

Supplementary Material

Implications of GW241011 for rotating exotic compact objects

COMPUTATION OF SPIN-INDUCED QUADRUPOLE MOMENTS

Overview of the formalism

In this section, we outline the procedure used to extract spin-induced quadrupole moments (SIQMs) from numerically computed spacetimes, following the prescription of Refs. [96, 97]. We begin with the line element for a stationary and axisymmetric spacetime in quasi-isotropic coordinates, adopting the notation of Butterworth and Ipser [98],

$$ds^2 = -e^{2\nu} dt^2 + r^2 \sin^2 \theta B^2 e^{-2\nu} (d\phi - \omega dt)^2 + e^{2(\zeta - \nu)} (dr^2 + r^2 d\theta^2). \quad (\text{S1})$$

Above, the quantities ν , B , ω , and ζ denote the metric functions of (r, θ) . The asymptotic behaviour of Eq. (S1) has been derived in Ref. [98] [see their Eqs. (12a)-(12c)]; below we include the first few leading terms

$$\nu \sim \left(-\frac{M}{r} + \frac{B_0 M}{3r^2} + \dots \right) + \left(\frac{\nu_2}{r^3} + \dots \right) P_2 + \dots, \quad (\text{S2})$$

$$\omega \sim \left(\frac{2J}{r^3} - \frac{6JM}{r^4} + \dots \right) \frac{dP_1(\cos\theta)}{d\cos\theta} + \left(\frac{\omega_2}{r^5} + \dots \right) \frac{dP_3(\cos\theta)}{d\cos\theta} + \dots \quad (\text{S3})$$

$$B \sim \sqrt{\frac{\pi}{2}} \left(1 + \frac{B_0}{r^2} \right) T_0^{1/2} + \dots, \quad (\text{S4})$$

where $P_l(\cos\theta)$ denote Legendre polynomials, $T_l^{1/2}$ Gegenbauer polynomials (see Eq.(10a) of Ref. [98]), M and J the first two multipole moments of the spacetime (i.e. mass-energy and angular momentum). As first pointed out in Ref. [96], care has to be taken when relating the asymptotic behavior of the metric coefficients of numerical spacetimes to the multipole moments of Hansen-Geroch [43, 99]. Using Ryan's coordinate-independent method of identifying the multipole moments [100], one finds that the quadrupole moment M_2 is given by [96]

$$M_2 = -\nu_2 - \frac{1}{3}M^3 - \frac{4}{3}B_0 M. \quad (\text{S5})$$

The *reduced* quadrupole is therefore

$$\kappa_2 = -\frac{M_2}{\chi^2 M^3}, \quad (\text{S6})$$

where $\chi = J/M^2$ is the dimensionless spin. In this work we compute the multipole moments using (S5); our methods in extracting the ν_2 and B_0 coefficients differ across different ECOs we consider. We therefore discuss their details separately.

Boson stars

We numerically compute boson star solutions using the relaxation algorithm described in Ref. [67]. We compute most of our spinning BS solutions using the resolution of $N_{\tilde{r}} \times N_\theta = 500 \times 50$, where $\tilde{r} = r/(1+r)$ denotes the compactified coordinate. Although for highly compact stars, we increase it even further to $N_{\tilde{r}} \times N_\theta = 500 \times 100$ and sometimes even $N_{\tilde{r}} \times N_\theta = 1000 \times 100$. The numerical error incurred in the computation of the reduced quadrupole at these resolutions varies between 0.3 – 1.1%, depending on the compactness of the model in question.

The metric of the BS spacetime in our case is assumed to be

$$ds^2 = -f dt^2 + l f^{-1} \{ g (dr^2 + r^2 d\theta^2) + r^2 \sin^2 \theta (d\phi - \Omega r^{-1} dt)^2 \}, \quad (\text{S7})$$

where l , f and Ω are functions of (r, θ) . Our form is similar to that of Eq. (S1), however it requires some metric reparametrizations to be matched exactly, e.g., $\nu = \ln(f)/2$ and $B = \sqrt{l}$. The required ingredients for the computation

of M_2 can then be easily extracted from Eqs. (S2)-(S4) using orthogonality conditions for Legendre and Gegenbauer polynomials

$$\nu_0 = -M = \frac{1}{2} \lim_{r \rightarrow \infty} r \int_0^\pi \nu(r, \theta) \sin \theta d\theta, \quad (\text{S8})$$

$$\nu_2 = \frac{5}{2} \lim_{r \rightarrow \infty} r^3 \int_0^\pi \nu(r, \theta) \frac{1}{2} (3 \cos^2 \theta - 1) \sin \theta d\theta, \quad (\text{S9})$$

$$B_0 = \lim_{r \rightarrow \infty} r^2 \int_0^\pi \sqrt{\frac{2}{\pi}} [B(r, \theta) - 1] \sin^2 \theta d\theta. \quad (\text{S10})$$

Our expressions in Eqs. (S8)-(S10) are also in agreement with Ref. [55].

Exotic fluid stars

As described in the main text, our EFS models are computed using the RNS package [77–79]. The metric takes the form of

$$ds^2 = -e^{\gamma+\rho} dt^2 + r^2 \sin^2 \theta e^{\gamma-\rho} (d\phi - \omega dt)^2 + e^{2\alpha} (dr^2 + r^2 d\theta^2), \quad (\text{S11})$$

where again γ, ρ, ω and α are functions of (r, θ) and one requires further metric reparametrizations to match it to the metric form of Butterworth and Ipser: $B = e^\gamma$ and $\nu = (\gamma + \rho)/2$. The SIQMs are extracted using Eq. (S5), as for BSs. However, here we utilize the integral expressions using the methods of Komatsu, Eriguchi and Hachisu [101] from Ref. [79] [see their Eqs. (27)-(29)] to compute ν_2 and B_0 coefficients. We use resolution of 481×301 for radial and angular grid-points and estimate the reduced quadrupole moment errors to be $\sim 1\%$.

GW MEASUREMENT OF THE SIQM PARAMETER

Waveform model and Bayesian inference

As already discussed in the main text, we perform parameter estimation on GW241011 using the IMRPHENOMXPHM waveform model [62, 63], which includes SIQM corrections in the inspiral phase at 2PN and 3PN orders. Explicit expressions for these terms are given in Eqs. (1.5a) and (1.5b) of the Supplementary Material of Ref. [35].

The marginalized posterior distribution of κ is obtained using Bayes' theorem,

$$p(\kappa|d, \mathcal{H}) = \int p(\vec{\theta}|d, \mathcal{H}) d\vec{\theta}, \quad (\text{S12})$$

where

$$p(\vec{\theta}|d, \mathcal{H}) = \frac{\pi(\vec{\theta}|\mathcal{H}) \mathcal{L}(d|\vec{\theta}, \mathcal{H})}{Z(\mathcal{H})}. \quad (\text{S13})$$

Here, $\vec{\theta}$ denotes the full parameter set (binary black hole parameters and κ_i , $i \in [1, 2]$), $\mathcal{L}(d|\vec{\theta}, \mathcal{H})$ the likelihood, $\pi(\vec{\theta})$ the prior, and Z the evidence.

The likelihood function $\mathcal{L}(d|\vec{\theta}, \mathcal{H})$ is constructed assuming stationary Gaussian noise and a model consistent with the Kerr black hole hypothesis $\mathcal{H}$. The likelihood function is evaluated between a minimum frequency $f_{\min}$ (typically 20 Hz for current-generation LIGO and Virgo detectors) and a maximum cutoff frequency f_{cut} [102–105]. Posterior distributions are sampled using the DYNesty nested sampler [106, 107] within the BILBY framework [108–110]. We assume uniform priors on κ_1 and κ_2 over the range $[-500, 500]$ [10]. In our notation, κ_1 (κ_2) corresponds to the primary (secondary) component of the binary, with $m_1 > m_2$. Alongside κ_1 (Fig. 1), we simultaneously infer κ_2 , shown in Fig. S1. Since the secondary in GW241011 is slowly spinning, the κ_2 posterior is uninformative, and we cannot draw conclusions about its nature.

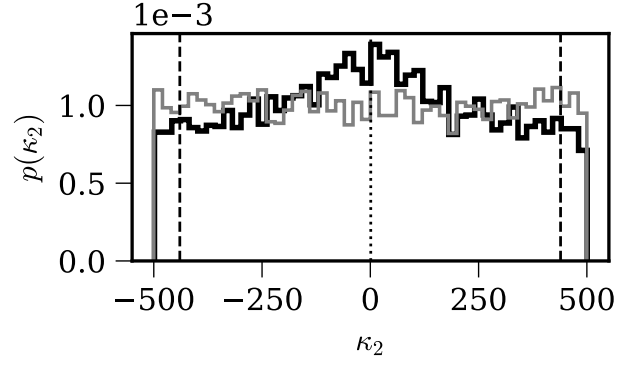


Figure S1. The one-dimensional posterior distribution of κ_2 obtained from GW241011 along with the prior distribution in grey.

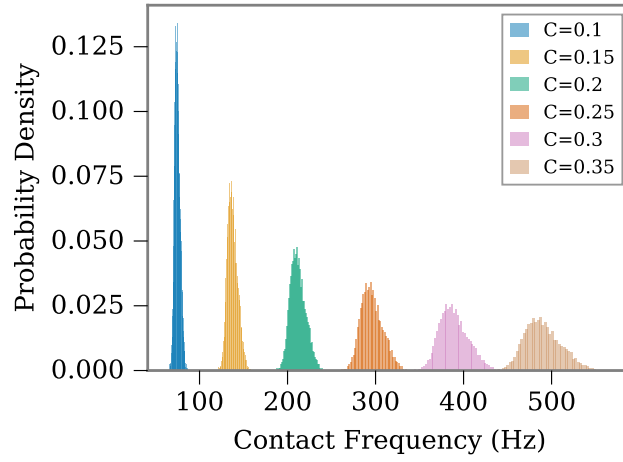


Figure S2. The approximate contact frequency f_{contact} for different compactnesses.

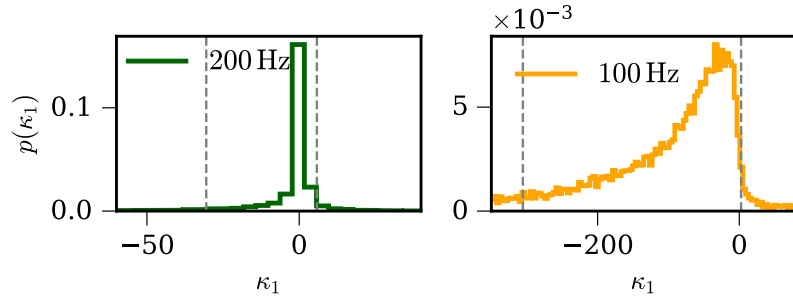


Figure S3. The one-dimensional posterior distribution of κ_1 obtained from GW241011 and estimated from $f_{\text{cut}} = 200$ (left panel) and 100 Hz (right panel).

Justification for the choice of the upper cut-off frequency, f_{cut}

The SIQM effects are implemented only in the inspiral portion of the waveform model. However, the likelihood evaluation in Fig. 1 of the main text extends up to f_{cut} , thereby including the post-inspiral regime. From a null-test perspective—where no deviations from the Kerr black hole hypothesis are assumed—this treatment is appropriate,

Repulsive		Solitonic		Axionic	
λ/μ^2	$C_{\max}$	σ	$C_{\max}$	f	$C_{\max}$
1000	0.053	0.04	0.284	0.008	0.356
450	0.078	0.05	0.339	0.01	0.384
250	0.106	0.08	0.412	0.05	0.174
150	0.164	0.1	0.421	0.1	0.168

Table S1. Estimates of the maximum compactness $C_{\max}$ compatible with the spin measurement of GW241011 for various BS models we have considered in the main text. We estimate $C_{\max}$ by considering the 95% contour of the κ - χ posterior, as illustrated in Figs. 2-3. Here, λ/μ^2 , σ , and f denote the coupling values for BS models with the repulsive, solitonic, and axionic potentials, respectively.

as it allows the full signal to be used in a model-agnostic consistency test. The resulting κ constraints are primarily informed by the inspiral phase, where the Kerr description of the binary components is expected to hold. As our waveform is parametrized in terms of explicit κ dependence, any potential deviation from the Kerr hypothesis would manifest as a shift of the posterior peak away from unity. Nevertheless, when translating these results to specific exotic compact object models, it is important to assess whether including the post-inspiral data introduces systematic biases.

To quantify these systematic effects, if present, we evaluate posterior distributions using different choices of the upper frequency cutoff in the likelihood computation. Specifically, we consider truncations at $f_{\text{cut}} = 529$ Hz, 489 Hz, 439 Hz, 400 Hz, 300 Hz, 200 Hz, and 100 Hz, as well as the full bandwidth used in the generic analysis ($f_{\text{cut}} = 1024$ Hz). This procedure allows us to test the robustness of the inferred κ constraints and identify potential biases introduced by including signal beyond the inspiral regime. The higher value of $f_{\text{cut}} = 489$ Hz is motivated by the inner most stable circular (ISCO) frequency of a Kerr BH [111, 112], derived from the maximum likelihood estimates of the GW241011 masses and spins, while lower truncation frequencies are motivated by the expected contact frequencies f_{contact} , at which the surfaces of two compact objects would come into contact. This value typically depends on the mass and compactness, and it can be *approximated* by $f_{\text{contact}} = (c^3/2G)(C^{3/2}/m_{\text{ECO}})$, where m_{ECO} is the mass of the ECO and C is its compactness [34]. Using the posterior of the primary mass of GW241011, in Fig. S2, we illustrate the contact frequency estimates for $C \in [0.1 - 0.35]$. The contact frequency spans a wide range, from ~ 89 Hz, insufficient for a meaningful constraint, to ~ 500 Hz, comparable to the Kerr black hole ISCO frequency. For further reference, in Table S1 we summarise the maximum compactness of various BS models compatible with the spin measurement of GW241011.

The black histogram in Fig. 1 of the main text corresponds to the analysis using the full bandwidth ($f_{\text{cut}} = 1024$ Hz), while the results for the other cutoff frequencies are shown as distinct colored histograms. For the lower cutoff frequencies, e.g., $f_{\text{cut}} = 100$ –200 Hz, the posterior distributions are noticeably broader, making direct comparison in Fig. 1 less clear; these low-frequency cases are therefore displayed separately in Fig. S3. Overall, the inferred parameters remain broadly consistent across all frequency choices (cf. Fig. S4, where representative posteriors are overplotted). However, the 100 Hz case, where the limited signal bandwidth leaves the binary parameters largely unconstrained, yields comparatively less tight κ_i estimates. As the cutoff frequency decreases, the κ posterior distribution becomes noticeably broader on the negative side. This asymmetry arises because the reduced signal strength weakens constraints on the binary masses and spins, thereby enhancing the mass-spin- κ degeneracy. For a positively spinning primary, this naturally leads to an extended, weakly constrained region toward negative κ values [37]. For the default broad priors placed on κ_1 , the resulting degeneracies can cause the posteriors on e.g., q and χ_{eff} , to appear inconsistent with the full-bandwidth results. We have verified that this apparent inconsistency is a by-product of the broad prior and sampling issues. If we restrict the prior boundaries or focus on the portion of the parameter space around $\kappa_1 = 1$, we find regions with significant likelihood support consistent with the full-bandwidth results.

Finally, we also plot the the coherent network signal-to-noise ratio (SNR) as a function of f_{cut} in Fig. S5. We see that there is consistent build-up of SNR far past the nominal values of f_{cut} of 200-300 Hz, suggesting that the objects do not come into contact at these frequencies.

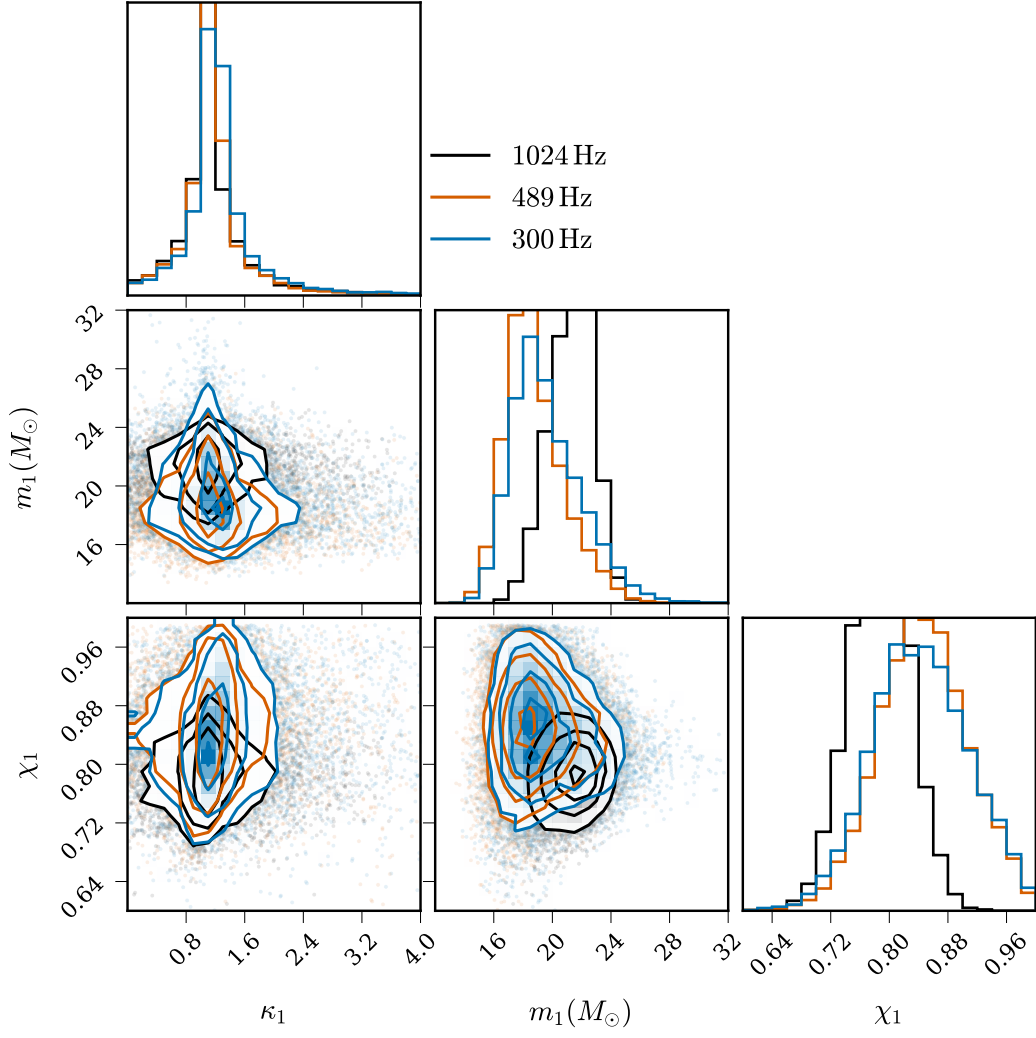


Figure S4. The correlation between κ , with primary mass and spin for different f_{cut} values.

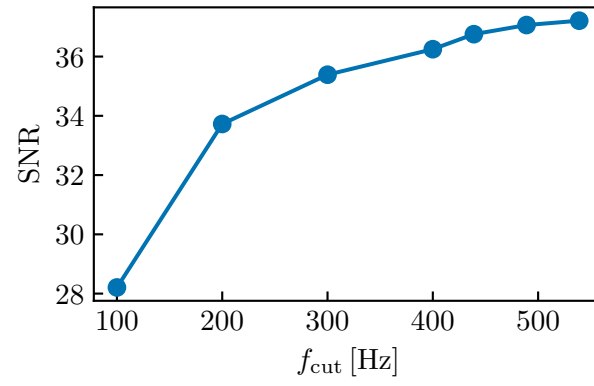


Figure S5. The SNR accumulated in the signal as a function of f_{cut} .

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