

Fermion-Selective Tests of New Physics with the Bound-Electron g Factor

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The use of high-precision measurements of the g factor of single-electron ions is considered as a detailed probe for physics beyond the standard model. The contribution of the exchange of a hypothetical force-carrying scalar boson to the g factor is calculated for the ground state of H-like ions and used to derive bounds on the parameters of that force. Similarly to the isotope shift, we employ the nuclide shift, i.e., the difference for elements with different proton and/or neutron numbers, in order to increase the experimental sensitivity to the new physics contribution. In particular we find, combining available measurements with current precision with different ions, that the coupling constant for the interaction between an electron and a proton can be constrained up to 3 orders of magnitude better than with the best current atomic data and theory.

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The g factors of the free electron and free muon have served as precision tests for quantum electrodynamics (QED), the standard model (SM) more broadly, and possible extensions of the SM [1–3]. Recent years have also seen rapid improvements in the experimental determination [4–8] and theoretical prediction [9–18] of the g factor of bound electrons. This allows for stringent tests of QED in strong fields and, to a lesser extent, other sectors of the SM such as nuclear structural effects [19], and can hence also be an avenue for the discovery of phenomena beyond the SM (BSM).

In the present Letter, we demonstrate the relevance of the g factor of bound electrons in the search for physics beyond the SM. Comparisons between the best available theoretical and experimental results, jointly with relative uncertainties, can provide competitive bounds for the new physics (NP) contribution to the g factor, induced, in this Letter, by a new massive scalar particle mediating a hypothetical force. With this method, we here constrain electron-neutron and electron-electron contributions independently. The same approach will also be used to constrain the electron-proton contribution with an orders-of-magnitude improvement, though not being fully independent of the other two. Therefore, we also put forward an approach that generalizes the well-known isotope shift to isolate the contribution to the g factor due to boson exchange between the bound electron and protons in the nucleus, the so-called “nuclide

shift,” i.e., the difference of g factors for elements with different proton and/or neutron numbers. This quantity can be accessed with very high precision with modern experimental methods [8], as we discuss below. We work in units of $c = 1$ and $\hbar = 1$.

Theory—We consider a scalar boson as a mediator of a hypothetical fifth force between electrons and nucleons. The Lagrangian involving the scalar only reads [20]

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 + y_e\phi\bar{\psi}_e\psi_e + \sum_N y_N\phi\bar{\psi}_N\psi_N, \quad (1)$$

where ϕ is the field associated to the scalar and the first two terms describe its kinematics and the subscripts e , $N \in \{n, p\}$ stand for electron and nucleon (neutron or proton), respectively. The ψ 's are the corresponding fermionic fields and the y 's are the coupling constants of this Yukawa theory. This induces three types of new possible interactions: electron-electron, electron-nucleon, and nucleon-nucleon. However, the latter is not considered in this Letter because it is included in the nuclear parameters extracted from experiments (e.g., the nuclear radius and mass), and also our atomic quantities are not particularly sensitive to their value.

In what follows, we consider an H-like ion, where only one $1s$ electron orbits around the nucleus. The nucleons will be treated separately as neutrons or protons, as both are present in the systems considered. We will label by ee , en , and ep the interactions of the electron with itself, the neutrons, and the protons, respectively. These three interactions contribute, among other quantities, to the g factor, which can be expressed as

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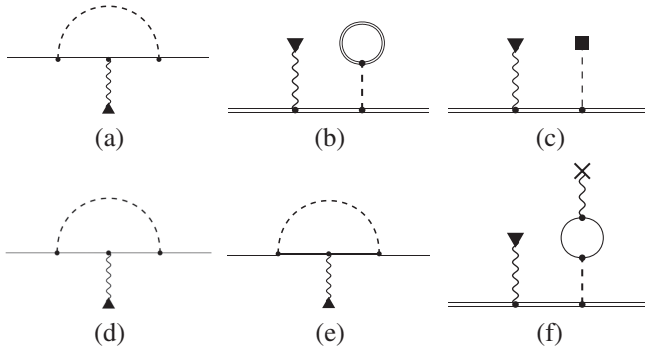


FIG. 1. First line: exemplary Feynman diagrams depicting perturbative scalar-boson corrections to the g factor of the electron from self-energy (a), vacuum polarization (b), and electron-nucleon interaction (c). Second line: bosonic vertex correction to the g factor due to free-electron SE (d), the diagram contributing to the first nonvanishing $Z\alpha$ order in bound electron SE (e) and VP (f). A double line represents a Coulomb-Dirac wave function or propagator. A wavy line terminated by a triangle denotes a photon from the external magnetic field. The dashed line terminated by a square denotes a scalar from the nucleons, while the dashed line alone represents the scalar propagator. A wavy line terminated with a cross denotes a Coulomb interaction with the nucleus.

$$\Delta g_\phi = \Delta g_\phi^{ee} + \Delta g_\phi^{en} + \Delta g_\phi^{ep}. \quad (2)$$

The first term of Eq. (2) encapsulates electron self-energy (SE) and vacuum polarization (VP), for which exemplary diagrams are shown in Figs. 1(a) and 1(b), while the second and the third terms correspond to Fig. 1(c). Any contribution of this new scalar must satisfy

$$\Delta g_\phi^i \leq g^{\text{exp}} - g^{\text{th}}, \quad (3)$$

where Δg_ϕ^i may represent any of the terms on the right-hand side of Eq. (2), $g^{\text{exp}} = g^{\text{SM}} + g^{\text{BSM}}$ and $g^{\text{th}} = g^{\text{SM}}$ are the experimentally measured and theoretical computed g factors, the former including all the SM and BSM contributions and the latter the SM ones only.

Let us constrain these three contributions independently. In order to do so we anticipate the following: (i) the electron-electron coupling can be effectively constrained via the free-electron g factor, and this can be transferred to the bound-electron contribution Δg_ϕ^{ee} ; (ii) Δg_ϕ^{en} can be limited through isotope shifts (see Ref. [21]); and (iii) for Δg_ϕ^{ep} we put forward a method that cancels the en contribution and exploits the accuracy found in the ee part.

The electron-electron interaction manifests itself in the SE and VP corrections. In the first case, the bound electron interacts with itself, while in the latter case it exchanges a boson with a virtual electron-positron pair. As we will see, it is sufficient to consider the corresponding free-electron diagram. It is in fact convenient to treat the binding effect on the electron as a perturbation to the free-electron case, as

the accuracy of the latter is better than the one of the former, and the binding correction is sufficiently small in the region of atomic numbers we are interested in. The corrections we calculated are up to order of $(Z\alpha)^2$, and they stem from one-loop vertex diagram.

The free SE g factor is derived from the one-loop correction of the QED vertex, with the virtual photon replaced by a scalar. Using the slash notation $\not{\phi} := a_\mu \gamma^\mu$, the Lagrangian reads [20]

$$\mathcal{L} = \bar{\psi}_e (i\not{\partial} - e\not{A} - m_e) \psi_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 + y_e \phi \bar{\psi}_e \psi_e, \quad (4)$$

where A_μ is the electromagnetic field and $F_{\mu\nu}$ is the corresponding strength field tensor. As the vertex $ie\gamma^\mu$ is corrected by QED (see Ref. [20]), it receives a correction from the scalar, too, as Fig. 1(d) shows, transforming into the general vertex function

$$ie\Gamma^\mu(p', p) = ie \left[F_1(q^2) \gamma^\mu - \frac{i}{2m} F_2(q^2) \sigma^{\mu\nu} q_\nu \right], \quad (5)$$

with $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, when sandwiched between two free Dirac states of momenta p' and p , and contracted with a photon of polarization $\epsilon^\mu(q)$. Here, $F_1(q^2)$ and $F_2(q^2)$ are the electron form factors and they are given by the sum of the contributions coming from the different propagators used to close the loops. While F_1 modifies the original electric charge, F_2 leads to a modification of the magnetic moment, thus $g = 2(1 + F_2(0))$. The one-scalar loop correction, analogous to QED calculations [20], reads

$$F_2^\phi(q^2) = \frac{y_e^2}{8\pi^2} \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \times \frac{1 - x_3^2}{(1 - x_3)^2 + x_1 x_2 \frac{q^2}{m_e^2} + x_3 \frac{m_\phi^2}{m_e^2}}, \quad (6)$$

and for an on-shell external photon ($q^2 = 0$), the integral can be expressed in a closed form as

$$F_2^\phi(0) = \frac{\alpha_{ee}}{2\pi} \left\{ \frac{3}{2} - r_m^2 - \frac{r_m(r_m^4 - 5r_m^2 + 4)}{\sqrt{r_m^2 - 4}} \times \left[\text{atanh}\left(\frac{r_m^2 - 2}{r_m \sqrt{r_m^2 - 4}}\right) - \text{atanh}\left(\frac{r_m}{\sqrt{r_m^2 - 4}}\right) \right] + r_m^2(r_m^2 - 3) \ln(r_m) \right\}, \quad (7)$$

where $r_m = m_\phi/m_e$ is the ratio of the scalar and electron mass, and $\alpha_{ee} = y_e^2/4\pi$. The correction to the g factor is then $\Delta g_\phi^{ee, \text{free}} = 2F_2^\phi(0)$.

For an H-like ion the only contributing diagram at leading order $(Z\alpha)^2$ is the one of Fig. 1(e). As explained in, e.g., Refs. [22,23], this corresponds to computing the expectation value of the electron anomalous magnetic moment Hamiltonian $H = a_e \mu_B \gamma^0 \mathbf{B} \cdot \mathbf{\Sigma}$ in a $1s$ external state, namely

$$\langle \psi_{1s} | H | \psi_{1s} \rangle = \mu_B B_z a_e^\phi \left[1 - \frac{(Z\alpha)^2}{6} + O((Z\alpha)^4) \right], \quad (8)$$

with μ_B being the Bohr magneton, B_z the projection of the magnetic potential onto the z axis, and $a_e^\phi = F_2^\phi(0)$ the anomalous magnetic moment induced by the scalar. The contribution to the g factor is

$$\Delta g_\phi^{ee} = \Delta g_\phi^{ee, \text{free}} \left(1 - \frac{(Z\alpha)^2}{6} \right), \quad (9)$$

plus corrections of order $(Z\alpha)^4$ and higher orders. To estimate the magnitude of $(Z\alpha)^4$ terms we consider one-loop SE and VP QED diagrams from Ref. [24], with care taken to account for the different boson propagator. At one-loop level, the only photon propagator is replaced by a scalar propagator, such that our estimation amounts to the correspondent perturbation term of Eq. (51) of Ref. [24], with the substitution $(\alpha/\pi) \rightarrow (\alpha_{\text{NP}}/\pi)$. It reads

$$\Delta g_{\phi, (Z\alpha)^4}^{ee} \simeq \Delta g_\phi^{ee, \text{free}} (Z\alpha)^4 \left[\frac{32}{9} \ln((Z\alpha)^{-2}) - \frac{247}{216} - \frac{9}{8} \ln(k_0) - \frac{8}{3} \ln(k_3) \right], \quad (10)$$

omitting even higher orders in $Z\alpha$, where, for $1s$ states $\ln(k_0) = 2.984128556$, and $\ln(k_3) = 3.272806545$. Contributions up to $(Z\alpha)^4$ in Eq. (10) are only small corrections to the free case for low and mid- Z nuclei ($< 5\%$ for $Z = 50$, and $\sim 0.1\%$ for $Z = 10$), which is sufficient for our purposes.

The best electron-neutron new physics exclusion limits in atomic precision spectroscopy originate from isotope shifts [8,25–27]. The correction to the g factor due to the new scalar mediator stems from the evaluation of the diagram in Fig. 1(c), and it can be calculated by taking the following derivative [12] of the expectation value of the Yukawa potential $V_\phi(\mathbf{r}) = -\alpha_{en}(A - Z)[e^{-m_\phi|\mathbf{r}|}/|\mathbf{r}|]$ experienced by the electron:

$$\begin{aligned} \Delta g_\phi^{en} &= \frac{\partial}{\partial m_e} \langle \psi_{1s} | V_\phi | \psi_{1s} \rangle \\ &= -\frac{4}{3} \alpha_{en} (A - Z) \frac{Z\alpha}{\gamma} \left(1 + \frac{m_\phi}{2Zam_e} \right)^{-2\gamma} \\ &\quad \times \left[3 - 2 \frac{(Z\alpha)^2}{1 + \gamma} - 2\gamma \left(1 + \frac{m_\phi}{2Zam_e} \right)^{-1} \right], \end{aligned} \quad (11)$$

where $\gamma = \sqrt{1 - (Z\alpha)^2}$, $\alpha_{en} = y_e y_n / 4\pi$ is the electron-neutron coupling, and $A - Z$ is the number of neutrons in the nucleus. The above equation reproduces the results of [21], where the correspondent exclusion plot for $y_e y_n$ can be found. The contribution of Δg_ϕ^{ee} fully cancels in an isotope shift in our approximation [see Eq. (9)].

As stated in Eq. (2), the total g -factor modification is given by the sum of three different contributions, and because of Eq. (3), one could naively think that, in order to obtain the electron-proton part, it may be sufficient to isolate the ep term in Eq. (2), however, given that each piece of the right-hand side of the latter equation undergoes a *lower-than* inequality, a difference among them is not mathematically allowed to obtain an upper bound for Δg_ϕ^{ep} . Either one relies on new methods to isolate Δg_ϕ^{ep} (that is what we propose later), or has to allow the least stringent scenario to take place, that is, $\Delta g_\phi^{ee} = \Delta g_\phi^{en} = 0$. We follow this latter idea anyway because it nonetheless imposes a strong constraint on $y_e y_p$. The potential between the electron and nuclear protons has the very same form as the previous neutron one, and so does the g -factor correction for which the only differences compared to Eq. (11) are the replacements $\alpha_{en} \rightarrow \alpha_{ep} = y_e y_p / 4\pi$ and $A - Z \rightarrow Z$, resulting in

$$\begin{aligned} \Delta g_\phi^{ep} &= -\frac{4}{3} \alpha_{ep} Z \frac{Z\alpha}{\gamma} \left(1 + \frac{m_\phi}{2Zam_e} \right)^{-2\gamma} \\ &\quad \times \left[3 - 2 \frac{(Z\alpha)^2}{1 + \gamma} - 2\gamma \left(1 + \frac{m_\phi}{2Zam_e} \right)^{-1} \right]. \end{aligned} \quad (12)$$

Along the lines of the isotope shift, where a cancellation of the electron-electron and electron-proton interactions takes place, here we put forward a method to cancel the neutron interaction and suppress the electronic self-interaction. From Eq. (9) we see that the ee contribution is proportional to the coupling constant via Eq. (7), and the part inside the round brackets depends on the atomic number Z . In Eqs. (11) and (12), we see a proportionality to the corresponding coupling constant, too, and since the two expressions have the same form except for the numbers $A - Z$ and Z , we rename them to $\Delta g_\phi^{en} = \alpha_{en}(A - Z)f(Z)$ and $\Delta g_\phi^{ep} = \alpha_{ep}Zf(Z)$, where $f(Z)$ is a function of Z including every other remaining term. Now, given two different ions, labeled 1 and 2, the g -factor corrections will vary depending on the number of protons and neutrons, namely

$$\begin{aligned} \Delta g_{\phi,1} &= \Delta g_\phi^{ee, \text{free}} \left(1 - \frac{(Z_1\alpha)^2}{6} \right) \\ &\quad + \alpha_{en}(A_1 - Z_1)f(Z_1) + \alpha_{ep}Z_1f(Z_1), \end{aligned} \quad (13)$$

$$\begin{aligned} \Delta g_{\phi,2} &= \Delta g_\phi^{ee, \text{free}} \left(1 - \frac{(Z_2\alpha)^2}{6} \right) \\ &\quad + \alpha_{en}(A_2 - Z_2)f(Z_2) + \alpha_{ep}Z_2f(Z_2). \end{aligned} \quad (14)$$

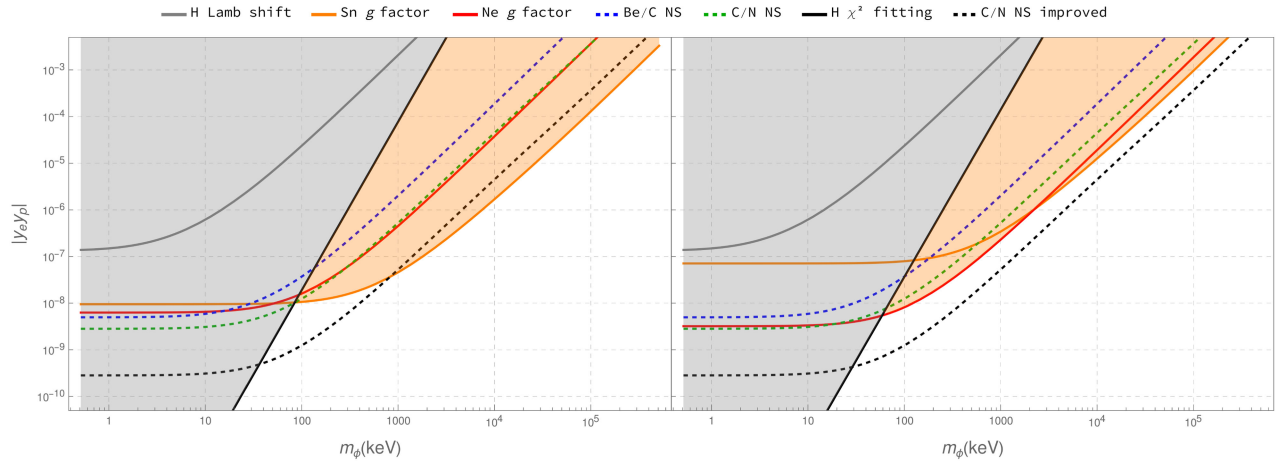


FIG. 2. Exclusion lines for electron-proton attractive (left) and repulsive (right) coupling. Solid lines are based on experimental and theoretical available data, whereas dashed ones are projections.

Subtracting from the former expression the latter multiplied by the quantity $x := (A_1 - Z_1/A_2 - Z_2)[f(Z_1)/f(Z_2)]$, dependent on m_ϕ , the neutron part is completely canceled, and the electron-electron part is suppressed, resulting in

$$\begin{aligned} \Delta g_{\phi,1-2}^{\text{NS}} &= \Delta g_{\phi,1} - x \Delta g_{\phi,2} \\ &= \Delta g_{\phi}^{\text{ee,free}} \left[\left(1 - \frac{(Z_1 \alpha)^2}{6} \right) - x \left(1 - \frac{(Z_2 \alpha)^2}{6} \right) \right] \\ &\quad + y \alpha_{ep} f(Z_1), \end{aligned} \quad (15)$$

where we defined $y = Z_1 - Z_2(A_1 - Z_1/A_2 - Z_2)$. The upper index NS stands for “nuclide shift.” Because an experiment can measure the g factor of two ions, or a difference between them (as in the case of isotope shifts [8]), we recast Eq. (15) into

$$\begin{aligned} \Delta g_{\phi,1-2}^{\text{NS}} &= \Delta g_{\phi,1} - \Delta g_{\phi,2} + (1-x) \Delta g_{\phi,2} \\ &= \Delta g_{\phi,1-2} + (1-x) \Delta g_{\phi,2}, \end{aligned} \quad (16)$$

where $\Delta g_{\phi,1-2} = \Delta g_{\phi,1} - \Delta g_{\phi,2}$ is the regular difference between the two g -factor contributions. A dedicated experiment needs to measure both the g -factor difference and the value for a single ion, and, as recently, isotope shift g -factor experiments were shown to be more precise [8] than those involving the g factor of a single ion, one can anticipate a similar behavior for an experiment based on this nuclide shift method. To this end, a value $x \simeq 1$ (such that $1 - x \ll 1$) is preferred to suppress inaccuracies due to $\Delta g_{\phi,2}$ in Eq. (16). The question arises as to which pair of isotopes works best (more details can be found in Supplemental Material [28]). In this Letter, we look for isotopes with a half-life of at least a few months and in the low-mid nuclear number range, such that $x \simeq 0.9$. We find that the pair that optimally satisfies these requests is $^{14}\text{C}^{5+}$, $^{14}\text{N}^{6+}$ for which $x \in [0.72, 0.98]$.

Note that the electronic g factor of an ion with nonzero nuclear spin can be expressed from the experimentally measurable g_F factors of the hyperfine levels [31] or from transition frequencies between magnetic sublevels [32]. Another suitable pair is $^9\text{Be}^{3+}$, $^{12}\text{C}^{5+}$ for which $x \in [0.24, 0.56]$; the latter does not fulfill the requirement for x , but it is still very interesting for our purposes and sets competitive bounds (see Supplemental Material [28] for more details). In order to give a proper bound on α_{ep} using this method, we need experimental and theoretical data. Since for some nuclides we have neither one nor the other, we derive projected bounds based on an estimation using data from the neon isotope shift experiment [7,8] from the beryllium theory [29], and from carbon and nitrogen theory [16]. The estimation of uncertainties is explained in Supplemental Material [28].

Results—We found that the limit for $y_e y_p$ through the free-electron $g - 2$ is already below 10^{-10} for small scalar masses m_ϕ , and in our NS it becomes even more suppressed.

According to this model, exclusion plots for the coupling constant $y_e y_p$ are presented in Fig. 2, where the parameter regions above the curves are excluded. For the sake of generality, we allow both the positive and negative signs of $y_e y_p$, translating into a repulsive and an attractive force, respectively. To draw the exclusion curves, we compared experimental and theoretical data, and assumed that their difference within the error bars can be explained by the new physics contribution, as in Eq. (3). The resulting bounds are obtained at one-sided 95% confidence level. The plots have been generated by making use of electronic wave functions corrected for a uniform spherical charge distribution of the nucleus, and the Yukawa potential has been convoluted with the same nuclear model. Nuclear rms radius values have been taken from [33]. Finite nuclear size corrections are practically negligible for our interests for light ions, e.g., for neon ($Z = 10$)

they are $\sim 1\%$ for $m_\phi \sim 1000m_e \sim 500\,000$ keV, already decreasing to $\sim 0.001\%$ for $m_\phi \sim 100m_e \sim 50\,000$ keV. However they grow significantly for tin ($Z = 50$), which features a $\sim 1\%$ correction for $m_\phi \sim 20m_e \sim 1000$ keV, rising to $\sim 42\%$ for $m_\phi \sim 1000m_e \sim 500\,000$ keV.

Solid lines are based on fully available data and depend on the sign of $y_e y_p$, dashed lines are projections, independent on $y_e y_p$. The gray ones come from the hydrogen $1s - 2s$ Lamb shift data [34,35], where we employed theory from [21]. The solid black curves are linear extrapolations of hydrogen data from Refs. [36,37], where the exclusion plots have been drawn through χ^2 fitting using many transitions in the hydrogen atom. It establishes a competitive bound in the mass range up to 30 keV, after which, hydrogen stops performing well. Under the stricter physical assumption that a muon couples to a proton with the same coupling constant as an electron, Ref. [36] reports an even better curve, although this case is not directly comparable to ours. Orange and red curves are consequence of Eq. (12). The orange curves stem from the mid- Z H-like tin g -factor measurement [38], for which the QED theory has recently been improved by a nonperturbative calculation of the two-loop self-energy corrections [18]; although g -factor experimental uncertainties remain approximately the same as the charge number increases, the same cannot be said for theoretical uncertainties, since certain higher-order corrections become more and more important with increasing Z . This is reflected in the red curves, for the low- Z neon ion, where a smaller theoretical error bar yields a better exclusion line [7] in the mid-low mass regime. However, the higher nuclear number of Sn remains a better probe for the low-range or high-mass regime. Next, we point to the blue and green dashed lines, for which the NS method has been employed. As mentioned above, to draw these lines we approximated the experimental error bar of the NS to be the same as that of the isotope shift in the experiment [8], and assumed theoretical error bars from [29] for Be, and [16] for C and N. Lacking values for some elements, we assume that QED theoretical and experimental values are equal. The combination of dashed lines with solid ones generates, in each plot, the most competitive bounds for the mid- and high-mass regime, enhancing bounds by at least 2 orders of magnitude for boson masses above 1 MeV. Finally, the black, dashed curves have been generated similarly to the green ones, with the assumption of a feasible improvement [30] of one order of magnitude in the experimental accuracy with respect to the isotope shift mentioned above [8] and assuming a matching theoretical improvement. The orange shaded area presents the improvement achieved with the available data compared to the best current bounds [36,37] shown by the gray shaded area. Further improvements could be obtained with the NS method, as indicated by the dashed lines.

Conclusions and outlook—In this Letter, we analyzed the contribution of a hypothetical scalar boson to the bound-electron g factor in hydrogenlike ions. Beside the electron self-interaction and electron-nucleon interactions, we put forward a method to selectively probe the electron-proton coupling in a semi-independent way that is otherwise not accessible in other spectroscopic studies. This results in a series of competitive exclusion plots of coupling strength against the mass of the scalar. An effective disentanglement of the three contributions has been performed by employing a g -factor shift between two ions with different nuclear charges, and identifying the optimal pair of nuclides. Projections of Fig. 2 follow from existing theoretical data and foreseeable near-future “nuclide shift” experiments generalizing the concept of the well-known isotope shift, with the reasonable assumption of having experimental error bars similar to those already achieved. With our concept, we were able to provide and predict exclusion curves for electron-proton interaction with a stringency comparable to that of the electron-neutron interaction, improving the best previous bounds up to 3 orders of magnitude in the regime of high boson masses above 100 keV. Near-future experimental advancements in Penning trap measurements, combined with foreseeable improved theoretical calculations of QED and nuclear structure effects, will further enhance the sensitivity of these tests.

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Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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