

Fully Charm-Tetraquark Production at Hadronic Collisions with Gluon Radiation Effects

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We report the first complete next-to-leading order QCD calculation for processes involving fully charm-tetraquark states, revealing that the renormalization constant of the color-singlet four charm quark operator is exactly unity at this order. We have investigated the possible quark configurations of the fully charm tetraquarks and expanded their states in the color symmetry-antisymmetry basis. By applying the transverse momentum dependent factorization formalism, large logarithms induced by soft and collinear gluon radiations are resummed to all orders in the expansion of the strong interaction coupling at the accuracy of next-to-leading logarithm. By combining LHCb data on the total cross section of the exotic hadron $X(6900)$ and CMS measurements of its spin parity, we extracted its nonperturbative but universal long-distance matrix element. The rapidity and transverse momentum distributions of the $X(6900)$ and its spin-zero partners are also predicted, which await further experimental verification.

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Introduction—Charm family hadrons, according to the color confinement principle, are believed to be capable of containing one, two, three, four or even more charm quarks. Although the J/ψ meson composed of a charm-anticharm pair was discovered 52 years ago [1,2] and the D meson composed of a charm-antilight pair two years later [3], it was not until 2020 that the LHCb experiment found the first evidence of a pure charm-tetraquark state, the $X(6900)$, in the J/ψ -pair invariant mass spectrum using a data sample corresponding to an integrated luminosity of 9 fb^{-1} thereby providing a signal for a hadron with multiple charm quarks [4].

Using a data sample focused on the small rapidity region, both the ATLAS and CMS collaborations have confirmed the narrow $X(6900)$ state and observed another narrow structure around 7100 MeV and a broad structure above twice the J/ψ mass [5,6].

Several particularly interesting aspects of these exotic states include (1) the spin-parity quantum numbers for all observed data are found to be compatible with the $J^{PC} = 2^{++}$ hypothesis from the very recent CMS analysis [7], which constrains the possible internal quark configuration for these exotic structures; (2) a Regge trajectory is predicted and observed from their mass spectrum [8,9], which gives hints of a radial or orbital excitation tetraquark

family; (3) independent observation from three LHC experiments with different rapidity region reveals that the production rate of the fully charm tetraquark state is slightly affected by low and medium rapidity, yet its signal relative to that of J/ψ pair is enhanced at medium and high transverse momentum [4].

Addressing and clarifying the above three points requires both a consistent theoretical framework and precise calculations. From the perspective of group theory, the hadron states can always be represented by a complete set of basis states. In terms of color space, one can choose either the color-symmetry-antisymmetry $6 \otimes \bar{6}/\bar{3} \otimes 3$ basis or the color-singlet-octet $1 \otimes 1/8 \otimes 8$ basis for local observations. In color-symmetry-antisymmetry basis, the two charm quarks can form the “good” axial-vector diquark with antisymmetric color $\bar{3}$ and symmetric spin-flavor configuration or “bad” scalar diquark with symmetric color 6 and antisymmetric spin-flavor configuration. The two charmed antiquarks behave similarly. Then charmed diquark and antidiquark carry color charge and form a color-charge-neutral and tightly bound hadron from a QCD confining attractive force. Interestingly, charmed diquark here is distinctly different from light-diquark system, due to the additional constraint of isospin symmetry present in the latter. In color-singlet-octet basis, the two charm quark-antiquark pairs can form a color-singlet charmonium or a color-octet cluster separately, then the two charmonia or colored clusters form a color-charge-neutral and loosely bound molecule hadron from a residual strong interaction.

A true physical system may reside in one of the basis states mentioned above, or in a superposition of these basis states. For example, we can expand the fully heavy tetraquark state as

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$$|T_{4Q}\rangle = \sum_{L=0} C_L^s |(\mathcal{Q}_1 \mathcal{Q}_2)_0; (\bar{\mathcal{Q}}_3 \bar{\mathcal{Q}}_4)_0; L\rangle + C_L^a |(\mathcal{Q}_1 \mathcal{Q}_2)_1; (\bar{\mathcal{Q}}_3 \bar{\mathcal{Q}}_4)_1; L\rangle, \quad (1)$$

where C_L^s and C_L^a are the symmetric and antisymmetric configuration coefficients with $\sum_{L=0} |C_L^s|^2 + |C_L^a|^2 = 1$, respectively. Orbital excitation effects are marked with L . The higher Fock states with gluon content are omitted here but can be included in a similar manner.

In this Letter, we calculate the differential and total cross sections for the hadronic production of S-wave fully charm-tetraquark states based on the color-symmetry-antisymmetry basis and nonrelativistic QCD (NRQCD) effective theory. Our calculations include contributions from all possible structures, such as $J^{PC} = 0^{++}$ or 2^{++} , as well as mixing effects arising from identical quantum numbers. At hadron colliders, gluon radiation effects are nonnegligible; therefore, we provide a complete treatment of QCD corrections up to the next-to-leading order (NLO). Meanwhile, we perform transverse momentum resummation at the next-to-leading logarithmic (NLL) accuracy to mitigate large logarithmic terms induced by soft and collinear gluon emissions in the low transverse momentum $P_\perp$ region.

QCD resummation formulas—Using the QCD factorization and transverse momentum resummation theory [10,11], the differential cross section of fully charm tetraquark at hadronic collisions can be factorized as

$$\begin{aligned} & \frac{d\sigma(p + p \rightarrow T_{4c} + X)}{dy dP_\perp^2} \\ &= \int \frac{d^2 b e^{i \vec{P}_\perp \cdot \vec{b}}}{(2\pi)^2} \sum_{ij} \int_{x_A}^1 \frac{d\xi_1}{\xi_1} \int_{x_B}^1 \frac{d\xi_2}{\xi_2} f_i\left(\frac{x_A}{\xi_1}, \mu\right) f_j\left(\frac{x_B}{\xi_2}, \mu\right) \\ & \times \frac{M^2}{S} \hat{\sigma}_{i'j'}^{(0)} \pi W^{NP}(b, M) C_{i'i}(\xi_1, b_*, \mu) C_{j'j}(\xi_2, b_*, \mu) \\ & \times e^{-\int_{c_1^2/b_*^2}^{c_2^2 M^2/\bar{\mu}^2} \left[A_{ij}(\alpha_s(\bar{\mu})) \ln\left(\frac{c_2^2 M^2}{\bar{\mu}^2}\right) + B_{ij}(\alpha_s(\bar{\mu})) \right]} \\ & + Y(P_\perp, M, x_A, x_B, \mu_F, \mu_R), \end{aligned} \quad (2)$$

where $x_{A(B)} = (M/\sqrt{S})e^{\pm y}$ with the collider energy $\sqrt{S}$, the tetraquark mass M , and rapidity y . $f_i(x_i, \mu)$ is the parton distribution function. The matching scale is chosen as $\mu = C_3/b_*$ with $b_* = b/\sqrt{1 + (b/b_{\max})^2}$ such that there are no large logs in matching kernel C_{ij} [12–48]. C_i are on the order of $\mathcal{O}(1)$, and we have $C_1 = C_3 = 2e^{-\gamma_E}$, $C_2 = 1$. The nonperturbative Sudakov factor $W^{NP}(b, M) = e^{-S_{NP}}$ with $S_{NP} = b^2(g_1 + g_2 \ln(M/(2Q_0)) + g_3 \ln(100x_1x_2))$ is employed with the known BLNY and SIYY form [32–34], where $x_i = x_{A,B}/\xi_i$, $g_1 = 0.181 \pm 0.005$, $g_2 = 0.167 \pm 0.01$, $g_3 = 0.003$ with $b_{\max} = 1.5 \text{ GeV}^{-1}$, $Q_0 = 1.55 \text{ GeV}$ for

quarks and $g_1 = 0.030 \pm 0.113$, $g_2 = 0.870 \pm 0.134$, $g_3 = -0.170 \pm 0.030$ with $b_{\max} = 0.5 \text{ GeV}^{-1}$, $Q_0 = 1.6 \text{ GeV}$ for gluon. The regular term is

$$\begin{aligned} Y(P_\perp, M, x_A, x_B, \mu_F, \mu_R) &= \sum_{ij} \int_{x_A}^1 \frac{d\xi_1}{\xi_1} \int_{x_B}^1 \frac{d\xi_2}{\xi_2} \\ & \times f_i\left(\frac{x_A}{\xi_1}, \mu_F\right) f_j\left(\frac{x_B}{\xi_2}, \mu_F\right) \\ & \times R_{ij}(P_\perp, M, x_A, x_B, \mu_F, \mu_R). \end{aligned} \quad (3)$$

The above factorization formula is believed to hold to all orders in α_s , and the short-distance perturbative coefficients can be calculated order by order in the perturbation QCD theory. We expand these coefficients as

$$\begin{aligned} A(\alpha_s(\mu)) &= \sum_n \left(\frac{\alpha_s(\mu)}{\pi}\right)^n A^{(n)}, \\ B(\alpha_s(\mu)) &= \sum_n \left(\frac{\alpha_s(\mu)}{\pi}\right)^n B^{(n)}, \\ C_{ij}(z, b, \mu) &= \sum_n \left(\frac{\alpha_s(\mu)}{\pi}\right)^n C_{ij}^{(n)}, \\ R_{ij}(P_\perp, M, x_A, x_B, \mu, \mu_R) &= \sum_n \left(\frac{\alpha_s(\mu)}{\pi}\right)^n R_{ij}^{(n)}. \end{aligned} \quad (4)$$

Up to NLO + NLL accuracy, the nontrivial results are

$$\begin{aligned} A_{gg}^{(1)} &= C_A, \quad A_{q\bar{q}}^{(1)} = C_F, \\ B_{ij}^{(1)} &= A_{ij}^{(1)} \ln\left(\frac{C_1^2 e^{2\gamma_E}}{4C_2^2}\right) + \delta_{ij}^{(1)}, \quad \text{where } ij = gg, q\bar{q}, \\ C_{gg}^{(0)}(z, b, \mu) &= C_{q\bar{q}}^{(0)}(z, b, \mu) = \delta(1-z), \\ C_{ij}^{(1)}(z, b, \mu) &= P_{ij}(z) \ln\left(\frac{2e^{-\gamma_E}}{b\mu}\right) + \frac{A_{ij}^{(1)}}{2} \delta(1-z) \\ & \times \left[-\frac{\pi^2}{6} - 2\ln^2\left(\frac{C_1 e^{\gamma_E}}{2C_2}\right) + \ln\left(\frac{C_1 b \mu e^{2\gamma_E}}{4C_2}\right) \right. \\ & \left. + Fin_{i\bar{j}, g_n}^{(1),(J)}\left(\frac{C_1}{C_2 b}, \mu\right) \right], \quad \text{where } i\bar{j} = gg, q\bar{q}, \\ C_{qg}^{(1)}(z, b, \mu) &= C_{q\bar{q}}^{(1)}(z, b, \mu) = P_{qg}(z) \ln\left(\frac{2e^{-\gamma_E}}{b\mu}\right), \\ C_{gq}^{(1)}(z, b, \mu) &= C_{g\bar{q}}^{(1)}(z, b, \mu) = P_{gq}(z) \ln\left(\frac{2e^{-\gamma_E}}{b\mu}\right), \end{aligned} \quad (5)$$

where $P_{ij}(z)$ is the parton splitting distribution function $\delta_{gg}^{(1)} = \frac{11}{2} - (2n_h + n_l)/3$, $\delta_{q\bar{q}}^{(1)} = 9 - (2n_h + 2n_l)/3$ with the heavy and light quark flavor numbers n_h and n_l . The expressions for $Fin_{i\bar{j}, g_n}^{(n),(J)}(M, \mu)$ and $R_{ij}^{(n)}$ are given

in Supplemental Material [49]. $\hat{\sigma}_{ij}^{(0)}$ is the leading-order (LO) partonic cross section with

$$\hat{\sigma}_{ij}^{(0)} = \frac{\pi}{M^4} \frac{1}{c_i c_j s_i s_j} \left| \mathcal{A}_{ij}^{(0)} \right|^2, \quad (6)$$

where the color and spin average factors are $c_g = N_c^2 - 1$, $c_q = c_{\bar{q}} = N_c$, $s_g = 2 - 2\epsilon$, and $s_q = s_{\bar{q}} = 2$. The Feynman amplitude squared from certain operator contribution can be written as

$$\left| \mathcal{A}_{ij}^{(n)} \right|_{O_{g_1, g_2}^J}^2 = \frac{M\pi^4 \alpha_s^4}{128 m_c^8} c_{ij, O_{g_1, g_2}^J}^{(n)} \langle 0 | O_{g_1, g_2}^J | 0 \rangle. \quad (7)$$

In the following, $m_c = M/4$ with the tetraquark mass $M = 6847_{-28-20}^{+44+48}$ MeV [6] is adopted as the charm quark mass. O_{g_1, g_2}^J are the NRQCD long-distance operators for tetraquarks with spin quantum number J ,

$$O_{g_1, g_2}^J = O_{g_1}^{(J)} \sum_X [T_{4c}^J + X] \langle T_{4c}^J + X | O_{g_2}^{(J)\dagger} \rangle. \quad (8)$$

Therein g_i can be either $\mathbf{6} \otimes \bar{\mathbf{6}}$ or $\bar{\mathbf{3}} \otimes \mathbf{3}$ in color symmetry-antisymmetry basis. Then we have [50,51]

$$\begin{aligned} \mathcal{O}_{\bar{\mathbf{3}}\mathbf{3}}^{(0)} &= -\frac{1}{\sqrt{3}} [\psi_a^T (i\sigma^2) \sigma^i \psi_b] [\chi_c^\dagger \sigma^i (i\sigma^2) \chi_d^*] \mathcal{C}_{\bar{\mathbf{3}}\mathbf{3}}^{ab;cd}, \\ \mathcal{O}_{\bar{\mathbf{3}}\mathbf{3}}^{(2;ij)} &= [\psi_a^T (i\sigma^2) \sigma^m \psi_b] [\chi_c^\dagger \sigma^n (i\sigma^2) \chi_d^*] \Gamma^{ij;mn} \mathcal{C}_{\bar{\mathbf{3}}\mathbf{3}}^{ab;cd}, \\ \mathcal{O}_{\mathbf{6}\bar{\mathbf{6}}}^{(0)} &= [\psi_a^T (i\sigma^2) \psi_b] [\chi_c^\dagger (i\sigma^2) \chi_d^*] \mathcal{C}_{\mathbf{6}\bar{\mathbf{6}}}^{ab;cd}, \end{aligned} \quad (9)$$

where $\mathcal{O}_{\bar{\mathbf{3}}\mathbf{3}}^{(2)} = \frac{1}{2} \epsilon^{*ij} \mathcal{O}_{\bar{\mathbf{3}}\mathbf{3}}^{(2;ij)}$ with the polarization tensor ϵ^{ij} , the rank-4 color tensor is defined as $\mathcal{C}_{\mathbf{6}\bar{\mathbf{6}}}^{cd;ef} = \frac{1}{2\sqrt{6}} (\delta_{ce} \delta_{df} + \delta_{cf} \delta_{de})$ and $\mathcal{C}_{\bar{\mathbf{3}}\mathbf{3}}^{cd;ef} = \frac{1}{2\sqrt{3}} (\delta_{ce} \delta_{df} - \delta_{cf} \delta_{de})$. The rank-4 Lorentz tensor is defined as $\Gamma^{\alpha\beta; \mu\nu} = \frac{1}{2} g^{\mu\alpha} g^{\nu\beta} + \frac{1}{2} g^{\mu\beta} g^{\nu\alpha} - 1/(D-1) g^{\mu\nu} g^{\alpha\beta}$.

The typical Feynman diagrams for fully charm-tetraquark hadronic production are plotted in Fig. 1. At LO, there are 62 and 4 tree-level nonvanishing Feynman diagrams contributing to $g + g \rightarrow T_{4c}$ and $q + \bar{q} \rightarrow T_{4c}$. At NLO, there are 2008 and 170 one-loop nonvanishing Feynman diagrams contributing to $g + g \rightarrow T_{4c}$ and $q + \bar{q} \rightarrow T_{4c}$. Also we need to consider the real corrections and find 618, 98, and 98 tree-level nonvanishing Feynman diagrams contributing to $g + g \rightarrow T_{4c} + g$, $q + \bar{q} \rightarrow T_{4c} + g$ and $q + g \rightarrow T_{4c} + q$, respectively. Dimensional regularization with $D = 4 - 2\epsilon$ is employed for both UV and IR divergences. We have generated and simplified the tree-level and one-loop amplitudes by using the mainstream packages [52–59]. In addition, the Ward identity is used to check our results.

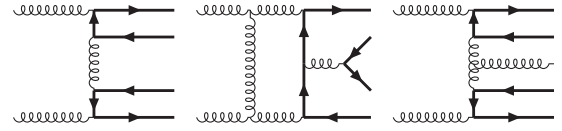


FIG. 1. Typical Feynman diagrams for the gluon-gluon fusion to fully charm tetraquark up to NLO. The thick black lines stand for the massive charm quarks. The hadronization process of the four charm quarks into a fully charm tetraquark is not displayed. The Feynman diagrams from the quark-antiquark annihilation or (anti)quark-gluon scattering can be plotted similarly.

At LO, the short-distance coefficients for gluon-gluon fusion are

$$c_{gg, O_{g_1, g_2}^J}^{(0)} = \left\{ \frac{15488}{27}, 2304, \frac{1408\sqrt{6}}{3}, \frac{200704}{27} \right\}, \quad (10)$$

for $O_{g_1, g_2}^J = O_{\mathbf{6}\bar{\mathbf{6}}, \mathbf{6}\bar{\mathbf{6}}}^0, O_{\bar{\mathbf{3}}\mathbf{3}, \bar{\mathbf{3}}\mathbf{3}}^0, O_{\mathbf{6}\bar{\mathbf{6}}, \bar{\mathbf{3}}\mathbf{3}}^0, O_{\bar{\mathbf{3}}\mathbf{3}, \bar{\mathbf{3}}\mathbf{3}}^2$, respectively. The result for $O_{\bar{\mathbf{3}}\mathbf{3}, \mathbf{6}\bar{\mathbf{6}}}^0$ is the same as for $O_{\mathbf{6}\bar{\mathbf{6}}, \bar{\mathbf{3}}\mathbf{3}}^0$. The nonvanishing contribution in quark-antiquark annihilation for the spin-2 tetraquark is $c_{q\bar{q}, O_{\bar{\mathbf{3}}\mathbf{3}, \bar{\mathbf{3}}\mathbf{3}}^2}^{(0)} = 32768/9$, while the result vanishes for the spin-0 tetraquark because of helicity conservation. We also find that the renormalization constants of all color singlet four charm quark operators in Eq. (9) are exactly unity at NLO, which has never been explored in prior literature.

Constraint on long-distance matrix elements for fully charm tetraquark—The production cross section of the $X(6900)$ relative to that of J/ψ pair, times the branching fraction $\mathcal{B}(X(6900) \rightarrow J/\psi + J/\psi)$, was measured to be $\mathcal{R} = [1.1 \pm 0.4(\text{stat}) \pm 0.3(\text{syst})]\%$ without any $P_{\perp}^{T_{4c}}$ cuts in the proton-proton collision data at $\sqrt{S} = 13$ TeV by the LHCb Collaboration [4]. Subsequently, they have measured the production cross section of the J/ψ pair to be $\sigma(2J/\psi) = 16.36 \pm 0.28(\text{stat}) \pm 0.88(\text{syst}) \text{ nb}$ at $\sqrt{S} = 13$ TeV with both J/ψ mesons in the transverse range $0 < P_{\perp}^{J/\psi} < 14$ GeV and rapidity range $2.0 < y^{J/\psi} < 4.5$ [60]. The related theoretical result for $\sigma(2J/\psi)$ appears in Refs. [61,62]. It is known that the spin parity of the $X(6900)$ is determined as 2^{++} from the CMS data. From the above analysis, only one of long-distance matrix elements (LDMEs) is required for a $J^{PC} = 2^{++}$ fully charm tetraquark at leading power, which can be uniquely determined from the preceding experimental data. Combining the data and the NLO + NLL QCD formula, we determine the $J^{PC} = 2^{++}$ LDMEs joint di- J/ψ decay branching ratio as

$$\begin{aligned} &\langle \mathcal{O}_{\bar{\mathbf{3}}\mathbf{3}}^{T_{4c}} \rangle ({}^5S_2, 2^{++}) \mathcal{B}(T_{4c}) \\ &= \langle 0 | O_{\bar{\mathbf{3}}\mathbf{3}, \bar{\mathbf{3}}\mathbf{3}}^2 | 0 \rangle \mathcal{B}(T_{4c}) \\ &= (2.22 \pm 0.80_{-0.57-0.00}^{+1.29+0.62}) \times 10^{-4} \text{ GeV}^9, \end{aligned} \quad (11)$$

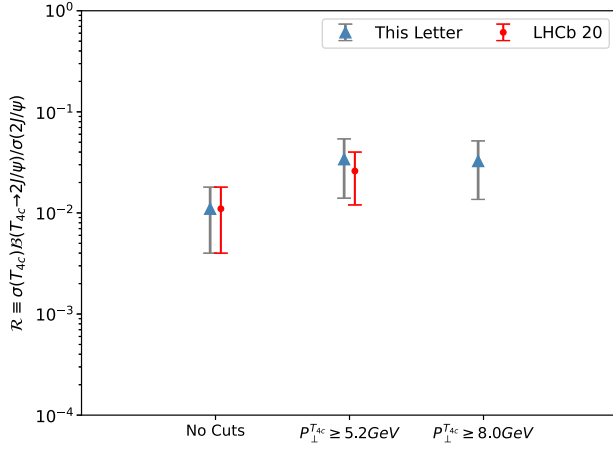


FIG. 2. NLO prediction for cross section of the $X(6900)$ relative to that of J/ψ pair, times the branching fraction $\mathcal{B}(X(6900) \rightarrow J/\psi + J/\psi)$. The LHCb data is from Ref. [4].

where the first column in uncertainties is from the existing LHCb measurements; the second column is from the scale uncertainty with $\mu_F = \mu_R \in \{M/2, M, 2M\}$ for $P_\perp$ -integrated observables and $\{\sqrt{(M/2)^2 + P_\perp^2}, \sqrt{M^2 + P_\perp^2}, \sqrt{(2M)^2 + P_\perp^2}\}$ for $P_\perp$ -differential cross sections; and the third column is from the parton distribution function sets (PDF4LHC21_mc [63], CT18NNLO [64], NNPDF23_nnlo_as_0116 [65]) with the LHAPDF package [66].

Due to the lack of experimental data for 0^{++} fully charm tetraquark, other LDMEs in the Letter are hard to determine reliably. As an order-of-magnitude estimate, using heavy quark spin symmetry, we can perform an estimation

$$\begin{aligned} \langle \mathcal{O}_{66}^{T_{4c}} \rangle ({}^1S_0, 0^{++}) &\approx \langle \mathcal{O}_{33}^{T_{4c}} \rangle ({}^1S_0, 0^{++}) \\ &\approx \langle \mathcal{O}_{\text{mix}}^{T_{4c}} \rangle ({}^1S_0, 0^{++}) \approx \frac{\langle \mathcal{O}_{33}^{T_{4c}} \rangle ({}^5S_2, 2^{++})}{2J+1} \Big|_{J=2} \\ &= (4.44 \pm 1.60_{-1.14}^{+2.58+1.24}) \times \frac{10^{-5}}{\mathcal{B}(T_{4c})} \text{GeV}^9. \end{aligned} \quad (12)$$

To test the predictive power of the NLO + NLL QCD formula in other kinematic regions, we plot the $\mathcal{R} \equiv \sigma(T_{4c})/\sigma(2J/\psi)\mathcal{B}(T_{4c} \rightarrow 2J/\psi)$ with different transverse momentum cuts in Fig. 2. From the plots, the ratio $\mathcal{R}$ increases when the lower limit of $P_\perp$ increases. Consider the $P_\perp^{2J/\psi} > 5.2$ GeV case, the NLO + NLL calculation gives $\mathcal{R}^{\text{theo.}} = [3.2 \pm 2.0]\%$, which shows good agreement with the LHCb data $\mathcal{R}^{\text{LHCb}} = [2.6 \pm 0.6(\text{stat}) \pm 0.8(\text{syst})]\%$ [4].

Differential and total production cross section—The soft and collinear gluon radiation induce singular structures specifically terms proportional to $1/P_\perp^2$ and $\ln(P_\perp^2/M^2)/P_\perp^2$ in the transverse momentum distribution, as shown in the divergence of NLO results with $P_\perp \ll M$ in Fig. 3. By applying the transverse momentum resummation

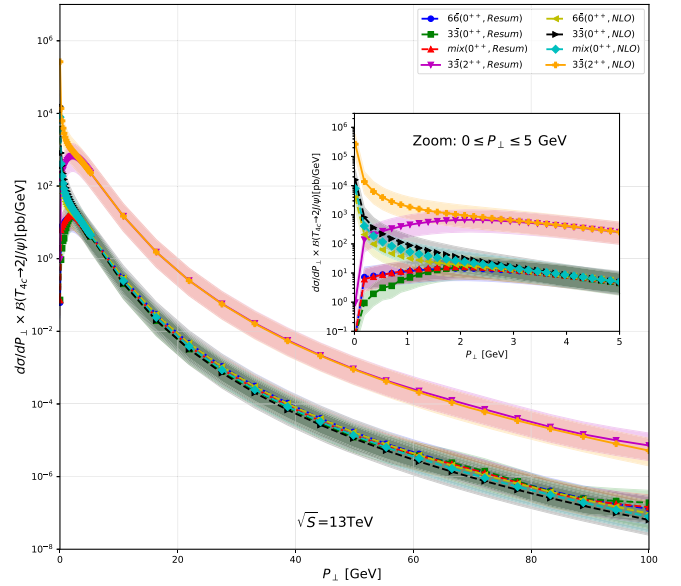


FIG. 3. NLO and Resummation(NLO + NLL) differential cross sections for the fully charm tetraquark with different quark configurations depending on the transverse momentum $P_\perp$, times the $2J/\psi$ decay branching fraction. In this Letter $X(6900)$ is treated as a 2^{++} state and its 0^{++} partner is also plotted. The band is from the theoretical uncertainty.

formalism [10–48], we improved the theoretical prediction of fully charm tetraquark on the kinematical distributions to sum over large logarithms $\ln(P_\perp^2/M^2)$ to all orders in the expansion of α_s at the NLO + NLL accuracy.

We also plot the rapidity distribution in Fig. 4 and the total cross section in Fig. 5 for different quark configurations. The differential cross section exhibits a relatively flat profile in rapidity, particularly across the low and intermediate rapidity ranges. From these plots, the 2^{++}

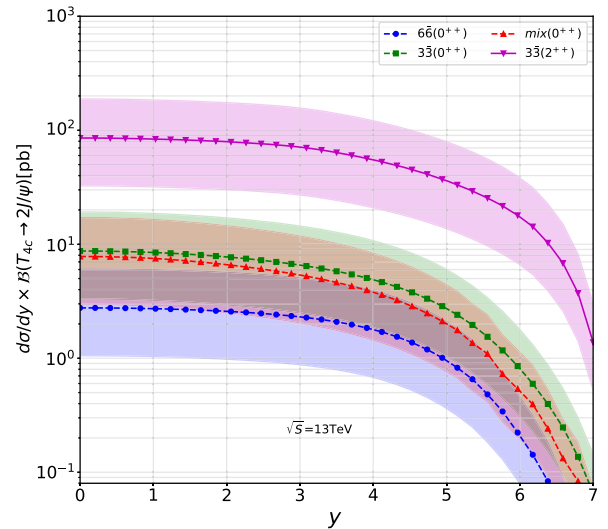


FIG. 4. NLO differential cross-sections for the fully charm tetraquark with different quark configurations depending on the rapidity y , times the $2J/\psi$ decay branching fraction.

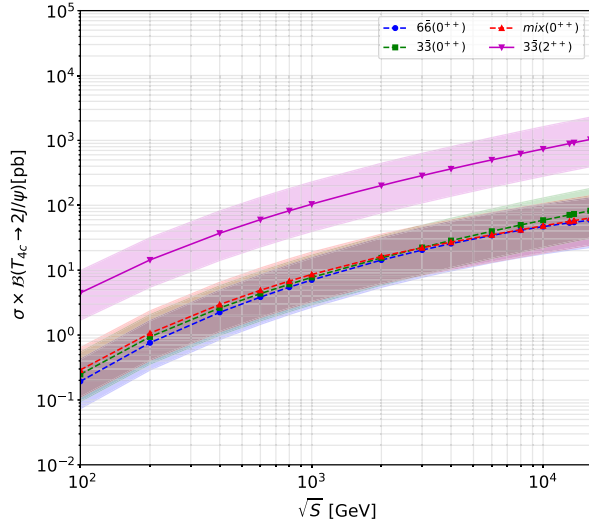


FIG. 5. NLO cross-section results for the fully charm tetraquark depending on the hadron-hadron center-of-mass energy $\sqrt{S}$, times the $2J/\psi$ decay branching fraction.

tetraquark cross section times the $2J/\psi$ decay ratio is significantly enhanced compared to that of the 0^{++} tetraquarks. This enhancement stems not only from differences in LDMEs, but also from the difference in short-distance coefficients, a finding which is also consistent with previous theoretical and experimental works [7,8,67–72]. Similar observation of enhancement of color antitriplet compared to other quark configurations is also shown in the doubly charmed tetraquark system [73].

Furthermore we consider the two-body decays of fully charm tetraquarks where the 0^{++} state will only produce an isotropic polar angular distribution, while the distribution for the 2^{++} state will be nontrivial. According to Jacob-Wick angular theory [74], we can always expand the polar angular distribution of the produced 2^{++} fully charm-tetraquark state into a J/ψ pair as $dN/d\cos\theta^* = \sum_{i=1}^4 L_i \cos[(i-1)\theta^*]$, where the coefficients L_i depend on the production and decay mechanisms and θ^* is the polar angle between one of J/ψ and the fully charm-tetraquark momenta. Fortunately, the CMS experiment has measured the polar angular distribution with 135 fb^{-1} , allowing us to fit these parameters. We show the results in Fig. 6; the parameters for 2^{++} have been fitted as $L_1 = 170.0(7.9)$, $L_2 = -8(13)$, $L_3 = -59(12)$, $L_4 = 9(13)$ with $\chi^2/d.o.f. = 1.8$, which determine the correlation between the production and decay of the fully charm-tetraquark state. Due to the complexity of the CMS analysis where all the three peaks are included, we defer a detailed discussion to future work. The polar angle θ^* distribution from the CMS experiment supports the $J^{PC} = 2^{++}$ assignment for the exotic structures in the J/ψ pair final states.

Conclusion—We would like to conclude this work with the following remarks. First, our calculation on fully charm tetraquark is built upon the fundamental principles of QCD

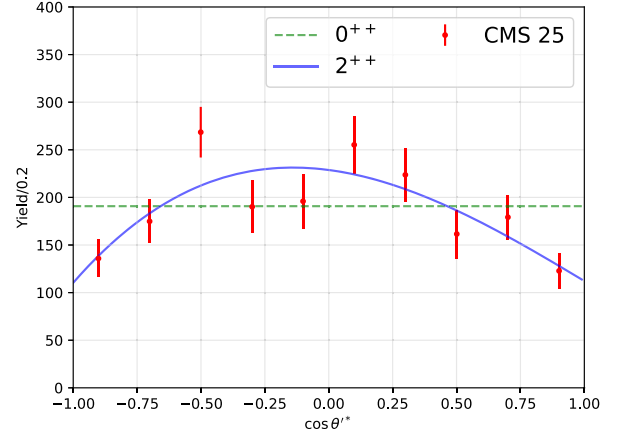


FIG. 6. Distribution of the polar angle θ^* between the J/ψ and the fully charm-tetraquark momenta. The data are from supplemental material of Ref. [7].

factorization and transverse momentum resummation, ensuring the rigor of the theoretical formalism and the thoroughness of the analysis. Second, we have for the first time constrained the nonperturbative but universal LDME of the $X(6900)$ in a model-independent way by utilizing one LHCb data point, and based on this, the predicted $\mathcal{R}(P_\perp > 5.2 \text{ GeV})$ value agrees with the remaining experimental data. This extraction can be also widely applied to other processes involving $X(6900)$ such as the production at electron-positron colliders. Third, for sufficiently large $P_\perp \gg M$, new factorization and resummation have been proposed in the literature [75]; we leave this part for future work. Since the cross section is dominated in the small and moderate transverse momentum region, the impact on the extraction of LDMEs is negligible. Fourth, although theoretical studies have been conducted on the spectrum of S-wave, P-wave, and higher excited fully charm-tetraquark states [76,77], the work presented in this Letter will provide an important foundation for systematically studying the production and inner structure of the family of fully charm-tetraquark states. The reason is that we consider the symmetry-based basis expansion method to be effective, and after factorizing the short-distance coefficients and the long-distance matrix elements, the short-distance coefficients can be calculated order by order in perturbation theory. With the increasing data samples collected by the LHC, we also hope that experimental colleagues will perform more precise measurements of (differential) cross sections, rapidity, and transverse momentum distributions for each exotic hadron in J/ψ -pair mass spectrum. Such efforts will provide further insights into clarifying the structural properties of these states, particularly regarding the existence of compact clustering mechanisms and the color confinement mechanism of four heavy quarks.

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Data availability—The data that support the findings of this article are openly available [49].

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