

Frustration-Free Control and Absorbing-State Transport in Entangled State Preparation

T. Dörstel^{1,2}, T. Iadecola^{3,4,5}, J. H. Wilson^{6,7} and M. Buchhold^{1,2}

¹*Universität Innsbruck, Institut für Theoretische Physik, Technikerstraße 21a, 6020 Innsbruck, Austria*

²*Universität zu Köln, Institut für Theoretische Physik, Zùlpicher Straße 77a, 50937 Cologne, Germany*

³*Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

⁴*Institute for Computational and Data Sciences, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

⁵*Materials Research Institute, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

⁶*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA*

⁷*Center for Computation and Technology, Louisiana State University, Baton Rouge, Louisiana 70803, USA*



(Received 13 November 2025; revised 3 February 2026; accepted 27 February 2026; published 25 March 2026)

We introduce frustration-free control, a measurement-feedback framework for quantum state preparation that extends frustration-free Hamiltonians to monitored stochastic dynamics. The protocol drives many-body systems into highly entangled target states, common dark states of all measurement projectors, through minimal local unitary corrections that realize an absorbing-state dynamics without postselection. We show that relaxation to the target state is governed by emergent transport of nonlocal charges, such as singlet excitations in SU(2)-symmetric dynamics. While measurement feedback annihilates compatible charge configurations, both measurement and scrambling unitaries induce charge transport and thus determine the convergence time. Mapping a baseline model of SU(N) SWAP measurements with local corrections to a solvable absorbing random walk yields a run-time scaling $t \sim L^z$ with transport exponent $z = 2$. Simulations of Motzkin and Fredkin chains reveal subdiffusive scaling $z \geq \frac{8}{3}$, confirming absorbing-state transport as a general organizing principle for deterministic entangled-state preparation and transport diagnostics in monitored quantum dynamics.

DOI: [10.1103/zp25-ppgk](https://doi.org/10.1103/zp25-ppgk)

Introduction—Engineering resourceful quantum many-body states is central to quantum information and quantum simulation. While Hamiltonian or unitary gate-based sequences remain a key paradigm [1–3], measurement-based protocols with feedback offer robust alternatives [4–10]. Mid-circuit measurements on modern platforms [11–13] enable measurement-induced dynamics [14–18], including measurement-induced entanglement or control transitions [19–29]. Feedback schemes targeting dark or absorbing states [30–35] achieve deterministic control by enforcing local constraints that render the desired state globally attractive.

Here we introduce *frustration-free control*, an absorbing-state-inspired protocol that prepares multipartite-entangled ground states of frustration-free Hamiltonians, and show its run-time is set by transport of local constraint violations. Limitations from decoherence and imperfect measurement-feedback are discussed below. It uses local projectors whose common kernel defines a target manifold of entangled dark states, and steers the system toward it via

projective measurements with outcome-conditioned local unitaries. The dark state reached from an initial state is dictated by the distribution of conserved charges in that state; in the SU(2)-symmetric setting studied here this yields Dicke states, while other frustration-free targets can be prepared analogously. Crucially, the protocol realizes an absorbing-state mechanism on single trajectories without postselection.

We uncover the physical mechanism underlying convergence: local measurement and feedback generate global entanglement via an emergent absorbing-state dynamics in which nonlocal excitations (e.g., singlet pairs) diffuse and annihilate. This transport picture establishes the scaling of convergence times and provides a framework to optimize and generalize frustration-free control. More generally, when measurement and feedback enforce local frustration-free constraints, absorbing-state transport of constraint violations controls convergence and guides protocol optimization.

Frustration-free control—Consider a quantum system with L degrees of freedom and a set of local projectors $\{\hat{P}_\alpha\}_{\alpha=1,\dots,L}$ satisfying $\hat{P}_\alpha^2 = \hat{P}_\alpha$, such that their common kernel defines a many-body dark state manifold $\{|\psi_D\rangle\}$ via

$$\hat{P}_\alpha|\psi_D\rangle = 0 \quad \forall \alpha. \quad (1)$$

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

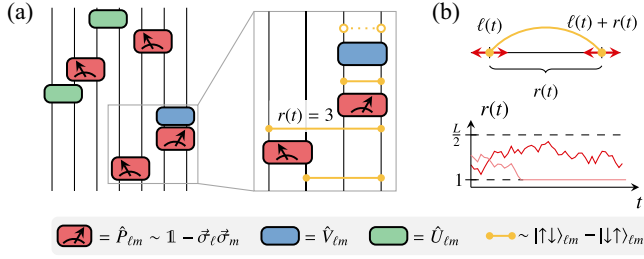


FIG. 1. Frustration-free control and absorbing random walks. (a) Schematic circuit driving a spin chain into the entangled ground state manifold of the Heisenberg ferromagnet. Local measurements (red) compatible with the target leave the state unchanged, while incompatible outcomes trigger unitary corrections (blue) steering towards the absorbing target. Scrambling unitaries (green) preserve the target and accelerate convergence. (b) $SU(2)$ -symmetric measurement and scrambling unitaries induce Brownian motion of singlet endpoints, here labeled as $\ell(t), \ell(t) + r(t)$. Feedback annihilates short singlets of length $r(t) = 1$. The diffusion-annihilation dynamics map to an absorbing random walk, displayed for two possible realizations (red, yellow) in the lower panel governed by the transport exponent z .

Consider a circuit where projectors $\hat{P}_\alpha$ are measured in a random spatiotemporal pattern [36] and feedback unitaries $\hat{V}_\alpha$ are applied, conditioned on the outcome. If the measurement yields 0, the state is locally compatible with the target; if it yields 1, a local unitary $\hat{V}_\alpha$ is applied, satisfying $[\hat{V}_\alpha, \hat{P}_\alpha] \neq 0$, to rotate the state toward the kernel of $\hat{P}_\alpha$; see Fig. 1. In addition, one can interleave unconditioned *scrambling* unitaries $\hat{U}_\alpha$ that preserve the target manifold $\hat{U}_\alpha|\psi_D\rangle = |\psi_D\rangle$, while redistributing local excitations and accelerating relaxation.

When all projectors commute, measurements are independent and target a product state, reachable in $\mathcal{O}(L)$ parallel operations. In contrast, noncommuting projectors $[\hat{P}_\alpha, \hat{P}_\beta] \neq 0$ generate interference between measurements and feedback, turning control into a genuine many-body process that builds entanglement. This motivates the term *frustration-free control*, analogous to frustration-free Hamiltonians $\hat{H} = \sum_\alpha \hat{P}_\alpha$ whose ground state minimizes competing local terms.

For noncommuting measurements, deviations from the target state appear as *nonlocal excitations*, such as singlets with well-defined end points [Fig. 1(b)]. Measurements and scrambling unitaries move these end points, while feedback annihilates adjacent pairs, leading to dynamics that map onto an *absorbing random walk* with relaxation time set by the transport exponent z . This mechanism, derived analytically for $SU(N)$ -symmetric SWAP measurements and verified numerically for Motzkin and Fredkin chains [37], provides a unifying description of convergence under feedback. As a concrete realization, we consider a spin- $\frac{1}{2}$ chain with $SU(2)$ -symmetric measurements and $SU(2)$ -symmetry-breaking feedback—previously studied

numerically in Ref. [38]—which admits an *exact solution* via mapping to a random walker with an absorbing impurity. Extending this construction to include scrambling unitaries, arbitrary measurement geometries, imperfections, and $SU(N)$ symmetry yields a complete analytical framework for frustration-free control governed solely by transport.

Quantum master equation and order parameter—The measurement-feedback protocol without scrambling defines a local quantum channel with trace-preserving Kraus maps:

$$\hat{\rho} \longrightarrow \sum_{\sigma=0,1} \hat{K}_{\alpha,\sigma} \hat{\rho} \hat{K}_{\alpha,\sigma}^\dagger; \quad \hat{K}_{\alpha,0} = \mathbb{1} - \hat{P}_\alpha, \quad \hat{K}_{\alpha,1} = \hat{V}_\alpha \hat{P}_\alpha. \quad (2)$$

Let projectors $\hat{P}_\alpha$ be measured with probability $1 - e^{-\gamma_\alpha t}$ in a time interval t . Taking the continuous-time limit $t \rightarrow dt$ and averaging over the stochastic application of measurement and feedback yields a Lindblad-type quantum master equation:

$$\partial_t \hat{\rho} = \sum_\alpha \sum_{\sigma=0,1} \gamma_\alpha \left(\hat{K}_{\alpha,\sigma} \hat{\rho} \hat{K}_{\alpha,\sigma}^\dagger - \frac{1}{2} \left\{ \hat{K}_{\alpha,\sigma}^\dagger \hat{K}_{\alpha,\sigma}, \hat{\rho} \right\} \right) = \mathcal{L} \hat{\rho}. \quad (3)$$

Each $|\psi_D\rangle$ is annihilated by all $\hat{K}_{\alpha,1}$ and left invariant by $\hat{K}_{\alpha,0}$. Hence, the protocol drives the system toward the dark-state manifold $\{|\psi_D\rangle\}$, which forms a stationary manifold of Eq. (3).

A useful metric to quantify the distance from the target state is the mean projector expectation value, $\mathbb{E}(\langle \hat{P}_\alpha \rangle) \equiv (1/L) \sum_\alpha \langle \hat{P}_\alpha \rangle$, with $\langle \hat{P}_\alpha \rangle \equiv \text{tr}(\hat{\rho} \hat{P}_\alpha)$. By construction, $\langle \hat{P}_\alpha \rangle \geq 0$ for any physical state $\hat{\rho}$, and $\langle \hat{P}_\alpha \rangle = 0$ if and only if $\hat{P}_\alpha \hat{\rho} = 0$. Thus, $\mathbb{E}(\langle \hat{P}_\alpha \rangle) = 0$ characterizes the global target state, while $\langle \hat{P}_\alpha \rangle$ serves as a local order parameter measuring the local deviation from the target. These local observables obey a closed evolution equation, derived from the master equation (3)

$$\partial_t \langle \hat{P}_\beta \rangle = \sum_\alpha \gamma_\alpha \langle \hat{P}_\alpha [\hat{V}_\alpha, \hat{P}_\beta] \hat{V}_\alpha^\dagger \hat{P}_\alpha - \frac{1}{2} [\hat{P}_\alpha, [\hat{P}_\alpha, \hat{P}_\beta]] \rangle. \quad (4)$$

The double commutator in Eq. (4) corresponds to dynamics without feedback ($\hat{V}_\alpha = \mathbb{1}$). It induces measurement-induced dephasing, typically driving $\langle \hat{P}_\beta \rangle$ toward its value in the maximally mixed state. In contrast, the first term captures the effect of feedback and is responsible for steering the system toward the target state. This control is effective when the unitaries $\hat{V}_\alpha$ act *off diagonally* with respect to the projectors, such that $\hat{P}_\beta \hat{V}_\alpha \hat{P}_\beta = \sqrt{(1-\lambda)} \hat{P}_\beta$ with $\lambda > 0$. In this case, feedback induces a decay of $\langle \hat{P}_\beta \rangle$ with a rate proportional to $\gamma_\beta \lambda$.

Spin- $\frac{1}{2}$ with SU(2) symmetry—To illustrate the feedback-control scheme introduced above, we consider a one-dimensional chain of L spin- $\frac{1}{2}$ degrees of freedom subject to SU(2)-symmetric measurements and SU(2)-symmetry-breaking feedback. Specifically, the measurements project onto eigenstates of the SWAP operator $\mathcal{S}_{\ell,m}$, which exchanges the quantum states of spins ℓ and m . Since $\mathcal{S}_{\ell,m}^2 = \mathbb{1}$, its eigenvalues are ± 1 , with the antisymmetric eigenvalue -1 corresponding to the maximally entangled singlet state $|s_{\ell,m}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{\ell,m}$. Using the Pauli vector $\vec{\sigma}_\ell = (\sigma_\ell^x, \sigma_\ell^y, \sigma_\ell^z)$ for spin ℓ , the projector onto the singlet sector is $\hat{P}_{\ell,m} = \frac{1}{2}(\mathbb{1} - \mathcal{S}_{\ell,m}) = \frac{1}{4}(\mathbb{1} - \vec{\sigma}_\ell \cdot \vec{\sigma}_m)$. Each SWAP measurement conserves all total spin components $S_{\text{tot}}^\nu = \frac{1}{2}\sum_\ell \sigma_\ell^\nu$ for $\nu = x, y, z$, and the total angular momentum $\vec{J}^2 = \sum_\nu (S_{\text{tot}}^\nu)^2 = \frac{1}{4}L(L+2) - \sum_{\ell \neq m} \hat{P}_{\ell,m}$, which quantifies the total number of singlets in the system.

The target manifold is defined by $\hat{P}_{\ell,m}|\psi_D\rangle = 0$ for all ℓ, m . It contains no singlets and exhibits maximal total spin $J = L/2$, implying that each bond $\hat{P}_{\ell,\ell+1} = 0$ is symmetric. This subspace hosts $L+1$ fully symmetric Dicke states—the ground states of the frustration-free ferromagnetic Heisenberg model. For any fixed S_{tot}^z , the corresponding Dicke state is unique. Since the protocol conserves S_{tot}^z , it converges within the initial S_{tot}^z sector to the corresponding Dicke state; here we present $S_{\text{tot}}^z = 0$ [39]. The initial S_{tot}^z fixes the target Dicke state and only changes nonuniversal prefactors (via the initial defect density), not the asymptotic runtime scaling with L .

To reach the target state, we implement frustration-free control using nearest-neighbor measurements $\hat{P}_{\ell,\ell+1}$ along with corresponding local corrections $\hat{V}_{\ell,\ell+1}$. These unitaries are required to (i) act strictly locally on spins ℓ and $\ell+1$, making use of the measurement outcome, and (ii) break the SU(2) symmetry, i.e., $[\hat{P}_{\ell,\ell+1}, \hat{V}_{\ell,\ell+1}] \neq 0$, in order to rotate the system toward the target state. Single-spin Pauli rotations generated by σ_ℓ^ν and $\sigma_{\ell+1}^\nu$ satisfy both requirements: they act locally and break SU(2) symmetry [40]. To preserve S_{tot}^z , we choose, without loss of generality, the feedback unitary $\hat{V}_{\ell,\ell+1} = \sigma_\ell^z$.

Absorbing random walk—In the absence of feedback, the singlet projectors $\hat{P}_{\ell,m}$ are conserved under SU(2)-symmetric dynamics. They represent nonlocal excitations—entangled Bell pairs—anchored between sites ℓ and m . Their evolution follows Eq. (4) with $\alpha = (\ell, \ell+1), \beta = (n, m)$. The double commutator yields

$$\begin{aligned} & \sum_\ell \langle [\hat{P}_{\ell,\ell+1}, [\hat{P}_{n,m}, \hat{P}_{\ell,\ell+1}]] \rangle \\ &= \frac{1}{2} \sum_\ell D_{n,\ell} \langle \hat{P}_{\ell,m} \rangle + \langle \hat{P}_{n,\ell} \rangle D_{\ell,m}, \end{aligned}$$

with $D_{\ell,\ell'} = \delta_{\ell,\ell'+1} + \delta_{\ell,\ell'-1} - 2\delta_{\ell,\ell'}$ the lattice Laplacian. In the absence of control, SU(2) is intact and the expectation value $\langle \hat{P}_{n,m} \rangle$ evolves diffusively, with both endpoints

of the singlet executing Brownian motion. The symmetry enforces conservation of the total singlet number and constrains the dynamics to local, diffusive rearrangements of singlet endpoints.

Control explicitly breaks the SU(2) symmetry, opening a decay channel for singlets with minimal spatial extent,

$$\sum_\ell \langle \hat{P}_{\ell,\ell+1} [\hat{V}_{\ell,\ell+1}, \hat{P}_{n,m}] \hat{V}_{\ell,\ell+1}^\dagger \hat{P}_{\ell,\ell+1} \rangle = -\lambda \langle \hat{P}_{n,m} \rangle \delta_{n,m\pm 1},$$

where $\lambda = 1 - |\langle s_{\ell,\ell+1} | \hat{V}_{\ell,\ell+1} | s_{\ell,\ell+1} \rangle|^2$ determines the decay [$\lambda = 1$ for $\hat{V}_{\ell,\ell+1} = \sigma_\ell^z$]. This describes an annihilation process for singlets with end points on adjacent sites, which are rotated into the symmetric subspace.

To close the equations, we exploit translation invariance and label singlets by center-of-mass $s = \frac{1}{2}(\ell + m)$ and relative distance $r = |\ell - m| \bmod (L/2)$. The total weight of singlets of length r , $\mathcal{P}_r = \sum_\ell \langle \hat{P}_{\ell,\ell+r} \rangle$, evolves as (discrete Laplacian D)

$$\partial_t \mathcal{P}_r = (D\mathcal{P})_r - 2\delta_{r,1} \mathcal{P}_r, \quad (5)$$

where we set $\gamma_\alpha \equiv 1$. This describes a one-dimensional, classical random walk of the relative coordinate, with an absorbing impurity located at $r = 1$. Hence, the total singlet weight $\sum_r \mathcal{P}_r$ decays diffusively, with a global relaxation timescale that scales as L^z , with the dynamical exponent $z = 2$.

We numerically simulate the full quantum evolution of the mean $\mathbb{E}(\langle \hat{P}_\alpha \rangle)$ and the variance $\text{var}(\langle \hat{P}_\alpha \rangle) \equiv (1/\sum_\psi) \sum_\psi \langle \psi | \sum_\beta (\hat{P}_\beta - \mathbb{E}(\langle \hat{P}_\alpha \rangle)) / L | \psi \rangle^2$ for system sizes up to $L = 24$, shown in Figs. 2(a) and 2(b). After a short nonuniversal transient [$t = \mathcal{O}(1)$], the mean exhibits hydrodynamic scaling $\mathbb{E}(\langle \hat{P}_\alpha \rangle) \sim t^{-1/z}$, followed by an exponential finite-size cutoff $\sim e^{-t/L^z}$ at $t \sim L^z$, consistent with the absorbing random-walk picture at matching L . The variance, taken over all simulated trajectories $|\psi\rangle$ and not captured by the linear quantum master equation in Eq. (3), shows pronounced fluctuations at the onset of the hydrodynamic regime, saturates to a system-size-independent plateau, and then decays exponentially on the same characteristic timescale $t \sim L^z$ as $\mathbb{E}(\langle \hat{P}_\alpha \rangle)$, indicating the absence of strong trajectory fluctuations.

Effect of unitary scrambling—We now consider the impact of additional, unconditioned *scrambling* unitaries $\hat{U}_\ell$, applied randomly in space and time with uniform rate $\gamma\kappa$. The gates are chosen from the set of SU(2)-symmetric unitaries of the form $\hat{U}_\ell = \exp(i\varphi \mathcal{S}_{\ell,\ell+1}) = \cos(\varphi) + i \sin(\varphi) \mathcal{S}_{\ell,\ell+1}$ [41], and averaging each update step $\hat{\rho} \rightarrow \hat{U}_\ell \hat{\rho} \hat{U}_\ell^\dagger$ uniformly over $\varphi \in [0, 2\pi]$ yields $\hat{\rho} \rightarrow \hat{\rho} - \frac{1}{4}[\mathcal{S}_{\ell,\ell+1}, [\mathcal{S}_{\ell,\ell+1}, \hat{\rho}]]$. With this process acting uniformly in space, the singlet dynamics in Eq. (5) is modified by a renormalization of the diffusion constant, $D \rightarrow (1 + \kappa)D$, i.e., scrambling enhances Brownian motion. Tuning κ

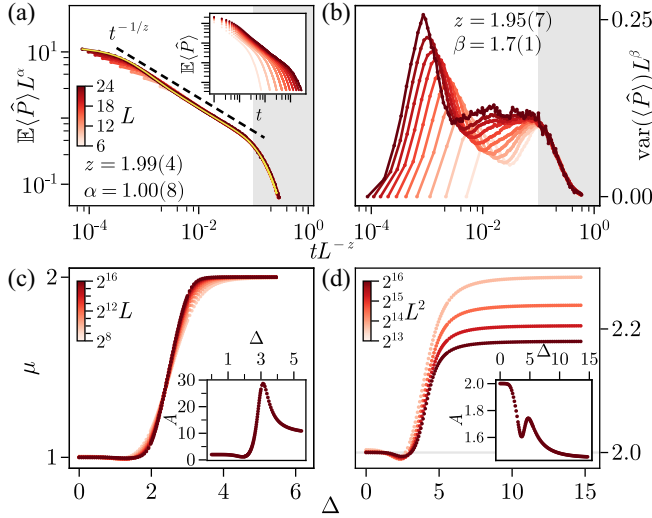


FIG. 2. Frustration-free control of the Heisenberg chain. (a) Exact simulations of the measurement-feedback protocol show a hydrodynamic regime with $\mathbb{E}(\langle \hat{P}_{\ell, \ell+1} \rangle) \sim t^{-1/z}$ and an asymptotic exponential decay $\sim \exp(-t/L^z)$ for system sizes $L \leq 24$, averaged over 10^5 trajectories (10^3 for $L = 24$). The yellow line is a single run of $\mathcal{P}_1$ in Eq. (5) for $L = 24$, the same initial state and no free parameters. (b) The variance of the order parameter exhibits strong fluctuations at the onset of the hydrodynamic regime [$t \sim O(L^0)$], plateaus during hydrodynamic evolution and is exponentially cutoff at $t \sim L^z$ (gray region); colors denote system size L as in (a). (c),(d) Scaling exponents $\mu(L)$ of the cutoff time $t \sim L^{\mu(L)}$ as functions of system size and long-range exponent Δ for absorbing random walks in one (c) and two (d) dimensions, obtained from solving Eq. (6). Inset: total decay rate as a function of Δ .

rescales the eigenvalues of the effective diffusion operator $(1 + \kappa)D$, accelerating convergence. For example, local scrambling at rate $\kappa \sim L^\alpha$ shifts the bare L^{-2} level spacing of D to $L^{\alpha-2}$ for κD , reducing relaxation times to $t \sim \max(L^{2-\alpha}, 1)$. Fast scrambling hence removes the diffusion bottleneck and turns a diffusion-limited absorbing process into annihilation limited.

Generalization to arbitrary geometries and $SU(N)$ —The control scheme naturally extends to different lattice geometries of the spins. Consider L^d spins in d dimensions at positions $\vec{\ell}, \vec{m} \in \mathbb{R}^d$, with $\hat{P}_{\vec{\ell}, \vec{m}} = \frac{1}{2}(\mathbb{1} - S_{\vec{\ell}, \vec{m}})$ projecting onto the antisymmetric state. Measuring $\hat{P}_{\vec{\ell}, \vec{m}}$ and applying local, magnetization-preserving feedback unitaries on one spin of each measured pair drives the system toward the fully symmetric subspace $\hat{P}_{\vec{\ell}, \vec{m}}|\psi\rangle = 0$, independent of geometry and the measurement sequence. The $SU(2)$ symmetry of the measurements, conserving the total spin and magnetization, remains intact. While this leaves the target state unchanged, the *approach* to it—i.e., the dynamical efficiency of the protocol—depends on the geometry and measurement sequence.

We consider a translationally invariant setup where pairs $(\vec{\ell}, \vec{\ell} + \vec{r})$ are measured with rate $\gamma_{|\vec{r}|}$. Since the projector algebra of $SU(2)$ SWAPs is independent of geometry, the previous analysis applies. Defining the “diffusion kernel” $\tilde{D}_{\vec{r}, \vec{s}} = \gamma_{|\vec{r}-\vec{s}|}$, the total weight of singlets of distance $\vec{r}$ follows (cf. [42])

$$\partial_t \mathcal{P}_{\vec{r}} = \sum_{\vec{s} \neq 0} (\tilde{D}_{\vec{r}, \vec{s}} - 2\gamma_{|\vec{r}|} \delta_{\vec{r}, \vec{s}}) \mathcal{P}_{\vec{s}} = -\sum_{\vec{s} \neq 0} \tilde{H}_{\vec{r}, \vec{s}} \mathcal{P}_{\vec{s}}. \quad (6)$$

Probability conservation requires $\sum_{\vec{s} \neq 0} \tilde{D}_{\vec{r}, \vec{s}} = 0$ for all $\vec{r}$, i.e., $\sum_{\vec{r} \neq 0} \gamma_{|\vec{r}|} = -\gamma_0$. We normalize the total rate such that $\gamma_0 = 1$, which implies an extensive number of $L^d dt$ measurements per time step dt . The asymptotic decay of the total weight is governed by the lowest eigenvalue $\tau > 0$ of the operator $\tilde{H}$, i.e., $\sum_{\vec{r}} \mathcal{P}_{\vec{r}} \sim \exp(-\tau t)$.

We consider an algebraic decay of measurement rates, $\gamma_{|\vec{r}|} = \Gamma_\Delta |\vec{r}|^{-\Delta}$ for $|\vec{r}| > 0$, with the range exponent Δ interpolating between nearest-neighbor ($\Delta \rightarrow \infty$) and all-to-all ($\Delta \rightarrow 0$) measurements and $\Gamma_\Delta > 0$ fixing the normalization for any finite $L < \infty$. Our goal is to determine the scaling of the decay rate $\tau(\Delta, L) = A(\Delta)L^{-\mu(\Delta)}$ with the total number of spins L^d . The limiting case $\Delta \rightarrow 0$ is exactly solvable [42], yielding $\tau = 2L^{-d}$ and $\mu(\Delta \rightarrow 0) = d$.

We analyze square lattices in $d = 1$ [Fig. 2(c)], and $d = 2$ [Fig. 2(d)]. For each Δ , we numerically extract the smallest eigenvalue $\tau(\Delta, L)$ of $\tilde{H}$, and compute the exponent $\mu(\Delta)$ via the $\square$ function $\mu(\Delta) \equiv \{\log[\tau(\Delta, 2L)] - \log[\tau(\Delta, L)] / \log(2)\}$. For $d = 1$, a sharp transition at a critical decay exponent $\Delta_c \approx 2.1$ separates the infinite-range regime ($\mu = 1$) from the short-range diffusive regime ($\mu = 2$). In $d = 2$, μ displays a narrow range $\mu \in [2, 2.3]$ and crossing points at $\mu = d$, such that the relaxation is governed by the infinite-range scaling.

For spin- $(N-1)/2$ states, the algebra of SWAPs $\mathcal{S}_{\ell, m} = \sum_{\alpha, \beta=1}^N |\alpha\beta\rangle\langle\beta\alpha|_{\ell, m}$ is independent of N . Hence, the control scheme readily generalizes to $SU(N)$: the SWAP projector dynamics, Eq. (6) and all subsequent analysis apply equally to general N . We stress that for $N > 2$, the symmetric subspace, i.e., the control target space, is strictly larger than the subspace of maximal total angular momentum [42].

Fredkin and Motzkin chains—We illustrate frustration-free control in kinetically constrained models, namely the Fredkin and Motzkin chains [43]. Both are frustration-free spin systems with ground-state manifolds exhibiting multipartite entanglement. Although their excitation sectors are not separated by a non-Abelian symmetry, they are particularly interesting because (i) their excitations, nonlocal charge configurations represented by *Motzkin words*, are balanced spin configurations with two well-defined end points, and (ii) their transport dynamics is subdiffusive, characterized by a dynamical exponent $z \geq \frac{8}{3}$ [44,45].

Frustration-free control is implemented as above: we measure local projectors $\hat{Q}_{\vec{\ell}}$ and apply feedback operations

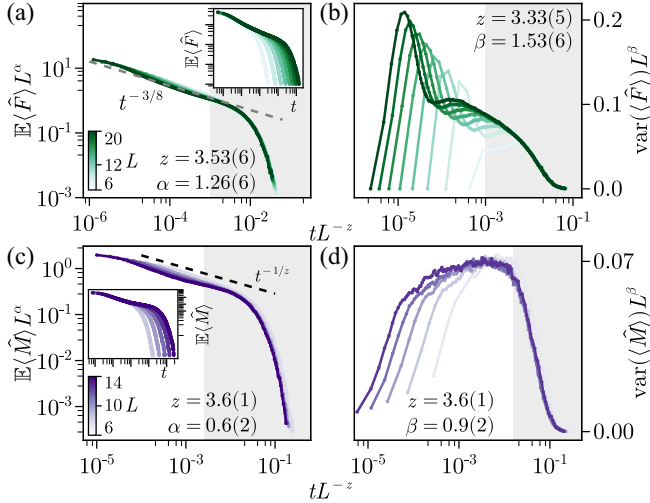


FIG. 3. Anomalous transport in frustration-free control. Measurement-feedback dynamics for (a),(b) the spin- $\frac{1}{2}$ Fredkin and (c), (d) spin-1 Motzkin chains exhibit an intermediate regime $\mathbb{E}(\langle \hat{Q}_\ell \rangle) \sim t^{-1/z}$ followed by exponential decay $\mathbb{E}(\langle \hat{Q}_\ell \rangle) \sim \exp(-t/L^z)$, signaling subdiffusive transport with $z > 2$. For the Motzkin chain, data up to $L \leq 14$ yield $z \approx 3.6$. The Fredkin chain shows a hydrodynamic regime $\mathbb{E}(\langle \hat{P}_\ell \rangle) \sim t^{-z}$ with $z = \frac{8}{3}$, crossing over to an asymptotic decay consistent with $z \gtrsim 3.3$ [46].

steering the system toward the target state $|\psi\rangle$ with $\hat{Q}_\ell|\psi\rangle = 0$. For the Fredkin dynamics, we consider a spin- $\frac{1}{2}$ chain with $\hat{Q}_\ell \equiv \hat{F}_\ell = (1 - \sigma_{\ell-1}^z)\hat{P}_{\ell,\ell+1} + \hat{P}_{\ell-1,\ell}(1 + \sigma_{\ell+1}^z)$, where $\hat{P}_{\ell,\ell+1}$ projects onto the singlet state of the two spins. For the Motzkin chain, we consider L spin-1 degrees of freedom and $\hat{Q}_\ell \equiv \hat{M}_\ell = |U\rangle\langle U|_{\ell,\ell+1} + |D\rangle\langle D|_{\ell,\ell+1} + |F\rangle\langle F|_{\ell,\ell+1}$, with $|U\rangle_{\ell,\ell+1} = (|\uparrow 0\rangle - |\uparrow \uparrow\rangle)/\sqrt{2}$, $|D\rangle_{\ell,\ell+1} = (|\downarrow 0\rangle - |\downarrow \downarrow\rangle)/\sqrt{2}$, and $|F\rangle_{\ell,\ell+1} = (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)/\sqrt{2}$. Upon positive measurement outcomes, local feedback is applied as $V_\ell = \exp[i(\pi/2)\hat{Z}_\ell]$, enforcing conservation of S_{tot}^z . For the Motzkin chain the absorbing state is unique within a fixed S_{tot}^z sector. For the Fredkin chain this holds for $S_{\text{tot}}^z \neq 0$, whereas for $S_{\text{tot}}^z = 0$ the absorbing manifold is twofold degenerate. The precise target state is then fixed by the initial overlap; here we choose a Néel initial state.

Numerical simulations of the order parameter $\mathbb{E}(\langle \hat{Q}_\ell \rangle)$ and its variance for both the Motzkin and Fredkin chains reveal a scaling collapse consistent with an absorbing random walk; see Fig. 3. As in the SWAP case, we identify an intermediate transport regime $O(L) < t < L^z$, where $\mathbb{E}(\langle \hat{Q}_\ell \rangle) \sim t^{-1/z}$, followed by an exponential cutoff $\mathbb{E}(\langle \hat{Q}_\ell \rangle) \sim \exp(-t/L^z)$. The Fredkin chain exhibits transport consistent with $z = \frac{8}{3}$ at intermediate times, while the scaling collapse of the exponential tail suggests $z \approx 3.3\text{--}3.5$, indicating a strong suppression of transport at long times. For the Motzkin chain, we find consistently large exponents, $z \approx 3.6$. While previous studies [44,46]

reported $z = \frac{8}{3}$ for the lowest-lying excitation of the Hamiltonian $\hat{H} = \sum_\ell \hat{Q}_\ell$, values $z > 3$ have been observed for higher excitations [46], which appear to govern the relaxation dynamics relevant for feedback control.

Imperfect control—Realistic settings are subject to imperfections such as noise or measurement errors. Detrimental noise channels, such as, e.g., single-qubit relaxation $\sim T_1^{-1}$ and dephasing $\sim T_2^{-1}$, do not commute with the measurement projectors and therefore interfere with the absorbing-state condition. To understand their effect we consider an elementary phenomenological model of reversible defect dynamics within the absorbing random walk framework: let random defects occur with rate η and create (annihilate) a singlet of length $|\vec{r}|$ with combinatorial probability $2/L$ ($4\mathcal{P}_{\vec{r}}$). This modifies Eq. (6):

$$\partial_t \mathcal{P}_{\vec{r}} = 2\eta - \sum_{\vec{s}} \left(\tilde{H}_{\vec{r},\vec{s}} + 4\eta \delta_{\vec{r},\vec{s}} \right) \mathcal{P}_{\vec{s}}. \quad (7)$$

The stationary state is set by a balance between decay and noise. It is maximally mixed $\mathcal{P}_{\vec{r}} = \frac{1}{2}$ if $\eta \gg \tilde{H}$, while it scales as $\mathcal{P}_{\vec{r}} \sim \eta L^z$ for $\eta \ll \tilde{H}$ and hence is governed by the dynamical exponent z . This is confirmed by exact numerical simulations of imperfect measurements, see Ref. [42], and allows probing z under realistic imperfections. Thus, high fidelity at size L requires $\eta \lesssim L^{-z}$, equivalently $L < L_0 \sim \eta^{-1/z}$.

Outlook—Frustration-free control prepares entangled states through measurement-feedback-driven annihilation of nonlocal charges. The dynamics is governed by charge transport, characterized by a global dynamical exponent z that sets the run-time and efficiency. On digital platforms, the SWAP measurement is realized via the SWAP test [47], compiled into two-qubit gates and one Toffoli gate; overhead is readout and feedforward latency [48]. Thus, the total physical runtime is the number of control cycles $t \sim L^z$ multiplied by a platform-dependent *constant* per-cycle duration (compiled joint measurement, readout, and conditional unitary), i.e., an overall prefactor not affecting the L^z scaling. Remarkably, scrambling unitaries preserving the target manifold do not hinder but instead accelerate convergence, and they typically require fewer resources per cycle than the compiled joint measurements; thus the dominant hardware overhead and error budget is set by the measurements.

Our framework is not restricted to frustration-free Hamiltonians: it applies whenever the target state or manifold is specified by locally checkable constraints that can be monitored and corrected by local updates, so that violations behave as mobile defects that are transported and annihilated (e.g., enforcing local Gauss-law constraints in lattice-gauge-theory simulators). For frustration-free ground-state targets, a standard coherent competitor is adiabatic preparation, whose sufficient conditions bound the runtime in terms of the inverse minimum many-body

gap along the path, typically scaling at least as $\sim 1/\Delta_{\min}^2$ [54]. For the ferromagnetic Heisenberg chain, the lowest magnon gap scales as $\Delta_{\min}(L) \sim 1 - \cos(\pi/L) \propto L^{-2}$ [55], which implies the corresponding adiabatic scaling estimate $\sim L^4$, whereas our protocol converges in $\sim L^2$ control cycles, assuming only local operations. A complementary measurement-based scheme yields polylog depth by exploiting global observables together with a polylog number of ancillas in a single shot [56]; in contrast, our strictly local protocol can recycle ancillas across repeated local checks.

Acknowledgments—T.D. and M.B. acknowledge support from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy Cluster of Excellence Matter and Light for Quantum Computing (ML4Q) No. EXC 2004/1 390534769, and by the DFG Collaborative Research Center (CRC) 183 Project No. 277101999—projects B01. M.B. acknowledges support from the Heisenberg programme of the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation), Project No. 549109008. T.I. and J.H.W. acknowledge support from the National Science Foundation under Grants No. DMR-2143635 (T.I.) and No. DMR-2238895 (J.H.W.). This work was initiated at Aspen Center for Physics, which is supported by National Science Foundation Grant No. PHY-2210452. The numerical simulations have been performed on the RAMSES cluster at RRZK, University of Cologne.

Data availability—All code for generating the figures and the underlying numerical data is available on Zenodo [57].

[1] D. Wecker, M. B. Hastings, and M. Troyer, *Phys. Rev. A* **92**, 042303 (2015).
 [2] M. P. Zaletel and F. Pollmann, *Phys. Rev. Lett.* **124**, 037201 (2020).
 [3] L. Causer, F. Jung, A. Mitra, F. Pollmann, and A. Gammon-Smith, *Phys. Rev. Res.* **6**, 033062 (2024).
 [4] S. Roy, J. T. Chalker, I. V. Gornyi, and Y. Gefen, *Phys. Rev. Res.* **2**, 033347 (2020).
 [5] S. Morales, Y. Gefen, I. Gornyi, A. Zazunov, and R. Egger, *Phys. Rev. Res.* **6**, 013244 (2024).
 [6] J. Langbehn, K. Snizhko, I. Gornyi, G. Morigi, Y. Gefen, and C. P. Koch, *PRX Quantum* **5**, 030301 (2024).
 [7] L. Piroli, G. Styliaris, and J. I. Cirac, *Phys. Rev. Lett.* **127**, 220503 (2021).
 [8] L. Piroli, G. Styliaris, and J. I. Cirac, *Phys. Rev. Lett.* **133**, 230401 (2024).
 [9] T.-C. Lu, L. A. Lessa, I. H. Kim, and T. H. Hsieh, *PRX Quantum* **3**, 040337 (2022).
 [10] A. Bhuiyan, H. Pan, and C.-M. Jian, [arXiv:2507.13437](https://arxiv.org/abs/2507.13437).
 [11] D. Bluvstein *et al.*, *Nature (London)* **626**, 58 (2023).

[12] M. Foss-Feig, A. Tikku, T.-C. Lu, K. Mayer, M. Iqbal, T. M. Gatterman, J. A. Gerber, K. Gilmore, D. Gresh, A. Hankin, N. Hewitt, C. V. Horst, M. Matheny, T. Mengle, B. Neyenhuis, H. Dreyer, D. Hayes, T. H. Hsieh, and I. H. Kim, [arXiv:2302.03029](https://arxiv.org/abs/2302.03029).
 [13] E. Bäumer, V. Tripathi, A. Seif, D. Lidar, and D. S. Wang, *Phys. Rev. Lett.* **133**, 150602 (2024).
 [14] A. Kitaev, *Ann. Phys. (Amsterdam)* **303**, 2 (2003).
 [15] E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, *J. Math. Phys. (N.Y.)* **43**, 4452 (2002).
 [16] D. Gottesman and I. L. Chuang, *Nature (London)* **402**, 390 (1999).
 [17] M. A. Nielsen, *Phys. Lett. A* **308**, 96 (2003).
 [18] R. Raussendorf and H. J. Briegel, *Phys. Rev. Lett.* **86**, 5188 (2001).
 [19] M. Szyniszewski, A. Romito, and H. Schomerus, *Phys. Rev. B* **100**, 064204 (2019).
 [20] B. Skinner, J. Ruhman, and A. Nahum, *Phys. Rev. X* **9**, 031009 (2019).
 [21] Y. Li, X. Chen, and M. P. A. Fisher, *Phys. Rev. B* **100**, 134306 (2019).
 [22] Y. Li, X. Chen, and M. P. A. Fisher, *Phys. Rev. B* **98**, 205136 (2018).
 [23] M. J. Gullans and D. A. Huse, *Phys. Rev. X* **10**, 041020 (2020).
 [24] A. Zabalo, M. J. Gullans, J. H. Wilson, S. Gopalakrishnan, D. A. Huse, and J. H. Pixley, *Phys. Rev. B* **101**, 060301 (2020).
 [25] A. Nahum, S. Roy, B. Skinner, and J. Ruhman, *PRX Quantum* **2**, 010352 (2021).
 [26] C.-M. Jian, Y.-Z. You, R. Vasseur, and A. W. W. Ludwig, *Phys. Rev. B* **101**, 104302 (2020).
 [27] T. Iadecola, S. Ganeshan, J. H. Pixley, and J. H. Wilson, *Phys. Rev. Lett.* **131**, 060403 (2023).
 [28] K. Klocke, D. Simm, G.-Y. Zhu, S. Trebst, and M. Buchhold, *Phys. Rev. B* **111**, 224301 (2025).
 [29] A. Lavasani, Y. Alavirad, and M. Barkeshli, *Phys. Rev. Lett.* **127**, 235701 (2021).
 [30] M. Gau, R. Egger, A. Zazunov, and Y. Gefen, *Phys. Rev. B* **102**, 134501 (2020).
 [31] M. Wampler and N. R. Cooper, *Phys. Rev. Res.* **7**, 033053 (2025).
 [32] P. Sierant and X. Turkeshi, *Phys. Rev. Lett.* **130**, 120402 (2023).
 [33] E. Chertkov, Z. Cheng, A. C. Potter, S. Gopalakrishnan, T. M. Gatterman, J. A. Gerber, K. Gilmore, D. Gresh, A. Hall, A. Hankin, M. Matheny, T. Mengle, D. Hayes, B. Neyenhuis, R. Stutz, and M. Foss-Feig, *Nat. Phys.* **19**, 1799 (2023).
 [34] L. Piroli, Y. Li, R. Vasseur, and A. Nahum, *Phys. Rev. B* **107**, 224303 (2023).
 [35] S. Roy, J. T. Chalker, I. V. Gornyi, and Y. Gefen, *Phys. Rev. Res.* **2**, 033347 (2020).
 [36] Structured patterns, such as Floquet circuits, may also be used, but the core strategy remains unchanged.
 [37] S. Chirame, F. J. Burnell, S. Gopalakrishnan, and A. Prem, *Phys. Rev. Lett.* **134**, 010403 (2025).
 [38] J. Hauser, Y. Li, S. Vijay, and M. P. A. Fisher, *Phys. Rev. B* **109**, 214305 (2024).

- [39] Note that we could equally well select the W -state by requiring $S_{\text{tot}}^z = \pm(L-1)$.
- [40] Any two-spin unitary either preserves $SU(2)$ or reduces to combinations involving single Pauli rotations and symmetric gates.
- [41] Any $SU(2)$ gate can be decomposed into a sequence of such local SWAP operations. Averaging over random realizations is therefore equivalent to using this elementary set.
- [42] End matter of this Letter.
- [43] O. Salberger and V. Korepin, *Rev. Math. Phys.* **29**, 1750031 (2017).
- [44] C. McCarthy, H. Singh, S. Gopalakrishnan, and R. Vasseur, *Phys. Rev. B* **111**, 184317 (2025).
- [45] H. Singh, B. A. Ware, R. Vasseur, and A. J. Friedman, *Phys. Rev. Lett.* **127**, 230602 (2021).
- [46] X. Chen, E. Fradkin, and W. Witczak-Krempa, *J. Phys. A* **50**, 464002 (2017).
- [47] H. Buhrman, R. Cleve, J. Watrous, and R. de Wolf, *Phys. Rev. Lett.* **87**, 167902 (2001).
- [48] Three-qubit Toffoli/controlled-SWAP primitives have been demonstrated in superconducting circuits [49], trapped-ion processors [50], neutral-atom (Rydberg) arrays [51], and photonic systems [52,53].
- [49] A. Fedorov, L. Steffen, M. Baur, M. P. da Silva, and A. Wallraff, *Nature (London)* **481**, 170 (2011).
- [50] T. Monz, K. Kim, W. Hänsel, M. Riebe, P. Schindler, M. Chwalla, M. Hennrich, A. S. Villar, and R. Blatt, *Phys. Rev. Lett.* **102**, 040501 (2009).
- [51] H. Levine, A. Keesling, G. Semeghini, A. Omran, T. T. Wang, S. Ebadi, H. Bernien, M. Greiner, V. Vuletić, H. Pichler, and M. D. Lukin, *Phys. Rev. Lett.* **123**, 170503 (2019).
- [52] R. B. Patel, J. Ho, F. Ferreyrol, T. C. Ralph, and G. J. Pryde, *Sci. Adv.* **2**, e1501531 (2016).
- [53] T. Ono, R. Okamoto, M. Tanida, H. F. Hofmann, and S. Takeuchi, *Sci. Rep.* **7**, 45353 (2017).
- [54] T. Albash and D. A. Lidar, *Rev. Mod. Phys.* **90**, 015002 (2018).
- [55] T. Koma and B. Nachtergaele, *Lett. Math. Phys.* **40**, 1 (1997).
- [56] J. Yu, S. R. Muleady, Y.-X. Wang, N. Schine, A. V. Gorshkov, and A. M. Childs, *Phys. Rev. Lett.* **136**, 030601 (2026).
- [57] T. Dörschel, T. Iadecola, J. H. Wilson, and M. Buchhold, Data for: Frustration-free control and absorbing-state transport in entangled state preparation, [10.5281/zenodo.17450833](https://zenodo.org/record/17450833) (2025).
- [58] L. Dell'Anna, *SciPost Phys.* **7**, 053 (2019).
- [59] A. G. Pronko, *Nucl. Phys. B* **1017**, 116963 (2025).
- [60] W. Munizzi and H. J. Schnitzer, *Phys. Rev. A* **109**, 012405 (2024).

End Matter

Absorbing random walk derivation—For projectors $\hat{P}_{\ell,m} = \frac{1}{2}(1 - \mathcal{S}_{\ell,m})$ defined via the spin- j SWAP operator $\mathcal{S}_{\ell,m} = \sum_{\alpha\beta} |\alpha\beta\rangle\langle\beta\alpha|_{\ell,m}$, the double commutator $[\hat{P}_{\ell,m}, [\hat{P}_{\ell,m}, \hat{P}_{s,n}]]$ vanishes if either $\{\ell, m\} = \{s, n\}$ or all indices are distinct. It is nontrivial only when one index pair coincides and the other does not. Using $\mathcal{S}_{\ell,m}^2 = 1$, $\mathcal{S}_{\ell,m}\mathcal{S}_{\ell,n}\mathcal{S}_{\ell,m} = \mathcal{S}_{m,n}$, and $[\hat{P}_{\ell,m}, [\hat{P}_{\ell,m}, \hat{P}_{\ell,n}]] = -\frac{1}{8}[\mathcal{S}_{\ell,m}, [\mathcal{S}_{\ell,m}, \mathcal{S}_{\ell,n}]]$, one finds,

$$[\hat{P}_{\ell,m}, [\hat{P}_{\ell,m}, \hat{P}_{\ell,n}]] = -\frac{1}{4}(\mathcal{S}_{\ell,n} - \mathcal{S}_{m,n}) = \frac{1}{2}(\hat{P}_{\ell,n} - \hat{P}_{m,n}). \quad (\text{A1})$$

This result is independent of the spatial dimension d and spin j . Introducing the relative coordinate $r = \text{mod}(m - \ell, L/2)$ and the center of mass (c.m.) coordinate $s = (\ell + m)/2$ via $\hat{P}_{\ell,m} = \hat{P}_{s-r/2, s+r/2} \equiv \hat{P}_{s,r}$ and collecting all nonvanishing terms in Eq. (4) of the main text, one obtains

$$\begin{aligned} \partial_t \hat{P}_{s,r} = & \frac{1}{2} \sum_{r' \neq 0} \gamma_{|r'|} \left(-2\hat{P}_{s,r}(2 - \delta_{r,-r'} - \delta_{r,r'}) \right. \\ & + \hat{P}_{s-r', r+r'} + \hat{P}_{s, r-r'} + \hat{P}_{s, r+r'} + \hat{P}_{s+r', r-r'} \Big) \\ & - \lambda(\gamma_{|r|} \hat{P}_{s,r} + \gamma_{|-r|} \hat{P}_{s+r, -r}). \end{aligned} \quad (\text{A2})$$

Summing both sides over s and introducing $\mathcal{P}_r = \sum_s \langle \hat{P}_{s,r} \rangle$, the center of mass coordinate s is eliminated

due to translational invariance. This yields the equation

$$\begin{aligned} \partial_t \mathcal{P}_r = & \sum_{r' \neq 0} \gamma_{|r'|} (\mathcal{P}_{r+r'} + \mathcal{P}_{r-r'} - \mathcal{P}_r(2 - \delta_{r,-r'} - \delta_{r,r'})) \\ & - 2\lambda\gamma_{|r|} \mathcal{P}_r, \end{aligned} \quad (\text{A3})$$

which due to $\gamma_{|r'|} = \gamma_{|-r'|}$ yields Eq. (6) in the main text. The contribution from the feedback $\hat{V}_{\ell,\ell'}$ reduces to a factor λ , as

$$\hat{P}_\alpha V_\alpha P_\alpha V_\alpha^\dagger P_\alpha = P_\alpha |\langle s_\alpha | V_\alpha | s_\alpha \rangle|^2 = P_\alpha \frac{1}{4} \text{tr}(V_\alpha)^2. \quad (\text{A4})$$

With this relation, the full contribution from the correction reduces to

$$\hat{P}_\alpha [\hat{V}_\alpha, \hat{P}_\beta] \hat{V}_\alpha^\dagger \hat{P}_\alpha = \delta_{\alpha,\beta} \left(1 - \frac{1}{4} \text{tr}(V_\alpha)^2 \hat{P}_\alpha \right) \equiv \delta_{\alpha,\beta} \lambda \hat{P}_\alpha, \quad (\text{A5})$$

where the $\delta_{\alpha,\beta}$ is a result of $\hat{P}_{\ell,\ell'} \sigma_\ell^i \hat{P}_{\ell,\ell'} = 0$ for $i = x, y, z$. Studying single Pauli corrections that yield $\lambda = 1$ is sufficient: any two-site unitary can be represented by $e^{i\phi_\nu \sigma_\ell^x \sigma_{\ell'}^y}$ multiplied with single-Pauli operations, whereas $|\langle s_{\ell,\ell'} | e^{i\phi_\nu \sigma_\ell^x \sigma_{\ell'}^y} | s_{\ell,\ell'} \rangle|^2 = 1$.

If we choose $\gamma_r = r^{-\Delta}$ with $\Delta \rightarrow \infty$, Eq. (6) is recovered. In the all-to-all measurement limit $\Delta = 0$, the equation can be simplified even further. The equal weight sum over all

terms yields $\sum_{r'} \mathcal{P}_{r+r'} = \sum_{r'} \mathcal{P}_{r-r'}$ and thus

$$\partial_t \mathcal{P}_r = \frac{2}{L^d - 1} \left(\sum_{r' \neq 0} \mathcal{P}_{r'} - (L^d - 1) \mathcal{P}_r \right). \quad (\text{A6})$$

A matrix with this structure can be diagonalized directly, where the eigenvalue with the largest absolute value reads $\tau = -2(L^d - 1)^{-1}$. These limits confirm Fig. 2 with $\mu(\Delta = 0) = d$ and $A(\Delta = 0) = 2$.

SU(2) charge dynamics—The Hilbert space of two spin- $\frac{1}{2}$ particles is spanned by the singlet and triplet states: $|s\rangle = \frac{1}{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$, $|t_1\rangle = |\uparrow\uparrow\rangle$, $|t_0\rangle = \frac{1}{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, and $|t_{-1}\rangle = |\downarrow\downarrow\rangle$. The operator $\hat{P}_{\ell,m} = \frac{1}{2}(\mathbb{1} - \mathcal{S}_{\ell,m})$ projects onto the singlet: $\hat{P}_{\ell,m}|s\rangle_{\ell,m} = |s\rangle_{\ell,m}$. Enforcing $\hat{P}_{\ell,\ell+1}|\psi\rangle = 0$ for all ℓ implies that every bond—and therefore every pair (ℓ, m) —is in a superposition of triplets. For a chain of length L , the $L+1$ states satisfying this condition are the permutation-symmetric Dicke states $|D_L^k\rangle$, which are symmetric superpositions of k spins up and $L-k$ spins down.

Any measurement $\hat{P}_{\ell,m}$ commutes with the total angular momentum $\vec{J}^2$ (see main text). Hence, the total number of singlets is conserved under measurement, and only the singlet end points are reshuffled. For example, for $|\psi\rangle = |\sigma\rangle_1 |s\rangle_{2,3}$, measuring a singlet on $(1,2)$ yields $\hat{P}_{1,2}|\psi\rangle \sim |s\rangle_{1,2} |\sigma\rangle_3$, effectively swapping spins $1 \leftrightarrow 3$. A triplet measurement gives $(\mathbb{1} - \hat{P}_{1,2})|\psi\rangle \sim |\psi\rangle - |\sigma\rangle_2 |s\rangle_{1,3}$. The correction $\hat{V}_{\ell,\ell+1} = \sigma_\ell^z$ rotates a singlet into a triplet, $\sigma_\ell^z |s\rangle_{\ell,m} = |t_0\rangle_{\ell,m}$, and thus, when applied immediately after a measurement $\hat{P}_{\ell,m}$, removes exactly one length-1 singlet.

Symmetric subspaces in SU(N)—Unlike the SU(2) spin- $\frac{1}{2}$ case, the maximally symmetric subspaces of SU(N) are not identical to the maximal- J subspaces, although the latter are contained within the former. Consider spin-1 [SU(3)]: two spins decompose into singlet, triplet, and quintet sectors with $\vec{J}^2 = 0, 1, 2$, respectively. Despite having minimal total angular momentum, the singlet $|s\rangle = \frac{1}{\sqrt{3}}(|1, -1\rangle + |-1, 1\rangle - |0, 0\rangle)$ is symmetric under particle exchange. For $j > \frac{1}{2}$, the maximal- J sector is therefore embedded in (but not identical to) the symmetric subspace. While measurements of SU(N) SWAP operators (with $N = 2j + 1$) project naturally into the symmetric subspace, isolating the maximal- J sector requires more elaborate measurement schemes and generally lacks a closed-form time evolution.

Motzkin words—The Fredkin operator $\hat{F}_\ell = (1 - \sigma_{\ell-1}^z) \hat{P}_{\ell,\ell+1} + \hat{P}_{\ell-1,\ell} (1 + \sigma_{\ell+1}^z)$ implements a similar logic to the CSWAP (Fredkin) gate. It can be rewritten as

$$F_\ell = \frac{1}{2}(1 - \mathcal{CS}_{\ell,\ell+1,\ell+2}) + \frac{1}{2}(1 - \sigma_{\ell+2}^x \mathcal{CS}_{\ell+2,\ell+1,\ell} \sigma_{\ell+2}^x),$$

where $\mathcal{CS}_{i,j,k}$ is the CSWAP gate that swaps sites j and k , if i is in the $|\uparrow\rangle$ state. These dynamics preserve the total magnetization S^z and a family of nonlocal charges known as Motzkin (or Dyck) words. Spin configurations are mapped to Motzkin paths by identifying spin-up and spin-down with ascending and descending steps, $\downarrow \rightarrow /$ and $\uparrow \rightarrow \backslash$, and spin-zero with a flat step, $0 \rightarrow -$. A configuration thus becomes a height profile, e.g., $|\uparrow\downarrow\uparrow\downarrow\downarrow\rangle \rightarrow / \backslash / \backslash \backslash$. Any configuration forming a closed profile, $/ \dots \backslash$, is a Motzkin word. Fredkin gates do not mix Motzkin with non-Motzkin words; they only shift the endpoints of matched parentheses. On three qubits, the Fredkin gate acting on computational-basis states has two outcomes: if no Motzkin word is present (e.g., $///$, $\backslash\backslash\backslash$, $\backslash//$, or $\\//$), the action vanishes; otherwise, it moves the Motzkin word and produces an antisymmetric superposition of the shifted and input states.

In the Motzkin model, Motzkin words are extended to include flat steps; otherwise, their interpretation and dynamics remain equivalent. See Refs. [58,59] for details on both chains and on Motzkin words.

Target states and entanglement scaling—The target states of the SU(N), Fredkin, and Motzkin measurement-feedback protocols are the common kernels of $\{\hat{P}_\alpha\}$, $\{\hat{F}_\alpha\}$, and $\{\hat{M}_\alpha\}$, respectively. Each protocol conserves the total magnetization $S_z = \sum_\ell \hat{Z}_\ell$. Since S_z is conserved, the protocol converges within each S_z sector; we focus on the $S_z = 0$ subspace, where the targets exhibit nontrivial multipartite entanglement.

In the SU(2) model, the target is the symmetric Dicke state $|D_L^{L/2}\rangle$. For a Dicke state $|D_L^k\rangle$, the entanglement entropy of a block of size ℓ is [60]

$$S_\ell(|D_L^k\rangle) = -\binom{L}{k}^{-1} \sum_{i=0}^{\min(\ell,k)} \binom{\ell}{i} \binom{L-\ell}{k-i} \times \ln \left[\binom{L}{k}^{-1} \binom{\ell}{i} \binom{L-\ell}{k-i} \right].$$

We confirm logarithmic entanglement growth for the target state; see Fig. 4(a).

For the Fredkin circuit at $S_z = 0$, two states satisfy $\hat{F}_\ell|\psi\rangle = 0$: the Dicke state $|D_L^{L/2}\rangle$ and the anomalous ground state $|A\rangle$ [43],

$$|A\rangle = \binom{L}{n}^{-1/2} \sum_{\omega \in \text{comb}(L)} (-1)^{m(\omega)} |\omega\rangle, \quad (\text{E1})$$

where $\text{comb}(L)$ denotes the unique permutations with $n = L/2$ up and n down spins, and $m(\omega)$ is the absolute

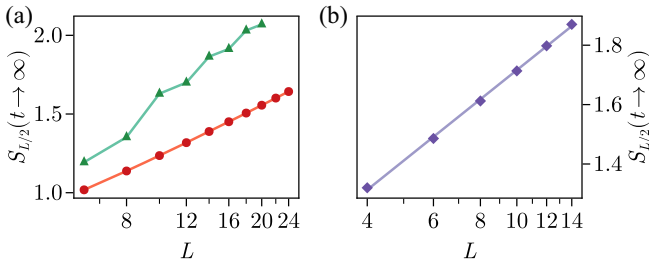


FIG. 4. Stationary-state half-chain entanglement. Numerical half-chain entanglement in the $S_z = 0$ sectors versus analytical expressions. (a) The spin- $\frac{1}{2}$ SU(2) model (circles) coincides with the entanglement of the Dicke state $S_{L/2}(|D_L^{L/2}\rangle)$ (lower solid line). The Fredkin model (triangles) converges to $|A\rangle - |D_L^{L/2}\rangle$ (upper solid line). (b) Consistent with analytical results for the open Motzkin chain [58], the stationary state of the periodic Motzkin chain exhibits logarithmic entanglement scaling (solid line: linear fit on a semi-log plot).

maximum height of configuration ω . Figure 4(a) shows that the controlled Fredkin circuit's stationary state is the antisymmetric superposition $|\psi\rangle = |A\rangle - |D_L^{L/2}\rangle$. The simulated entanglement entropy matches that of $|\psi\rangle$. The oscillatory increase likely reflects the enhanced number of Dyck paths for system sizes divisible by four, as also seen in open-boundary Fredkin chains [58].

For the Motzkin circuit, the target is the ground state of the periodic Motzkin chain $H_{\text{Motzkin, pbc}} = \sum_i M_{i,i+1} + M_{L,1}$, conjectured to be a generalized spin-1 Dicke state [59]. We also find logarithmic entanglement scaling with system size; see Fig. 4(b).

Confirming the imperfect control picture—To validate the phenomenological noise model of Eq. (7), we simulate a circuit with measurement imperfections in the SU(2) setup of the main text. We assume an apparatus that occasionally misreports outcomes. To implement a defect rate η following the phenomenological model, we take singlet outcomes to be falsely flagged—interpreted as triplets and not corrected—with probability $p_{s,\text{false}} = \frac{1}{2}(1 + e^{-\eta})$. Conversely, triplet outcomes are (incorrectly) corrected with probability $p_{t,\text{false}} = \frac{1}{2}(1 - e^{-\eta})$. The simulations reproduce the predicted scaling of $\vec{\mathcal{P}}(t \rightarrow \infty)$;

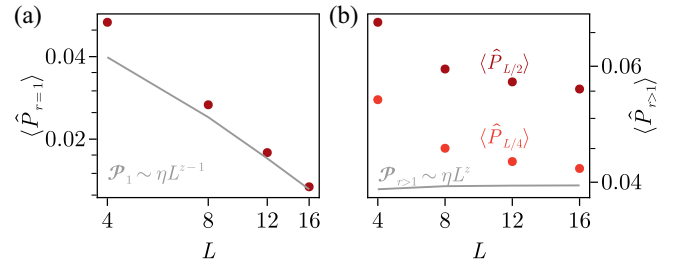


FIG. 5. Imperfect control. Scaling of the stationary singlet expectation $\mathcal{P}_r$ with noise rate $\eta = L^{-2}$ versus exact simulations. The gray lines show $M^{-1}\vec{\eta}$. (a) Length-1 singlets scale as $\mathcal{P}_1 \sim \eta L^{z-1}$; (b) longer singlets obey $\mathcal{P}_r \sim \eta L^z$ for $r > 1$. Dots display the stationary singlet expectations for the corresponding r , exhibiting the same scaling.

see Fig. 5. Singlets of length $r = 1$ scale as $\mathcal{P}_1 \sim \eta L^{z-1}$, while longer ones scale as $\mathcal{P}_r \sim \eta L^z$, yielding an average scaling $\sim L^z$ for the total singlet number.

Scaling of the diffusion operator versus decay rate in 1D—The spectrum of the translationally invariant diffusion operator $D_{\vec{r}}$ follows from the Fourier transform of the algebraic kernel $D_{\vec{r}} \sim |\vec{r}|^{-\Delta}$. In momentum space,

$$D_q \sim \begin{cases} q^2, & \text{for } \Delta > 3, \\ q^2 \log(q), & \text{for } \Delta = 3, \\ q^{\Delta-1}, & \text{for } 1 < \Delta < 3. \end{cases} \quad (\text{G1})$$

For a finite system of size L , momentum modes have spacing $\delta q \sim 1/L$, so the excitation gap follows directly. For a localized absorbing impurity at $\vec{r} = 1$, this gap sets the decay rate. However, when the dispersion becomes long-ranged, the impurity also acquires algebraic spatial support. In the infinite-range limit, the decay rate scales as $\sim 1/L$, independent of the diffusion-operator dispersion. Thus, as the impurity range increases, the dynamics cross over from diffusion-limited to decay-limited absorbing random walks. At the transition in Fig. 2, the system therefore switches from diffusion- to decay-limited behavior, rather than from short- to long-range diffusion.