

Supplement: Gardening on the Moon: An Advection-Diffusion Model to Guide the Search for Supernova Debris in the Lunar Regolith

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SUPPLEMENTAL MATERIAL

S1. An ADE Model of Gardening

S1.1. Impact Flux and Geometric Scaling

We assume that the impactor population bombarding the lunar surface has a cumulative size-frequency distribution (CSFD) of the form $N(> \mathcal{D}) = a\mathcal{D}^{-b}$. This gives the rate at which craters of diameter $\mathcal{D}$ or more are created per unit area. We use the flux from secondary impacts, which dominate gardening following [1–3] (see Table I for numerical values). We define the resulting vertical evolution of the regolith using three coupled geometric processes: excavation, compaction, and burial.

S1.2. Excavation and Compaction Zones

Following [3], we define the characteristic gardening depth for a mechanism i as:

$$D_i(t) = h_i \left(\frac{\pi a b t c_i^2}{4(b-2)\lambda} \right)^{\frac{1}{b-2}}, \quad (1)$$

where h_i is the vertical reach and c_i is the horizontal interaction scale. We assume a transient crater profile $z = H(1 - (r/R)^2)$ with a depth to diameter ratio of $H = 0.2$ (Figure 1).

Excavation (e): In the upper third of the crater ($\xi \in [0, 1/3]$), we assume

$$h_e = \frac{1}{3}H = \frac{1}{3}(0.2) \approx 0.067, \quad c_e = \sqrt{\frac{5}{6}} \approx 0.913. \quad (2)$$

Compaction (c): In the lower two-thirds of the crater ($\xi \in [1/3, 1]$), we assume

$$h_c = \frac{2}{3}H = \frac{2}{3}(0.2) \approx 0.133, \quad c_c = \sqrt{\frac{1}{3}} \approx 0.577. \quad (3)$$

The smaller interaction radius c_c reflects the parabolic tapering of the crater bowl at depth.

S1.3. Mass Conservation and Burial

We use the physical model of burial developed in [3] and illustrated in Fig. 1, ensuring volumetric consistency between material removed from the bowl of a crater and

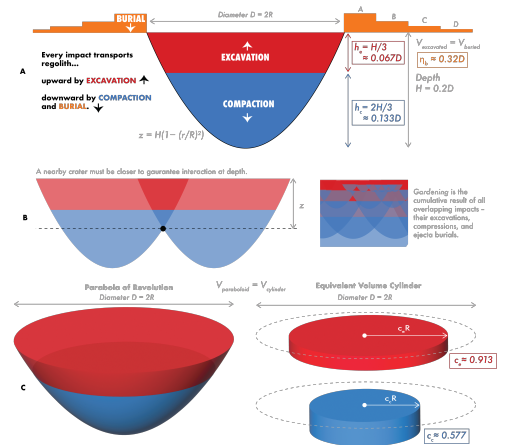


FIG. 1. Schematic of the impact geometry underlying the gardening model.

relocated as ejecta. Here, we adapt this treatment for our four discrete depositional annuli (labeled A through D). By equating a deposited volume fraction $P_{V,k}$ to the geometric volume of its receiving annulus, we derive the dimensionless burial scale $h_{b,k}$:

$$h_{b,k} = \frac{P_{V,k}}{18(j_k^2 - i_k^2)} \quad (4)$$

where i_k and j_k are the inner and outer radii of the annulus in units of the transient crater radius R . Unlike the bulk excavation and compaction zones, the net burial is a superposition of transport from multiple spatial interaction scales $c_{b,k} = \sqrt{j_k^2 - i_k^2}$.

By substituting these scales into the process-specific transport law, Eq. (1), the total burial depth $D_B(t)$ is the sum of contributions from all ejecta annuli:

$$\begin{aligned} D_B(t) &= \sum_k D_{b,k}(t) \\ &= \underbrace{\left[\sum_{k \in \{A,B,C,D\}} \frac{h_{b,k}}{c_{b,k}^{\frac{2}{-b+2}}} \right]}_{\text{Geometric Efficiency}} \underbrace{\left(\frac{4(b-2)\lambda}{\pi abt c_i^2} \right)^{\frac{1}{b-2}}}_{\text{Probabilistic Frequency}} \quad (5) \end{aligned}$$

This allows us to define an effective burial efficiency,

$$\eta_b(b) = \sum_{k \in \{A,B,C,D\}} \frac{h_{b,k}}{c_{b,k}^{\frac{2}{-b+2}}}, \quad (6)$$

representing the net vertical reach of the combined ejecta blanket. To evaluate $\eta_b(b)$ explicitly in terms of the fundamental parameters of the annuli, we substitute $h_{b,k} = \frac{P_{V,k}}{18(j_k^2 - i_k^2)}$ and $c_{b,k} = \sqrt{j_k^2 - i_k^2}$ into the summation:

$$\eta_b(b) = \sum_{k \in \{A,B,C,D\}} \frac{\frac{P_{V,k}}{18(j_k^2 - i_k^2)}}{\left(\sqrt{j_k^2 - i_k^2} \right)^{\frac{2}{-b+2}}} \quad (7)$$

Consolidating the terms in the denominator, the $(j_k^2 - i_k^2)$ factors combine as $(j_k^2 - i_k^2)^1 \cdot (j_k^2 - i_k^2)^{\frac{1}{-b+2}} = (j_k^2 - i_k^2)^{\frac{-b+3}{-b+2}}$. This yields the final closed-form solution for the effective burial efficiency:

$$\eta_b(b) = \frac{1}{18} \sum_{k \in \{A,B,C,D\}} \frac{P_{V,k}}{(j_k^2 - i_k^2)^{\frac{-b+3}{-b+2}}} \quad (8)$$

The dimensionless inner radii i_k , outer radii j_k , and depositional volume fractions $P_{V,k}$ for each annulus are detailed in Table I. For a secondary-dominated impactor population ($b \approx -4$ [3]), evaluating this summation using these parameters yields $\eta_b \approx 0.032$, which we use as the standard burial efficiency in our advection calculations.

S1.4. Net Advection and Turnover Equilibrium

The net vertical transport depth $D_z(t)$ represents the characteristic progression of the gardening front. As shown in main text Eq. (1), it is the sum of process-specific depths:

$$D_z(t) = D_B(t) + D_C(t) - D_E(t). \quad (9)$$

We can now derive the integrated form of main text Eq. (1) by substituting the process-specific transport depths. For compaction and excavation, we use the scaling law from Eq. (1), and for burial, we use the effective efficiency $\eta_b(b)$ derived in Eq. (5):

$$\begin{aligned} D_z(t) &= \left[\sum_k \frac{h_{b,k}}{c_{b,k}^{\frac{2}{-b+2}}} \right] \left(\frac{4(b-2)\lambda}{\pi abt} \right)^{\frac{1}{b-2}} \\ &+ \left[\frac{h_c}{c_c^{\frac{2}{-b+2}}} \right] \left(\frac{4(b-2)\lambda}{\pi abt} \right)^{\frac{1}{b-2}} \\ &- \left[\frac{h_e}{c_e^{\frac{2}{-b+2}}} \right] \left(\frac{4(b-2)\lambda}{\pi abt} \right)^{\frac{1}{b-2}}. \quad (10) \end{aligned}$$

Factoring out the common probabilistic frequency term yields the final analytic expression for the gardening front defined in the main text via Eq. (1):

$$D_z(t) = \underbrace{\left[\eta_b(b) + \frac{h_c}{c_c^{\frac{2}{-b+2}}} - \frac{h_e}{c_e^{\frac{2}{-b+2}}} \right]}_{\text{Geometric Efficiency}} \underbrace{\left(\frac{4(b-2)\lambda}{\pi abt} \right)^{\frac{1}{b-2}}}_{\text{Probabilistic Frequency}}.$$

To define the advection of the regolith column, we take the time derivative of the gardening front to find the vertical advection velocity, $v_z(t)$:

$$v_z(t) = \frac{\partial D_z}{\partial t} = -\frac{D_z(t)}{(-b+2)t}. \quad (11)$$

This result completes the analytical link between the impact flux and the transport coefficients used in our model.

At the depth scales sampled by surface operations (upper few meters), impact gardening is dominated by secondary impacts [3]. Every primary meteorite impact generates hundreds of thousands of smaller secondaries, resulting in a power law flux of $b \approx 4$ [3]. For secondary impact-dominated gardening, burial and compaction dominate transport, leading to net downward advection ($v_z, D_z > 0$).

Exhumation only dominates when the production function is shallow ($b < b_{crit} \approx 3.25$), favoring the excavation of deep material over burial. This balance explains the dual nature of regolith evolution: while shallow-sloped primary impacts ($b \approx 2.7$ [5]) effectively mine the underlying bedrock to grow the total regolith column, the steep power-law flux of secondary impacts dominates the downward advection of surface-correlated material into the upper centimeter and meter depths they primarily affect.

TABLE I. Model Parameters, Transport Coefficients, and Units

Category	Symbol	Value/Expression	Units	Description
Impact Flux	a	1.6×10^{-5}	$\text{cm}^{b-2} \text{ yr}^{-1}$	Crater production scaling constant
	b	4	–	Crater production power-law slope
	λ	4.605	–	Poisson threshold ($P = 99\%$)
Geometry	h_e, c_e	0.067, 0.913	–	Excavation vertical and horizontal scales
	h_c, c_c	0.133, 0.577	–	Compaction vertical and horizontal scales
	i_k	[1.0, 1.5, 2.0, 4.0]	R	Inner radii of burial annuli $k \in \{A, B, C, D\}$
	j_k	[1.5, 2.0, 4.0, 11.0]	R	Outer radii of burial annuli $k \in \{A, B, C, D\}$
	$P_{V,k}$	[0.24, 0.19, 0.05, 0.005]	–	Volume fraction per annulus (S1.3)
	$\eta_b(b)$	≈ 0.032 (for $b = -4$)	–	Effective burial efficiency (Eq. (5))
Transport	$v_z(t)$	$\partial D_z / \partial t$	cm yr^{-1}	Advection (gardening) velocity
	$\kappa(t)$	z_κ^2 / t	$\text{cm}^2 \text{ yr}^{-1}$	Diffusion coefficient
	Γ	0.1077	–	Gardening efficiency factor (S2)
	σ_{global}	0.24	–	Empirical error factor
Physics	$T_{1/2}({}^{60}\text{Fe})$	2.62	Myr	${}^{60}\text{Fe}$ half-life [4]
	$T_{1/2}({}^{244}\text{Pu})$	80.0	Myr	${}^{244}\text{Pu}$ half-life [4]
	$\mathcal{F}$	Table II	at cm^{-2}	Total SN isotope fluence

S2. Derivation of Transport Coefficients

The continuum evolution of a concentration $C(z, t)$ in the lunar regolith is governed by the Advection-Diffusion Equation (ADE):

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z} \left(\kappa(t) \frac{\partial C}{\partial z} \right) - v_z(t) \frac{\partial C}{\partial z} - \lambda_{\text{decay}} C + S(z, t) \quad (12)$$

where $\kappa(t)$ is the time-dependent diffusion coefficient and $v_z(t)$ is the vertical advection velocity. These coefficients are derived from the integrated gardening front $D_z(t)$ defined in main text Eq. (1).

The vertical advection velocity represents the instantaneous rate of advance of the gardening front. We derive this by taking the time derivative of the net vertical transport depth:

$$v_z(t) = \frac{\partial D_z}{\partial t} = \frac{\partial}{\partial t} \left[\eta_{\text{total}}(b) \left(\frac{4(b-2)\lambda}{\pi a b t} \right)^{\frac{1}{b-2}} \right] \quad (13)$$

where $\eta_{\text{total}}(b)$ is the combined geometric efficiency of burial, compaction, and excavation. Because $D_z \propto t^{-1/(b+2)}$, the power-rule differentiation with respect to t yields the velocity expression used in our model:

$$v_z(t) = \frac{1}{b-2} \frac{D_z(t)}{t}. \quad (14)$$

Defining depth z as positively downward, the direction of transport is dictated by the impact population slope b . For a secondary-dominated population ($b \approx 4$), the leading term $-(-b+2)^{-1}$ is positive (≈ 0.5), resulting in the net downward advection ($v_z > 0$) of surface-delivered materials.

While $D_z(t)$ and $v_z(t)$ describe the bulk advective progression of the regolith column, impact gardening is fundamentally stochastic. The regolith is continuously

churned by numerous smaller craters that do not independently drive the net advective front but dominate local mixing. We capture this stochastic mixing as a diffusive process, where the time-dependent diffusion coefficient $\kappa(t)$ scales with the square of a characteristic mixing length $z_\kappa(t)$:

$$\kappa(t) = \frac{z_\kappa(t)^2}{t}. \quad (15)$$

The scale of this local mixing is physically coupled to the net transport depth $D_z(t)$ via two dimensionless geometric properties of an average impact event: the total vertical reach (H) and the gardening efficiency (Γ). The total vertical reach characterizes the combined vertical scale of crater formation. We partition this total depth into the excavation zone ($h_e \approx 0.067$) and the underlying sub-crater compaction zone ($h_c \approx 0.133$). Their sum precisely corresponds to the transient depth-to-diameter ratio ($d/D \approx 0.2$ [6]) of simple lunar craters, such that $H = h_e + h_c = 0.2$.

The gardening efficiency (Γ) captures the spatial variance of impact-driven vertical displacements across all depositional and formational zones. We derive Γ by taking the root-sum-square of the individual vertical geometric scales (h_e , h_c , and the effective burial efficiency η_b), weighted by the mean horizontal interaction fraction $\bar{c} = 1/\sqrt{2} \approx 0.707$:

$$\Gamma = \bar{c} \sqrt{h_e^2 + h_c^2 + \eta_b(b)^2}. \quad (16)$$

For our secondary-dominated parameters ($h_e = 0.067$, $h_c = 0.133$, and $\eta_b(-4) \approx 0.032$), this yields a dimensionless gardening efficiency of $\Gamma \approx 0.108$. The characteristic mixing length $z_\kappa(t)$ is then defined by the ratio of this spatial variance to the total vertical reach, scaled by the current advection depth:

$$z_\kappa(t) = \frac{\Gamma}{H} D_z(t) \approx \frac{0.108}{0.2} D_z(t) = 0.54 D_z(t). \quad (17)$$

Substituting this relationship into our definition of $\kappa(t)$ yields the final analytical expression for the time-varying diffusion coefficient:

$$\kappa(t) = \left(\frac{\Gamma}{H} \right)^2 \frac{D_z(t)^2}{t}. \quad (18)$$

This formulation physically links the rate of diffusive spreading to the advective progression of the regolith column, with both transport coefficients (κ, v_z) naturally decaying as t increases and the gardening front moves deeper into the regolith.

S3. Concentration with Depth

The continuum evolution of a concentration $C(z, t)$ in the lunar regolith is governed by the conservation of mass. We evaluate the rate of change of concentration at any depth z by taking the negative divergence of the vertical mass flux $J(z, t)$, combined with internal sinks (decay) and sources:

$$\frac{\partial C}{\partial t} = -\frac{\partial J}{\partial z} - \lambda_{\text{decay}} C + S(z, t) \quad (19)$$

where $\lambda_{\text{decay}} = 1/\tau$ is the radioactive decay rate (for stable maturity metrics, $\lambda_{\text{decay}} = 0$) and $S(z, t)$ represents the spatiotemporal source term, combining continuous surface delivery, episodic deposition, and in-situ cosmogenic production.

The total vertical mass flux $J(z, t)$ consists of two distinct physical transport mechanisms driven by impact gardening:

1. **Advective Flux (J_{adv}):** The bulk, directed transport of the regolith column due to the net progression of the gardening front. This is governed by the time-varying velocity $v_z(t)$, resulting in $J_{\text{adv}} = v_z(t)C$.
2. **Diffusive Flux (J_{diff}):** The stochastic, bidirectional mixing caused by numerous smaller impacts above the main advective front. This flux is proportional to the concentration gradient, such that $J_{\text{diff}} = -\kappa(t) \frac{\partial C}{\partial z}$.

Substituting these flux components ($J = J_{\text{adv}} + J_{\text{diff}}$) into the mass conservation equation yields the general form of the Advection-Diffusion Equation (ADE) presented in the main text as Eq. (4):

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z} \left(\kappa(t) \frac{\partial C}{\partial z} \right) - \frac{\partial}{\partial z} (v_z(t)C) - \lambda_{\text{decay}} C + S(z, t). \quad (20)$$

Because our transport coefficients $\kappa(t)$ and $v_z(t)$ are derived from the integrated impact history (Eqs. (11) and main text Eq. (3)), they are strictly time-dependent and spatially uniform across the defined regolith column (i.e.,

$\frac{\partial \kappa}{\partial z} = 0$ and $\frac{\partial v_z}{\partial z} = 0$). This allows us to pull the coefficients outside the spatial derivatives, recovering the final working form of the equation used to calculate our depth profiles:

$$\frac{\partial C}{\partial t} = \kappa(t) \frac{\partial^2 C}{\partial z^2} - v_z(t) \frac{\partial C}{\partial z} - \lambda_{\text{decay}} C + S(z, t). \quad (21)$$

When solving this equation numerically, we treat the vacuum-regolith interface ($z = 0$) as a flux boundary.

S3.1. Continuous Influx (Maturity)

For the calculation of I_s/FeO , we assume $\lambda = 0$ and $S(z, t) = \frac{\phi_m}{\rho(z)} \delta(z)$, where ϕ_m is the constant maturity influx rate and $\rho(z)$ is the density. In this context, $C(z, t)$ represents the accumulated maturity (normalized to FeO) resulting from surface exposure.

S3.2. Sporadic Pulses and Radionuclides:

The model generalizes to any radionuclide. For an isotope with a half-life $t_{1/2}$, the decay constant is $\lambda = \ln(2)/t_{1/2}$. The total source $S(z, t)$ is normalized by the abundance of a target element (e.g., wt% FeO for ^{60}Fe) as follows:

$$S(z, t) = \frac{\Phi(t)}{\rho(z)} \delta(z) + P_0 \cdot \exp\left(-\frac{\xi(z)}{\Lambda}\right), \quad (22)$$

where P_0 is the normalized surface production rate by galactic cosmic rays (GCR), which attenuates with mass-depth $\xi(z) = \int_0^z \rho(z') dz'$, and Λ is the attenuation length.

The external surface flux $\Phi(t)$ for radionuclides such as ^{60}Fe is modeled as a summation of discrete events, each represented by a Gaussian distribution in time:

$$\Phi(t) = \phi_{bg} + \sum_i \frac{\mathcal{F}_i}{\sigma_i \sqrt{2\pi}} \exp\left(-\frac{(\tau - t_i)^2}{2\sigma_i^2}\right), \quad (23)$$

where τ is the look-back time, $\mathcal{F}_i$ is the event magnitude (fluence) or background flux (ϕ_{bg}), σ_i is the temporal width, and t_i is the time of the peak flux.

S3.3. Isotope Source Parameters

The age and width of the more recent live-isotope pulse are taken from [7], and its ^{60}Fe fluence is taken from [8]. The parameters of the earlier ^{60}Fe pulse are taken from the data of [8], using the updated analysis described in [9]. The ^{244}Pu fluences and continuous flux are taken from [8]. The isotope source scenarios and flux parameters that we consider are shown in Table II. Fig. 2 shows

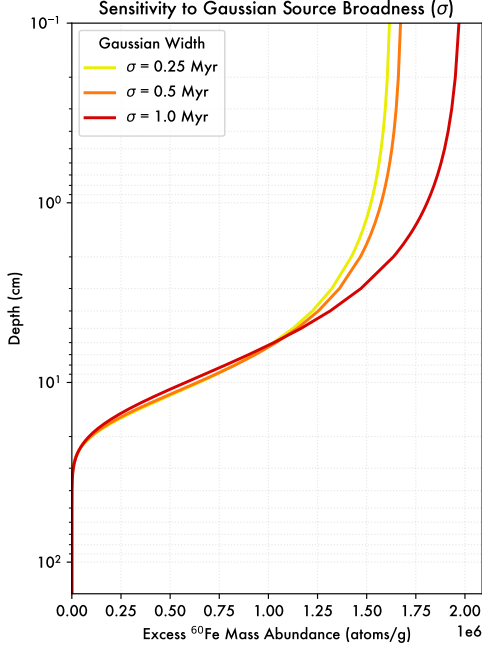


FIG. 2. Illustration of the sensitivity of the calculated ^{60}Fe density profile to the Gaussian width assumed for the more recent pulse pulse.

the sensitivity of the calculated ^{60}Fe density profile to the Gaussian width assumed for the more recent pulse. The variation in the density profile is $\sim 20\%$ at depths $\lesssim 1$ cm and much smaller in the tail at depths $\gtrsim 10$ cm.

S.3.4. Cosmogenic Production and Gardening Steady State

To evaluate the background concentration of radionuclides against which episodic supernova signals must be identified, we model the steady-state profile resulting from in-situ production by galactic cosmic rays (GCR) and impact-driven redistribution. This background is established by the balance between depth-dependent production, radioactive decay, and vertical transport.

Depth-Dependent Production Rate

The volumetric production rate $S_{\text{cosmo}}(z, t)$ (atoms $\text{cm}^{-3} \text{yr}^{-1}$) is governed by the attenuation of the cosmic ray flux as it penetrates the regolith. We model this as a function of the cumulative mass depth $\xi(z)$ (g cm^{-2}) to account for the depth-varying density of the soil [10, 11]:

$$S_{\text{cosmo}}(z, t) = P_0 \cdot \rho(z) \cdot \exp\left(-\frac{\xi(z)}{\Lambda}\right) \quad (24)$$

where P_0 is the surface production rate per unit mass (normalized to ≈ 0.039 atoms $\text{g}^{-1} \text{yr}^{-1}$ for ^{60}Fe , based

on the cosmogenic background flux estimated in [12, 13]), and $\Lambda = 160 \text{ g cm}^{-2}$ selected as the attenuation length for galactic cosmic ray (GCR) spallation and capture reactions [10, 11].

Steady-State and Equilibrium

The evolution of the cosmogenic component is integrated using the unified Advection-Diffusion Eq. (4) in the main text. Unlike the episodic supernova pulses, which are introduced as surface delta functions $\delta(z)$, the cosmogenic source is continuous and distributed throughout the regolith column.

In simulations we integrate the system for a simulation time ($T_{\text{sim}} = 20\text{--}50$ Myr) significantly longer than the mean life of the isotope ($\tau(^{60}\text{Fe}) \approx 3.78$ Myr) so that profile reaches a secular equilibrium where the total inventory is governed by $\int S(z)dz = \lambda \int C(z)dz$, and the gardening transport parameters (v_z, κ) have reached their long-term average behavior.

Normalization to Site Chemistry

Empirical measurements of ^{60}Fe are typically reported as activities relative to nickel ($^{60}\text{Fe}/\text{Ni}$). While stable iron within the regolith does not exceed ^{58}Fe and cannot produce ^{60}Fe , we use the local weight fraction of iron oxide (w_{FeO}) as a proxy for Ni, justified by the broad correlation between Fe and Ni abundances on the Moon [14–16]. This approach facilitates a direct comparison between model outputs and diverse site data (e.g., *Apollo* 12 basalts vs. *Apollo* 16 highlands):

$$C_{\text{norm}}(z) = \frac{C(z)}{\rho(z) \cdot w_{\text{FeO}}} \quad (25)$$

This normalization expresses the concentration in units of atoms g^{-1} per 1 wt% FeO. By doing so, the cosmogenic floor becomes a site-independent baseline, allowing for the clear identification of excess ^{60}Fe delivered by interstellar pulses regardless of local major-element chemistry.

S4. Empirical Uncertainty and Model Stochasticity

To account for the inherent stochasticity of lunar regolith gardening, we define an empirical uncertainty envelope based on the characteristic deviation of observed maturity profiles from mean-field model predictions. We utilize the digitized I_s/FeO depth profiles from [17] across the *Apollo* 15, 16, and 17 landing sites as proxies for long-term gardening effects.

The empirical error calculation follows a three-step normalization and aggregation procedure:

TABLE II. Isotope Source Histories and Flux Parameters

Isotope	Scenario	Peak (t_i)	Width (σ_i)	Fluence/Flux ($\mathcal{F}_i/\phi_{bg}$)
^{60}Fe	SN Pulses	2.3 Mya	0.47 Myr	$3.6 \pm 0.2 \times 10^7$ at/cm ²
		7.3 Mya	0.25 Myr	$1.1 \pm 0.2 \times 10^7$ at/cm ²
^{244}Pu	H1: Sporadic	2.3 Mya	0.47 Myr	1.7×10^3 at/cm ²
		7.3 Mya	0.25 Myr	5.0×10^2 at/cm ²
	H2 and H3: Continuous	—	—	246 at cm ⁻² Myr ⁻¹

1. **Residual Calculation:** For each reference core s , the mean-field transport model $M(z)$ was solved for the specific exposure age of the site and interpolated to the exact measurement depths z_i of the experimental data $D(z_i)$. Residuals $\epsilon_{i,s}$ were calculated on a normalized scale relative to the profile maximum:

$$\epsilon_{i,s} = \frac{D(z_{i,s})}{\max(D_s)} - \frac{M(z_{i,s})}{\max(M_s)}. \quad (26)$$

2. **Global Variance:** A site-agnostic standard deviation, σ_{global} , was derived from the combined residuals of all reference profiles. This parameter captures the universal noise introduced by discrete impact events that deviate from the continuous diffusion-advection approximation:

$$\sigma_{global} = \sqrt{\frac{1}{N} \sum_{i,s} \epsilon_{i,s}^2} \approx 0.24. \quad (27)$$

3. **Model Application:** The uncertainty in forward-modeled species (e.g., ^{60}Fe and ^{244}Pu) is expressed as a 1σ envelope. Because the gardening intensity scales with the total concentration, the absolute error is defined proportional to the peak concentration C_{max} of the resulting profile:

$$C_{err}(z) = C_{model}(z) \pm (\sigma_{global} \times C_{max}). \quad (28)$$

This approach ensures that the model provides a statistically realistic representation of the vertical distribution while acknowledging that individual core samples are single realizations of a highly variable impact history.

S5. Space Weathering and ^{60}Fe

Pitting and spall by micro-craters and amorphization of the crystal structure by solar wind irradiation systematically alter the external microstructure of mineral grains [e.g., 18–20]. A primary consequence of this alteration is the precipitation of nanophase metallic iron (np-Fe⁰ $\equiv I_s$) within regolith grain rims, which determines the measured I_s/FeO maturity index (Fig. 1 in the main text) and is known to concentrate in the finest, highly mature fractions of the soil closest to the lunar

surface [18, 21]. Because supernova-derived ^{60}Fe is delivered externally via interstellar dust, it may preferentially accumulate on these damaged, highly reactive grain exteriors [13].

Conversely, the in-situ cosmogenic ^{60}Fe background is produced primarily within the grain interior through cosmic-ray spallation on stable Ni [13]. Space weathering actively drives chemical and isotopic fractionation; impact-induced evaporation from continuous micrometeorite bombardment causes the preferential loss of lighter isotopes and elements like Ni, a mechanism recently evidenced in young lunar drill cores [22]. Therefore, the extensive structural damage and the generation of chemically reactive np-Fe⁰ rims in the finest mature regolith provide enhanced capacity for capturing recent exogenic ^{60}Fe , while the simultaneous weathering-induced loss of cosmogenic ^{60}Fe and Ni limits in-situ cosmogenic production, ultimately revealing a pronounced supernova signature.

S6. Regolith Density and Volumetric Abundance

To facilitate a direct comparison between laboratory-measured specific activities—typically reported as activity per unit mass of nickel—and our numerical transport model, we transformed the observed abundances into a volumetric number density, $n(z)$, with units of atoms/cm³. A critical factor in this transformation is the depth-dependency of the lunar bulk density, $\rho(z)$. The lunar regolith is characterized by a “fluffy” surface layer that compacts with depth due to overburden pressure.

Following the empirical results from the Lunar Reconnaissance Orbiter (LRO) Diviner Thermal Radiometer [23, 24], we modeled the regolith density profile using the exponential H-parameter formula:

$$\rho(z) = \rho_d - (\rho_d - \rho_s)e^{-z/H}. \quad (29)$$

where $\rho_s = 1.3 \text{ g/cm}^3$ represents the surface density, $\rho_d = 1.9 \text{ g/cm}^3$ is the compacted deep density, and $H = 7.0 \text{ cm}$ is the characteristic scaling depth. The number density of a specific supernova-derived isotope at depth z is then derived using:

$$n(z) = \left(\frac{A(z) \cdot C_{\text{Ni}}(z)}{\lambda_{\text{decay}}} \right) \rho(z) \cdot \Psi. \quad (30)$$

In this expression, $A(z)$ is the measured activity (e.g., dpm per kg-Ni), $C_{\text{Ni}}(z)$ is the clean nickel concentration ($\mu\text{g/g}$), λ_{decay} is the isotopic decay constant, and Ψ is a composite conversion factor (10^{-9}) required to reconcile mass and volume scales.

This depth-dependent correction accounts for the increased density of the lower regolith, scaling the soil's capacity for supernova fluences and preventing artificially inflated number densities near the surface.

S7. Numerical Implementation

All numerical modeling, data processing, and visualization were implemented using the Python programming language. The core mathematical operations and array manipulations were handled using the NumPy library.

To solve the one-dimensional advection-diffusion equation governing the vertical transport of isotopes in the lunar regolith, we discretized the spatial derivatives along a non-uniform 1D depth grid using NumPy, with high resolution (0.1 cm spacing) in the top 2 cm to capture sharp surface boundary dynamics, relaxing to 1.0 cm spacing down to a depth of 5 m.

The advective velocity (v_z) in our model emerges analytically from the time-derivative of the net transport depth, representing a competition between downward drivers (burial and compaction) and upward exhumation. Because lunar regolith gardening is dominated by a steep size-frequency distribution of secondary impactors ($b \approx 4$), this derivative mathematically dictates a net downward advection of material in the upper meter ($v_z > 0$). To respect this physical directionality and stably model the downward advective front, we implemented a first-order upwind differencing scheme. By calculating the advective flux based strictly on the upstream (shallower) regolith layers, this method prevents numerical dispersion.

The diffusion term, representing the isotropic mixing of regolith by frequent, small impacts, was discretized using standard second-order central differencing. Cosmogenic production and exogenic inputs (astrophysical deposition events) were treated as explicit source terms. The discrete astrophysical events were introduced dynamically at the surface boundary as a time-dependent flux modeled as Gaussian pulses in time.

The resulting system of coupled ordinary differential equations was integrated over the multi-million-year simulation time using the `solve_ivp` function from the SciPy library. We selected the `Radau` method (an implicit Runge-Kutta scheme) to handle the stiffness of the system caused by the stark contrast between continuous,

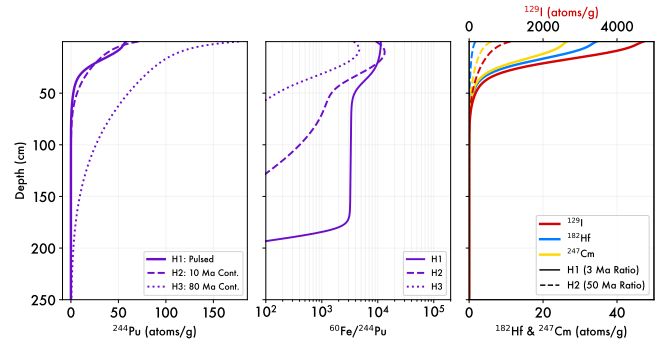


FIG. 3. A linear-scale version of Figure 3 in the main text.

slow background processes and highly transient astrophysical pulses. To guarantee that the adaptive time-stepper did not skip over these geologically brief deposition events, we enforced a strict maximum step size of 10^5 years. Combined with strict absolute (10^{-7}) and relative (10^{-4}) tolerances, this configuration ensures mass conservation and temporally resolves the exact onset and decay of the modeled ^{60}Fe pulses.

The ingestion of observational *Apollo* sample datasets and normalization to local FeO weight percentages, and statistical handling were managed using the Pandas library. All graphical outputs, error bound calculations, and model-data profile visualizations were subsequently generated using Matplotlib.

S8. Linear Version of Plot for Sample Strategy

We provide a linear-scale version of the depth-distribution predictions for isotopes of interest to support future sample collection planning and analysis (Figure 3).

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