarrow 0} \text{Im} \chi_{XX}^R(\omega)/\omega$ as χ_{XX}^R is the part of the influence kernel restricted to the electronic sector. Indeed, the electron wind and the electronic drag correspond to the static real part and the low-frequency imaginary part of the same retarded response function χ_{XX}^R .

1. Quasistatic limit

For subsonic motion $v_d \ll c_s$, the strain field of a moving dislocation differs from its static counterpart only by corrections of order $(v_d/c_s)^2$ [44], so the electronic sector sees an adiabatically drifting source. In this regime, the only velocity dependence that survives at leading order is the Doppler phase $e^{-iq_x v_d t}$ multiplying the Fourier components of the Kanzaki-induced displacement. The configurational force is then obtained by evaluating the operator Eq. (85) in the

stationary nonequilibrium state generated by the electric field, retaining only the $q_x = 0$ component that survives the time average.

Using $G^< = i f A$ and the quasiparticle spectral form, the force reduces to the momentum integral

$$\langle F_X \rangle = \sum_n \int \frac{d^3k}{(2\pi)^3} V_n(\mathbf{k}) f_n(\mathbf{k}), \quad (89)$$

where the vertex V_n is the band-level form of the Kanzaki deformation potential defined in Sec. II C. The distribution $f_n(\mathbf{k})$ is the drifted steady state solution of the Keldysh kinetic equation derived in Sec. IV B,

$$f_n(\mathbf{k}) = f_0(\varepsilon_{n\mathbf{k}}) - \hbar(\mathbf{v}_n \cdot \delta \mathbf{k})(-\partial_\varepsilon f_0)_{\varepsilon_{n\mathbf{k}}}, \quad \delta \mathbf{k} = \frac{e\tau_{\text{tr}}}{\hbar} \mathbf{E}. \quad (90)$$

Substituting Eq. (90) into Eq. (89) and keeping only terms linear in $\mathbf{E}$ yields the electron wind force,

$$\langle F_X \rangle = -eE \tau_{\text{tr}} N(E_F) \langle V(\mathbf{k}) v_x(\mathbf{k}) \rangle_{\text{FS}}, \quad (91)$$

where $N(E_F)$ is the density of states per spin at the Fermi level. Dividing by the Burgers vector gives the configurational wind stress,

$$\tau_e = \frac{\langle F_X \rangle}{b} = -\frac{eE \tau_{\text{tr}}}{b} N(E_F) \langle V(\mathbf{k}) v_x(\mathbf{k}) \rangle_{\text{FS}}. \quad (92)$$

Eliminating the field in favour of the macroscopic current using the Drude conductivity computed within the same conserving approximation,

$$J = \sigma E, \quad \sigma = e^2 \tau_{\text{tr}} N(E_F) \langle v_x^2 \rangle_{\text{FS}},$$

gives the current-driven form,

$$\tau_e = \eta_1 \frac{J}{b} = -\frac{J}{be} \frac{\langle V v_x \rangle_{\text{FS}}}{\langle v_x^2 \rangle_{\text{FS}}}, \quad (93)$$

where

$$\eta_1 = -\frac{1}{e} \frac{\langle V(\mathbf{k}) v_x(\mathbf{k}) \rangle_{\text{FS}}}{\langle v_x^2(\mathbf{k}) \rangle_{\text{FS}}}$$

is defined for computational convenience. Equation (93) is the quasistatic limit of the mixed contour kernel χ_{XX}^R at $\omega = 0$ and $q_x \rightarrow 0$. All refinements in later sections, including thermal corrections, field-induced anisotropies, and magnetic modifications, are obtained as controlled variations of Eq. (93).

The electron wind stress τ_e represents the stress equivalent driving term associated with the steady state electronic wind force per unit dislocation length. Eliminating E in favour of the current through the Kubo conductivity computed in the same conserving ladder ensures that the self-energy corrections entering τ_{tr} and the vertex renormalization cancel exactly, as required by the Ward identity linking charge conservation and momentum transfer. The resulting expression Eq. (93) is therefore the fully Keldysh consistent, current controlled form of the wind stress.

Equation (93) corresponds to the $\omega = 0$, $q_x \rightarrow 0$ limit of the mixed stress-stress response function

$$\begin{aligned} \Pi_{FX}^<(\omega, q_x) &= -i \sum_n \int d^3k d\varepsilon V_n(\mathbf{k}) \\ &\times G_n^R(\varepsilon + \omega, \mathbf{k} + q_x) V_n(\mathbf{k}) G_n^<(\varepsilon, \mathbf{k}), \end{aligned} \quad (94)$$

whose static component gives the configurational force, while the velocity-dependent and dissipative terms that appear in Sec. IV B 2 correspond, respectively, to $\text{Re } \Pi^R$ and $\text{Im } \Pi^R$ at finite Doppler frequency.

The electron wind stress derived in Eq. (93) represents the configurational component of the momentum flux transferred from the drifting electronic fluid to the dislocation core. In microscopic terms, an applied current density $\mathbf{J}$ slightly displaces the Fermi distribution in $\mathbf{k}$ -space, producing a net imbalance in quasiparticle momentum. Because the dislocation distorts the lattice locally through the Kanzaki field, electrons experience a position-dependent potential $V(\mathbf{r})$ that scatters them anisotropically. The statistical average of this scattering asymmetry gives rise to a steady force on the defect. This is the electron wind: a mechanical stress of electronic origin acting on the dislocation's configuration rather than on the lattice as a whole.

Equation (93) is linear in the applied current and odd under $\mathbf{J} \rightarrow -\mathbf{J}$. Reversing the current reverses the direction of the induced shear τ_e and thus the Peach-Koehler force on every segment of the dislocation network. In contrast to dissipative drag coefficients, which depend on even powers of velocity or field, the electron wind is a *configurational drive*: it breaks microscopic time-reversal symmetry through the nonequilibrium steady state distribution but does not dissipate energy. In that sense, τ_e plays a role analogous to the Lorentz force in electron transport, entering the dislocation kinetics as an anti-symmetric term in the configurational equations of motion.

Although the total force per unit length is $f_{\text{PK}} = b \tau_e$, the resolved shear τ_e itself is independent of the sign of the Burgers vector or the line direction. This follows because the Kanzaki vertex $V(\mathbf{k})$ is proportional to b and changes sign together with it, while the definition of the resolved shear divides by the same b . Consequently, a 180° rotation of the crystal about the dislocation line, which reverses the sense of the Burgers vector and the slip-plane normal simultaneously, leaves τ_e invariant. The electron wind, therefore, constitutes a genuine material shear field that couples identically to positive and negative dislocations. What distinguishes the two signs is not the electronic stress but the resulting direction of glide under that stress: dislocations of opposite sign move in opposite directions on the same slip system, contributing additively to the macroscopic plastic shear according to Orowan's relation, $\dot{\gamma} = b \rho_m \bar{v}$.

The underlying cause of the electron wind lies in the momentum conservation between electrons and the ionic lattice. In a uniform conductor, the electronic momentum gained from the electric field is dissipated through phonon and impurity scattering, maintaining a steady current. In the presence of a dislocation, the lattice distortion itself provides a nonuniform scattering center that can exchange momentum with the drifted electronic cloud. The resulting imbalance in momentum flux across the core is equivalent to a shear stress acting on the dislocation line. This effect is entirely elastic at the microscopic level (no net energy is dissipated unless the dislocation moves) and thus τ_e enters as a conservative configurational force in the dislocation's equation of motion.

A frequent misconception, traceable to early continuum treatments, is that the electron wind force cannot produce macroscopic plastic strain because it would translate

dislocation loops without expanding them. That conclusion follows only in the limit of an isolated, perfectly symmetric loop in a spatially uniform τ_e . In any real crystal the local Peach-Koehler force includes contributions from both the electron wind and the heterogeneous internal stress field created by other dislocations and external loads,

$$\mathbf{f}_{\text{PK}}(\mathbf{r}) = [\boldsymbol{\sigma}_{\text{int}}(\mathbf{r}) + \boldsymbol{\sigma}_{\text{appl}}(\mathbf{r}) + \boldsymbol{\sigma}_e] \cdot \mathbf{b} \times \mathbf{t}(\mathbf{r}).$$

Because $\boldsymbol{\sigma}_{\text{int}}$ varies along each line, the superposed $\boldsymbol{\sigma}_e$ alters the local resolved shear differently on opposite sides of a loop, breaking the perfect symmetry assumed in the classical argument. The current-induced stress thereby changes the local glide velocity and the net plastic strain rate $\dot{\gamma} = \rho b v (\tau_{\text{appl}} + \tau_{\text{int}} + \tau_e)$. Hence the electron wind bias modifies the evolution of the entire dislocation network, accelerating motion in one subset of segments and retarding it in another. Even when its magnitude is only a few MPa, this bias can measurably lower the effective flow stress in high-current experiments—the essential phenomenon of electroplastic softening.

The appearance of τ_e in the configurational balance laws places electroplasticity on the same microscopic footing as electronic transport: both arise from the same Keldysh stress-stress response function evaluated in different limits. The electron wind is therefore not an ad hoc body force but a predictive consequence of the coupling between quasiparticle momentum and defect-induced strain. Its invariance under sign reversal of the Burgers vector, its oddness in current, and its insensitivity to temperature under fixed current conditions all follow naturally from this microscopic structure. Within that framework the “electron wind” is the lowest order nonequilibrium correction to the elastic energy functional, establishing a quantitative bridge between quantum electronic response and mesoscale dislocation dynamics.

All results in this subsection thus follow from a single contour expression for $\langle F_X \rangle$ evaluated with different analytic continuations: the quasistatic wind stress here corresponds to the zero-frequency limit of the Keldysh stress-stress correlator, while the next subsections extend the same object to finite $\omega = q_x v_d$ and finite temperature. It is useful to insert representative metallic parameters into Eq. (93). For a representative current density $J = 10^9 \text{ A m}^{-2}$ and a Burgers vector $b \approx 2.9 \times 10^{-10} \text{ m}$ (fcc Al), the scale $J/b \approx 3.4 \text{ MPa}$ so that $\tau_e \approx \eta_1 \times 3.4 \text{ MPa}$. Even for $|\eta_1| \sim 0.1$ this yields a few tenths of a MPa—small compared to typical Peierls stresses, but potentially large enough to measurably bias glide when the lattice is already flowing.

The quasistatic reduction used here exploits the hierarchy $\omega_{\text{glide}} \ll \gamma_{\text{el}}$, so the electronic response is sampled only in the low-frequency limit relevant for the mobility law. Quantitatively, ω_{glide} is bounded by $\omega_{\text{glide}} \sim v_d/b$; even for $v_d \sim 10^3 \text{ m s}^{-1}$ and $b \sim 2.5 \times 10^{-10} \text{ m}$ one has $\omega_{\text{glide}} \sim 4 \times 10^{12} \text{ s}^{-1}$. By contrast, typical room-temperature transport times $\tau_{\text{tr}} \sim 10^{-14} \text{ s}$ imply $\gamma_{\text{el}} \sim \tau_{\text{tr}}^{-1} \sim 10^{14} \text{ s}^{-1}$, hence $\omega_{\text{glide}}/\gamma_{\text{el}} \sim 10^{-2}$ even in this extreme limit. Outside this adiabatic regime, meaning drives with significant spectral weight on femtosecond scales comparable to τ_{tr} , one retains the full ω dependence of the electronic kernel and couples it to the time-dependent $f_n(\mathbf{k}, t)$ within the same Keldysh framework, without altering the structure of the influence functional.

Current modified electronic drag. A uniform current modifies the electronic drag coefficient through the same kernel that defines the equilibrium viscosity. The current dependence of the electronic drag follows from the same long-wavelength kernel introduced in Eq. (67), since the viscous back-reaction is determined entirely by the low-frequency slope of its imaginary part $\mathcal{B}_{el} = -[\partial_\omega \text{Im } \mathcal{D}_{el}^R(\omega)]_{\omega \rightarrow 0}$ given in Eq. (72). The structure of the kernel is unchanged. This is because all geometric information is contained in the Kanzaki line vertex $\Lambda_i(\mathbf{q})$, while the electronic part enters through the stress correlator $\Pi_{ij}^R(\mathbf{q}, \omega)$, which we shall evaluate with the nonequilibrium distribution.

The imaginary part of the stress bubble has, in the quasiparticle representation developed in Sec. III,

$$\text{Im } \Pi_{ij}^R(\mathbf{q}, \omega) = \omega \int \frac{d^3k}{(2\pi)^3} v_i(\mathbf{k}) v_j(\mathbf{k}) \mathcal{W}(\varepsilon_{\mathbf{k}}), \quad (95)$$

where all dependence on the electronic distribution enters through

$$\mathcal{W}(\varepsilon_{\mathbf{k}}) = \int d\varepsilon A^2(\varepsilon, \mathbf{k}) [-\partial_\varepsilon f(\varepsilon)].$$

The spectral width entering $A(\varepsilon, \mathbf{k})$ is the quasiparticle broadening already contained in the retarded Green function through the imaginary part of the electronic self energy,

$$G_n^R(\varepsilon, \mathbf{k}) = \frac{1}{\varepsilon - \varepsilon_{n\mathbf{k}} + i \text{Im } \Sigma^R(\varepsilon, \mathbf{k})}.$$

Writing $\gamma_{\mathbf{k}}(\varepsilon) = -2 \text{Im } \Sigma^R$ gives the standard Lorentzian spectral function

$$A(\varepsilon, \mathbf{k}) = \frac{1}{\pi} \frac{\gamma_{\mathbf{k}}(\varepsilon)/2}{(\varepsilon - \varepsilon_{\mathbf{k}})^2 + [\gamma_{\mathbf{k}}(\varepsilon)/2]^2}.$$

This width is not an auxiliary prescription but an intrinsic part of the Keldysh propagator. Indeed, the Keldysh component is $G^K = A(1 - 2f)$, and therefore any quantity involving $G^<$ or G^K necessarily carries the same $A(\varepsilon, \mathbf{k})$ determined by the microscopic damping. In the present calculation, the current-dependent terms introduce higher energy derivatives of the Fermi function. These derivatives must act on the combination $A^2(\varepsilon, \mathbf{k})$ that appears inside the stress bubble. A finite spectral width is essential to make these manipulations well defined, since A^2 becomes a smooth function of energy only when $\gamma_{\mathbf{k}} > 0$, as dictated by the self energy. In this sense the Lorentzian regularizes the energy integrals in a manner that follows directly from the analytic structure of the Keldysh Green functions rather than from any additional assumption.

Using the drifted steady state distribution Eq. (84) gives

$$\begin{aligned} -\partial_\varepsilon f(\varepsilon_{\mathbf{k}}) &= -\partial_\varepsilon f_0(\varepsilon_{\mathbf{k}}) + (\mathbf{v}_{\mathbf{k}} \cdot \delta \mathbf{k}) \partial_\varepsilon^2 f_0(\varepsilon_{\mathbf{k}}) \\ &\quad - \frac{1}{2} (\mathbf{v}_{\mathbf{k}} \cdot \delta \mathbf{k})^2 \partial_\varepsilon^3 f_0(\varepsilon_{\mathbf{k}}) + O(E^3), \end{aligned} \quad (96)$$

with $\delta \mathbf{k} = e\tau_v \mathbf{E}/\hbar$. The linear term vanishes in a centrosymmetric metal (as every integrand that is odd under $\mathbf{k} \rightarrow -\mathbf{k}$ vanishes when integrated over the Brillouin zone), so the leading current dependence is quadratic. Inserting this form into Eq. (95) and integrating by parts with respect to energy transfers the derivatives of f_0 onto the smooth Lorentzian A^2 . The resulting expression is controlled by a narrow energy window of width set by the quasiparticle damping and

is finite in the limit in which the width tends to zero. To express the result in terms of the current density rather than the electric field it is convenient to use the Drude relation, obtained in the same RTA. For a uniform field one has $J = \sigma E$ with $\sigma = e^2 \tau_v N(E_F) \langle v_x^2 \rangle_{\text{FS}}$ as in Sec. IV B, while the drift vector $\delta \mathbf{k} = e\tau_v \mathbf{E}/\hbar$ appearing in the distribution implies $(\mathbf{v}_{\mathbf{k}} \cdot \delta \mathbf{k})^2 \propto E^2$. Combining these two relations allows one to rewrite the quadratic field dependence in terms of $(J/J_0)^2$, where the characteristic current scale $J_0 = nev_F$ is defined up to numerical factors that are absorbed into the dimensionless coefficient $\eta_2(\hat{\mathbf{J}})$.

Performing these steps and inserting the result into Eq. (72) leads to the

$$\frac{\mathcal{B}_{el}(J)}{\mathcal{B}_{el}(0)} = 1 - \eta_2(\hat{\mathbf{J}}) \left(\frac{J}{J_0} \right)^2 + O((J/J_0)^4), \quad (97)$$

where

$$\eta_2(\hat{\mathbf{J}}) = \frac{\langle V^2(\mathbf{k}) (\hat{\mathbf{J}} \cdot \mathbf{v}_{\mathbf{k}})^2 / v_F^3(\mathbf{k}) \rangle_{\text{FS}}}{3 \langle V^2(\mathbf{k}) / v_F(\mathbf{k}) \rangle_{\text{FS}}}, \quad J_0 = nev_F. \quad (98)$$

For an isotropic Fermi surface $\eta_2 = 1/3$ [81]. The negative sign reflects the physical reduction of the phase space available for dissipative electron-hole transitions when the Fermi surface is drifted.

Quantitatively, this correction is small. For simple metals with $n \sim 10^{29} \text{ m}^{-3}$ and $v_F \sim 10^6 \text{ m s}^{-1}$, the characteristic current scale $J_0 = nev_F \sim 10^{16} \text{ A m}^{-2}$. Even for laboratory currents of $J \sim 10^{12} \text{ A m}^{-2}$, the relative change $(J/J_0)^2$ is below 10^{-10} . The intrinsic current-induced reduction of the electronic drag is thus negligible under accessible conditions. The principal current-driven effect remains the configurational electron wind stress derived in Eq. (93), while $\mathcal{B}_{el}(J)$ may safely be replaced by its equilibrium value in practical modeling.

2. Velocity dependence of the electron wind and the electronic drag

The quasistatic treatment used in Sec. IV B 1 above evaluates the lattice response in the $\omega \rightarrow 0$ limit and assumes a stationary dislocation. This approximation remains valid at low glide velocities, but as the dislocation approaches the speeds of sound its strain field contracts along the direction of motion [82–85]. Such contraction modifies the local elastic field that couples to the electrons, thereby invalidating the separation between the electronic and kinematic parts implicit in Eq. (93). The effective electron-dislocation interaction must then be corrected to account for the elastodynamic amplification of the strain, encoded in the retarded lattice Green function $D^R(\mathbf{q}, \omega = q_x v_d)$, which enhances the slip vertex that enters the electronic kernel even though the electronic states themselves remain at zero frequency. Here we show that this coupling acts only to renormalize the strain vertex $V_n(\mathbf{k})$, leaving the structure of the Keldysh electronic block unchanged, and that the resulting correction can be represented by a velocity-dependent vertex function. This analysis therefore requires a careful use of the limit procedures we have so far performed for $q_x \rightarrow 0$, $\omega \rightarrow 0$ for the electronic kernel, as they may appear to conflict with the finite velocity Doppler contraction $\omega = v_d q_x$ of the strain field.

For a straight dislocation with $X(t) = v_d t$, the screened Kanzaki vertex entering the electronic kernel becomes

$$\Lambda_i(\mathbf{q}; v_d) = D_{ij}^R(\mathbf{q}, \omega = q_x v_d) F_j^K(\mathbf{q}), \quad (99)$$

$$\Lambda_i^{(0)}(\mathbf{q}) = D_{ij}^R(\mathbf{q}, 0) F_j^K(\mathbf{q}), \quad (100)$$

and the ratio $\Lambda_i(\mathbf{q}; v_d)/\Lambda_i^{(0)}(\mathbf{q})$ reduces, in the long-wavelength limit, to the purely kinematic amplification factor familiar from the Frank-Eshelby solution in elastodynamics [82,83],

$$\Lambda_i(\mathbf{q}; v_d) = \mathcal{A}_\lambda(\beta, \theta) \Lambda_i^{(0)}(\mathbf{q}), \quad \beta = v_d/c_\lambda(\hat{\mathbf{q}}),$$

$$\mathcal{A}_\lambda(\beta, \theta) = \frac{1}{\sqrt{1 - \beta^2 \cos^2 \theta}} + O(qa_{\text{core}}), \quad (101)$$

with θ the angle between $\mathbf{q}$ and the glide direction. This factor originates entirely from the lattice dynamics and survives the $q_x \rightarrow 0$ reduction that defines the long-wavelength electronic kernel, because the Doppler shift is taken inside D^R before the electronic limit $\omega \rightarrow 0$ is applied.

After inserting Eq. (101) into the electronic part of the influence kernel, the wind and drag inherit a simple multiplicative renormalization. Writing the angular and branch average of $\mathcal{A}_\lambda(\beta, \theta)$ with the same weight that defines the static vertex as $\mathcal{A}(v_d)$, one obtains

$$\tau_e(J, v_d) = \mathcal{A}(v_d) \tau_e(J, 0), \quad (102)$$

$$B_{el}(v_d) = \mathcal{A}^2(v_d) B_{el}(0), \quad (103)$$

with $\tau_e(J, 0)$ and $B_{el}(0)$ the quasi static expressions derived in Eq. (97). The linear amplification of the wind and the quadratic amplification of the drag follow directly from the single and double insertions of the vertex in the corresponding kernels.

For small velocities, a Taylor expansion of the Frank-Eshelby factor gives

$$\mathcal{A}(v_d) = 1 + \alpha \frac{v_d^2}{c_T^2} + O(v_d^4/c_T^4), \quad (104)$$

where

$$\alpha = \frac{\sum_\lambda \int \frac{d\Omega_{\hat{\mathbf{q}}}}{4\pi} |e^\lambda(\hat{\mathbf{q}}) \cdot \mathbf{F}^{\text{K,ac}}(\hat{\mathbf{q}})|^2 \frac{1}{2} \cos^2 \theta}{\sum_\lambda \int \frac{d\Omega_{\hat{\mathbf{q}}}}{4\pi} |e^\lambda(\hat{\mathbf{q}}) \cdot \mathbf{F}^{\text{K,ac}}(\hat{\mathbf{q}})|^2},$$

so the numerical value of α is fixed entirely by the slip geometry through the same core-projected weighting that enters the static Kanzaki vertex.

Near transonic conditions the amplification factor Eq. (104) ceases to be reliable. The divergence of the Frank-Eshelby expression arises from the assumption $\omega_\lambda(\mathbf{q}) = c_\lambda(\hat{\mathbf{q}})q$ for c_λ the speed of sound in the relevant branch, which is valid only in the long-wavelength limit. This is the source of the "sound barrier" effects that have been discussed at length in studies of the linear elastodynamics of high speed dislocations [86], and repeatedly disproven first in harmonic lattice models [87], then molecular dynamics simulations [88], and finally experimentally [89]. The physical argument against this elastodynamic sound barrier is simple: once the Doppler frequency $q_x v_d$ approaches the regime where the true lattice dispersion curves away from

its linear form, the continuum approximation is no longer adequate and the strain amplification inherited from the propagator must be regularized by the discrete spectrum.

The Kanzaki vertex couples to the displacement field through $\Lambda_i = D_{ij}^R f_j^K$, so the velocity dependence must be introduced at the level of the Green function itself. With the exact branch dispersion $\omega_\lambda(\mathbf{q})$ one has

$$\mathcal{A}_\lambda(v_d; \mathbf{q}) = \frac{D_\lambda^R(\mathbf{q}, \omega = q_x v_d)}{D_\lambda^R(\mathbf{q}, 0)} = \frac{\omega_\lambda^2(\mathbf{q})}{\omega_\lambda^2(\mathbf{q}) - (q_x v_d)^2},$$

which indeed reduces to the Frank-Eshelby factor in the acoustic limit $\omega_\lambda = c_\lambda q$. When $q_x v_d$ samples the part of the Brillouin zone where the dispersion departs from linearity, one may expand

$$\omega_\lambda(\mathbf{q}) = c_\lambda(\hat{\mathbf{q}})q + \gamma_\lambda(\hat{\mathbf{q}})q^3 + O(q^5),$$

which captures the leading lattice correction along a given direction. Inserting this into the ratio above yields the lattice regularized amplification,

$$\mathcal{A}_\lambda(v_d; \mathbf{q}) \simeq \frac{1 + 2\gamma_\lambda(\hat{\mathbf{q}})q^2/c_\lambda(\hat{\mathbf{q}})}{1 - \beta^2 \cos^2 \theta + 2\gamma_\lambda(\hat{\mathbf{q}})q^2/c_\lambda(\hat{\mathbf{q}})},$$

$$\beta = \frac{v_d}{c_\lambda(\hat{\mathbf{q}})},$$

which replaces the continuum form once $q_x v_d$ approaches $\omega_\lambda^{\text{max}}$ on the branch. The term proportional to q^2 prevents the artificial divergence present in the long-wavelength expression and enforces a velocity-induced amplification-induced contraction once the Doppler frequency nears the band maximum. Since the Kanzaki vertex enters linearly in the electronic kernel, the corresponding velocity dependence of the electron wind inherits this regularized amplification, while the electronic drag carries its square.

3. Finite-temperature corrections

Electronic wind. The finite-temperature behavior of the electron wind follows directly from the finite-temperature form of the collision integral already introduced in the Keldysh kinetic equation of Sec. IV B 1. Once the electronic propagators are expressed in terms of the nonequilibrium distribution $f = f_0 - \tau_{\text{tr}} eE v_x \partial_\varepsilon f_0$, the steady configurational force reduces, as in Eq. (88), to the Fermi surface average

$$\langle F_X \rangle = \int \frac{d^3k}{(2\pi)^3} V(\mathbf{k}) f(\mathbf{k}),$$

so all temperature dependence enters through $\tau_{\text{tr}}(T)$ and the thermal broadening of $-\partial_\varepsilon f_0$.

The relaxation time $\tau_{\text{tr}}(T)$ follows from the same collision integral that enters the kinetic Eq. (83). Its temperature dependence is fixed by the structure of the particle-hole phase space appearing in the Keldysh convolution of the two electronic lines. For a quasiparticle of energy $\varepsilon_{\mathbf{k}}$ the relevant part of the collision integral contains

$$\int d\omega A(\varepsilon_{\mathbf{k}} - \omega, \mathbf{k}') A(\varepsilon_{\mathbf{k}}, \mathbf{k}) [n_B(\omega) + f_0(\varepsilon_{\mathbf{k}} - \omega)(1 - f_0(\varepsilon_{\mathbf{k}}))].$$

At low temperature, both spectral functions restrict the energy transfer to $|\omega| \lesssim k_B T$, while the combination of Fermi and Bose weights behaves as $\omega/(1 - e^{-\beta\omega}) \sim \omega$ in this window.

The collision integral therefore contains an energy integral of the form

$$\int_0^{k_B T} d\omega \omega^2,$$

the first factor of ω coming from the statistical weights and the second from the fact that only energies within ω of the Fermi surface participate. This gives a contribution proportional to T^3 , and the remaining angular integration over momentum transfer suppresses forward scattering through the factor $(1 - \cos \theta) \sim q^2$. Since $q \sim \omega/v_F$ in the long-wavelength limit appropriate for Fermi liquids, this produces an additional factor of ω^2 , so that the full transport rate scales as

$$\tau_{tr}^{-1}(T) \propto \int_0^{k_B T} d\omega \omega^2 \left(\frac{\omega}{v_F} \right)^2 \propto T^2.$$

Thus, the familiar quadratic temperature law of Fermi liquid transport arises directly from the ω dependence of the Keldysh collision kernel and the phase space available for small particle-hole excitations near the Fermi surface.

Substituting the drifted distribution into the force expression yields, to linear order in the field

$$\langle F_X \rangle = -eE \int \frac{d^3 k}{(2\pi)^3} V(\mathbf{k}) v_x(\mathbf{k}) \tau_{tr}(\varepsilon_{\mathbf{k}}, T) [-\partial_\varepsilon f_0(\varepsilon_{\mathbf{k}}, T)],$$

and therefore the wind stress at fixed electric field is

$$\tau_e(E, T) = -\frac{eE}{b} \int \frac{d^3 k}{4\pi^3} V(\mathbf{k}) v_x(\mathbf{k}) \tau_{tr}(\varepsilon_{\mathbf{k}}, T) [-\partial_\varepsilon f_0(\varepsilon_{\mathbf{k}}, T)]. \quad (105)$$

Since $\tau_{tr} \propto T^{-2}$ while $-\partial_\varepsilon f_0 \propto T^{-1}$ after rescaling energies by $k_B T$, the overall amplitude $\tau_e(E, T)$ decreases rapidly with temperature at fixed E : the driven anisotropy of the distribution cannot be maintained once inelastic broadening becomes large.

In experimental situations, the current density J is the controlled variable. In this case, the field is $E = J/\sigma(T)$, where the conductivity $\sigma(T) = e^2 \int d^3 k / (4\pi^3) v_x^2 \tau_{tr} [-\partial_\varepsilon f_0]$ contains precisely the same transport time τ_{tr} appearing in Eq. (105). Inserting $E = J/\sigma(T)$ into the force expression eliminates all explicit factors of τ_{tr} , leaving

$$\tau_e(J, T) = -\frac{J}{be} \frac{\int d\varepsilon [-\partial_\varepsilon f_0(\varepsilon, T)] N(\varepsilon) \langle V v_x \rangle_\varepsilon}{\int d\varepsilon [-\partial_\varepsilon f_0(\varepsilon, T)] N(\varepsilon) \langle v_x^2 \rangle_\varepsilon}. \quad (106)$$

The only remaining temperature dependence lies in the mild broadening of $-\partial_\varepsilon f_0$, which produces a correction of order $(k_B T/E_F)^2$ and is negligible for metallic Fermi energies. Thus, at fixed current the electron wind stress is essentially temperature independent.

Electronic drag. Indeed, temperature affects the electron wind and electronic drag through fundamentally different mechanisms, a distinction that emerges naturally from our Keldysh formalism. The electron wind represents a steady momentum transfer from current carrying electrons to the dislocation (a first order response proportional to the applied current). The electronic drag, conversely, arises from the dislocation's motion exciting particle-hole pairs near the Fermi surface, an inherently dissipative process controlled by the available phase space for such excitations. At low

temperatures, this phase space is restricted by Pauli blocking, leading to the characteristic T^2 Fermi liquid scaling for electronic drag [90]. Meanwhile, the wind force exhibits a surprising temperature independence at fixed current density, arising from a precise cancellation between the temperature-dependent transport relaxation time and conductivity.

Here we examine how the fact the drag arises from the dissipative part of the retarded response explains its temperature dependence. We start from the stress kernel defined in Sec. III, Eq. (77), and recall that the drag coefficient is determined by its low-frequency slope, Eq. (72). Thus, only the imaginary part of the electronic bubble $\Pi_{ij}^R(\mathbf{q}, \omega)$ contributes. On the Keldysh contour this bubble has the standard structure we gave in Eq. (79), which expresses the retarded bubble as a difference of $G^>G^<$ and $G^<G^>$ contractions. To extract its imaginary part in the regime $\mathbf{q} \rightarrow 0$ and $\omega \ll E_F$ one inserts the spectral forms of the contour propagators,

$$G^< = i f A, \quad G^> = -i(1 - f)A,$$

with A the single particle spectral function defined earlier in Eq. (37). After substituting these into Eq. (79) and taking the limit $\mathbf{q} \rightarrow 0$, the imaginary part of the bubble reduces to a particle-hole phase space factor,

$$\text{Im } \Pi_{ij}^R(\mathbf{q} \rightarrow 0, \omega) = -\pi \int \frac{d^3 k}{(2\pi)^3} \Gamma_i(\mathbf{k}) \Gamma_j(\mathbf{k}) \omega \rho_{ph}(\mathbf{k}, \omega, T), \quad (107)$$

where

$$\rho_{ph}(\mathbf{k}, \omega, T) = \int d\varepsilon A(\varepsilon, \mathbf{k}) A(\varepsilon + \omega, \mathbf{k}) [f(\varepsilon) - f(\varepsilon + \omega)].$$

This quantity expresses the number of electron-hole pairs with energy ω that can be created near $\mathbf{k}$.

For a normal Fermi liquid the spectral functions are sharply peaked at the quasiparticle pole, so expanding $f(\varepsilon + \omega) - f(\varepsilon)$ for small ω yields

$$\rho_{ph}(\mathbf{k}, \omega, T) = \frac{1}{v_F(\mathbf{k})} \frac{\omega^2 + (\pi k_B T)^2}{E_F^2} \quad (\omega, T \ll E_F).$$

This form [91] is universal, as it reflects nothing more than the curvature of the Fermi surface and the restriction that only quasiparticles within $k_B T$ of the Fermi level participate in low-energy excitations.

Substituting this form into Eq. (107) and taking the frequency derivative at $\omega=0$ gives

$$\partial_\omega \text{Im } \Pi_{ij}^R|_{\omega=0} = -\pi \int \frac{d^3 k}{(2\pi)^3} \Gamma_i(\mathbf{k}) \Gamma_j(\mathbf{k}) \frac{(\pi k_B T)^2}{E_F^2 v_F(\mathbf{k})}.$$

Contracting with the Kanzaki vertex as in Eq. (B5) then yields the finite-temperature drag coefficient,

$$B_{el}(T) = \frac{\pi^3}{2} N(E_F) \left\langle \frac{V^2(\mathbf{k})}{v_F(\mathbf{k})} \right\rangle_{FS} \left(\frac{k_B T}{E_F} \right)^2, \quad (108)$$

which is the characteristic T^2 law of a Fermi liquid. It originates entirely from the particle-hole phase space and not from the quasiparticle lifetime, which enters only through the smooth regularization of the spectral functions. In Sec. VB we discuss the practical implications of this thermal dependence, which we must emphasize is merely local.

Equation (108) thus complements the current controlled result for the electron wind in Eq. (106): the wind remains essentially temperature independent at fixed current, whereas the viscous damping grows, locally, as T^2 .

C. Magnetic field modification

We now address the effect of a static magnetic field on the two electronic contributions that couple the dislocation to the electronic sector. The configurational electron wind is obtained from the stationary expectation value $\langle F_X \rangle$ in the current driven steady state, while the electronic drag is controlled by the low-frequency slope of the same retarded stress kernel $\mathcal{D}_{el}^R(\omega)$ that we gave in Sec. III. Throughout this subsection, we work in the semiclassical regime $\omega \rightarrow 0$ with $\omega_c \tau_1 \lesssim 1$, keep the Kanzaki geometry and static screening fixed through the line vertex $\Lambda_i(\mathbf{q})$ defined earlier, and use a conserving ladder so that the Ward identities linking self energies and vertices remain satisfied.

1. Electron wind at fixed current

The microscopic force operator conjugate to the glide coordinate is $F_X = \sum_{\mathbf{k}} V(\mathbf{k}) c_{\mathbf{k}}^\dagger c_{\mathbf{k}}$, with $V(\mathbf{k})$ the screened acoustic strain vertex obtained from the Kanzaki vertex in Sec. II C. The steady configurational wind follows from $\langle F_X \rangle$ evaluated with the nonequilibrium distribution in the presence of a uniform electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$.

In the presence of a static electric field $\mathbf{E}$ and a transverse magnetic field $\mathbf{B}$, the stationary Keldysh kinetic equation acquires an additional term describing the Lorentz rotation of quasiparticle momenta. The Wigner-space form of the contour equation,

$$\mathbf{v} \cdot \nabla_{\mathbf{r}} f + e \mathbf{E} \cdot \nabla_{\mathbf{k}} f + e(\mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{k}} f = \mathcal{I}_{\text{coll}}[f],$$

reduces in a spatially uniform steady state to a balance between the electric drive, the collision integral, and the magnetic precession of the momentum direction. Linearizing around f_0 and projecting onto the Fermi surface gives

$$(\hat{\mathcal{C}} + \omega_c \hat{\mathcal{L}}_B) \delta f = e \mathbf{E} \cdot \mathbf{v} (-\partial_{\varepsilon} f_0), \quad \omega_c = \frac{eB}{m^*}, \quad (109)$$

where $\hat{\mathcal{C}}$ is the elastic collision operator on the Fermi surface and $\hat{\mathcal{L}}_B$ generates the rigid rotation of $\hat{\mathbf{k}}$ about $\hat{\mathbf{B}}$. The source term on the right hand side is dipolar in momentum space, so only the $\ell = 1$ angular harmonic contributes. It is convenient to expand $\delta \hat{f}$ into spherical harmonics,

$$\delta f(\hat{\mathbf{k}}) = \sum_{m=-1}^1 \psi_{1m} Y_{1m}(\hat{\mathbf{k}}),$$

with the quantization axis along $\hat{\mathbf{B}}$. In this basis, the Lorentz operator and the collision operator act diagonally on the $\ell = 1$ sector,

$$\hat{\mathcal{L}}_B Y_{1m} = im Y_{1m}, \quad \hat{\mathcal{C}} Y_{1m} = \tau_1^{-1} Y_{1m},$$

where τ_1 is the transport time associated with the dipolar harmonic (the same transport time that appeared in the relaxation

time approximation in Sec. IV B 1). Equation (109) then gives

$$\psi_{1m} = \frac{S_{1m}}{\tau_1^{-1} + im\omega_c}, \quad (110)$$

with S_{1m} the components of the dipolar source proportional to the Cartesian components of $\mathbf{E}$.

Reassembling the harmonics gives the standard vector form

$$\delta f(\mathbf{k}) = e\tau_1 \frac{\mathbf{v} \cdot (\mathbf{E} + \tau_1 \omega_c \hat{\mathbf{B}} \times \mathbf{E})}{1 + (\omega_c \tau_1)^2} (-\partial_{\varepsilon} f_0)_{\varepsilon_{\mathbf{k}}}, \quad (111)$$

which displays explicitly the Hall rotation of the nonequilibrium dipole in momentum space. Substituting this into the expression for the configurational force yields

$$\tau_e(\mathbf{E}, \mathbf{B}) = \frac{e}{b} \int \frac{d^3 k}{4\pi^3} V(\mathbf{k}) \mathbf{v} \cdot \frac{\tau_1 (\mathbf{E} + \tau_1 \omega_c \hat{\mathbf{B}} \times \mathbf{E})}{1 + (\omega_c \tau_1)^2} (-\partial_{\varepsilon} f_0), \quad (112)$$

where division by b converts the force per unit length into an effective configurational shear stress.

The electric current obtained from the same nonequilibrium distribution has the form

$$\begin{aligned} \mathbf{J}(\mathbf{E}, \mathbf{B}) &= e^2 \int \frac{d^3 k}{4\pi^3} \mathbf{v} \mathbf{v} \cdot \frac{\tau_1 (\mathbf{E} + \tau_1 \omega_c \hat{\mathbf{B}} \times \mathbf{E})}{1 + (\omega_c \tau_1)^2} (-\partial_{\varepsilon} f_0) \\ &= \sigma(B) \cdot \mathbf{E}, \end{aligned} \quad (113)$$

which defines the magnetoconductivity tensor $\sigma(B)$. For a general band structure and Kanzaki vertex the tensor $\sigma(B)$ and the rank two object

$$\mathbf{K}(B) = e^2 \int \frac{d^3 k}{4\pi^3} V(\mathbf{k}) \mathbf{v} \mathbf{v} \frac{\tau_1}{1 + (\omega_c \tau_1)^2} (-\partial_{\varepsilon} f_0) \quad (114)$$

differ by anisotropic weightings of the Fermi surface. Expressing the wind stress at fixed current in terms of the resistivity tensor $\rho(B) = \sigma^{-1}(B)$ gives

$$\tau_e(\mathbf{J}, \mathbf{B}) = \frac{1}{be} \mathbf{K}(B) : \rho(B) \cdot \mathbf{J}, \quad (115)$$

which is the general Keldysh form of the wind at fixed current in a magnetic field.

In the special case of an isotropic metal with a spherical Fermi surface and an angle independent scalar vertex $V(\mathbf{k}) \rightarrow V_{\text{FS}}$, the same angular averages appear in $\mathbf{K}(B)$ and $\sigma(B)$ so that

$$\mathbf{K}(B) = \frac{V_{\text{FS}}}{e} \sigma(B).$$

Inserting this relation into Eq. (115) leads to the exact cancellation of all magnetic denominators and Hall rotations,

$$\tau_e(\mathbf{J}, \mathbf{B}) = -\frac{V_{\text{FS}}}{be} \frac{\mathbf{J} \cdot \hat{\mathbf{x}}}{\langle v_x^2 \rangle_{\text{FS}}} \equiv \eta_1 \frac{J_x}{b}, \quad \eta_1 = -\frac{\langle V v_x \rangle_{\text{FS}}}{e \langle v_x^2 \rangle_{\text{FS}}}, \quad (116)$$

where $\hat{\mathbf{x}}$ denotes the glide direction. In this isotropic and conserving limit the wind stress at fixed current is therefore strictly independent of magnetic field: the Lorentz force merely rotates the nonequilibrium dipole on the Fermi surface, but does not alter its magnitude, so the configurational momentum transfer mediated by $V(\mathbf{k})$ remains unchanged.

In general, one cannot expect $\mathbf{K}(B)$ [Eq. (114)] and $\sigma(B)$ [Eq. (113)] to be proportional, because both tensors have the same Lorentz denominator, but differ through the factor $V(\mathbf{k})$ inside the angular average. The wind stress at fixed current, Eq. (115), therefore need not be as simple as that given for the isotropic case with Eq. (116), and need revert to the more general form (115). To understand its field dependence, we expand both the $\mathbf{K}(B)$ and $\tau(B)$ tensors in spherical harmonics on the Fermi surface. Writing the velocity dyad and the vertex as

$$\mathbf{v} \mathbf{v} = \sum_{\ell m} a_{\ell m} Y_{\ell m}, \quad V(\mathbf{k}) = \sum_{\ell m} v_{\ell m} Y_{\ell m},$$

so that

$$\mathbf{K}(B) = e^2 \tau_1 \sum_{\ell m} \frac{a_{\ell m} v_{\ell m}}{1 + (\omega_c \tau_1)^2},$$

$$\sigma(B) = e^2 \tau_1 \sum_{\ell m} \frac{a_{\ell m}}{1 + (\omega_c \tau_1)^2}.$$

In an isotropic metal only the $\ell = 0$ harmonic of both $a_{\ell m}$ and $v_{\ell m}$ survives, giving

$$\mathbf{K}(B) = \frac{v_{00}}{e} \sigma(B) \implies \tau_e = \frac{1}{be} \frac{v_{00}}{e} \mathbf{J} \cdot \hat{\mathbf{x}},$$

so all $(\omega_c \tau_1)^2$ factors cancel identically and we obtain Eq. (116).

When anisotropy is present, higher harmonics $\ell \geq 2$ contribute. Writing $a_{\ell m} = a_{\ell m}^{(0)} + \delta a_{\ell m}$ and $v_{\ell m} = v_{\ell m}^{(0)} + \delta v_{\ell m}$ with the superscript (0) denoting the isotropic part, one finds

$$\mathbf{K}(B) = \frac{v_{00}}{e} \sigma(B) + e^2 \tau_1 \sum_{\ell \geq 2, m} \frac{\delta v_{\ell m} a_{\ell m}^{(0)} + \delta v_{\ell m} \delta a_{\ell m}}{1 + (\omega_c \tau_1)^2}. \quad (117)$$

The first term reproduces the isotropic cancellation, while the remaining harmonics do not match any component of $\sigma(B)$ and therefore survive the contraction in Eq. (115). Expanding the Lorentz factor gives

$$\frac{1}{1 + (\omega_c \tau_1)^2} = 1 - (\omega_c \tau_1)^2 + O((\omega_c \tau_1)^4), \quad (118)$$

so the field-dependent contribution to τ_e becomes

$$\delta \tau_e(B) = -(\omega_c \tau_1)^2 \frac{1}{be} \sum_{\ell \geq 2, m} \delta v_{\ell m} a_{\ell m}^{(0)} + O((\omega_c \tau_1)^4). \quad (119)$$

Thus, the field dependence of $\tau_e(\mathbf{J}, \mathbf{B})$ arises entirely from the $\ell \geq 2$ anisotropic harmonics of the Fermi surface velocity and of the Kanzaki vertex. The leading correction is quadratic in $\omega_c \tau_1$ and is multiplied by dimensionless anisotropy coefficients $\delta v_{\ell m} a_{\ell m}^{(0)}$. The exact cancellation in Eq. (116) is therefore a special property of the isotropic limit; any departure from spherical symmetry produces controlled magnetoconfigurational corrections of order $(\omega_c \tau_1)^2$.

2. Electronic viscous drag in a magnetic field

The magnetic field dependence of the electronic viscous drag follows directly from the electronic part of the retarded kernel, Eq. (59), with the stress bubble replaced by its field-dependent counterpart $\Pi^R ij(\mathbf{q}, \omega; B)$. The drag coefficient at

fixed transverse direction $\hat{\mathbf{q}} \perp$ remains determined by the low-frequency slope of the kernel, Eq. (72), or equivalently by the long-wavelength reduction in Eq. (74) upon the substitution $\Pi^R ij \rightarrow \Pi^R ij(\mathbf{q}, \omega; B)$. The Kanzaki vertex $\Lambda_i(\mathbf{q})$ and the $q_x = 0$ selection imposed by the slip geometry are unchanged, so all magnetic field structure is encoded in the quadrupolar part of the stress bubble.

In the field free limit, Appendix B shows that the contraction with the shear projector $\mathcal{P}_{ij}(\hat{\mathbf{q}} \perp)$ isolates the $\ell = 2$ sector of the electronic stress, so that the imaginary part of $\Pi^R ij$ is governed by the relaxation of a quadrupolar angular distortion aligned with the screened homogeneous strain vertex. Choosing the quantization axis along $\hat{\mathbf{q}} \perp$, this shear component corresponds to the $m = 0$ element of a rank two spherical tensor built from $Y_{2m}(\hat{\mathbf{k}})$ and its associated tensor harmonics. A finite magnetic field introduces a second distinguished axis $\hat{\mathbf{B}}$; in the $\ell = 2$ representation the relative orientation of the strain axis $\hat{\mathbf{q}} \perp$ and the field is described by a rotation through the polar angle $\theta = \angle(\hat{\mathbf{q}} \perp, \hat{\mathbf{B}})$, whose action on the quadrupolar tensor is generated by the Wigner matrix $d_{m0}^2(\theta)$. Thus, the same tensor structure of the stress bubble that controls the zero field drag reappears in the magnetic case as a rotated quadrupole in the $\ell = 2$ representation.

As in the analysis of the electron wind in Sec. IV C 1, the effect of the magnetic field on the stress bubble is captured by the Keldysh kinetic equation on the Fermi surface. Restricting Eq. (109) to vanishing electric field and to the $\ell = 2$ sector gives, for a uniform B ,

$$(\hat{\mathcal{C}}\ell = 2 + \omega_c \hat{\mathcal{L}}B), \quad \phi_2 = \mathcal{S}_2(\hat{\mathbf{q}} \perp), \quad (120)$$

where $\phi_2(\hat{\mathbf{k}})$ is the $\ell = 2$ component of the nonequilibrium correction that mediates the stress response and $\mathcal{S}_2$ is the quadrupolar source generated by the homogeneous strain vertex entering the bubble. Expanding

$$\phi_2 = \sum_{m=-2}^2 \phi_{2m} Y_{2m}(\hat{\mathbf{k}}), \quad (121)$$

$$\hat{\mathcal{L}}BY_{2m} = imY_{2m}, \quad (122)$$

$$\hat{\mathcal{C}}\ell = 2Y_{2m} = \tau_2^{-1}Y_{2m}, \quad (123)$$

one obtains, in the frame where the quantization axis is aligned with $\hat{\mathbf{B}}$,

$$\phi_{2m} = \frac{\mathcal{S}_2, d^2 m 0(\theta)}{\tau_2^{-1} + im\omega_c}, \quad (124)$$

with τ_2 the quadrupolar relaxation time and $d^2 m 0(\theta)$ the Wigner rotation matrix that rotates the $m = 0$ quadrupole aligned with $\hat{\mathbf{q}} \perp$ into the basis defined by $\hat{\mathbf{B}}$. In other words, the coefficients ϕ_{2m} are precisely the rotated components of the shear stress quadrupole that enters the tensor structure of the electronic bubble.

The dissipative part of the stress response that appears in Eq. (74) is proportional to ω , Re, ϕ_{2m} in the hydrodynamic limit, so that each m channel contributes a Lorentz narrowed factor $\tau_2^{-1}/[\tau_2^{-2} + m^2 \omega_c^2]$. Cancelling the common prefactors with the zero field result gives an orientation resolved drag

ratio

$$\begin{aligned} \frac{\mathcal{B}_{\text{el}}(B; \theta)}{\mathcal{B}_{\text{el}}(0)} &= \sum_{m=-2}^2 |d^2 m 0(\theta)|^2 \frac{1}{1 + m^2 (\omega_c \tau_2)^2} \\ &= \left(\frac{3 \cos^2 \theta - 1}{2} \right)^2 + \frac{3 \sin^2 \theta \cos^2 \theta}{1 + (\omega_c \tau_2)^2} \\ &\quad + \frac{3 \sin^4 \theta}{4[1 + 4(\omega_c \tau_2)^2]}, \end{aligned} \quad (125)$$

with small field expansion

$$\frac{\mathcal{B}_{\text{el}}(B; \theta)}{\mathcal{B}_{\text{el}}(0)} = 1 - 3(\omega_c \tau_2)^2 \sin^2 \theta + O((\omega_c \tau_2)^4). \quad (126)$$

At fixed $\hat{\mathbf{q}} \perp$ the static factor $|\Lambda(\mathbf{q})|^2$ cancels identically in Eq. (125), so that all field dependence resides in the quadrupolar kinetic resolvent. If one averages over the transverse belt with an anisotropic $|\Lambda|^2$ set by core multipoles, the experimentally observed ratio is the $|\Lambda|^2$ weighted average of Eq. (125). In the isotropic s wave limit the collision operator is diagonal with $\tau_2 = \tau_1$, and the azimuthal average of Eq. (125) reproduces the familiar Drude suppression factor $[1 + (\omega_c \tau_{\text{tr}})^2]^{-1}$ for the viscous drag.

3. Thermal and disorder dependencies

In the isotropic limit the wind evaluated at fixed current is field independent by Eq. (116). Its temperature dependence is then inherited entirely from the zero field conductivity and does not carry additional information about the magnetic field. The nontrivial thermal dependence resides in the magnetic reduction of the viscous drag, encoded in the ratio Eq. (125).

For fixed B and fixed orientation θ the factor $\mathcal{B}_{\text{el}}(B; \theta)/\mathcal{B}_{\text{el}}(0)$ depends on temperature only through the quadrupolar relaxation time $\tau_2(T)$ that enters $\omega_c \tau_2$. The static ingredients of the stress bubble, namely the Kanzaki vertex and the screened interaction, are essentially temperature independent in the regime of interest; they cancel in the ratio and do not modify the field dependence. The entire thermal evolution of the magnetic correction is therefore carried by the kinetic resolvent of the $\ell = 2$ channel.

In a Fermi liquid one may parametrize the quadrupolar relaxation rate as

$$\tau_2^{-1}(T) = \tau_{2,\text{imp}}^{-1} + a, T^2, \quad (127)$$

where $\tau_{2,\text{imp}}$ is the static disorder contribution and a is an interaction coefficient that includes the appropriate umklapp weight. At fixed B this implies a monotonically decreasing product $\omega_c \tau_2(T)$ as temperature rises, from its interaction dominated value at low T toward the disorder limited value set by $\tau_{2,\text{imp}}$. Consequently the angular reduction factor in Eq. (125) is largest at low temperature and is gradually suppressed as T increases, driving $\mathcal{B}_{\text{el}}(B; \theta)/\mathcal{B}_{\text{el}}(0)$ smoothly toward unity without any change in the Kanzaki geometry or static screening.

The small field expansion Eq. (126) makes the temperature dependence of the magnetic correction explicit,

$$\frac{\mathcal{B}_{\text{el}}(B; \theta)}{\mathcal{B}_{\text{el}}(0)} - 1 \simeq -3, [\omega_c \tau_2(T)]^2 \sin^2 \theta, \quad (128)$$

so that the leading magnetodrag contribution scales as $B^2 \tau_2(T)^2$ with an anisotropic prefactor fixed by the $\ell = 2$ tensor structure of the stress bubble. Any observed temperature dependence of the magnetic suppression of the drag thus provides a direct probe of the quadrupolar relaxation time, while the steady momentum transfer encoded in the wind remains field independent when expressed at fixed current.

D. Combined electric and magnetic fields

The combined effect of an electric current and a weak magnetic field on both the configurational wind and the viscous drag is encoded in the same Keldysh kinetic equation that underlies the zero field theory and the separate B -dependent corrections, Eq. (109). The electric field enters through the drift term $-e\mathbf{E} \cdot \nabla_{\mathbf{k}}$, the magnetic field through the Lorentz generator $\omega_c \hat{\mathcal{L}}_B$, while the source $\mathcal{S}(\hat{\mathbf{q}}_{\perp})$ and the collision operator $\hat{\mathcal{C}}$ remain those of the conserving approximation used throughout. The question is whether, within this common framework, the current and the field generate genuinely new interference terms, or whether they only renormalize the same angular channels that already control the pure E and pure B responses.

1. Electron wind under simultaneous J and B

For the configurational wind, the analysis of Sec. IV C 1 already shows that the Lorentz rotation acts on the same $\ell = 1$ harmonics that determine the magnetoconductivity tensor $\sigma(B)$. The wind at fixed electric field $\mathbf{E}$ acquires the usual Hall admixture through the $(\hat{\mathcal{C}}_{\ell=1} + \omega_c \hat{\mathcal{L}}_B)^{-1}$ resolvent, but when the comparison is made at fixed current the same resolvent appears in the definition of $\sigma(B)$ and cancels exactly. This is the content of Eq. (116): in an isotropic conserving approximation the wind evaluated at fixed $\mathbf{J}$ is identical to its zero field form and the combined presence of J and B does not introduce any new structure at linear order. Corrections that explicitly mix B and J first appear at order $(\omega_c \tau_1)^2 (J/J_0)^2$, where the conserving form of the kernel ceases to protect the cancellation, and lie beyond the accuracy of the present treatment.

2. Electronic drag under simultaneous J and B

The viscous drag is governed by the quadrupolar sector of the stress bubble. In the absence of current, Sec. IV C 2 shows that the screened homogeneous strain vertex excites an $\ell = 2$ quadrupole whose angular dependence is transported by the Wigner matrix $d_{m0}^2(\theta)$ and whose magnetic field response is controlled by the quadrupolar relaxation time τ_2 through the Lorentz resolvent $(\hat{\mathcal{C}}_{\ell=2} + \omega_c \hat{\mathcal{L}}_B)^{-1}$. The resulting drag ratio is given by Eq. (125), with the small field expansion Eq. (126). A finite current does not alter this angular tensor structure. Its only effect at the level of the conserving approximation is to renormalize the $\ell = 2$ collision operator by drifting the Fermi sea, which may be parameterized as a current-dependent relaxation rate $\tau_2^{-1} \rightarrow \tau_2^{-1}(J) = \tau_2^{-1}[1 + \eta_2(\hat{\mathbf{J}})(J/J_0)^2 + \dots]$. To leading order in B and J the drift correction $\delta \hat{\mathcal{C}}_J$ commutes with $\hat{\mathcal{L}}_B$, so that the combined current and field response of the $\ell = 2$ kinetic resolvent factorizes.

This means that all expressions derived for the magnetic reduction of the drag remain valid at finite current upon the simple substitution $\tau_2 \rightarrow \tau_2(J)$. In particular, at fixed orientation θ one may write

$$\frac{\mathcal{B}_{\text{el}}(B, J; \theta)}{\mathcal{B}_{\text{el}}(0, 0)} = \frac{\mathcal{B}_{\text{el}}(B, 0; \theta)}{\mathcal{B}_{\text{el}}(0, 0)} [1 - \eta_2(\hat{\mathbf{J}})(J/J_0)^2 + O((J/J_0)^4)], \quad (129)$$

where the first factor is the purely magnetic drag ratio given by Eq. (125) and the bracket encodes the current-induced renormalization of the quadrupolar time. In the small field limit this reduces to a multiplicative modification of Eq. (126),

$$\frac{\mathcal{B}_{\text{el}}(B, J; \theta)}{\mathcal{B}_{\text{el}}(0, 0)} - 1 \simeq -3[\omega_c \tau_2(J)]^2 \sin^2 \theta, \quad (130)$$

with all the angular dependence dictated by the $\ell = 2$ tensor structure of the stress bubble.

The combined action of current and field therefore separates cleanly between the two mechanisms. The wind, controlled by the $\ell = 1$ component of the kinetic resolvent and protected by its relation to the conductivity tensor, is insensitive to B at fixed current to the order considered. The drag, controlled by the $\ell = 2$ component, senses both the curvature of electronic trajectories generated by the magnetic field and the redistribution of phase space generated by the drift, but only through the single microscopic time $\tau_2(T, J)$ that already governs its zero field and pure B limits. From a practical point of view, the temperature and current dependence of the magnetic suppression of the drag provide a direct probe of the quadrupolar relaxation rate, without contamination from the configurational wind.

E. Core effects and their role in the electronic drag and electronic wind

The coupling of a moving dislocation to the electron gas enters the Keldysh kernels only through the screened slip vertex $\Lambda_i(\mathbf{q})$ that sits at the ends of the electronic bubble $\Pi_{ij}^R(\mathbf{q}, \omega)$. The vertex we have used so far is a linear functional of the Kanzaki force distribution, which neglects the structure of the dislocation core for a Volterra description of the dislocation. As demonstrated by Gurrutxaga-Lerma and Verschuereen [49,59], the dislocation core can be also treated as a local Kanzaki force distribution that is superposed to the Volterra description; further, the core Kanzaki forces can be expanded into core multipoles, that may be superposed additively to study the core's near field. Hence the question: what should one expect core effects to be like on the electronic drag and wind?

Within the present Kanzaki representation, such *sensitivity* to the core means sensitivity to the short-range completion of the Volterra source, encoded by the localized term $F^{\text{core}}(\mathbf{q})$ in Eq. (8). The Volterra sector is fixed by the slip disregistry and controls the nonanalytic long-wavelength structure of the screened projector, whereas any microscopic core model enters only through analytic multipoles supported at $r \lesssim a_{\text{core}}$. A direct benchmark of residual core dependence is therefore obtained by varying a minimal multipole completion (for example a dipole or quadrupole ansatz with radius a_{core}) and monitoring the induced spread in the predicted wind coefficient and electronic drag prefactor. As shown below,

the leading wind is governed by the Volterra contribution, while core structure can only renormalize the drag through controlled higher-order terms.

Consistent with this representation, the discrete Kanzaki drive is decomposed into Volterra and core contributions as we outlined in Sec. II:

$$F_i^K(\mathbf{q}, t) = [F_i^V(\mathbf{q}) + F_i^{\text{core}}(\mathbf{q})] \Psi(\mathbf{q}; X(t)), \quad (131)$$

where the Volterra part F^V encodes the slip discontinuity of the ideal dislocation and F^{core} parametrizes short-range core physics, as in Eq. (8). The electronic sector couples to this source through the line vertex

$$\Lambda_i(\mathbf{q}) = G_{ij}(\mathbf{q}, 0) f_j^K(\mathbf{q}), \quad (132)$$

already defined in Eq. (53). Linearity in f^K immediately implies a decomposition

$$\Lambda_i(\mathbf{q}) = \Lambda_i^V(\mathbf{q}) + \Lambda_i^{\text{core}}(\mathbf{q}), \quad (133)$$

with Λ^V and Λ^{core} obtained from the Volterra and core pieces of the Kanzaki field, respectively. All core corrections to the electronic drag and to the electron wind are encoded in Λ^{core} and enter the observables only through this superposition.

The defining distinction between Volterra and core contributions is their behavior at small wave vector. The Volterra source carries the slip discontinuity across the glide plane and is nonanalytic in the continuum limit, producing the familiar $1/q$ singularity in $\Lambda^V(\mathbf{q})$ discussed in Sec. II. By contrast, $F_i^{\text{core}}(\mathbf{q})$ is a strictly local contribution, analytic at $\mathbf{q} = \mathbf{0}$, and admits a multipole expansion

$$F_i^{\text{core}}(\mathbf{q}) = F_i^{(0)} + i q_\alpha M_{i\alpha} - \frac{1}{2} q_\alpha q_\beta Q_{i\alpha\beta} + O(q^3), \quad (134)$$

where $F^{(0)}$ is the resultant force on the core, $M_{i\alpha}$ the dipole moment, and $Q_{i\alpha\beta}$ the quadrupole. Inserting this into Eq. (132) and using the long-wavelength form of the lattice Green function with metallic screening shows that each multipole generates an *analytic* contribution $\Lambda_i^{\text{core}}(\mathbf{q})$ at small q . The monopole term is suppressed by translational invariance of the acoustic branch, so that $\Lambda_i^{\text{core}}(\mathbf{q})$ starts at order q in the long-wavelength expansion, whereas $\Lambda_i^V(\mathbf{q}) \propto 1/q$. It follows that the entire nonanalytic $1/q$ structure required to produce a finite homogeneous coupling is carried by the Volterra sector; the core multipoles only add regular corrections.

This analytic structure has two immediate consequences. First, the homogeneous band vertex that drives the electron wind, Eq. (56), involves the longitudinal projection $\Lambda_{\parallel}(\mathbf{q})$ and an explicit factor of q ,

$$V(\mathbf{k}; \mathbf{q}) = i q W(\mathbf{k}; \hat{\mathbf{q}}, q) \Lambda_{\parallel}(\mathbf{q}). \quad (135)$$

Using Eq. (133), one may write $V = V^V + V^{\text{core}}$ with $V^V \sim i q \Lambda_{\parallel}^V \sim q \cdot (1/q)$ finite as $q \rightarrow 0$ and $V^{\text{core}} \sim i q \Lambda_{\parallel}^{\text{core}} = O(q^2)$ vanishing in the same limit. The homogeneous stress vertex $V_n(\mathbf{k})$ that enters the Keldysh kinetic equation and the current controlled expression for τ_e is therefore determined entirely by the Volterra piece. Core multipoles do not modify the leading homogeneous vertex in the $q \rightarrow 0$ limit, but only generate higher order gradient corrections in $q_{\perp} a_{\text{core}}$ which are parametrically small when the electronic mean free path exceeds the core radius. In ordinary metals, this separation is large: room-temperature electronic mean free paths are

typically $\ell \sim 10\text{--}50$ nm, whereas compact dislocation cores have $a_{\text{core}} \sim 1\text{--}5$ Å, so that $(a_{\text{core}}/\ell)^2 \sim 10^{-6}\text{--}10^{-4}$, confirming that curvature-induced corrections to η_1 are parametrically negligible under metallic transport conditions. Consequently, the configurational wind at fixed current retains the form derived in Sec. IV B, up to corrections of order $(q_{\perp} a_{\text{core}})^2$ that are negligible in the long-wavelength regime. This is the same ordering underlying the neglect of the $O(q^2)$ electronic curvature vertices in Sec. II C. In both cases, the relevant small parameter is fixed by the wave-vector support of the screened Kanzaki projector, which for compact cores remains in the long-wavelength sector where higher order gradients are parametrically suppressed. Otherwise, the core structure does not alter the existence, sign, or leading magnitude of the electron wind force; it changes only higher order spatial structure in the coupling.

Second, the electronic drag coefficient is obtained from the retarded kernel Eq. (67), and its low-frequency slope is given by Eq. (74). Substituting the decomposition Eq. (133) into Eq. (74) and collecting terms yields

$$B_{\text{el}}(T, J, B) = B_{\text{el}}^V(T, J, B) [1 + \delta_C^{(B)}], \quad (136)$$

with

$$\delta_C^{(B)} = \frac{\sum_{\mathbf{q}_{\perp}} (|\Lambda^{\text{core}}|^2 + 2 \text{Re} \Lambda^{V*} \Lambda^{\text{core}}) S(\mathbf{q}_{\perp}; T, J, B)}{\sum_{\mathbf{q}_{\perp}} |\Lambda^V|^2 S(\mathbf{q}_{\perp}; T, J, B)}, \quad (137)$$

where

$$S(\mathbf{q}_{\perp}; T, J, B) = -[\partial_{\omega} \text{Im} \Pi^R(\mathbf{q}_{\perp}, \omega; T, J, B)]_{\omega \rightarrow 0}. \quad (138)$$

The function $S(\mathbf{q}_{\perp}; T, J, B)$ contains the entire dependence on temperature, current, and magnetic field through the electronic stress bubble, including the T^2 Fermi liquid scaling of the relaxation rate, the J^2 renormalization of the quadrupolar time, and the B^2 suppression governed by the $\ell = 2$ Lorentz resolvent. By contrast, $\delta_C^{(B)}$ is a purely *static* dimensionless number: it depends on the multipole parameters $M_{i\alpha}$ and $Q_{i\alpha\beta}$, on the Thomas-Fermi length, and on the core radius through $|\Lambda^{\text{core}}(\mathbf{q})|^2$ and the interference term $\Lambda^{V*} \Lambda^{\text{core}}$, but it carries no additional powers of T , J , or B .

Because $\Lambda^{\text{core}}(\mathbf{q})$ is analytic and concentrated at large wave vectors of order a_{core}^{-1} , the sums entering Eq. (137) are dominated by the ultraviolet part of the Brillouin zone. In the regime $k_{\text{TF}} a_{\text{core}} \gg 1$, metallic screening removes the long-wavelength sensitivity before the core radius is resolved, and one may approximate the screening factor by its $q \rightarrow 0$ value within the support of $|\Lambda^{\text{core}}|^2$. The resulting correction $\delta_C^{(B)}$ is then parametrically small, of order $(a_{\text{core}} k_{\text{TF}})^{-4}$ for typical metals. Its only role is to renormalize the *magnitude* of the electronic drag, including its magnetic reduction and its temperature and current dependencies, by a core specific prefactor in Eq. (136). The T^2 , J^2 , and B^2 scalings derived earlier from the Volterra theory remain unchanged.

From the Keldysh point of view this separation is enforced by symmetry. The retarded stress kernel $\mathcal{D}^R(\omega; \mathbf{J})$ can be expanded at small ω and small current into parts that are odd and even in $\mathbf{J}$. The odd in $\mathbf{J}$ sector is associated with the antisymmetric part of the stress response and generates a genuine configurational drive that reverses with current polarity (the electron wind), whereas the even in $\mathbf{J}$ sector is

tied to the symmetric correlators that determine conductivity and Joule heating and therefore only renormalizes the viscous drag. In our conserving approximation the dislocation enters the latter solely through the Kanzaki scattering vertex that also controls its contribution to the resistivity, and no additional odd in $\mathbf{J}$ term appears in $\mathcal{D}^R$ beyond the wind channel already identified.

Putting these results together, the dislocation core enters the Keldysh description in a remarkably constrained way. The Volterra sector, with its nonanalytic $1/q$ structure, fixes the homogeneous stress vertex and hence the leading electron wind. The core multipoles are analytic in $\mathbf{q}$ and therefore invisible in the $q \rightarrow 0$ limit that defines this vertex. Their effect is confined to a finite ultraviolet renormalization of the electronic drag through the dimensionless factor $1 + \delta_C^{(B)}$, without modifying the underlying temperature, current, or field dependencies. This shows that the configurational wind is largely insensitive to microscopic core structure at leading order, whereas the viscous drag provides a more direct probe of core physics through its overall magnitude rather than through its scaling laws. By contrast, the dislocation mobility law is intrinsically core sensitive [92]; thus, whereas we have just shown the configurational force exerted by a steady electric current is fixed by the Volterra sector and is therefore essentially independent of the detailed core structure, the resulting velocity response of the dislocation will remain governed by the core.

F. Joule heating and dislocations in the Keldysh framework

A dc current flowing through a crystal dissipates power density $p = \mathbf{J} \cdot \mathbf{E} = \rho_{ij} J_i J_j$ and thereby raises the electronic and lattice temperatures. In a perfect crystal, this is entirely governed by the current-current correlator K_{JJ}^R that defines the conductivity tensor. When dislocations are present the same Keldysh diagrams acquire Kanzaki insertions $\Lambda(\mathbf{q})$ in the electron self energy, and it is natural to ask whether this leads to additional local Joule heating beyond the usual resistivity increment and, in particular, whether any polarity dependence can appear.

Keldysh formulation of the heating rate. Within the coupled electron, phonon, and dislocation system, the microscopic energy balance follows from the Keldysh Dyson equations for the electronic Green functions and self energies. The total power transferred from electrons to the lattice can be expressed in terms of the electronic stress bubble and the phonon propagator,

$$p = \frac{i}{2\hbar} \int \frac{d^3 q d\omega}{(2\pi)^4} \hbar\omega \text{Tr}[\Pi^K(\mathbf{q}, \omega) D^A(\mathbf{q}, \omega) - \Pi^R(\mathbf{q}, \omega) D^K(\mathbf{q}, \omega)], \quad (139)$$

where $\Pi^{R,K}$ are built from electronic Green functions that already contain electron phonon and electron dislocation self energies, and $D^{R,A,K}$ describe the lattice. The dislocation density n_d enters through the dislocation part of the electronic self energy, which in turn generates a correction $\delta\Pi_d$ to the stress bubble. To first order in n_d one may write

$$\Pi^{R,K} = \Pi_0^{R,K} + n_d \delta\Pi_d^{R,K}, \quad (140)$$

and obtain the corresponding change in Joule power as

$$\delta p = \frac{i n_d}{2\hbar} \int \frac{d^3 q d\omega}{(2\pi)^4} \hbar\omega \text{Tr}[\delta \Pi_d^K D_0^A - \delta \Pi_d^R D_0^K]. \quad (141)$$

Diagrammatically this is a sum of bubble graphs with one dislocation insertion per loop. Crucially, every term in Eq. (141) is *quadratic* in the response to the external field: either in the current (through D^K and Π^K) or, equivalently, in combinations of G^K and the Keldysh self energies. This guarantees that δp is even in $\mathbf{J}$. Any odd in $\mathbf{J}$ contribution would require a single insertion of the external field into the force kernel and therefore corresponds to a configurational drive of the kind generated by the wind, not to Joule heating.

Relation to the conductivity and the role of the Kanzaki vertex. The same dislocation self energy that appears in Eq. (140) also generates the dislocation contribution to the current-current correlator K_{JJ}^R , obtained by attaching current vertices to the electron lines instead of stress vertices. Through the Kubo formula,

$$\sigma_{ij}(\omega) = \frac{1}{\omega} \text{Im} K_{JJ,ij}^R(\omega), \quad (142)$$

this yields a correction $\delta\sigma_{ij}$ to the conductivity tensor that is linear in n_d and quadratic in the Kanzaki vertex $\Lambda(\mathbf{q})$. In particular, at the level of the conserving approximation used throughout, the dislocation enters only through the same scattering channel that governs its contribution to the resistivity: there is no new term in the retarded configurational kernel $\mathcal{D}^R$, and no separate source of heating associated with the core beyond its role in the static scattering amplitude encoded in $\Lambda(\mathbf{q})$.

Combining Eq. (141) with Eq. (142), and taking the dc limit, the total power can be written in the standard form

$$p = p_0 + \delta p = \rho_{ij} J_i J_j, \quad \rho_{ij} = \rho_{0,ij} + \delta\rho_{ij}, \quad (143)$$

where ρ_{ij} is the full resistivity tensor and $\delta\rho_{ij}$ is obtained from $\delta\sigma_{ij}$ by expanding $\rho = \sigma^{-1}$ to first order in n_d . The even in J character of the correction is now transparent: p depends on the dislocations only through $\delta\rho_{ij}$, which is independent of the sign of $\mathbf{J}$, and through the quadratic contraction $J_i J_j$.

In the isotropic case, $\rho_{0,ij} = \rho_0 \delta_{ij}$ and $\delta\rho_{ij} = \delta\rho \delta_{ij}$, the dislocation contribution can be summarized by an effective transport cross section σ_d defined through

$$\delta\sigma = -n_d \sigma_d, \quad (144)$$

where σ_d is an integral over the Fermi surface involving $|\Lambda(\mathbf{k}-\mathbf{k}')|^2$ and the usual transport factor $(1 - \cos \theta_{\mathbf{k}\mathbf{k}'})$ [cf. Eq. (144)]. Expanding $\rho = (\sigma_0 + \delta\sigma)^{-1}$ to first order then gives

$$p \simeq \rho_0 J^2 [1 + n_d \rho_0 \sigma_d], \quad (145)$$

which is the scalar expression used in the following discussion. The prefactor $1 + n_d \rho_0 \sigma_d$ is typically close to unity for realistic dislocation densities, so dislocations act as a modest additional Joule channel whose strength is set entirely by the same Kanzaki vertex that controls their contribution to the resistivity and to the electronic drag.

Feedback into the drag kernels and absence of configurational bias. The power Eq. (145) feeds back into the drag problem only through the local temperature field. Using the

geometry factor $\mathcal{G}_{\text{geom}}(\mathbf{r})$ introduced earlier, the defect enhanced temperature increment can be written schematically as

$$\Delta T(\mathbf{r}) \simeq \frac{\rho_0 J^2}{\kappa} \mathcal{G}_{\text{geom}}(\mathbf{r}) [1 + n_d \rho_0 \sigma_d], \quad (146)$$

with κ the thermal conductivity. Substituting this into the Keldysh expressions for the electronic and phononic drag kernels gives a purely even in J renormalization of B_{ph} and an exponentially small change in B_{el} , in direct correspondence with the zero defect heating analysis.

Equations (139)–(141) place the long-standing Joule heating in electroplasticity argument on a quantitative basis. The local heating rate follows from the same current-current correlator that controls conductivity, with all defect effects entering through the Kanzaki vertex $\Lambda(\mathbf{q})$ in the electronic self-energies. The result is even in J and purely dissipative, producing only a temperature-dependent renormalization of B_{ph} and an exponentially small correction to B_{el} . No odd-in- J configurational drive arises, confirming that Joule heating, even when locally enhanced by dislocation scattering, cannot explain current-polarity-dependent electroplastic effects.

Our microscopic conclusion is that Joule heating and electroplastic softening seem to be distinct phenomena that share a common cause, the applied current. The defect term $n_d \rho_0 \sigma_d$ quantifies the additional dissipation generated by electron-dislocation scattering and enters identically in both the conductivity and the drag kernels. Its effect is to raise the local lattice temperature by an amount

$$\delta T_d(\mathbf{r}) \simeq \frac{n_d \rho_0 \sigma_d J^2}{\kappa} \mathcal{G}_{\text{geom}}(\mathbf{r}),$$

so that any local softening must originate from the temperature dependence of the phonon bath parameters within a radius where δT_d is appreciable. For realistic conductivities and dislocation densities this increment is subkelvin even at $J \sim 10^9 \text{ A m}^{-2}$, implying that macroscopic softening observed in experiments cannot arise from this channel. The Keldysh structure makes clear that no new configurational term appears: δT_d enters only through the even in J occupation factors of the Keldysh propagators, not through the retarded kernels that generate forces. Consequently, the local power enhancement may modify relaxation rates but cannot bias the dislocation motion with respect to current polarity. Within the accuracy of the weak-field expansion the dislocation thus behaves as a passive scatterer for Joule heating, not as a source of localized electronic energy release. Any genuine athermal drive must instead arise from the odd in J component of the stress response, that is, from the electron wind channel treated in Sec. IV.

V. CALCULATING THE ELECTRON WIND AND ELECTRONIC DRAG COEFFICIENTS: DEFORMATION POTENTIALS AND FERMI SURFACE AVERAGES

The electron wind and drag coefficients derived in Secs. II–IV are functionals of two microscopic objects: the deformation potential tensors $\Gamma_{ij}(\mathbf{k})$ and $\Xi_{je}(\mathbf{k})$ that encode the coupling of electrons to homogeneous strain and to strain gradients, respectively. The purpose of this section is to

describe how these tensors are obtained from first principles DFT (and from multiple scattering theory) and how they enter the transport coefficients through Fermi surface averages.

A direct evaluation of the full Keldysh kernels with all microscopic structure retained would be prohibitively expensive. In practice there is a strong separation of length scales which allows the response to factorize. The dislocation produces a long-wavelength strain field, governed by the Kanzaki force f^K propagated by the lattice Green function, while the electronic vertices that describe the response on the Fermi surface vary on the atomic scale. At low frequency this permits the electronic drag to be written in the factorized form

$$B_{el} = \frac{\pi}{\hbar} \int \frac{d^2 q_{\perp}}{(2\pi)^2} \frac{|\Lambda(\mathbf{q}_{\perp}, 0)|^2}{q_{\perp}} \left\langle \frac{W^2(\mathbf{k}; \hat{\mathbf{q}}, q_{\perp})}{v_F(\mathbf{k})} \delta(\hat{\mathbf{v}}_{\mathbf{k}} \cdot \hat{\mathbf{q}}) \right\rangle_{\text{FS}}, \quad (147)$$

where $\Lambda(\mathbf{q})$ is the screened Kanzaki vertex defined in Sec. II, and the Fermi surface average

$$\langle A(\mathbf{k}) \rangle_{\text{FS}} = \frac{\int \frac{d^3 k}{(2\pi)^3} A(\mathbf{k}) \delta(\epsilon_{\mathbf{k}} - E_F)}{\int \frac{d^3 k}{(2\pi)^3} \delta(\epsilon_{\mathbf{k}} - E_F)} \quad (148)$$

is taken with respect to the interacting Fermi surface obtained from the band structure. Equation (147) follows from the general expression for $\mathcal{D}_{el}^R$ in Eq. (B5) and its low-frequency slope in Eq. (74), after resolving the on shell constraints and isolating the long-wavelength part of the Kanzaki coupling. All geometrical information about the dislocation and screening resides in $|\Lambda(\mathbf{q}_{\perp}, 0)|^2$, whereas the electronic structure enters only through the Fermi surface average of W^2/v_F .

As discussed in Sec. II, the electronic vertex admits a controlled long-wavelength expansion,

$$W(\mathbf{k}; \hat{\mathbf{q}}, q) = \hat{q}_i \hat{q}_j \Gamma_{ij}(\mathbf{k}) + i q_{\ell} \Xi_{j\ell}(\mathbf{k}) \Pi_{jm}^{\text{ac}}(\hat{\mathbf{q}}) \hat{q}_m + O(q^2), \quad (149)$$

with Π^{ac} the acoustic projector. The first term describes the coupling to a homogeneous strain aligned with $\hat{\mathbf{q}}$ and therefore controls the far field part of the drag and of the wind; it is the only piece that survives in the $q \rightarrow 0$ limit and is the natural object for comparison with standard deformation potential theory. The gradient term, proportional to $\Xi_{j\ell}$, captures the leading correction due to strain inhomogeneity and becomes important in the near core region where the lattice distortion varies on the scale of the Burgers vector. In the following we show how $\Gamma_{ij}(\mathbf{k})$ and $\Xi_{j\ell}(\mathbf{k})$ can be extracted from first principles calculations and how their Fermi surface averages enter the coefficients η_1 , B_{el} and their magnetic extensions.

A. Ab initio methodology

1. Homogeneous deformation potential Γ_{ij}

The tensor Γ_{ij} quantifies the shift in electronic energy under uniform strain. We computed it using CASTEP [93] with the PBE functional [94] for both fcc aluminium (lattice constant $a = 4.05$ Å) and hcp zinc ($a = 2.665$ Å, $c/a = 1.856$), using a plane wave cutoff of 400 eV on Monkhorst-Pack meshes [95] of 32^3 (Al) and $36 \times 36 \times 18$ with a 392.0 eV cutoff (Zn) with an energy tolerance of 10^{-8} eV.

Small homogeneous strains $\epsilon_{ii} = \pm \epsilon$ with $\epsilon \in [10^{-5}, 10^{-3}]$ were applied by scaling lattice vectors along each crystallographic direction. The deformation potential

follows from the Fermi level shift under strain,

$$\Gamma_{ii} = \lim_{\epsilon \rightarrow 0} \frac{E_F(+\epsilon) - E_F(-\epsilon)}{2\epsilon}. \quad (150)$$

For aluminium, Richardson extrapolation to zero strain yields the slopes $\Gamma_{xx} = \Gamma_{yy} = \Gamma_{zz} = -7.76$ eV, with off-diagonal components below 0.001 eV. Independent verification via the KKR method using spin-polarized relativistic-KKR (SPR-KKR) [96] gave -7.62 ± 1.4 eV for the Fermi level shifts, in good agreement with our DFT results. For zinc, we obtain $\Gamma_{xx} = \Gamma_{yy} = -7.4685$ eV and $\Gamma_{zz} = -10.1262$ eV, with $\Gamma_{xy} = 0.831$ eV and Γ_{xz} , Γ_{yz} below 0.001 eV. The negative sign indicates that compression lowers the energy of bonding states at the Fermi surface, as expected for sp -hybridized metals.

2. Strain gradient vertex $\Xi_{j\ell}$

The gradient coupling $\Xi_{j\ell}$ describes the linear response of band energies to strain gradients in the long-wavelength limit. In practice, we extract $\Xi_{j\ell}$ in two complementary ways, depending on the symmetry of the material and the numerical robustness of a frozen phonon treatment.

Frozen phonon approach. For anisotropic systems such as Zn, it is natural to work directly with frozen phonon supercells, which provide the full band structure response to long-wavelength acoustic modulations without recourse to perturbation theory. In this case, we impose displacement patterns of the form

$$\mathbf{u}(\mathbf{R}) = u_0 \hat{\mathbf{e}}_j \sin(\mathbf{q}_{\ell} \cdot \mathbf{R}), \quad (151)$$

with $u_0 = 0.005$ – 0.01 Å the displacement amplitude, $\hat{\mathbf{e}}_j$ the polarization vector (longitudinal or transverse), and $\mathbf{q}_{\ell} = \pm(2\pi/L_{\ell}) \hat{\mathbf{e}}_{\ell}$ an acoustic wave vector for a supercell of linear dimension L_{ℓ} primitive cells along direction ℓ . Each frozen phonon configuration is solved self consistently with CASTEP at the same level of theory as the ground state, yielding perturbed eigenvalues $\epsilon_{n\mathbf{k}}(\pm \mathbf{q}_{\ell})$ on the supercell Brillouin zone. The total electronic response to the modulation is given in a finite difference frozen phonon gradient g^{FP} , given by

$$g_{n\mathbf{k}}^{\text{FP}} = \frac{\epsilon_{n\mathbf{k}}(+\mathbf{q}_{\ell}) - \epsilon_{n\mathbf{k}}(-\mathbf{q}_{\ell})}{2u_0}, \quad (152)$$

which contains both homogeneous and gradient contributions. The associated strain field is $\epsilon_{ij}(\mathbf{R}) = \frac{1}{2}[\partial_i u_j(\mathbf{R}) + \partial_j u_i(\mathbf{R})]$, which for the sinusoidal pattern oscillates as $\epsilon_{ij}(\mathbf{R}) \propto \cos(\mathbf{q}_{\ell} \cdot \mathbf{R})$. The homogeneous contribution predicted by $\Gamma_{ij}(\mathbf{k})$ is therefore

$$g_{n\mathbf{k}}^{\Gamma}(\ell, j) = \Gamma_{ij}(\mathbf{k}) \epsilon_{ij} |\mathbf{q}_{\ell}|, \quad \epsilon_{ij} = \frac{1}{2} \frac{q_{\ell,j} \hat{e}_i + q_{\ell,i} \hat{e}_j}{|\mathbf{q}_{\ell}|}, \quad (153)$$

with ϵ_{ij} the strain amplitude. Subtracting this from the frozen phonon gradient and dividing by $|\mathbf{q}_{\ell}|$ isolates the true gradient coupling,

$$\Xi_{j\ell}(\mathbf{k}, n) = \frac{g_{n\mathbf{k}}^{\text{FP}}(\ell, j) - g_{n\mathbf{k}}^{\Gamma}(\ell, j)}{|\mathbf{q}_{\ell}|}, \quad (154)$$

which we evaluate on a reference $\mathbf{k}$ grid by interpolation and average over bands within a small energy window around the Fermi level. For Zn we performed such calculations on

a $6 \times 6 \times 3$ a supercell with commensurate number of k -points, for acoustic wave vectors aligned with the basal and c directions. The resulting tensor is strongly anisotropic, with $\Xi_{xx} = -6.2$ eV, $\Xi_{yy} = -7.7$ eV, and $\Xi_{zz} = -7.9$ eV, and off-diagonal components $\Xi_{xy} = -3.1$ eV, $\Xi_{xz} = -0.03$ eV, and $\Xi_{yz} = +0.02$ eV. The predominantly negative values indicate that strain gradients couple to bonding states in the same manner as homogeneous compression, with the variation in magnitude across components reflecting the anisotropy of the hcp crystal structure.

DFPT approach. For aluminium, which is cubic and has a highly degenerate Fermi surface, the same frozen phonon strategy proved numerically unreliable. Cosine displacement patterns at commensurate supercell wave vectors $\mathbf{q}_\ell$ couple each Bloch state at $\mathbf{k}$ predominantly to symmetry related partners at $\mathbf{k} \pm \mathbf{q}_\ell$; in the presence of band degeneracies the first order shift of the band center can vanish, while the true response is carried by off diagonal matrix elements within the degenerate manifold. In practice, we find that for reasonable supercell sizes, the central difference $\varepsilon_{nk}(+\mathbf{q}_\ell) - \varepsilon_{nk}(-\mathbf{q}_\ell)$ at the Fermi surface is dominated by numerical noise, even though the total energy and charge density respond appreciably to the phonon. This makes an eigenvalue based frozen phonon extraction of $\Xi_{j\ell}$ in Al intrinsically ill-conditioned.

For Al, we therefore switch to a density functional perturbation theory [97] treatment of electron-phonon coupling, implemented through the QUANTUM ESPRESSO [98–100] and EPW [101,102]. The procedure is as follows. A ground-state calculation with the same parameters as the original CASTEP one provides a well converged charge density and Bloch manifold on a coarse $\mathbf{k}$ mesh. Phonon perturbations are then computed on a commensurate $\mathbf{q}$ mesh by DFPT, yielding dynamical matrices and phonon polarizations for the three acoustic branches at small $|\mathbf{q}|$. A non self consistent calculation on a denser commensurate $\mathbf{k}$ grid is used together with maximally localized Wannier functions to interpolate both the band energies and the electron-phonon matrix elements $g_{nm}^v(\mathbf{k}, \mathbf{q})$ to very fine $\mathbf{k}$ and $\mathbf{q}$ meshes. For each small acoustic wave vector $\mathbf{q}_\ell$ and branch v we project the matrix elements onto a fixed Cartesian displacement pattern and write

$$g_{nm}^v(\mathbf{k}, \mathbf{q}_\ell) \simeq \Gamma_{ij}(\mathbf{k}, n) \varepsilon_{ij}^v(\mathbf{q}_\ell) + i q_\ell \Xi_{j\ell}(\mathbf{k}, n) u_j^v(\mathbf{q}_\ell), \quad (155)$$

where $\varepsilon_{ij}^v(\mathbf{q}_\ell)$ and $u_j^v(\mathbf{q}_\ell)$ are the DFPT strain and displacement associated with the acoustic mode. The homogeneous part $\Gamma_{ij}\varepsilon_{ij}^v$ is known from the separate homogeneous strain calculation and is subtracted explicitly; the residual matrix element is then divided by $i q_\ell u_j^v$ and the limit $|\mathbf{q}_\ell| \rightarrow 0$ is taken by a linear fit in $|\mathbf{q}_\ell|$ along the three Cartesian directions. This yields $\Xi_{j\ell}(\mathbf{k}, n)$ directly in the perturbative framework, without relying on delicate eigenvalue differences between displaced supercells. After Fermi surface averaging, the resulting tensor in Al is isotropic to within numerical uncertainty, with $\Xi_{xx} \simeq \Xi_{yy} \simeq \Xi_{zz} = 4.31 \pm 1.91$ eV and off-diagonal components below 0.01 eV, as expected from cubic symmetry and consistent with the earlier KKR estimates.

Fermi surface averaging. The $\Gamma_{ij}(\mathbf{k})$ and $\Xi_{j\ell}(\mathbf{k})$ fields were evaluated on the Fermi surface using tetrahedron integration on the k -grids within energy windows of ± 0.05 eV (Al) and ± 0.08 eV (Zn). Fermi velocities were obtained from finite-difference band gradients, and Fermi surface averages

of the form

$$\left\langle \frac{A(\mathbf{k})}{v_F(\mathbf{k})} \right\rangle_{\text{FS}} = \frac{\sum_{\mathbf{k}} A(\mathbf{k}) \delta(\varepsilon_{\mathbf{k}} - E_F) / v_F(\mathbf{k})}{\sum_{\mathbf{k}} \delta(\varepsilon_{\mathbf{k}} - E_F)} \quad (156)$$

were evaluated by Gaussian broadening with $\eta = 0.01$ eV (Al) and $\eta = 0.01$ eV (Zn).

The Kanzaki forces appropriate to $\frac{1}{2}\langle 110 \rangle \{111\}$ edge dislocations in fcc aluminium and $\frac{1}{3}\langle 11\bar{2}0 \rangle \{0001\}$ dislocations in hcp zinc were modeled by resolving the $f_i^K = D_{ij}[u_j]$ representation [49]. For edge dislocations, this renders horizontal force doublets across the slip plane of magnitude almost exactly $f_x^K \pm \mu b/2$ and a vertical net component acting solely on the line site $f_y^K = \mu b$, with computed elastic constants $\mu = 26$ GPa ($b = 2.86$ Å) for Al and $\mu = 38$ GPa ($b = 2.67$ Å) for Zn [49,59]. The dynamical matrix $D_{ij}(\mathbf{q})$ was obtained via DFPT dynamical matrix obtained from 12^3 (Al) and $8 \times 8 \times 6$ (Zn) phonon meshes. Screening was included by multiplying $\Lambda_i(\mathbf{q})$ by the Thomas-Fermi dielectric function $\varepsilon^{-1}(\mathbf{q}, 0) = [1 + (k_{\text{TF}}^2/q^2)]^{-1}$, with $k_{\text{TF}} = 1.7 \times 10^{10} \text{ m}^{-1}$ (Al) and $k_{\text{TF}} = 1.9 \times 10^{10} \text{ m}^{-1}$ (Zn).

B. Transport coefficients

1. Electron wind

The current normalized coefficient is

$$\eta_1 = -\frac{1}{e} \frac{\langle V(\mathbf{k}) v_x(\mathbf{k}) \rangle_{\text{FS}}}{\langle v_x^2(\mathbf{k}) \rangle_{\text{FS}}},$$

so that $\tau_e = \eta_1 J / b$.

For the classical $\frac{1}{2}\langle 110 \rangle \{111\}$ edge dislocation in fcc aluminium, the antisymmetric part of the Kanzaki form factor across the glide plane introduces the signed geometric weights discussed earlier. Using the first principles homogeneous and gradient deformation potentials extracted in Sec. V, the substitution of the full vertex into Eq. (93) yields

$$\eta_1^{(\text{Al})} = 1.48 \times 10^{-7} \text{ eV s/m}.$$

For reference, the η_1 value obtained from the vertex reliant on Γ_{ij} alone was $\eta_1^\Gamma = 1.24 \times 10^{-7} \text{ eV s/m}$, so the Ξ_{ij} gradient tensor introduces a correction to the electron wind force of about 10%. At a representative current density $J = 10^9 \text{ A m}^{-2}$ the wind stress becomes

$$|\tau_e^{(\text{Al})}| = |\eta_1^{(\text{Al})}| \frac{J}{b} \approx 2.4 \text{ MPa}.$$

This figure is of a similar magnitude to that of the Peierls barrier in pure fcc Al. Thus, although on its own the current may not be able to overcome the barrier, it will certainly contribute considerably to the driving of plastic slip once this has begun under the action of mechanical loading.

The zinc calculation follows the same expression without modification. The basal system $\frac{1}{3}\langle 11\bar{2}0 \rangle \{0001\}$ requires only a change of slip direction and normal. The deformation potentials were computed for the same ground-state grid as the velocities, and the full vertex $V = b(\Gamma + \Xi)$ assembled from the homogeneous and gradient contributions. The Fermi surface contraction in Eq. (93) then yields

$$\eta_1^{(\text{Zn})} = -2.343 \times 10^{-7} \text{ eV s/m}.$$

The corresponding η_1 with the Γ_{ij} strain vertex alone was $\eta_1^\Gamma = 2.215 \times 10^{-7}$ eV s/m, Ξ_{ij} thereby affording a correction of 5.8% to the electron wind. This value lies within the expected range for an hcp metal with a comparatively sharp Fermi surface and larger deformation potentials than aluminium. At $J = 10^9$ A m⁻² the corresponding wind stress is

$$|\tau_e^{(\text{Zn})}| \approx 4.8 \text{ MPa.}$$

The two materials are therefore treated under an identical microscopic definition of the wind coefficient. Their numerical values differ solely due to the electronic structure encoded in the Fermi surface contractions and the distinct strength and sign structure of their deformation potential components.

2. Electronic drag

In the $q \rightarrow 0$ and $q_x \rightarrow 0$ limits the odd part of W vanishes and the even projector selects the homogeneous vertex. The zero temperature drag reduces to

$$B_{\text{el}}^{(0)} = \frac{\pi}{\hbar} G_\Lambda \left\langle \frac{\Gamma^2(\mathbf{k})}{v_F(\mathbf{k})} \delta(\hat{\mathbf{v}}_{\mathbf{k}} \cdot \hat{\mathbf{q}}) \right\rangle_{\text{FS}}, \quad (157)$$

$$G_\Lambda = \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{|\Lambda(\mathbf{q}_\perp, 0)|^2}{q_\perp}, \quad (158)$$

with Thomas-Fermi screening included in Λ as above.

For the $\frac{1}{2}\langle 110 \rangle \{111\}$ edge dislocation in aluminium, the homogeneous deformation potential extracted gives

$$\left\langle \frac{\Gamma^2}{v_F} \right\rangle_{\text{FS}} = (8.33 \pm 0.30) \times 10^{-43} \text{ J}^2 \text{ s m}^{-1},$$

and the screened Kanzaki form factor yields the geometric factor $G_\Lambda = 0.127527 \text{ \AA}$. This leads to the zero temperature electronic drag

$$B_{\text{el}}^{(0)} = (7.92 \pm 0.2) \times 10^{-6} \text{ Pa s}, \quad (159)$$

which is consistent with the classical measurement estimate by Hikata *et al.* [103], who reported $B_{\text{el}} = 1.4 \times 10^{-6} \text{ Pa s}$ for aluminium. As we shall see, temperature dependence (and experimental uncertainty) may help understand the discrepancy.

The zinc calculation follows the same structure but uses the discrete hcp Kanzaki forces constructed directly from full lattice Green function and the band resolved deformation potential discussed in Sec. V. For the basal $\frac{1}{3}\langle 11\bar{2}0 \rangle \{0001\}$ edge dislocation, the discrete force field produces a well conditioned geometric projector with

$$G_\Lambda = 0.13367 \text{ \AA},$$

about two orders of magnitude larger than the aluminium value because of the stronger screening and the more extended near core field in the hcp geometry. The Fermi surface contraction computed on the same 9600 k-point grid as used for the wind is

$$\left\langle \frac{\Gamma^2}{v_F} \right\rangle_{\text{FS}} = 6.05 \times 10^{-44} \text{ J}^2 \text{ s m}^{-1},$$

reflecting the larger deformation potentials and reduced Fermi velocity on the basal sheets. Substitution into Eq. (74) gives

$$B_{\text{el}}^{(0)} = 2.67 \pm 0.22 \times 10^{-7} \text{ Pa s}, \quad (160)$$

This scaling follows directly from the relative magnitudes of G_Λ and the Fermi surface contraction, both of which are significantly larger for zinc. The drag in zinc is therefore dominated by the strong inhomogeneous deformation potential and the larger geometrical weight of the basal Kanzaki field. Experimental estimates of the electronic drag coefficient for Zn have never been made; although Jassby and Vreeland [104] reported values of $3.2\text{--}3.5 \times 10^{-5} \text{ Pa s}$ for the drag coefficient in Zn at cryogenic temperatures 44 K, it seems clear that this data is polluted by phonon drag effects too, as the figure corresponds too closely for comfort with the values of the net drag coefficient for Zn reported at room temperature $\approx 3 \times 10^{-5} \text{ Pa s}$ [105].

Thermal dependencies. The thermal dependence of the electronic drag coefficient is considerably weaker than that of, e.g., the phonon sector's (q.v., Ref. [77]). To quantify it, we calculated the $B_{\text{el}}(T)$ coefficient [see Eq. (108)] from first principles as follows. At finite temperature, the δ function in Eq. (147) is replaced by the thermally broadened derivative of the Fermi function. More precisely, we introduce the finite-temperature weight

$$w(\varepsilon; T) = -\frac{\partial f(\varepsilon; T)}{\partial \varepsilon} = \frac{1}{4k_B T} \text{sech}^2\left(\frac{\varepsilon}{2k_B T}\right), \quad (161)$$

with

$$f(\varepsilon; T) = \frac{1}{1 + e^{\varepsilon/k_B T}},$$

and define the thermal deformation potential functional

$$\mathcal{I}(T) = \frac{\sum_{n,\mathbf{k}} \frac{\Gamma_{n\mathbf{k}}^2}{v_{F,n\mathbf{k}}} w(\varepsilon_{n\mathbf{k}} - E_F; T)}{\sum_{n,\mathbf{k}} w(\varepsilon_{n\mathbf{k}} - E_F; T)}. \quad (162)$$

The sums run over the Brillouin zone mesh and over all bands in a small window about the Fermi level, using the same eigenvalues $\varepsilon_{n\mathbf{k}}$, velocities $\mathbf{v}_{n\mathbf{k}}$ and slip projected deformation potentials $\Gamma_{n\mathbf{k}}$ that enter the zero temperature calculation. The ratio in Eq. (162) is a normalized thermal Fermi surface average of Γ^2/v_F with respect to the weight $w(\varepsilon; T)$, and is insensitive to the overall k-space measure.

Within this framework, the finite-temperature drag coefficient can be written as

$$B_{\text{el}}(T) = \frac{\pi}{\hbar} G_\Lambda \mathcal{I}(T) = B_{\text{el}}^{(0)} \frac{\mathcal{I}(T)}{\mathcal{I}(0)}. \quad (163)$$

The geometric factor G_Λ drops out of the ratio, so that the entire temperature dependence is governed by the electronic functional $\mathcal{I}(T)$. In practice, we evaluate $\mathcal{I}(T)$ on the same k-mesh used for the zero temperature drag, for a set of temperatures T_i up to a small fraction of the Fermi scale, and form the normalized ratio

$$R(T) = \frac{\mathcal{I}(T)}{\mathcal{I}(0)} = \frac{B_{\text{el}}(T)}{B_{\text{el}}^{(0)}}. \quad (164)$$

This offers the exact values of $B_{\text{el}}(T)$, which we may compare to those afforded by the Sommerfeld expansion of the Fermi weight implies

$$R(T) = 1 + \alpha_2 \left(\frac{k_B T}{E_F}\right)^2 + O(T^4), \quad (165)$$

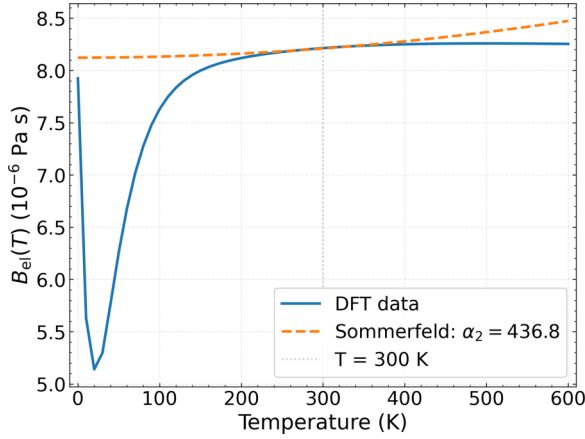


FIG. 1. Calculated vs Sommerfeld expansion (at 300 K) of the thermal dependence of the electronic drag coefficient for an edge dislocation in Al.

with a dimensionless coefficient α_2 determined by the curvature of the Fermi surface and the energy dependence of the deformation potential. We obtain α_2 by fitting $R(T)$ to the form (165) in the low-temperature regime $k_B T \ll E_F$, using the numerically computed values of $\mathcal{I}(T)$ and $\mathcal{I}(0)$ from Eq. (162). Since G_Λ cancels in (163), this procedure yields a parameter free prediction for the thermal correction to B_{el} that is entirely controlled by the microscopic band velocities and deformation potentials already computed in Sec. V.

Figures 1 and 2 show the computed thermal dependence of the drag coefficient for Al and Zn, respectively. As can be seen, the thermal dependence of the electronic drag coefficient is considerably weaker than that of the phonon sector (q.v., Ref. [77]). In our formulation, the temperature enters only through the thermally broadened Fermi surface average [Eq. (162)] so that $B_{el}(T) = B_{el}(0) I(T)/I(0)$. At strictly zero temperature the kernel $-\partial_\epsilon f$ collapses to a δ function and $I(0)$ samples only the few $\mathbf{k}$ points that happen to fall within a very narrow energy window around E_F . For $T > 0$ the kernel broadens over an energy scale of order $k_B T$ and $I(T)$ becomes a shell average of Γ^2/v_F over a neighbourhood of the

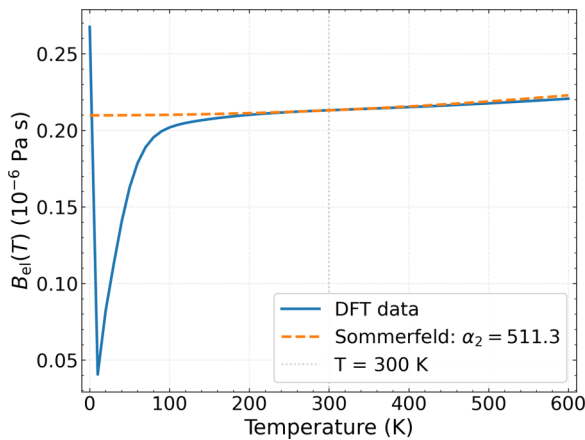


FIG. 2. Calculated vs Sommerfeld expansion (at 300 K) of the thermal dependence of the electronic drag coefficient for a basal edge dislocation in Zn.

Fermi level. In the Zn calculation this shell average increases slightly as the window is widened from $|\epsilon - E_F| \lesssim 10$ meV to ~ 50 – 100 meV, so that $I(T)$ and hence $B_{el}(T)$ initially rise and display a shallow maximum around $T \simeq 50$ – 100 K. At higher temperatures the kernel samples states where Γ^2/v_F is again smaller and $B_{el}(T)$ decreases toward a plateau. This weak non monotonicity is therefore a consequence of the combination of discrete Fermi surface sampling at $T = 0$ and the mild energy dependence of Γ^2/v_F near E_F , and it affects $B_{el}(T)$ only at the level of order 10^{-2} up to $T = 500$ K. The electronic drag may thus be regarded as effectively temperature independent on experimental scales, with the observed temperature dependence of the total drag dominated by phonons. Within the accuracy of the present first principles calculation, the electronic drag may therefore be regarded as temperature independent on experimental scales, and any pronounced temperature dependence of the total dislocation drag must be attributed to the phonon sector rather than to the electronic contribution.

The same computational strategy applies to the current driven wind coefficient. The zero temperature expression in Eq. (93) can be rewritten in terms of the total slip projected vertex V_{nk} (including both homogeneous and gradient contributions, cf. Sec. V) as

$$\eta_1^{(0)} = -\frac{1}{e} \frac{\langle V_{nk} v_{\parallel, nk} \rangle_{FS}}{\langle v_{\parallel, nk}^2 \rangle_{FS}}, \quad (166)$$

with $v_{\parallel, nk} = \hat{\mathbf{b}} \cdot \mathbf{v}_{nk}$ the velocity component along the Burgers direction. At finite temperature we again replace the sharp Fermi surface by the thermally broadened weight $w(\epsilon; T)$ defined above, and introduce the transport functionals

$$\mathcal{N}(T) = \frac{\sum_{n, \mathbf{k}} V_{nk} v_{\parallel, nk} w(\epsilon_{nk} - E_F; T)}{\sum_{n, \mathbf{k}} w(\epsilon_{nk} - E_F; T)}, \quad (167)$$

$$\mathcal{D}(T) = \frac{\sum_{n, \mathbf{k}} v_{\parallel, nk}^2 w(\epsilon_{nk} - E_F; T)}{\sum_{n, \mathbf{k}} w(\epsilon_{nk} - E_F; T)}. \quad (168)$$

These are thermal Fermi surface averages of the numerator and denominator in the wind functional, evaluated on the same Brillouin zone mesh and within the same energy window as in the $T = 0$ calculation. The finite-temperature wind coefficient then follows as

$$\eta_1(T) = -\frac{1}{e} \frac{\mathcal{N}(T)}{\mathcal{D}(T)} = \eta_1^{(0)} \frac{\mathcal{N}(T)/\mathcal{D}(T)}{\mathcal{N}(0)/\mathcal{D}(0)}, \quad (169)$$

so that the entire temperature dependence is again contained in the dimensionless ratio

$$S(T) = \frac{\eta_1(T)}{\eta_1^{(0)}} = \frac{\mathcal{N}(T)/\mathcal{D}(T)}{\mathcal{N}(0)/\mathcal{D}(0)}. \quad (170)$$

In practice, we evaluate $\mathcal{N}(T)$ and $\mathcal{D}(T)$ numerically on the DFT $\mathbf{k}$ -mesh for a discrete set of temperatures T_i , using the same V_{nk} and velocities $\mathbf{v}_{nk}$ as in the $T = 0$ wind calculation, and form $S(T_i)$ from Eq. (170).

In the low-temperature regime $k_B T \ll E_F$ the Sommerfeld expansion gives a quadratic correction,

$$S(T) = 1 + \alpha_2 \left(\frac{k_B T}{E_F} \right)^2 + O(T^4), \quad (171)$$

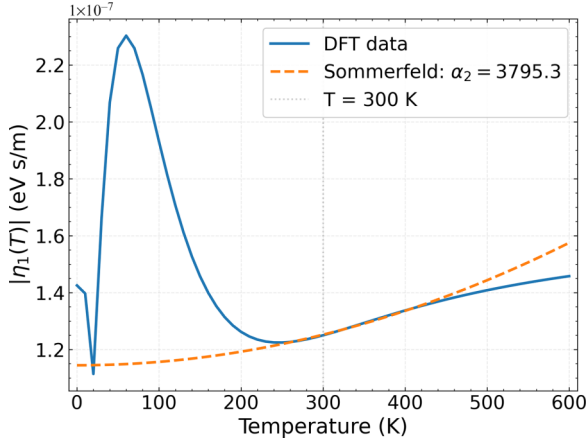


FIG. 3. Calculated vs Sommerfeld fit (at 300 K) of the thermal dependence of the η_1 coefficient for an edge dislocation in Al.

with a dimensionless coefficient α_2 governed by the energy dependence of $V_{n\mathbf{k}}$ and $v_{\parallel,n\mathbf{k}}$ in a narrow neighbourhood of the Fermi level. In the present work, we do not impose Eq. (171) as a global fit over the full temperature range; instead we use it as a local parametrization of the numerically computed $S(T)$ either in the strict low-temperature regime or around a representative temperature such as $T = 300$ K, by matching the quadratic coefficient to $S(T)$ at that point. In this way, the Sommerfeld coefficients for both for the drag and for the wind can be viewed as convenient measures of the curvature of the exact $B_{\text{el}}(T)$ and $\eta_1(T)$ curves near the temperatures of interest, rather than as asymptotic laws that must hold at all T .

The resulting temperature evolution of η_1 for Al and Zn is shown in Figs. 3 and 4. In both cases the variation of η_1 with T is extremely weak, and the residual curvature reflects only subtle material specific structure in the Fermi surface averages without altering the overall scale of the wind coefficient. On experimental temperature ranges η_1 can therefore be regarded as effectively constant, so that current-induced configurational forces are governed by a single, nearly temperature independent electronic parameter. This implies that the strength of the electron wind, and hence the magnitude of electroplastic

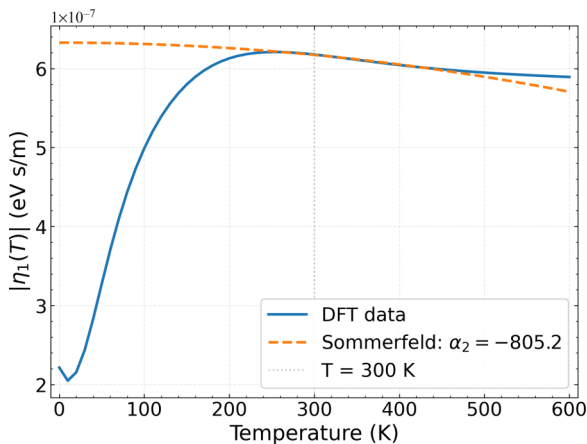


FIG. 4. Calculated vs Sommerfeld fit of the thermal dependence of the electronic drag coefficient for a basal edge dislocation in Zn.

softening at fixed current density, is set primarily by the underlying electronic structure rather than by details of the thermal environment.

C. Crossover temperature between electronic and phonon drag in Al and Zn

Having computed the electronic drag coefficient from first principles, we can determine the temperature T^* below which electronic dissipation becomes comparable to phonon scattering. This crossover is defined by

$$B_{\text{el}}(T^*) = B_{\text{ph}}(T^*). \quad (172)$$

From the Keldysh analysis with first principles vertices, the electronic drag coefficient is obtained as

$$B_{\text{el}}(T) = B_{\text{el}}^{(0)} \frac{\mathcal{I}(T)}{\mathcal{I}(0)}, \quad (173)$$

with $B_{\text{el}}^{(0)} = (7.92 \pm 0.2) \times 10^{-6}$ Pa s for the $\frac{1}{2}\langle 110 \rangle\{111\}$ edge dislocation in aluminium and $\mathcal{I}(T)$ the thermally broadened Fermi surface functional defined in Sec. V. In practice $\mathcal{I}(T)/\mathcal{I}(0)$ departs from unity only at the level of 10^{-2} up to several hundred kelvin, so that $B_{\text{el}}(T)$ is essentially temperature independent on experimental scales. By contrast, the phonon drag coefficient follows from the phonon Keldysh kernel as discussed in Ref. [77]. In the low-temperature Debye regime $T \ll \Theta_D$ one finds [32,43]

$$B_{\text{ph}}(T) \propto T^5, \quad (174)$$

because Bose statistics restrict the phase space for acoustic phonon emission, whereas at higher temperatures $T \gtrsim \Theta_D/3$ classical equipartition yields

$$B_{\text{ph}}(T) \propto T. \quad (175)$$

The crossover between electronic and phonon drag will therefore occur in the low-temperature Debye regime, where the phonon contribution is strongly suppressed.

For the purpose of estimating T^* it is convenient to use the low-temperature power laws as effective parametrizations and to calibrate their prefactors at a reference temperature T_0 . We write

$$B_{\text{el}}(T) \simeq \beta T^2, \quad B_{\text{ph}}(T) \simeq D T^5, \quad (176)$$

with β and D material specific coefficients, and determine β/D from the ratio at some T_0 in the regime where the Debye forms apply. Introducing

$$R_0 := \frac{B_{\text{el}}(T_0)}{B_{\text{ph}}(T_0)}, \quad (177)$$

Equation (176) gives

$$R_0 = \frac{\beta}{D} \frac{1}{T_0^3}. \quad (178)$$

At the crossover, Eq. (172) implies $\beta T^{*2} = D T^{*5}$, hence

$$T^{*3} = \frac{\beta}{D} = T_0^3 R_0, \quad (179)$$

and therefore the crossover temperature can be expressed directly in terms of R_0 as

$$T^* = T_0 R_0^{1/3}. \quad (180)$$

This representation avoids any attempt to interpret $B_{\text{el}}^{(0)}$ or $B_{\text{ph}}(0)$ as literal power law prefactors and instead uses the *ab initio* ratio at a temperature where both drags are known.

For fcc aluminium, our microscopic calculation of the electronic drag yields a room temperature value $B_{\text{el}}(300 \text{ K}) \simeq 3 \times 10^{-5} \text{ Pa s}$ once the thermally broadened functional $\mathcal{I}(T)$ of Sec. V is evaluated at $T = 300 \text{ K}$. The corresponding phonon contribution at room temperature, computed from the phonon Keldysh kernel in Ref. [77], is $B_{\text{ph}}(300 \text{ K}) \simeq 4.2 \times 10^{-1} \text{ Pa s}$. The *ab initio* ratio at $T_0 = 300 \text{ K}$ is therefore

$$R_0^{(\text{Al})} = \frac{B_{\text{el}}(300 \text{ K})}{B_{\text{ph}}(300 \text{ K})} \simeq \frac{3 \times 10^{-5}}{4.2 \times 10^{-1}} \approx 7 \times 10^{-5}. \quad (181)$$

Substituting Eq. (181) into Eq. (180) gives

$$T_{\text{Al}}^* \simeq 300 \text{ K} (7 \times 10^{-5})^{1/3} \approx 1.2 \times 10^1 \text{ K}, \quad (182)$$

so that a physically consistent estimate based on the present first principles coefficients is

$$T_{\text{Al}}^* \approx 10\text{--}15 \text{ K}, \quad (183)$$

well within the cryogenic regime, and within the range of quoted experimental values [18,106]. At $T = 300 \text{ K}$, Eq. (181) shows that phonon scattering dominates the viscous drag by roughly four orders of magnitude, reinforcing the conclusion that electronic dissipation plays no direct role in electroplastic effects at ambient conditions.

An analogous analysis can be carried out for zinc by combining the *ab initio* electronic drag $B_{\text{el}}^{(0)}$ obtained in Sec. V with the phonon drag from Ref. [77]. The resulting ratios $R_0^{(\text{Zn})}$ are again very small, and Eq. (180) then yields crossover temperatures in the sub 10 K range, compatible with and generally below the 0.1 to 10 K window inferred from Brailsford's phenomenological estimates [18]. In both fcc Al and hcp Zn the electronic drag therefore becomes comparable to phonon drag only in deep cryogenic conditions.

The hierarchy between electronic and phonon drag is in fact generic. The magnitude of $B_{\text{el}}^{(0)}$ is controlled by Fermi surface averages of deformation potentials and we would expect it to typically lie around 10^{-6} Pa s in pure transition metals, whereas phonon drag coefficients at room temperature fall in the range 10^{-4} to 10^{-3} Pa s across fcc and bcc systems [77]. Combining these magnitudes with the generic T^2 and T^5 (or T) scalings shows that $B_{\text{el}}(300 \text{ K})/B_{\text{ph}}(300 \text{ K})$ is typically between 10^{-3} and 10^{-4} , and Eq. (180) then places the crossover temperature below 100 K even in the most favourable case. The *ab initio* aluminium calculation, Eq. (181), sits at the lower end of this range and pushes T^* down to the $\sim 10 \text{ K}$ scale. This universality ensures that, throughout the temperature window relevant for electroplastic experiments, the electronic contribution to the viscous drag is parametrically small compared with the phonon contribution.

A direct consequence of the mobility law is that any nonequilibrium electron wind term accelerates plastic flow at fixed applied stress through the Orowan relation. Since $\dot{\gamma} = \rho b v_d$, an increment in v_d produced by a current driven configurational force increases the plastic strain rate at constant resolved shear stress τ . In a controlled stress loading protocol this enhanced $\dot{\gamma}$ relaxes the elastic strain more rapidly, so the measured shear stress must drop to maintain equilibrium

with the imposed boundary conditions. The magnitude of the drop is set by the same electronic contribution that enters v_d , because the back stress required to stabilize the system simply compensates the additional wind-induced configurational drive. This provides a clear macroscopic signature of the nonequilibrium electronic forcing and is entirely independent of any microscopic detail of the dislocation core. Within the present microscopic framework, the cryogenic character of T^* and the large room temperature separation between phonon and electronic drag imply that electroplastic effects in metals must arise from the configurational wind force rather than from any appreciable reduction of the viscous drag.

The same separation between configurational and dissipative sectors implies an immediate experimental protocol. Because the wind contribution is odd under $J \rightarrow -J$ whereas the viscous drag is even, the antisymmetric component of the measured glide velocity,

$$\frac{1}{2}[v_d(+J) - v_d(-J)],$$

isolates the wind coefficient η_1 directly, while the symmetric component probes the total drag. Current-polarity-dependent glide asymmetry in in situ microscopy (e.g., Ref. [28]) therefore constitutes a direct observable of the odd kernel derived here, largely insensitive of any specific core model.

In addition, an external magnetic field suppresses the electronic drag through orbital deflection while leaving the configurational wind unchanged to leading order. Measurements of $v_d(J, B)$ combining current reversal and field variation therefore separate even and odd sectors of the response, providing a magnetoplastic analog of magnetoresistance and enabling parameter free comparison with the present first principles kernels.

VI. CONCLUSIONS

We have addressed the longstanding problem of predicting electron wind forces and electronic drag on dislocations from first principles by embedding a moving dislocation in a conserving Keldysh field theory and integrating out the electronic and phononic baths. Within this framework configurational wind and viscous drag do not arise as independent mechanisms but appear as the odd and even components of a single retarded stress correlation kernel built from a common deformation potential vertex. This unification removes the conceptual ambiguity that has surrounded electron-dislocation coupling for many decades and places both effects on the same many-body footing as electronic transport coefficients such as conductivity, Hall response, and magnetoresistance.

The formalism yields several structural results that are fixed by symmetry and topology rather than by microscopic detail. The shear channel theorem that we have proven shows that, for gliding dislocations, the electronic drag couples only to the shear component of the electronic stress, while the bulk channel makes no contribution because the glide geometry and the $q_x = 0$ character of the Kanzaki strain field project out longitudinal distortions. This is enforced by the topological disregistry of the displacement field of the dislocation itself, and therefore applies to any crystal with a well defined Burgers vector and slip plane. The electron wind force

is rigorously polar: under spatial inversion the driving force reverses while the underlying lattice remains unchanged. Slip systems with opposite Burgers vectors therefore experience opposite wind forces under the same current, which provides a microscopic mechanism for current driven texture evolution and anisotropic hardening in electroplasticity. At the same time the dislocation core structure is shown to contribute only subleading corrections through higher multipoles of the Kanzaki projector, while the dominant contributions to wind and drag are controlled by the long range Volterra strain. This vindicates descriptions of plastic slip based on continuum elasticity, and shows that atomistic core resolution is not required for the quantitative prediction of electronic contributions to dislocation mobility.

The quantitative content of the theory follows from closed statistical mechanical expressions for both electronic drag and electron wind in terms of Fermi surface averages of deformation potentials, band velocities and the screened Kanzaki projector. These expressions contain no phenomenological relaxation times or adjustable masses, and we have thus shown that wind and drag, long treated as phenomenological fit parameters, can instead be computed directly as functionals of the electronic structure, thereby making them accessible via *ab initio* calculations. Using DFT, we have then evaluated them for representative systems, here fcc aluminium and hcp zinc; the resulting electronic drag at room temperature is several orders of magnitude smaller than the phonon drag, whereas the associated wind stresses at experimentally accessible current densities are of the same order as measured electroplastic softening stresses. This establishes that electronic drag cannot account for current-induced reductions in flow stress, while the configurational wind force has precisely the correct magnitude and symmetry to do so. The weak dependence of the wind coefficient on temperature, together with the very low crossover temperature at which electronic and phonon drag become comparable, reinforces this picture: throughout the temperature range of practical interest the electronic contribution to the viscous drag is negligible, while the configurational force remains controlled by a single, almost temperature independent electronic parameter. These results also provide experimentally testable signatures that go beyond fitting a mobility law. Because the wind contribution is odd under $J \rightarrow -J$ while the viscous response is even, current reversal isolates the configurational drive through the antisymmetric part of the measured glide response, while the symmetric part determines the drag. Moreover, the predicted magnetic-field suppression of the electronic drag, with the wind essentially unchanged to leading order, offers a magnetoplastic analog of magnetoresistance that separates dissipative and configurational sectors in a single protocol. Together, current-polarity tests and field-dependent measurements furnish a direct, parameter-free route for confronting the *ab initio* kernels with in situ observations of glide asymmetry and stress relaxation.

Extension of the Keldysh-Kanzaki construction to finite magnetic field reveals a magnetoplastic effect that is closely analogous to magnetoresistance in transport. The magnetic field suppresses electronic drag through cyclotron bending of electronic trajectories, so that the drag decreases quadratically with the cyclotron frequency and transport time, while the

configurational wind coefficient remains unchanged to leading order because it is controlled by a different symmetry channel of the same kernel. This parity separation, with drag even in field and wind effectively field independent, provides a sharp experimental diagnostic for electronic contributions to magnetoplasticity and predicts a measurable field dependence of dislocation mobility in sufficiently clean metals at laboratory magnetic fields.

The broader significance of this work lies in the bridge it establishes between quantum transport theory, crystal plasticity, and materials design. Dislocation mobility, electroplasticity, and magnetoplasticity are now expressed in terms of exactly the same deformation potential vertices and correlation functions that underpin electronic transport theory, so that electronic structure calculations can be used to predict both electronic and mechanical response on equal footing. The resulting parameter free coefficients can be fed directly into discrete dislocation dynamics, enabling quantitative predictions of current driven plastic flow without empirical adjustment. At the same time the Keldysh-Kanzaki formalism provides a rigorous quasi equilibrium reference against which proposed non equilibrium mechanisms such as transient electron-phonon pumping, strong field Fermi surface reconstruction or magnetically assisted depinning must be tested. Any electroplastic effect that significantly exceeds the stresses and drag corrections predicted here is therefore a direct indicator of physics beyond quasi equilibrium linear response and motivates explicit time-dependent extensions of the present theory. The construction is also exportable: by treating other extended defects as Kanzaki sources in the same conserving field theory, the mobility of grain boundaries under electromigration, magnetic domain walls under spin currents, or crack tips in semiconductors subject to electric fields can all be addressed within a common framework where configurational forces and dissipation emerge as different symmetry components of bath correlation functions. Combined with the analogous Keldysh-Kanzaki treatment of phonon drag, this yields a unified microscopic theory of dislocation mobility over the full temperature range.

In summary, we have transformed electronic dislocation drag and electron wind forces from phenomenological transport coefficients into first principles, computable quantities rooted in nonequilibrium quantum field theory. Our Keldysh framework establishes that plastic flow at the mesoscale is controlled by the same quantum correlation functions that govern electronic transport, and that their values can be predicted from electronic structure calculations with no adjustable parameters. This closes the gap between materials physics and crystal plasticity, opening a route to predictive theories of strength and ductility that trace macroscopic mechanical response back to microscopic quantum processes via *ab initio* calculations that can now predict the mechanical response of metals under combined electrical and mechanical loading, enabling materials optimization strategies that were previously inaccessible.

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DATA AVAILABILITY

The data that support the findings of this article are available from the author upon reasonable request.

APPENDIX A: SHEAR CHANNEL SELECTION THEOREM FOR DISLOCATION DRAG

1. Proposition

Let $X : \mathbb{R} \rightarrow \mathbb{R}$ be a glide history that is piecewise C^1 with bounded velocity and with $\dot{X}(t)$ nonzero except on a set of measure zero. Define the shift spectrum

$$S_X(\omega, \mathbf{q}) = \int_{-\infty}^{\infty} dt e^{i\omega t} e^{-iq_x X(t)}. \quad (\text{A1})$$

Consider a retarded quadratic functional

$$\mathcal{K}^R(\omega) = \sum_{\mathbf{q}} \mathcal{F}^*(\mathbf{q}) K^R(\mathbf{q}, \omega) \mathcal{F}(\mathbf{q}), \quad (\text{A2})$$

where $\mathcal{F}(\mathbf{q})$ is smooth and even in q_x , and $K^R(\mathbf{q}, \omega)$ is causal, even in q_x , real at $\omega = 0$, and analytic in ω near $\omega = 0$. Then

$$\begin{aligned} -\partial_{\omega} \text{Im } \mathcal{K}^R(\omega)|_{\omega=0} \\ = \sum_{\mathbf{q}_{\perp}} [-\partial_{\omega} \text{Im } K^R(\mathbf{q}_{\perp}, q_x=0, \omega)|_{\omega=0}] |\mathcal{F}(\mathbf{q}_{\perp}, 0)|^2 \Xi[X], \end{aligned} \quad (\text{A3})$$

with the history factor

$$\Xi[X] = 2\pi \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \frac{1}{|\dot{X}(t)|}. \quad (\text{A4})$$

In the special case of steady glide $X(t) = v_d t$ one has $\Xi[X] = 2\pi/|v_d|$ and hence

$$\begin{aligned} -\partial_{\omega} \text{Im } \mathcal{K}^R(\omega)|_{\omega=0} \\ = \frac{2\pi}{|v_d|} \sum_{\mathbf{q}_{\perp}} [-\partial_{\omega} \text{Im } K^R(\mathbf{q}_{\perp}, 0, \omega)|_{\omega=0}] |\mathcal{F}(\mathbf{q}_{\perp}, 0)|^2. \end{aligned} \quad (\text{A5})$$

2. Proof

Introduce a symmetric time window $w_T(t) = \Theta(T/2 - |t|)$ and the windowed spectrum

$$S_{X,T}(\omega, \mathbf{q}) = \int dt w_T(t) e^{i\omega t} e^{-iq_x X(t)}. \quad (\text{A6})$$

By causality and evenness in q_x ,

$$K^R(\mathbf{q}, \omega) = K_0(\mathbf{q}) + i\omega C(\mathbf{q}) + O(\omega^2), \quad (\text{A7})$$

with K_0, C real and even in q_x . Using $F^K(\mathbf{q}, \omega) = \mathcal{F}(\mathbf{q}) S_{X,T}(\omega, \mathbf{q})$ one obtains

$$\text{Im } \mathcal{K}^R(\omega) = \omega \sum_{\mathbf{q}} |\mathcal{F}(\mathbf{q})|^2 |S_{X,T}(\omega, \mathbf{q})|^2 C(\mathbf{q}) + O(\omega^3), \quad (\text{A8})$$

hence

$$-\partial_{\omega} \text{Im } \mathcal{K}^R(\omega)|_{\omega=0} = \sum_{\mathbf{q}} |\mathcal{F}(\mathbf{q})|^2 \lim_{\omega \rightarrow 0} |S_{X,T}(\omega, \mathbf{q})|^2 C(\mathbf{q}). \quad (\text{A9})$$

To evaluate $|S_{X,T}|^2$ without squared δ s, write

$$\begin{aligned} |S_{X,T}(\omega, \mathbf{q})|^2 \\ = \int dt \int dt' w_T(t) w_T(t') e^{i\omega(t-t')} e^{-iq_x [X(t)-X(t')]} \end{aligned} \quad (\text{A10})$$

Integrating over q_x first gives

$$\begin{aligned} \int \frac{dq_x}{2\pi} |S_{X,T}(\omega, \mathbf{q})|^2 \\ = \int dt \int dt' w_T(t) w_T(t') e^{i\omega(t-t')} \delta(X(t) - X(t')). \end{aligned} \quad (\text{A11})$$

For past histories that are locally monotone on the scale of the window, the identity

$$\delta(X(t) - X(t')) = \sum_{t': X(t')=X(t)} \frac{\delta(t-t')}{|\dot{X}(t')|} \quad (\text{A12})$$

reduces almost everywhere to $\delta(t-t')/|\dot{X}(t)|$, so that

$$\int \frac{dq_x}{2\pi} |S_{X,T}(\omega, \mathbf{q})|^2 = \int dt w_T(t)^2 \frac{1}{|\dot{X}(t)|}. \quad (\text{A13})$$

Since $w_T^2 = w_T$ for the rectangular window,

$$\lim_{\omega \rightarrow 0} \int \frac{dq_x}{2\pi} |S_{X,T}(\omega, \mathbf{q})|^2 = \int_{-T/2}^{T/2} dt \frac{1}{|\dot{X}(t)|}. \quad (\text{A14})$$

Returning to the $\mathbf{q}$ sum, smoothness and evenness of $\mathcal{F}$ and C allow Taylor expansion in q_x about $q_x = 0$. Dominated convergence then yields

$$\begin{aligned} \lim_{\omega \rightarrow 0} \int \frac{dq_x}{2\pi} |S_{X,T}(\omega, \mathbf{q})|^2 |\mathcal{F}(\mathbf{q})|^2 C(\mathbf{q}) \\ = \left[\int_{-T/2}^{T/2} dt \frac{1}{|\dot{X}(t)|} \right] \sum_{\mathbf{q}_{\perp}} |\mathcal{F}(\mathbf{q}_{\perp}, 0)|^2 C(\mathbf{q}_{\perp}, 0). \end{aligned} \quad (\text{A15})$$

Finally, divide both sides by T and take $T \rightarrow \infty$. This defines

$$\Xi[X] = 2\pi \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \frac{1}{|\dot{X}(t)|} \quad (\text{A16})$$

and gives the stated result once one recognizes $C(\mathbf{q}_{\perp}, 0) = -\partial_{\omega} \text{Im } K^R(\mathbf{q}_{\perp}, 0, \omega)|_{\omega=0}$. For steady glide, the integrand is constant and $\Xi[X] = 2\pi/|v_d|$. ■

3. Remarks

The argument shows that, after the q_x integration and the low-frequency limit, the dissipative slope depends only on the $q_x = 0$ value of the smooth factors for any admissible history, not only for uniform motion. No squared δ appears, and the history factor is the long-time average of $2\pi/|\dot{X}|$. If $\dot{X}$ vanishes on a set of nonzero measure the factor diverges, signaling that in the strict quasistatic limit the spectral weight accumulates at arbitrarily small q_x . In practice this is regularized by the smallest nonzero measuring frequency or by a microscopic momentum cutoff. The evenness in q_x of $\mathcal{F}$ and K^R is used only to eliminate odd contributions near $q_x = 0$ and to justify the Taylor replacement ($q_x \rightarrow 0$) under the $\mathbf{q}$ sum.

Physically speaking, the local monotonicity of $X(t)$, it essentially excludes pathological histories where the dislocation reverses direction infinitely often in a finite time or sits pinned for a macroscopic interval. The former case is trivially unphysical; the latter merely excludes from the discussion situations where the dislocation does not move, and therefore experiences no drag. Finally, the use of dominated convergence when pulling limits through the q_x integral and the $\mathbf{q}$ sum relies on the boundedness of coefficient $C(\mathbf{q})$ and the smoothness of the form factor $\mathcal{F}(\mathbf{q})$; again, these are natural in the physical setting, as $C(\mathbf{q})$ would represent the dissipative spectrum, and $\mathcal{F}(\mathbf{q})$ the spatial structure of the source.

4. Equivalence with the $\mathcal{P}$ - δ split

For steady glide the windowed Fourier transform gives

$$S_{X,T}(\omega, \mathbf{q}) = e^{i(\omega - q_x v_d)T/2} \frac{2 \sin[(\omega - q_x v_d)T/2]}{\omega - q_x v_d}. \quad (\text{A17})$$

As $T \rightarrow \infty$ this converges in the sense of distributions to

$$\mathcal{P}\left(\frac{1}{\omega - q_x v_d}\right) - i\pi \delta(\omega - q_x v_d). \quad (\text{A18})$$

At fixed small ω the imaginary part is the $-i\pi \delta(\omega - q_x v_d)$ term, and taking $\omega \rightarrow 0$ forces $q_x = 0$. In the equivalent spatial representation one uses

$$(q_x + i0^+)^{-1} = \mathcal{P}\left(\frac{1}{q_x}\right) - i\pi \delta(q_x). \quad (\text{A19})$$

For an even envelope in q_x the principal value contribution drops out, and in the low-frequency limit the remaining spectral weight projects the smooth factors onto $q_x = 0$, in agreement with the derivation above. The conclusion holds for both phonon and electronic kernels because the Kanzaki form factor supplies the same $(q_x + i0^+)^{-1}$ structure and the low-frequency reduction projects the smooth factors onto $q_x = 0$ whenever K^R is even in q_x by symmetry.

APPENDIX B: ELECTRONIC DRAG IN THE LOW-FREQUENCY LIMIT

In the electronic sector the microscopic stress tensor operator $\hat{\sigma}_{ij}(\mathbf{r}, t)$ gives rise to the four-index retarded stress-stress correlator

$$\Pi_{ij,kl}^R(\mathbf{q}, \omega) = -i \int_0^\infty dt e^{i\omega t} \langle [\hat{\sigma}_{ij}(\mathbf{q}, t), \hat{\sigma}_{kl}(-\mathbf{q}, 0)] \rangle. \quad (\text{B1})$$

Hermiticity of the stress operator implies the usual symmetry of retarded functions,

$$\Pi_{ij,kl}^R(\mathbf{q}, \omega)^* = \Pi_{ij,kl}^R(\mathbf{q}, -\omega), \quad (\text{B2})$$

so that the real part is even and the imaginary part odd in ω . In particular,

$$\text{Im } \Pi_{ij,kl}^R(\mathbf{q}, 0) = 0, \quad (\text{B3})$$

and the Taylor expansion about $\omega = 0$ has no constant term,

$$\text{Im } \Pi_{ij,kl}^R(\mathbf{q}, \omega) = \omega A_{ij,kl}(\mathbf{q}) + O(\omega^3), \quad (\text{B4})$$

where $A_{ij,kl}(\mathbf{q})$ is real and even in $\mathbf{q}$.

The electronic contribution to the drag kernel that appears in the influence functional is

$$\mathcal{D}_{\text{el}}^R(\omega) = \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \Pi_{ij}^R(\mathbf{q}, \omega) \Lambda_j(\mathbf{q}), \quad (\text{B5})$$

where

$$\Lambda_i(\mathbf{q}) = G_{ij}(\mathbf{q}, 0) F_j^K(\mathbf{q}) \varepsilon^{-1}(\mathbf{q}, 0) \Psi(\mathbf{q}) \quad (\text{B6})$$

is the screened Kanzaki slip vertex obtained from the discrete harmonic lattice and the slip form factor, and $\Pi_{ij}^R(\mathbf{q}, \omega)$ is the effective two-index stress kernel entering the glide problem. Operationally, Π_{ij}^R is the projection of the full four-index correlator Eq. (B1) onto the strain pattern generated by the vertex,

$$\Pi_{ij}^R(\mathbf{q}, \omega) = \mathcal{M}_{ij,kl}(\hat{\mathbf{q}}) \Pi_{kl,mn}^R(\mathbf{q}, \omega) \mathcal{N}_{mn}(\hat{\mathbf{q}}), \quad (\text{B7})$$

where $\mathcal{M}$ and $\mathcal{N}$ are geometric tensors (built from the glide geometry and the Kanzaki construction) that encode how the electronic stress couples to the slip-induced strain. The detailed form of $\mathcal{M}$ and $\mathcal{N}$ is not needed here; what matters is their behavior in the long-wavelength limit, which is controlled by symmetry.

1. Small-wave-vector dominance and electronic hydrodynamics

The Brillouin zone sum in Eq. (B5) is dominated by long-wavelength contributions $|\mathbf{q}| \ll k_F$. This follows from the structure of the vertex Eq. (B6). In the small- q regime the acoustic projection of the lattice Green function behaves as

$$G_{ij}(\mathbf{q}, 0) \simeq -\frac{\Pi_{ij}^{\text{ac}}(\hat{\mathbf{q}})}{\rho c^2(\hat{\mathbf{q}}) q^2}, \quad (\text{B8})$$

with $\Pi_{ij}^{\text{ac}}(\hat{\mathbf{q}}) = \delta_{ij} - \hat{q}_i \hat{q}_j$ the acoustic projector and $c(\hat{\mathbf{q}})$ the appropriate sound speed. Static metallic screening gives $\varepsilon^{-1}(\mathbf{q}, 0) = q^2/(q^2 + k_{\text{TF}}^2)$. The screened slip vertex therefore has the schematic form

$$\Lambda_i(\mathbf{q}) \sim \frac{\Pi_{ij}^{\text{ac}}(\hat{\mathbf{q}})}{q^2 + k_{\text{TF}}^2} F_j^K(\mathbf{q}) \Psi(\mathbf{q}), \quad (\text{B9})$$

with $\Psi(\mathbf{q})$ the discrete slip form factor. The near-core regularization implies that $\Psi(\mathbf{q})$ decays once $|\mathbf{q}| \gtrsim a_{\text{core}}^{-1}$, while screening suppresses $|\mathbf{q}| \gg k_{\text{TF}}$. As a result

$$|\Lambda_i(\mathbf{q})|^2 \text{ is strongly peaked for } |\mathbf{q}| \lesssim q_c, \quad (\text{B10})$$

$$q_c \sim \max(k_{\text{TF}}, a_{\text{core}}^{-1}), \quad (\text{B11})$$

and the combination $(q_x)^2 |\Lambda(\mathbf{q})|^2$ decays rapidly outside this region. Since in simple metals $k_{\text{FA}} \gg 1$, one has $q_c \ll k_F$ and

the dominant wave vectors lie deep in the long-wavelength sector where the conduction electrons may be treated as an isotropic hydrodynamic fluid.

In this regime rotational invariance fixes the tensor structure of $A_{ij,kl}$ in Eq. (B4) uniquely in terms of a bulk and a shear viscosity,

$$A_{ij,kl}(\mathbf{q} \rightarrow 0) = \zeta_e(T_e) \mathcal{P}_{ij,kl}^{(\text{bulk})} + \eta_e(T_e) \mathcal{P}_{ij,kl}^{(\text{shear})}, \quad (\text{B12})$$

with the projectors

$$\mathcal{P}_{ij,kl}^{(\text{bulk})} = \frac{1}{3} \delta_{ij} \delta_{kl}, \quad \mathcal{P}_{ij,kl}^{(\text{shear})} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) - \frac{1}{3} \delta_{ij} \delta_{kl}. \quad (\text{B13})$$

These projectors describe the decomposition of the *electronic* stress response into irreducible representations of the rotation group. The lattice discreteness and slip geometry enter only through the vertex and the geometric tensors $\mathcal{M}$ and $\mathcal{N}$ in Eq. (B7); for $|\mathbf{q}| \ll k_F$ the electronic sector itself is well described by continuum hydrodynamics. The shear viscosity is given by the Kubo relation

$$\eta_e(T_e) = - \lim_{\omega \rightarrow 0} \frac{1}{\omega V} \text{Im} \Pi_{xy,xy}^R(\mathbf{q}=0, \omega), \quad (\text{B14})$$

and contains the full temperature and scattering dependence of the electronic stress relaxation.

2. Vertex contraction and projection onto the shear channel

Contracting Eq. (B4) with the tensors that encode the glide geometry gives the effective two-index kernel

$$\text{Im} \Pi_{ij}^R(\mathbf{q}, \omega) = \omega B_{ij}(\hat{\mathbf{q}}) + O(\omega^3), \quad (\text{B15})$$

with

$$B_{ij}(\hat{\mathbf{q}}) = \mathcal{M}_{ij,kl}(\hat{\mathbf{q}}) A_{kl,mn}(\mathbf{q} \rightarrow 0) \mathcal{N}_{mn}(\hat{\mathbf{q}}), \quad (\text{B16})$$

and the electronic drag kernel becomes

$$\text{Im} \mathcal{D}_{\text{el}}^R(\omega) = \omega \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) B_{ij}(\hat{\mathbf{q}}) \Lambda_j(\mathbf{q}) + O(\omega^3). \quad (\text{B17})$$

Using the hydrodynamic decomposition Eq. (B12) one may write B_{ij} as

$$B_{ij}(\hat{\mathbf{q}}) = \zeta_e(T_e) B_{ij}^{(\text{bulk})}(\hat{\mathbf{q}}) + \eta_e(T_e) B_{ij}^{(\text{shear})}(\hat{\mathbf{q}}), \quad (\text{B18})$$

where $B^{(\text{bulk})}$ and $B^{(\text{shear})}$ are dimensionless, real tensors that depend only on the direction $\hat{\mathbf{q}}$ and on the slip geometry through $\mathcal{M}$ and $\mathcal{N}$. The bulk and shear viscosities therefore enter the drag kernel only through the amplitudes set by these geometric tensors.

The glide geometry now provides the selection rule. In the continuum, long-wavelength limit the Kanzaki construction for a crystallographic dislocation produces an isochoric slip discontinuity: the displacement jump across the slip plane is tangential and the associated strain field has vanishing trace when averaged over a region large compared to the core. Uniform hydrostatic strain, by contrast, changes the volume but does not introduce resolved shear on the slip system. These two deformations belong to orthogonal irreducible representations of the rotation group (scalar versus traceless tensor). As a result, the mixed response at long wavelength between the glide coordinate X and a homogeneous hydrostatic strain

$u_h = \frac{1}{3} u_{kk}$ vanishes:

$$\left. \frac{\partial^2 E_{\text{el}}}{\partial X \partial u_h} \right|_{u=0} = 0. \quad (\text{B19})$$

Equivalently, the tensors $\mathcal{M}$ and $\mathcal{N}$ in Eq. (B7) have no overlap with the bulk projector in the long-wavelength limit,

$$\mathcal{M}_{ij,kl}(\hat{\mathbf{q}}) \mathcal{P}_{kl,mn}^{(\text{bulk})} \mathcal{N}_{mn}(\hat{\mathbf{q}}) = 0, \quad |\mathbf{q}| \rightarrow 0, \quad (\text{B20})$$

so that the bulk viscous contribution is projected out of B_{ij} . Inserting Eqs. (B12) and (B20) into Eq. (B18) yields

$$B_{ij}(\hat{\mathbf{q}}) = \eta_e(T_e) \mathcal{P}_{ij}(\hat{\mathbf{q}}), \quad (\text{B21})$$

where $\mathcal{P}_{ij}(\hat{\mathbf{q}})$ is a dimensionless two-index projector onto the shear component that couples to glide. It is transverse to $\mathbf{q}$, traceless, and depends only on the geometry of the slip system and on the direction of $\mathbf{q}$.

Combining Eqs. (B17) and (B21) shows that the bulk viscosity drops out of the effective kernel. The low-frequency imaginary part of the electronic contribution reduces to

$$\begin{aligned} \text{Im} \mathcal{D}_{\text{el}}^R(\omega) &= -\omega \eta_e(T_e) \sum_{\mathbf{q}} (q_x)^2 \Lambda_i^*(\mathbf{q}) \mathcal{P}_{ij}(\hat{\mathbf{q}}) \Lambda_j(\mathbf{q}) \\ &\quad + O(\omega^3), \end{aligned} \quad (\text{B22})$$

or, equivalently, one may write the projected two-index correlator as

$$\text{Im} \Pi_{ij}^R(\mathbf{q}, \omega) = -\omega \eta_e(T_e) \mathcal{P}_{ij}(\hat{\mathbf{q}}) + O(\omega^3). \quad (\text{B23})$$

Equation (B23) makes explicit the two points used in the main text. First, causality and time reversal invariance force $\text{Im} \Pi_{ij}^R$ to vanish at $\omega = 0$, so that only the linear term in ω survives. Second, once the hydrostatic component has been eliminated by the glide geometry and the small-wave-vector dominance, the low-frequency electronic drag kernel is proportional to the shear viscosity $\eta_e(T_e)$ alone. Bulk viscosity does not couple to the glide coordinate in this limit.

APPENDIX C: EVALUATION OF THE ELECTRONIC VERTICES IN SPR-KKR

This Appendix details the computation of the homogeneous deformation potential Γ_{ij} and the strain gradient vertex $\Xi_{j\ell}$ using the relativistic multiple-scattering Green-function formalism implemented in SPR-KKR. Both quantities were evaluated for fcc aluminium within the PBE exchange-correlation functional, using the same computational settings to ensure internal consistency and to allow comparison with the plane wave DFT results reported in Sec. V.

1. Homogeneous deformation potential Γ_{ij}

The tensor Γ_{ij} quantifies the linear shift of electronic eigenenergies with respect to homogeneous strain. Within SPR-KKR, it was obtained by imposing small symmetric distortions of the cubic lattice and monitoring the change of the Fermi energy after full self-consistency. Calculations were performed in the atomic-sphere approximation using a $32 \times 32 \times 32$ Monkhorst-Pack $\mathbf{k}$ -mesh, angular momentum cutoff $l_{\text{max}} = 3$, and an energy contour discretized with 32 complex

points. The Fermi level was determined from the integrated density of states with a Gaussian smearing of 0.1 eV.

For each independent strain component, the lattice vectors were rescaled by $1 \pm \varepsilon$, with ε in the range 10^{-5} – 10^{-3} . The Fermi-level shift $\Delta E_F(\varepsilon)$ was extracted from the self-consistent potential. The deformation potential was then computed as the central finite difference

$$\Gamma_{ij}(\varepsilon) = \frac{E_F(+\varepsilon_{ij}) - E_F(-\varepsilon_{ij})}{2\varepsilon}. \quad (\text{C1})$$

The internal convergence was checked by halving ε , refining the $\mathbf{k}$ -mesh to 40^3 , and recomputing the potential from an unconstrained restart. The relative variation of Γ_{xx} was below 3%, confirming the numerical linearity of the strain response so long as $\varepsilon < 10^{-3}$.

2. Strain gradient vertex $\Xi_{j\ell}$

The strain gradient vertex $\Xi_{j\ell}$ describes the sensitivity of the electronic bands to acoustic strain gradients in the long-wavelength limit. Although the SPR-KKR code does not contain an explicit DFPT module, the quantity can be obtained directly from frozen phonon supercell calculations exploiting the linear relation

$$\Xi_{j\ell}(\mathbf{k}) = \left. \frac{\partial \varepsilon_{n\mathbf{k}}}{\partial q_\ell} \right|_{q \rightarrow 0}.$$

To this end, we constructed supercells commensurate with acoustic modulations of wave vector $\mathbf{q}_\ell = (\pm 2\pi/L_\ell)\hat{\mathbf{e}}_\ell$, where $L_\ell = 3$ or 4 primitive translations. Atomic displacements followed the pattern

$$\mathbf{u}(\mathbf{R}) = u_0 \hat{\mathbf{e}}_j \sin(\mathbf{q}_\ell \cdot \mathbf{R}), \quad u_0 = 5 \times 10^{-3} - 10^{-2} \text{ \AA}.$$

For each direction ℓ and sign of $\mathbf{q}_\ell$, self-consistent SPR-KKR calculations were carried out using the same energy contour,

potential parameters, and $\mathbf{k}$ -mesh density (folded to the supercell Brillouin zone). The eigenvalues $\varepsilon_{n\mathbf{k}}(\pm \mathbf{q}_\ell)$ were unfolded onto the primitive $\mathbf{k}$ -mesh by phase matching of the Bloch states. The derivative in Eq. (C2) was then formed as a central difference,

$$\Xi_{j\ell}(\mathbf{k}) = \frac{\varepsilon_{n\mathbf{k}}(+\mathbf{q}_\ell) - \varepsilon_{n\mathbf{k}}(-\mathbf{q}_\ell)}{2|\mathbf{q}_\ell|}. \quad (\text{C2})$$

The response was projected onto longitudinal and transverse acoustic polarizations using dynamical-matrix projectors $\Pi_{jm}^{\text{ac}}(\hat{\mathbf{q}}_\ell)$ obtained from independent DFPT calculations. Equivalent directions $\langle 100 \rangle$, $\langle 110 \rangle$, and $\langle 111 \rangle$ were symmetrized to suppress residual anisotropy.

Convergence with respect to both u_0 and $|\mathbf{q}_\ell|$ was verified by halving their magnitudes; deviations in Ξ_{xx} remained below 10%. Increasing the unfolded $\mathbf{k}$ -mesh to 40^3 changed Ξ_{xx} by less than 5%. Numerical isotropy was confirmed by evaluating

$$\Delta_{\text{aniso}} = \frac{\sqrt{\sum_{i \neq j} \Xi_{ij}^2}}{\frac{1}{3} \sum_i \Xi_{ii}} < 0.02,$$

implying that $\Xi_{j\ell}$ is effectively scalar for fcc aluminium.

3. Fermi surface contractions and consistency

The Fermi surface contractions entering Eqs. (147) and (98) were computed directly within SPR-KKR by integrating the relevant vertex products over the complex-energy contour,

$$\left\langle \frac{A(\mathbf{k})}{v_F(\mathbf{k})} \right\rangle_{\text{FS}} = \frac{\sum_{\mathbf{k}} A(\mathbf{k}) \delta(\varepsilon_{\mathbf{k}} - E_F)/v_F(\mathbf{k})}{\sum_{\mathbf{k}} \delta(\varepsilon_{\mathbf{k}} - E_F)},$$

using Gaussian smearing with width 0.01 eV. Here $A(\mathbf{k})$ represents either Γ^2 or $\Gamma \Xi$.

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$$\delta H_{el} = \int d^3r [\hat{\sigma}_{ij}(\mathbf{r}) \varepsilon_{ij}(\mathbf{r}) + \hat{\mu}_{ij\ell}(\mathbf{r}) \partial_\ell u_j(\mathbf{r})],$$

then the corresponding operators are

$$\begin{aligned} \hat{\sigma}_{ij}(\mathbf{q}) &= \sum_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}/2}^\dagger \Gamma_{ij}(\mathbf{k}) c_{\mathbf{k}-\mathbf{q}/2}, \\ \hat{\mu}_{ij\ell}(\mathbf{q}) &= \sum_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}/2}^\dagger \Xi_{ij\ell}(\mathbf{k}) c_{\mathbf{k}-\mathbf{q}/2}. \end{aligned}$$

The quantity $\hat{\mu}_{ij\ell}$ may be regarded as a moment-stress or hyperstress operator associated with the electronic response to inhomogeneous strain. In the long-wavelength limit it is sometimes useful to combine the two vertices into a single effective coupling

$$\Gamma_{ij}(\mathbf{k}) + iq_\ell \Xi_{ij\ell}(\mathbf{k}),$$

but conceptually the homogeneous stress and the gradient induced hyperstress remain distinct and only their particular acoustic projections enter the scalar line vertex W and the band vertex V used later in the text.

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$$\begin{aligned} \langle V^2(\mathbf{k}) (\hat{\mathbf{J}} \cdot \mathbf{v}(\mathbf{k}))^2 / v_F^3 \rangle_{\text{FS}} &= \frac{V^2}{v_F} \frac{1}{3}, \\ 3 \langle V^2(\mathbf{k}) / v_F \rangle_{\text{FS}} &= 3 \frac{V^2}{v_F}, \end{aligned}$$

so that $\eta_2^{\text{iso}} = (1/3)/3 = 1/9$. If one prefers to normalize η_2 so that the isotropic limit gives 1/3, the prefactor 3 in the denominator of Eq. (98) can be absorbed into the definition of η_2 .

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