

String Duals of Two-Dimensional Yang-Mills Theory and Symmetric Product Orbifolds

Shota Komatsu^{1,*} and Pronobesh Maity^{2,†}

¹*Department of Theoretical Physics, CERN, 1211 Meyrin, Switzerland*

²*Laboratory for Theoretical Fundamental Physics, EPFL, Rte de la Sorge, Lausanne, Switzerland*



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We propose a bosonic string dual to large N chiral Yang-Mills theory in two dimensions at finite 't Hooft coupling. The worldsheet theory is a β - γ system deformed by a chiral Polchinski-Strominger term. We reproduce the partition function on a torus, cylinder three-point amplitudes, and the area law for Wilson loops. We also present candidate string duals to symmetric product orbifolds for general seed conformal field theories with $c < 24$ and their $T\bar{T}$, $J\bar{T}$ deformations. The results hint at interrelations among confining, nonrelativistic, and matrix strings and $\text{AdS}_3/\text{CFT}_2$.

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Introduction—The idea of reformulating gauge theory in terms of strings has a long history, originating with 't Hooft's large N expansion [1] and loop equations [2,3]. A major early success came from “noncritical string theory” [4–6]; dualities between simple large N matrix models and string theory in less than one dimension. The idea later culminated in the AdS/CFT correspondence [7], offering concrete examples of dualities between supersymmetric gauge theories and string theory in anti-de Sitter (AdS) spacetime.

A common feature in these dualities is the emergence of an extra spatial direction—often called “holographic” or “Liouville”—linked to an additional scalar mode on the string worldsheet. While this is often viewed as a hallmark of gauge-string duality, recent lattice studies of confining flux tubes [8–12] challenge this paradigm, suggesting that the extra scalar is absent for Yang-Mills theory in three and four dimensions. Relatedly, effective field theories for confining flux tubes [13–15] are formulated without an extra scalar, although they are not UV complete. This leads to a question: Is there a UV-complete string dual of a confining gauge theory without an extra scalar?

In this Letter, we answer yes, presenting a dual for the simplest confining gauge theory; chiral two-dimensional (2D) Yang-Mills (YM) theory. As a by-product, we also propose candidate duals to symmetric product orbifolds for any seed conformal field theories (CFTs) with $c < 24$ and their $T\bar{T}$ and $J\bar{T}$ deformations. Notably, none of these constructions require supersymmetry, unlike many top-down examples of holography.

String dual of chiral 2D YM theory—Two-dimensional YM theory has been a subject of interest due to its confining nature and solvability. Importantly, its partition function at large N factorizes into chiral and antichiral parts, each of which receives $1/N$ corrections. In addition, there are nonfactorizable corrections, $Z_{2\text{D YM}} = Z_{\text{chiral}} Z_{\text{anti}} + \mathcal{O}(1/N^2) + \dots$.

Key developments in the past include the following. (i) References [16,17] showed that the $1/N$ expansion can be interpreted as a genus expansion of string theory, although an explicit worldsheet theory reproducing the expansion was not constructed. (ii) Cordes *et al.* [18] identified a dual of the zero-coupling limit of 2D YM theory as topological string. However, it remains to be understood how to generalize it to finite coupling. See [19] for a recent proposal on the deformation of Gromov-Witten theory. (iii) A “rigid topological string” action was proposed as a candidate dual to 2D YM theory at finite coupling [20]. Because of the unconventional structure of the world sheet theory, however, its analysis poses technical challenges. This approach has recently been revisited, with detailed studies focusing on the zero-coupling regime [21]. (iv) It was shown that topological string on certain Calabi-Yau manifolds reproduce the partition function of the q -deformed 2D YM theory at finite coupling [22,23]. See also Refs. [24–26] for further analyses including the undeformed limit.

Below, we present a bosonic string dual of chiral 2D YM theory that allows computation of observables beyond the partition function, including scattering amplitudes—unlike earlier proposals. However, it requires a flat target space: $\mathbb{R}^{1,1}$, torus, or cylinder.

World sheet action: Our proposed dual for $U(N)$ 2D YM theory at large N with 't Hooft coupling $\lambda = g_{\text{YM}}^2 N$ is

$$S = \int \frac{d^2 z}{2\pi} [\beta \bar{\partial} X^+ + \bar{\beta} \partial X^- + \frac{q}{2} \mathcal{L}_{\text{CLD}} + \frac{\lambda\pi}{2} (\partial X^+ \bar{\partial} X^- - \bar{\partial} X^+ \partial X^-)] + bc \text{ ghost}, \quad (1)$$

*Contact author: shota.komatsu@cern.ch

†Contact author: pronobesh.maity@epfl.ch

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where $X^\pm$ are light-cone coordinates of the target space $X^\pm = X^1 \pm X^0$ and $\mathcal{L}_{\text{CLD}}$ is a “chiral composite linear dilaton” action,

$$\mathcal{L}_{\text{CLD}} = 2\partial\bar{\partial}\varphi + \hat{R}\varphi, \quad \varphi = \log\left(\frac{\partial X^+ \bar{\partial} X^-}{R^2}\right), \quad (2)$$

where R is the length scale of the target space and $\hat{R}$ is the worldsheet Ricci scalar. A few comments are in order: (i) The anomaly cancellation sets $q = 1$ (see below). (ii) The β - γ system shows up also in nonrelativistic string theory [27–29]. Our theory can be viewed as its noncritical generalization. (iii) $\mathcal{L}_{\text{CLD}}$ resembles the Polchinski-Strominger term [13] in the Polyakov formalism [15], differing only by the lack of $\bar{\partial}X^+ \partial X^-$ in φ . (iv) Despite high nonlinearity due to $\mathcal{L}_{\text{CLD}}$, the path integral can be evaluated explicitly by first integrating out β 's, which sets X^+ (X^-) to be (anti)holomorphic. (v) Similar actions were proposed in the studies of lightcone gauge string field theory [30], Green-Schwarz string in the semi-lightcone gauge [31], and $T\bar{T}$ deformation [32]. We discuss the relationships in Supplemental Material [33] and note that the last one does not exhibit conformal invariance in its current form.

From the action (1), the operator product expansion (OPE) can be derived [34],

$$\begin{aligned} \beta(z)X^+(w) &\sim -\frac{1}{z-w}, & \bar{\beta}(\bar{z})X^-(\bar{w}) &\sim -\frac{1}{\bar{z}-\bar{w}} \\ \beta(z)\beta(w) &\sim 2q\partial_z\partial_w\left[\frac{1}{(z-w)^2}\frac{1}{\partial_z X^+(z)\partial_w X^+(w)}\right]. \end{aligned} \quad (3)$$

The stress tensor of the matter sector reads

$$T(z) = -\beta\partial X^+(z) + 2q\{X^+, z\}, \quad (4)$$

with $\{X^+, z\}$ being the Schwarzian derivative. The OPE of stress tensors can be computed from (3) as

$$T(z)T(w) = \frac{1+12q}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{z-w}. \quad (5)$$

This shows that the matter action in (1) has central charge $c = 2 + 24q$ and $q = 1$ is needed for the conformal anomaly cancellation.

Torus partition function: Let us take the target space to be Euclidean torus $\mathbb{T}_{\text{TS}}^2$ with modulus ζ and size R ,

$$\begin{aligned} X^+ \rightarrow X = X^1 + iX^2, & & X^- \rightarrow \bar{X} = X^1 - iX^2, \\ (X \sim X + 2\pi R \sim X + 2\pi R\zeta), & & \end{aligned} \quad (6)$$

and study the genus-1 worldsheet partition function $\mathcal{Z}_{\text{ws}}(R, \zeta)$ on this background. Below, we sketch the computation relegating the details to Supplemental Material [33].

First, integrating out oscillator modes of β cancels the bc -ghost contributions and sets X ($\bar{X}$) to be (anti)holomorphic except for zero modes. This forces X to be a map from the worldsheet torus $\mathbb{T}_{\text{ws}}^2$ to $\mathbb{T}_{\text{TS}}^2$, characterized by four winding numbers, $m^{1,2}$ and $w^{1,2}$,

$$\frac{X(z, \bar{z})}{R} = \frac{(M - \bar{\tau}W)z + (\tau W - M)\bar{z}}{\tau - \bar{\tau}}. \quad (7)$$

Here $M = m^1 + \zeta m^2$ and $W = w^1 + \zeta w^2$, and z parametrizes $\mathbb{T}_{\text{ws}}^2$ as $z \sim z + 2\pi \sim z + 2\pi\tau$.

Second, the integral of zero modes of β 's produces the δ function that localizes the world sheet to “covering maps” from $\mathbb{T}_{\text{ws}}^2$ to $\mathbb{T}_{\text{TS}}^2$, leading to the following expression for $\mathcal{Z}_{\text{ws}}$:

$$\mathcal{Z}_{\text{ws}} = \sum_{\{m^a, w^a\}} \frac{\zeta_2}{|\bar{W}|^2} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \delta^{(2)}\left(\tau - \frac{M}{W}\right) e^{-S_{m,w}}, \quad (8)$$

where $\zeta_2 = \text{Im}\zeta$, $\tau_2 = \text{Im}\tau$, and $\mathcal{F}$ is the fundamental domain of the torus moduli and $S_{m,w}$ is an on-shell action,

$$S_{m,w} = \frac{\lambda\pi^2 R^2}{2\tau_2} |\bar{W}\tau - \bar{M}|^2. \quad (9)$$

To evaluate the moduli integral (8), we extend the integration domain to the upper half plane using part of the sum over $\{m^a, w^a\}$ [35], resulting in

$$\mathcal{Z}_{\text{ws}} = \sum_{K=1}^{\infty} \frac{e^{-\frac{K\lambda A}{2}}}{2K} \sum_{ad=K} \sum_{a,d \in \mathbb{Z}_+} (a+d), \quad (10)$$

where A is the area of $\mathbb{T}_{\text{TS}}^2$. This precisely matches the free energy of chiral 2D YM theory at large N [6,17,36]. Note that the zero-winding sector $K = 0$ does not contribute thanks to the CLD action, as will be explained around (25).

Three-point amplitudes: Because of the absence of local dynamics, wave functions of chiral 2D YM theory on S^1 are given simply by class functions of the holonomy U [36]. In particular, a convenient basis at large N is the multitrace basis $\{|\sigma\rangle\}$,

$$\Upsilon_\sigma(U) = \langle U|\sigma\rangle = \prod_{\ell} (\text{tr}(U^\ell))^{k_\ell}, \quad (11)$$

where k_ℓ denotes the number of cycles of length ℓ in $\sigma \in S_r$ with $r = \sum_{\ell} \ell k_\ell$. As discussed in [37], $|\sigma\rangle$ corresponds to a multistring state with k_ℓ strings with winding number ℓ . It can be expressed with string creation operators as

$$|\sigma\rangle = \prod_{\ell} (a_\ell^\dagger)^{k_\ell} |\Omega\rangle, \quad (12)$$

where $[a_k, a_l^\dagger] = k\delta_{k,l}$. The Hamiltonian on S^1 of radius R is then given by [37]

$$H = H_{\text{free}} + H_{\text{int}}, \quad H_{\text{free}} = \pi\lambda R \sum_m a_m^\dagger a_m,$$

$$H_{\text{int}} = \frac{\pi R \lambda}{N} \sum_{n,m} a_{m+n}^\dagger a_m a_n + a_m^\dagger a_n^\dagger a_{m+n}. \quad (13)$$

Viewing H_{int} as the interaction Hamiltonian, one can compute the perturbative S matrix of winding strings. In particular, the $2 \rightarrow 1$ amplitude reads

$$S = -i \frac{4\lambda\pi^2 R}{N} w_1 w_2 w_3 \delta(E_1 + E_2 - E_3) \delta_{w_1+w_2, w_3}, \quad (14)$$

where w_k and $E_k = \lambda\pi R w_k$ are the winding number and the energy of the k th string, respectively.

We now reproduce it from genus-0 amplitudes in string theory (1) on a Lorentzian cylinder $X^\pm \sim X^\pm + 2\pi R$. The vertex operator for the w_k -winding state is [27,34]

$$\mathcal{V}_k(z_k) = e^{i w_k R \int^{z_k} (\beta dz - \bar{\beta} d\bar{z})} e^{-i E_k X^0}. \quad (15)$$

Inserting them in the path integral and integrating out β 's, we find that (X^+, X^-) localizes to the Mandelstam map $(\rho, \bar{\rho})$,

$$X^+(z) \mapsto \rho(z) = -i \sum_k w_k R \log(z - z_k). \quad (16)$$

As a result, we obtain the following expression for the three-point amplitude:

$$G_3 = g_s \left\langle \prod_{k=1}^3 \mathcal{V}_k(z_k) \right\rangle$$

$$= i g_s \mathcal{N}_{S^2} 2\pi \delta \left(\sum_k E_k \right) 2\pi R \delta_{\sum_k w_k} e^{-\Gamma[\rho]} \prod_{i < j} |z_{ij}|^2, \quad (17)$$

where $\mathcal{N}_{S^2}$ is the normalization coming from the sphere partition function and the path-integral measure and $\Gamma[\rho]$ is the renormalized on-shell action for $(q/2) \int \mathcal{L}_{\text{CLD}}$ [30,38],

$$e^{-\Gamma[\rho]} = \prod_{k=1}^n (w_k)^{2q} \left| \sum_{k=1}^n w_k z_k \right|^{-4q} \left| \prod_{l=1}^{n-2} \frac{\partial^2 \rho(Z_l)}{R} \right|^q, \quad (18)$$

where Z_l 's are the ‘‘interaction points,’’ where $\partial\rho$ vanishes. Here, we absorbed the divergence around the interaction points by renormalizing g_s , as in the light cone string field theory [30,38,39]. Setting $n = 3$ and $q = 1$ gives $e^{-\Gamma} = w_1 w_2 w_3 \prod_{i < j} |z_{ij}|^{-2}$. We thus have

$$G_3 = i(2\pi)^2 R g_s \mathcal{N}_{S^2} w_1 w_2 w_3 \delta \left(\sum_k E_k \right) \delta_{\sum_k w_k}. \quad (19)$$

Continuing the third state from the in- to out-state by $E_i \rightarrow -E_i$ and $w_i \rightarrow -w_i$, (19) reproduces (14) under the

identification of parameters [40],

$$g_s = \frac{\lambda R^2}{2N}, \quad \mathcal{N}_{S^2} = \frac{2}{R^2}. \quad (20)$$

Wilson loop: We now consider a disk amplitude for the theory on $\mathbb{R}^2$. It is convenient for the analysis to take the ‘‘winding gauge’’ [41]

$$\partial X = \bar{\partial} \bar{X} = 1. \quad (21)$$

In this gauge, the disk amplitude of a string ending on a curve $\mathcal{C} \subset \mathbb{R}^2$ is given by evaluating the on-shell action. The result reads

$$\mathcal{Z}_{D^2} \propto e^{-\frac{iA}{2}}, \quad (22)$$

reproducing the area law for the Wilson loop in 2D YM theory.

Symmetric product orbifolds—

Action and partition function: We next provide evidence that a generalization of (1) is a potential dual to symmetric product orbifolds for arbitrary seed CFTs. The worldsheet action reads

$$S_{\text{sym}} = \int \frac{d^2 z}{2\pi} \left[\beta \bar{\partial} X + \bar{\beta} \partial \bar{X} + \frac{q}{2} \mathcal{L}_{\text{CLD}} \right] + S_{\text{seed}}$$

$$- B \int \frac{d^2 z}{4\pi} (\partial X \bar{\partial} \bar{X} - \bar{\partial} X \partial \bar{X}) + bc \text{ ghost}, \quad (23)$$

with $q = (24 - c)/24$, where c and S_{seed} are the central charge and the action of the seed CFT.

The genus-1 partition function on a torus ($X \sim X + 2\pi R \sim X + 2\pi R \zeta$) can be evaluated as in 2D YM theory thanks to the localization,

$$\mathcal{Z}_{\text{ws}} = \sum_{a,d=1}^{\infty} \frac{p^{ad}}{ad} \sum_{b=0}^{d-1} \mathcal{Z}_{\text{seed}} \left(\frac{b+a\zeta}{d}, \frac{b+a\bar{\zeta}}{d} \right), \quad (24)$$

where $p = e^{B \text{ area}/(2\pi)}$ and $\mathcal{Z}_{\text{seed}}$ is the torus partition function of the seed CFT. This agrees precisely with the grand-canonical free energy of the symmetric product orbifold [42]. Note that the duality relates string theory to a grand-canonical ensemble of CFTs rather than a theory with a fixed central charge, as was the case with supersymmetric tensionless $\text{AdS}_3/\text{CFT}_2$ [43,44].

Holomorphic map vs covering map: Let us briefly discuss higher genus corrections. Because of the β integrals, the string path integral always localizes to holomorphic maps ρ . However, holomorphic maps, in general, admit extra branch points $[\partial\rho(Z_l) = 0]$ while the partition function of symmetric product orbifolds is given by a sum over covering maps [45]; holomorphic maps without such branch points.

This discrepancy can be resolved for seed CFTs with $c < 24$. Evaluating the on-shell action of $\mathcal{L}_{\text{CLD}}$ around the branch point Z_I , we get

$$\exp \left[-\frac{q}{2} \int \frac{d^2 z}{2\pi} \mathcal{L}_{\text{CLD}} \right] \supset |2\epsilon_I \partial^2 \rho(Z_I)|^q, \quad (25)$$

where ϵ_I is a radius of a small circle around Z_I excised for the regularization. This shows that, in the limit $\epsilon_I \rightarrow 0$, the maps with branch points do not contribute as long as $q > 0$, i.e., $c < 24$. The same mechanism removes the zero-winding sector in the partition functions (10) and (24).

Comparison to $\text{AdS}_3/\text{CFT}_2$: The bosonic action for string on AdS_3 supported by k units of NSNS flux reads [46]

$$S_{\text{AdS}_3} = k \int \frac{d^2 z}{4\pi} \left(4\partial\Phi\bar{\partial}\Phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - e^{-2\Phi}\beta\bar{\beta} - \frac{\hat{R}\Phi}{k} \right). \quad (26)$$

For $k = 1$, it was argued in [47] that the path integral localizes to classical solutions near the boundary given by $\Phi = -\log \epsilon - \frac{1}{2} \log \partial\gamma\bar{\partial}\bar{\gamma}$. After this substitution and appropriate rescalings, the action (26) coincides with our proposed action (23). Thus, our action can be viewed as a theory where the (nondynamical) Liouville mode Φ is integrated out. A key difference is that our action (23) always exhibits localization and produces the correct partition function for any seed CFT with $c < 24$, while the result of [47] holds only for specific supersymmetric seed CFTs.

$T\bar{T}$ and $J\bar{T}$ deformations—We now further extend our action (23) to describe symmetric product orbifolds of $T\bar{T}$ - and $J\bar{T}$ -deformed CFTs. Technical details are explained in Supplemental Material [33].

$T\bar{T}$ deformation: We conjecture that symmetric product orbifolds of $T\bar{T}$ -deformed CFT [48] is dual to the following action:

$$S_{T\bar{T}} = S_{\text{sym}} - \frac{\mu}{\pi} \int \frac{d^2 z}{2\pi} \beta\bar{\beta}, \quad (27)$$

where S_{sym} is given by (23). The simplest way to understand the connection to the $T\bar{T}$ deformation is to employ the winding gauge (21). In this gauge, the worldsheet stress tensor for S_{sym} is

$$T(z) = -\beta\partial X + 2q\{X, z\} + T_{\text{seed}} \rightarrow -\beta + T_{\text{seed}}, \quad (28)$$

where T_{seed} is the stress tensor of the seed CFT. Imposing the Virasoro constraint $T(z) = 0$, we get $\beta = T_{\text{seed}}$. Thus, the deformation term in (27) can be identified with the operator $T_{\text{seed}}\bar{T}_{\text{seed}}$. (i) A related observation connecting the deformation of nonrelativistic string and $T\bar{T}$ deformations

has been made in [49–51]. (ii) Similar actions were obtained by expanding deformed AdS_3 actions near the AdS boundary [52,53]. A key difference is that the boundary expansion in [52,53] comes with an extra Liouville mode and hence the dual CFT inevitably includes the linear dilaton as part of the seed CFT. On the other hand, our action can be defined for more general seed CFT. Another important point is that the full deformed AdS_3 action before the boundary expansion is not dual to the symmetric product orbifold (cf. Ref. [54]) and the boundary expansion only captures a subsector of the full theory in a certain limit. (iii) In the special case of the symmetric product orbifold of $T\bar{T}$ -deformed theory with four free bosons and four free fermions, [55] analyzed the corresponding worldsheet dual using tensionless $\text{AdS}_3/\text{CFT}_2$. They reproduced the partition function and identified the connection between the worldsheet deformation and the $T\bar{T}$ operator.

The relation to the $T\bar{T}$ deformation can be verified more directly by computing the torus partition function. To do so, we integrate out β and $\bar{\beta}$ to obtain a Polyakov-type action

$$S_{T\bar{T}} = \int \frac{d^2 z}{2\mu} \bar{\partial}X\partial\bar{X} + \dots \quad (29)$$

and use the winding gauge $\partial X = \bar{\partial}\bar{X} = 1$; see Supplemental Material for details [33]. The result reads

$$\mathcal{Z}_{\text{ws}} = \sum_{K>0} p^K \mathcal{T}_K[\mathcal{Z}_+] + p^{-K} \mathcal{T}_K[\mathcal{Z}_-], \quad (30)$$

where $\mathcal{T}_K$ is the Hecke operator [42],

$$\mathcal{T}_K[f] = \frac{1}{K} \sum_{ad=K} \sum_{0 \leq b < d, d>0} f\left(dR, \frac{a\zeta + b}{d}, \frac{a\bar{\zeta} + b}{d}\right), \quad (31)$$

and $\mathcal{Z}_{\pm}$ are the partition functions with winding number ± 1 ,

$$\mathcal{Z}_{\pm} = \frac{\pi R^2 \zeta_2}{\mu} \int_{\mathcal{H}_+} \frac{d^2 \tau}{(\tau_2)^2} e^{-\frac{\pi^2 R^2}{\mu \tau_2} |\zeta \mp \tau|^2} \mathcal{Z}_{\text{seed}}(\tau, \bar{\tau}), \quad (32)$$

where $\mathcal{H}_+$ is the upper half plane. Crucial differences from symmetric product orbifolds (24) are the contribution from negative-winding sectors and the absence of localization of the worldsheet moduli.

Expanding $\mathcal{Z}_{\text{seed}}$ in (32), $\mathcal{Z}_+$ reproduces the standard $T\bar{T}$ -deformed spectrum [48],

$$E_{(+)}(\mu) = \frac{\pi R}{\mu} \left(-1 + \sqrt{1 + \frac{\mu^2 P^2}{\pi^2 R^2} + \frac{2\mu E}{\pi R}} \right), \quad (33)$$

$$P_{(+)}(\mu) = P,$$

where E and P are undeformed energy and momentum. On the other hand, the expansion of $\mathcal{Z}_-$ gives

$$E_{(-)}(\mu) = \frac{\pi R}{\mu} \left(1 + \sqrt{1 + \frac{\mu^2 P^2}{\pi^2 R^2} + \frac{2\mu E}{\pi R}} \right),$$

$$P_{(-)}(\mu) = P. \quad (34)$$

In the undeformed limit $\mu \rightarrow 0$, $E_{(-)}$ diverges as $\propto 1/\mu$. Thus, negative-winding sectors can be viewed as nonperturbative corrections to the full partition function, as was noted also in [56].

$J\bar{T}$ deformation: The discussion around (28) suggests that the $J\bar{T}$ deformation [57] corresponds to deforming the worldsheet action by $\int d^2z \bar{\beta} J$. For concreteness, let us assume that the seed CFT contains a compact boson of radius r ,

$$S_{\mathbb{S}^1} = \int \frac{d^2z}{4\pi} \partial y \bar{\partial} y, \quad y \sim y + 2\pi r. \quad (35)$$

We conjecture that the dual worldsheet action is given by

$$S_{J\bar{T}} = S_{\text{sym}} - \frac{\mu}{\pi} \int \frac{d^2z}{2\pi} \bar{\beta} \partial y. \quad (36)$$

Unlike the $T\bar{T}$ deformation, the action is linear in β 's. Thus, integrating out β 's gives a δ function that localizes the worldsheet moduli. As a result, we get

$$\mathcal{Z}_{\text{ws}} = \sum_{K>0} p^K \mathcal{T}_K[\mathcal{Z}_+] + p^{-K} \mathcal{T}_K[\mathcal{Z}_-], \quad (37)$$

with

$$\mathcal{Z}_{\pm}(R, \zeta, \bar{\zeta}) = \pm \sum_{m,w} \frac{\mathcal{Z}_{\text{seed}}^{(m,w)} \left(\pm \zeta, \frac{\pm \bar{\zeta} \mp \frac{\mu_r m}{\pi R w}}{1 \mp \frac{\mu_r \zeta}{\pi R w}} \right)}{1 + \frac{\mu_r}{\pi R} \frac{m \mp w \zeta}{\zeta - \bar{\zeta}}}, \quad (38)$$

where $\mathcal{Z}_{\text{seed}}^{(m,w)}$ is the seed CFT partition function in the (m, w) -winding sector of y . Expanding $\mathcal{Z}_{\text{seed}}^{(m,w)}$ in $\mathcal{Z}_{\pm}$ (38), we obtain the spectrum,

$$E_{R(+)}(\mu) = \frac{\pi^2}{\mu^2} \left[\left(R - \frac{\mu Q}{\pi} \right) - \sqrt{\left(R - \frac{\mu Q}{\pi} \right)^2 - \frac{2\mu^2 R}{\pi^2} E_R} \right]$$

$$P_{(+)}(\mu) = P, \quad (39)$$

and

$$E_{R(-)}(\mu) = \frac{\pi^2}{\mu^2} \left[\left(R - \frac{\mu Q}{\pi} \right) + \sqrt{\left(R - \frac{\mu Q}{\pi} \right)^2 - \frac{2\mu^2 R}{\pi^2} E_R} \right]$$

$$P_{(-)}(\mu) = P, \quad (40)$$

where $E_R = (E - P)/2$ and $Q = (n/r) + (wr/2)$ is the $U(1)$ charge. The positive-winding spectrum (39) agrees with the $J\bar{T}$ -deformed spectrum obtained in the literature [58–60], while the negative-winding spectrum (40) gives nonperturbative corrections.

Conclusion—We proposed string duals to 2D chiral Yang-Mills theory and to symmetric product orbifolds of general seed CFTs with $c < 24$, including their $T\bar{T}$ and $J\bar{T}$ deformations. We presented supporting evidence, but further tests—such as higher-point amplitudes in 2D YM theory and correlation functions in symmetric orbifolds—are desirable [34,61].

It is also important to clarify connections with other proposals for 2D YM duals, particularly the topological string dual to q -deformed YM theory [23]. Such comparisons may also lead to improved formulations of our composite linear dilaton action.

A notable feature of our proposed duals for $T\bar{T}$ and $J\bar{T}$ deformations is the inclusion of negative-winding sectors. These correspond to alternative sign choices in square roots and give nonperturbative corrections to the partition function. In nonrelativistic string, they correspond to modes that decouple in the nonrelativistic limit. As noted in [56], such modes are crucial for the S duality of the partition function. Understanding their role in $T\bar{T}$ deformations—possibly via resurgence analysis [62–64]—may be an interesting future direction.

Our proposal shares similarities with matrix string conjecture [65,66], which relates the discrete light cone quantization (DLCQ) of nonperturbative type IIA superstring to 2D $\mathcal{N} = 8$ super-Yang-Mills (SYM) theory. In this context, the perturbative IIA string corresponds to the IR limit of SYM theory, described by a symmetric product orbifold of supersymmetric $\mathbb{R}^8$. Meanwhile, the T dual of the DLCQ string [67,68] is nonrelativistic string [27–29] described by a β - γ system. Together, these imply a duality between the symmetric orbifold of $\mathbb{R}^8$ and nonrelativistic type IIA string—a supersymmetric counterpart of our proposal. In addition, similarities between matrix string and 2D YM theory were discussed in interesting papers [69,70]. Exploring these webs of dualities (cf. Ref. [71]) further would be highly illuminating.

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