

Charging Quantum Batteries with Chiral Squeezing

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We propose a quantum-battery charger based on a driven bosonic Kitaev chain (BKC), where chiral squeezing converts passive input fluctuations into ordered, nonpassive battery states. While a coherent input pulse exhibits phase-sensitive chiral transport, the charging dynamics is dominated by bidirectionally propagating fluctuations that are amplified and squeezed into orthogonal quadratures at opposite chain ends. In contrast to conventional phase-preserving amplifiers, our scheme stores largely extractable energy and achieves a worklike signal-to-noise ratio (SNR) near unity, even in the presence of thermal noise and moderate symmetry-preserving disorder.

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Introduction—We address a key challenge in quantum batteries, namely, how to inject energy while converting unavoidable input fluctuations into useful, extractable work without sacrificing operational performance. Quantum batteries (QBs) are devices designed to store and retain energy in their internal—both classical and genuinely quantum—degrees of freedom [1–6]. Over the past decade, substantial progress has been achieved, including the design of charging protocols [7–16], the development of strategies to mitigate energy loss [17–24], and the exploration of reservoir-engineering [25–31] and topological effects [32] as resources for performance enhancement. From an experimental perspective, a variety of QB architectures have been proposed across different physical implementations [23,33–37]. Most QB charging schemes optimize how much energy can be stored or how fast [38–46]. However, energy fluctuations constitute an equally important aspect that must be carefully accounted for [47–60]. A key open question is therefore how to exploit input fluctuations for charging while keeping the extractable energy large compared with the energy fluctuations of the battery state. This issue is highly nontrivial because conventional energy amplification is typically accompanied by substantial added noise, making it difficult to combine a large charging output with a high signal-to-noise ratio (SNR) [61,62]. It is thus desirable to identify charging platforms in which both the coherent input and

input fluctuations are processed such that amplification leads to high extractable work.

In this Letter, we show that a driven bosonic Kitaev chain (BKC) provides such a platform that can charge batteries; see Fig. 1. The BKC is a paradigmatic one-dimensional (1D) bosonic lattice in which nearest-neighbor hopping is supplemented by parametric-pump terms that create and annihilate excitations in pairs [63]. The pairing breaks excitation-number conservation and gives rise to chiral transport. This unusual structure enables a series of extraordinary phenomena absent in conventional passive lattices, including phase-sensitive and topologically protected directional amplification [63–65], exceptionally enhanced

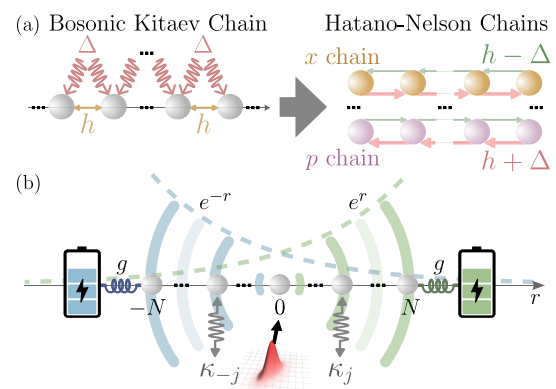


FIG. 1. (a) A bosonic Kitaev chain (BKC) with nearest-neighbor hopping h and parametric pairing Δ , which is equivalent to two decoupled Hatano-Nelson chains for the x and p quadratures with opposite chiralities. (b) Schematic of the chiral charger: a driven-dissipative BKC of length $2N + 1$, with local losses κ_j and chiral squeezing weights $e^{\pm r}$, is excited at the central site $j = 0$ by a short coherent pulse. The two end sites ($j = \pm N$) are coupled to the batteries through coupling g .

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quantum sensing [66], and realization of bosonic Majorana zero modes [67]. With the required ingredients now accessible in various platforms, such as superconducting circuits [68] and optomechanical systems [69], the BKC has become a controllable and robust platform for engineering energy transport in driven bosonic networks [70].

While previous studies have focused on BKC dynamics driven by coherent inputs, here we investigate how continuously injected reservoir fluctuations can participate in the charging process alongside a coherent pulse. We find that a coherent input pulse with a well-defined initial phase undergoes phase-sensitive chiral transport, whereas the fluctuation sector propagates bidirectionally and is amplified and squeezed into orthogonal quadratures toward opposite ends of the chain. Due to its finite duration and the dissipation accumulated across the chain, the coherent pulse contributes negligibly to the stored battery energy, so that the overall charging dynamics is dominated by the fluctuation sector. More surprisingly, the active BKC dynamics converts the originally passive fluctuations into ordered, nonpassive battery energies. Our protocol achieves a nearly unity worklike SNR and remains robust against thermal noise and symmetry-preserving disorder. In this Letter, we set the Planck constant to $\hbar = 1$.

Model—We model the BKC as a uniform 1D bosonic chain of length $2N + 1$, with nearest-neighbor hopping h and parametric-pump induced two-particle pairing Δ , as shown in Fig. 1(a). Let ω_C denote the bare resonance frequency of the BKC sites. We take the parametric pump to be resonant at $2\omega_C$ and work in a frame rotating at ω_C . The resulting rotating-frame chain Hamiltonian reads

$$\hat{H}_C = \frac{1}{2} \sum_{j=-N}^{N-1} (i\hbar\hat{a}_{j+1}^\dagger\hat{a}_j + i\Delta\hat{a}_{j+1}^\dagger\hat{a}_j^\dagger + \text{H.c.}), \quad (1)$$

where $\hat{a}_j$ ($\hat{a}_j^\dagger$) annihilates (creates) a bosonic excitation on the j th chain site, satisfying $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{jk}$. By introducing the position and momentum quadratures, $\hat{x}_j = (\hat{a}_j + \hat{a}_j^\dagger)/\sqrt{2}$ and $\hat{p}_j = i(\hat{a}_j^\dagger - \hat{a}_j)/\sqrt{2}$, the Hamiltonian can be rewritten as

$$\hat{H}_C = \frac{1}{2} \sum_j [-(h - \Delta)\hat{x}_{j+1}\hat{p}_j + (h + \Delta)\hat{p}_{j+1}\hat{x}_j]. \quad (2)$$

In the absence of dissipation and external drive, the Heisenberg equations of motion derived from Eq. (2) read $\dot{\hat{x}}_j = [(h + \Delta)\hat{x}_{j-1} - (h - \Delta)\hat{x}_{j+1}]/2$ and $\dot{\hat{p}}_j = [(h - \Delta)\hat{p}_{j-1} - (h + \Delta)\hat{p}_{j+1}]/2$. These equations show that the BKC decomposes into two decoupled Hatano-Nelson chains [71] for the x and p quadratures, with opposite nonreciprocal couplings, as illustrated in Fig. 1(a), where the pink (thick) and green (thin) arrows are swapped for the x and p dynamics. As a result, an injected wave packet with a *well-defined* initial phase exhibits a phase-dependent chirality: the x component couples

preferentially to the right neighbor, whereas the p component couples preferentially to the left neighbor.

We emphasize that $\hat{H}_C$ is an effective rotating-frame Hamiltonian that contains parametric pump terms. These terms, which are proportional to Δ , represent an external *active* resource that underlies the chirality and amplification of the dynamics. We therefore account for the corresponding energetic cost through the pump work defined in the End Matter.

For open boundaries, a lossless BKC is dynamically stable when $|h| > |\Delta|$. In contrast, for $|h| < |\Delta|$, the dynamics becomes unstable and no well-defined wave transport exists [see Note 1 of Supplemental Material (SM) [72]]. In what follows, we restrict ourselves to the stable regime, where one can introduce the squeezing parameter $r = \frac{1}{2} \ln [(h + \Delta)/(h - \Delta)]$ and an effective hopping amplitude $\tilde{h} = \sqrt{h^2 - \Delta^2} = h/\cosh r$. The chiral transport is reflected in the opposite squeezing for the x and p components: they propagate toward opposite ends of the chain with amplitudes weighted by $e^{\pm jr}$; see Eq. (3) below.

As shown in Fig. 1(b), in this Letter we consider a BKC of length $2N + 1$, and couple its two ends to storage modes (“batteries”). The batteries are modeled as two identical harmonic modes with Hamiltonian $\hat{H}_{\text{bat}} = \sum_\alpha \hat{H}_{\text{bat},\alpha} = \sum_\alpha \Omega \hat{b}_\alpha^\dagger \hat{b}_\alpha$ ($\alpha \in \{L, R\}$), where $\Omega = \omega - \omega_C$ is the detuning between the battery frequency ω and the BKC bare frequency ω_C . Hereafter, we focus on the resonant case $\Omega \equiv 0$, such that $\hat{H}_{\text{bat}}$ vanishes in the rotating frame. The BKC-battery interaction is described by $\hat{H}_{\text{int}} = g_L(t)(\hat{x}_{-N}\hat{x}_L + \hat{p}_{-N}\hat{p}_L) + g_R(t)(\hat{x}_N\hat{x}_R + \hat{p}_N\hat{p}_R)$, with the battery quadratures defined as $\hat{x}_\alpha = (\hat{b}_\alpha + \hat{b}_\alpha^\dagger)/\sqrt{2}$ and $\hat{p}_\alpha = i(\hat{b}_\alpha^\dagger - \hat{b}_\alpha)/\sqrt{2}$. The total Hamiltonian in the rotating frame is given by $\hat{H}_{\text{tot}} = \hat{H}_C + \hat{H}_{\text{int}} + \hat{H}_{\text{drive}}$, where $\hat{H}_{\text{drive}} = \sqrt{\kappa_0}(p_{\text{in}}\hat{x}_0 - x_{\text{in}}\hat{p}_0)$ describes a coherent drive applied to the central site ($j = 0$), with effective driving amplitudes $\sqrt{\kappa_0}x_{\text{in}}$ and $\sqrt{\kappa_0}p_{\text{in}}$ for the two quadrature components. The coherent drive is assumed to be resonant with the BKC sites, so that its carrier-frequency oscillation is removed in the rotating frame.

Equations of motion—Assuming that every chain site is subject to (uniform) losses, modeled by Markovian reservoirs, the quantum Langevin equations for the chain quadratures are given by

$$\begin{aligned} \dot{\hat{x}}_j &= \frac{\tilde{h}}{2} (e^r \hat{x}_{j-1} - e^{-r} \hat{x}_{j+1}) - \frac{\kappa_j}{2} \hat{x}_j - \sqrt{\kappa_0} x_{\text{in}}(t) \delta_{j,0} \\ &\quad + \sqrt{\kappa_j} \hat{\xi}_j^{(x)}(t) + g_R(t) \hat{p}_R \delta_{j,N} + g_L(t) \hat{p}_L \delta_{j,-N}, \\ \dot{\hat{p}}_j &= \frac{\tilde{h}}{2} (e^{-r} \hat{p}_{j-1} - e^r \hat{p}_{j+1}) - \frac{\kappa_j}{2} \hat{p}_j - \sqrt{\kappa_0} p_{\text{in}}(t) \delta_{j,0} \\ &\quad + \sqrt{\kappa_j} \hat{\xi}_j^{(p)}(t) - g_R(t) \hat{x}_R \delta_{j,N} - g_L(t) \hat{x}_L \delta_{j,-N}. \end{aligned} \quad (3)$$

Here κ_j is the decay rate of the j th chain site and $\hat{\xi}_j^{(\beta)}(t)$ ($\beta = x, p$) is the corresponding quadrature noise operator. The noise is δ -correlated, $\frac{1}{2}\langle\{\hat{\xi}_j^{(\beta)}(t), \hat{\xi}_k^{(\beta')}(t')\}\rangle = (n_{\text{th}} + \frac{1}{2})\delta_{jk}\delta_{\beta\beta'}\delta(t-t')$ with n_{th} being the thermal occupation, and has by definition zero mean, $\langle\hat{\xi}_j^{(\beta)}(t)\rangle = 0$. The dynamics of the right battery is described by

$$\begin{aligned}\dot{\hat{x}}_R &= -\frac{\gamma}{2}\hat{x}_R + g_R(t)\hat{p}_N + \sqrt{\gamma}\hat{\zeta}_R^{(x)}(t), \\ \dot{\hat{p}}_R &= -\frac{\gamma}{2}\hat{p}_R - g_R(t)\hat{x}_N + \sqrt{\gamma}\hat{\zeta}_R^{(p)}(t),\end{aligned}\quad (4)$$

with similar equations ($R \rightarrow L$ and $N \rightarrow -N$) for the left battery. The batteries are subject to dissipation at rate γ , with associated noise operators $\hat{\zeta}_{R/L}^{(x/p)}$, which also have zero mean and obey similar δ -type correlations. For simplicity, we hereafter assume $g_R(t) \equiv g_L(t) = g(t)$, meaning that the batteries are detached/attached simultaneously with the same coupling strength.

In our protocol, we coherently drive only the center site $j = 0$ with amplitude $\mathcal{A}$, phase θ , and a Gaussian envelope $f(t) = \exp\{-[(t-t_0)^2/2\sigma^2]\}$, so that $x_{\text{in}}(t) = \mathcal{A}\cos\theta f(t)$, $p_{\text{in}}(t) = \mathcal{A}\sin\theta f(t)$, where t_0 is the pulse-center time and σ sets the pulse width. Because the quantum Langevin equations for the x and p quadratures decouple and exhibit opposite chiralities, the x component of the injected wave packet propagates predominantly to the right, whereas the p component propagates predominantly to the left. Consequently, a coherent input field with a well-defined initial phase exhibits phase-sensitive chiral transport [63]. For fields *without* a definite phase, the transport remains bidirectional. As we show below, however, the right-moving and left-moving components of an incoherent signal are amplified and squeezed into orthogonal phase-space directions. This chiral squeezing mechanism allows the active BKC dynamics to convert otherwise passive fluctuations into ordered, nonpassive energies.

Chiral charging—We first use the stored energy and the ergotropy as the central figures of merit of our charging protocol [77,78]. Since $\hat{b}_\alpha^\dagger\hat{b}_\alpha$ is invariant under the rotating-frame transformation, the stored energy can be evaluated directly from the rotating-frame moments as $\hat{E}_\alpha \equiv \omega\hat{b}_\alpha^\dagger\hat{b}_\alpha = \omega(\hat{x}_\alpha^2 + \hat{p}_\alpha^2 - 1)/2$ and $E_\alpha(\hat{\rho}) \equiv \text{Tr}\{\hat{\rho}\hat{E}_\alpha\}$ ($\alpha = L, R$). The ergotropy quantifies the maximum amount of work that can be extracted from a quantum state via a unitary process, and is defined as [79]

$$\mathcal{W}_\alpha(\hat{\rho}) \equiv E_\alpha(\hat{\rho}) - E_\alpha(\hat{\rho}_{\text{pa}}), \quad (5)$$

where $\hat{\rho}_{\text{pa}}$ is the passive state associated with $\hat{\rho}$, obtained by assigning the eigenvalues of $\hat{\rho}$ (in descending order) to the eigenstates of $\hat{E}_\alpha$ (in ascending order).

Figure 2 shows representative charging dynamics. To illustrate the chiral features, we first plot in Fig. 2(a) the

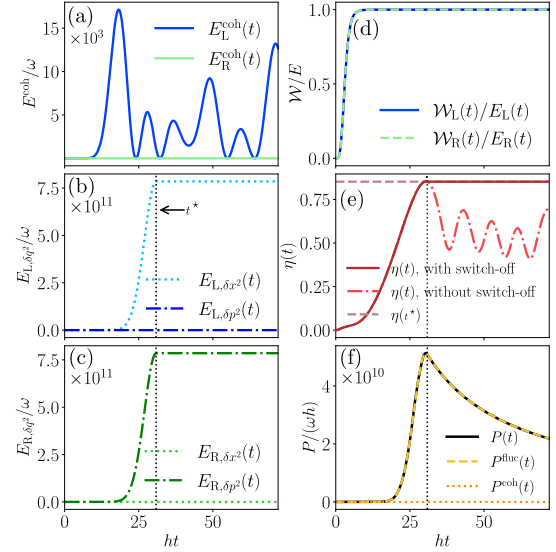


FIG. 2. Charging dynamics. (a) Coherent contributions to the left- and right-battery stored energies, E_L^{coh} and E_R^{coh} . (b) Quadrature-resolved fluctuation contributions to the left-battery stored energy, $E_{L,\delta x^2}$ and $E_{L,\delta p^2}$. (c) Same as (b) for the right battery, $E_{R,\delta x^2}$ and $E_{R,\delta p^2}$. (d) Ergotropy-energy ratios $\mathcal{W}_L(t)/E_L(t)$ and $\mathcal{W}_R(t)/E_R(t)$. (e) Overall charging efficiency η , with and without switch off at $t = t^*$. (f) Average charging power $P(t)$ and its coherent and fluctuation contributions. The vertical dotted lines indicate the switch-off time $t = t^*$. Parameters: $N = 5$, $\Delta/h = 11/12$, $g/h = 5/12$, $\kappa_j/h = 8.3 \times 10^{-4}$ ($j = -N, \dots, N$), $n_{\text{th}} = 0$, $\gamma = 0$, $\mathcal{A}/\sqrt{h} = 2.887$, $\theta = \pi/2$, $h\sigma = 1.2$, and $ht_0 = 1.2$.

coherent contributions to the stored energies, E_L^{coh} and E_R^{coh} , originating solely from the injected Gaussian pulse with initial phase $\theta = \pi/2$. These curves are obtained by solving the mean-value parts of Eqs. (3) and (4) (see Note 2 of SM [72]). As expected from the phase-sensitive chirality, the coherent wave packet is transported predominantly toward the left and is eventually loaded into the left battery (with oscillations reflecting the dynamics of the battery-chain coupling), whereas its contribution to the right battery remains negligible. However, this directional coherent contribution remains small and shows no appreciable amplification in the batteries. For the finite-duration pulse considered here, the dissipation accumulated across the chain dominates over the amplification.

In Figs. 2(b) and 2(c), we show the stored energies originating from the fluctuations, which are obtained from the second moments of Eqs. (3) and (4). We refer to them as the *fluctuation contributions* to the stored energies. For these plots, we switch off the setup at $t = t^*$, i.e., the time at which the charging efficiency reaches its maximum; see Fig. 2(e). The behaviors of the coherent and fluctuation contributions are qualitatively different. Unlike the finite coherent pulse, the fluctuation sector is continuously driven by the Markovian input-noise fields at all BKC sites. Thus, the same dissipation channels that attenuate the propagating mean field also inject vacuum (or thermal) fluctuations

into the covariance dynamics. The parametric pump then reshapes and amplifies these fluctuations into ordered, nonpassive energy, allowing the fluctuation contribution to become orders of magnitude larger than the coherent one. Since the coherent pulse enters only the first-moment dynamics, removing it leaves the covariance dynamics unchanged. In our protocol, the coherent pulse primarily serves to illustrate the phase-sensitive chiral transport established in previous studies of the BKC.

In contrast to the injected Gaussian pulse, the fluctuations are amplified in both directions. However, the components $E_{\alpha,\delta x^2}$ and $E_{\alpha,\delta p^2}$, which quantify the energies stored in the two orthogonal battery quadratures [72], are strongly direction dependent. The fluctuation energy in the left battery is carried almost entirely by the x sector, while that in the right battery is dominated by the p sector. More interestingly, the complementary quadratures of the two batteries are squeezed below the vacuum level: specifically, we have $\langle \delta p_L^2(t^*) \rangle < 1/2$ and $\langle \delta x_R^2(t^*) \rangle < 1/2$ (see Note 2 of SM [72]). Consistent with this strongly anisotropic fluctuation structure, the stored energies are almost fully extractable. As shown in Fig. 2(d), the ratios $\mathcal{W}_{L(R)}(t)/E_{L(R)}(t)$ approach unity after a short transient. This transient reflects the time required for the chiral dynamics to convert initially passive fluctuation energy into strongly squeezed, nonpassive battery states [72].

We next quantify the overall charging performance (including both the coherent and fluctuation contributions), through the charging efficiency $\eta(t)$ and average charging power $P(t)$. The former is defined as the ratio between the total energy stored in the two batteries and the active energy cost of the protocol, while the latter quantifies the total energy obtained by the batteries per unit charging time; details are given in the End Matter. As shown in Fig. 2(e), when the chain is kept on, $\eta(t)$ first increases and reaches a maximum at $t = t^*$ (see Note 3 of SM [72]), after which it decreases due to the continued chain-battery energy exchange and the accumulation of energy cost. This maximum therefore identifies an operational switch-off time. We thus implement a finite-time charging protocol by switching off the setup at $t = t^*$. As shown in Fig. 2(f), the average charging power likewise increases monotonically until the setup is switched off at t^* and is dominated by the fluctuation contribution. Moreover, we find that $P(t^*)$ exhibits a strongly *superextensive* growth with the half-chain length N , substantially exceeding the linear scaling $P \propto N$ over the investigated range (see Note 2 of SM [72]).

High signal-to-noise ratio—To quantify how the extractable work compares with the energy fluctuations of the charged battery state, we introduce a worklike SNR,

$$\text{SNR}^{\text{work}}(t^*) = \frac{\mathcal{W}(t^*)}{\sigma_E(t^*)}, \quad (6)$$

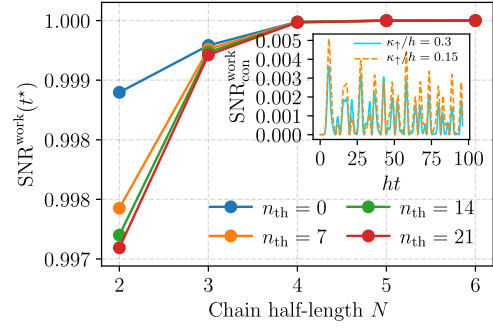


FIG. 3. Worklike signal-to-noise ratio $\text{SNR}^{\text{work}}(t^*)$, evaluated at $t = t^*$, as a function of the chain half-length N for several thermal occupancies n_{th} . Inset: signal-to-noise ratio $\text{SNR}^{\text{work}}_{\text{con}}(t)$ for the conventional phase-preserving amplifier (see Note 4 of the SM [72]), with $n_{\text{th}} = 0$ and two representative on-site gain rates. All other parameters are the same as in Fig. 2.

where $\mathcal{W}(t^*) = \mathcal{W}_L(t^*) + \mathcal{W}_R(t^*)$ and $\sigma_E(t^*) = \sqrt{\text{Var}[\hat{E}_{\text{bat}}](t^*)}$ is the standard deviation of the battery energy in the same state, with $\hat{E}_{\text{bat}} = \hat{E}_L + \hat{E}_R$. As we show below, our BKC-based charger exhibits another key advantage: unlike conventional phase-preserving amplifiers, such as a 1D tight-binding chain with on-site gain, it enables a nearly unity $\text{SNR}^{\text{work}}(t^*)$.

Figure 3 shows $\text{SNR}^{\text{work}}(t^*)$ as a function of the BKC half-length N for several values of the thermal occupancy n_{th} (applied uniformly to all chain sites). We find that $\text{SNR}^{\text{work}}(t^*)$ rapidly approaches unity as N increases. Even for moderate system sizes, $\mathcal{W}(t^*)$ becomes comparable to $\sigma_E(t^*)$, yielding an asymptotically size-independent SNR. Moreover, the curves for different n_{th} nearly coincide for large N . Only a slight difference can be observed for the smallest N , while the dependence on n_{th} becomes negligible for $N \gtrsim 4$. This indicates that, at the optimal time, the injected thermal noise does not qualitatively degrade the work-to-fluctuation performance.

The result in Fig. 3 does not mean that usable work can be extracted from a thermal bath alone. Indeed, a thermal (Gibbs) state is passive and has vanishing ergotropy. In our protocol, extractable work is generated by *actively* converting the incoming fluctuations through the externally pumped dynamics. In other words, the BKC acts as an active Gaussian processor that can convert vacuum or thermal fluctuations into nonpassive battery states. Moreover, σ_E should not be viewed as a work resource by itself, since passive states can exhibit substantial energy fluctuations while having zero ergotropy. In an active Gaussian charging process such as ours, instead, the same pump-driven dynamics that generates nonpassivity (and hence ergotropy) can also amplify energy fluctuations, so that $\mathcal{W}(t^*)$ and $\sigma_E(t^*)$ covary [80]. In the End Matter, we examine the scaling of these two quantities separately, showing that the near-unity $\text{SNR}^{\text{work}}(t^*)$ results from the

correlated (nearly linear) scaling of the extractable work and the battery-energy fluctuation.

This favorable work-to-fluctuation behavior clearly distinguishes our protocol from conventional phase-preserving gain media, where amplification is inevitably accompanied by added quantum noise. In a tight-binding chain with uniform on-site gain [72], for example, the added fluctuations are predominantly isotropic and contribute mainly as passive, thermal-like energy. This energy contribution therefore increases the battery-energy fluctuation while providing little ergotropy. Increasing the gain does not overcome this limitation, because the coherent contribution and the added fluctuations acquire the same leading amplification factor (see Note 4 of SM [72]). Accordingly, the inset of Fig. 3 shows SNRs of only $\sim 10^{-3}$ for two representative on-site gain rates $\kappa_{\uparrow}$. By contrast, the chiral squeezing mechanism of our BKC scheme (encoded in the drift matrix) converts amplified fluctuations into nonpassive energy, avoiding this passive-noise penalty.

Topological protection—We now examine the robustness of the worklike SNR against specific imperfections. Figure 4 quantifies this robustness by showing $\text{SNR}^{\text{work}}(t^*)$ in the presence of static disorder in both the hopping amplitudes (left panel) and the parametric pump strength (right panel). For each disorder strength, we sampled 50 random realizations and summarize the corresponding distribution by plotting the median together with the 16–84 percentile band (which contains the central 68% of the realizations) [81]. For comparison, we also show the mean, $\overline{\text{SNR}}^{\text{work}}$, together with error bars (one standard deviation), i.e., $\overline{\text{SNR}}^{\text{work}} \pm \text{STD}$.

We find that SNR^{work} remains close to its disorder-free value over a broad range of disorder strengths. This robustness can be understood from the fact that both types of disorder preserve the BKC structure and do not mix the x - and p -quadrature sectors. The chiral dynamics remains protected by the *point-gap topology* [82,83] of the x - and p -sector HNCs: their complex spectra remain separated

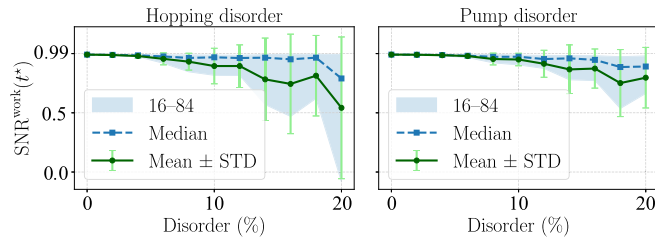


FIG. 4. $\text{SNR}^{\text{work}}(t^*)$ versus static hopping disorder (left) and pump disorder (right), for a fixed chain half length $N = 5$. For each disorder strength, 50 random realizations are sampled. The green solid curves with error bars show the mean $\pm$ standard deviation, the blue dashed curves denote the median, and the shaded regions indicate the 16th–84th percentile range. All other parameters are the same as in Fig. 2.

from the reference point (here set by the BKC on-site energy), so that the winding numbers cannot change under weak disorder [84]. The slightly stronger degradation under hopping disorder is mainly because it directly perturbs the field transport and enhances reflections, whereas the pump disorder primarily modifies the local squeezing and amplification.

The hopping and pump disorders considered above are examples of *symmetry-preserving* disorder. For symmetry-breaking disorder, however, the protocol becomes more fragile. A representative example is random on-site detuning disorder, incorporated through perturbations $\sum_j \delta\omega_j \hat{a}_j^\dagger \hat{a}_j$ in Eq. (1). Physically, such random on-site energies mix the x and p quadratures and thereby degrade the transport chirality. For sufficiently strong detunings, this quadrature mixing can also trigger an instability, leading to uncontrolled amplification during the charging process [63,70]. In practice, however, these effects can be kept under control by operating with shorter chains and/or weaker parametric pumps. A detailed analysis of the impact of on-site detuning disorder is provided in Note 5 of SM [72].

Conclusions—In summary, we have proposed a quantum-battery charger based on a driven BKC. While a coherent input pulse with a well-defined phase exhibits phase-sensitive chiral transport, the input fluctuations propagate bidirectionally and are reshaped into orthogonal quadratures at opposite ends of the chain. We showed that these amplified and squeezed fluctuations dominate the total stored energies and ergotropies of the batteries, thereby enabling fluctuation-driven quantum charging beyond the coherent channel alone. Moreover, the resulting protocol achieves a nearly unity worklike SNR at the optimal charging time, in sharp contrast to conventional phase-preserving gain media, and remains robust against thermal noise and moderate symmetry-preserving disorder. Our proposal is conceptually distinct from previous chiral charging schemes (see End Matter) and could be implemented in several experimentally relevant platforms, including parametric superconducting resonators [68] and optomechanical arrays [69].

Looking ahead, our results suggest a design principle for quantum batteries: input fluctuations need not only be a source of noise, but can be reshaped into useful nonpassive energy. This idea may extend beyond the specific BKC implementation to other nonreciprocal or topological bosonic networks. A natural next step is to explore whether going beyond Gaussian dynamics, for example, through weak nonlinearities or conditional operations, can further enhance or control this fluctuation-to-work conversion and its robustness.

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Data availability—The numerical code used to generate the data and figures in this Letter is available at [85]. The repository contains the simulation scripts, plotting routines, and parameter files required to reproduce the results presented in the main text and Supplemental Material [72].

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End Matter

Charging efficiency and power—Here we provide the energetic bookkeeping used to define the charging efficiency in Figs. 2(e) and 2(f). For each battery $\alpha = \{L, R\}$, the stored energy is

$$E_\alpha(t) = \frac{1}{2} \omega \mu_\alpha^T \mu_\alpha + \frac{1}{2} \omega \{ \text{Tr}[V_\alpha(t)] - 1 \}, \quad (\text{A1})$$

where $\omega = \omega_C$ is the battery frequency (in the laboratory frame), $\mu_\alpha = (\langle \hat{x}_\alpha \rangle, \langle \hat{p}_\alpha \rangle)^T$ and V_α are the first-moment vector and covariance matrix of the corresponding battery mode. The total stored energy is $E_{\text{bat}}(t) = E_L(t) + E_R(t)$. The (active) energy cost of our charging protocol has two contributions,

$$E_{\text{cost}}(t) = E_{\text{pulse}}(t) + E_{\text{pump}}(t).$$

The first term is the work supplied by the coherent Gaussian pulse applied to the central site (for details see Note 2 of SM [72]),

$$E_{\text{pulse}}(t) = \int_0^t dt' P_{\text{pulse}}(t'), \quad (\text{A2})$$

$$P_{\text{pulse}}(t) = -\omega_C \sqrt{\kappa_0} [x_{\text{in}}(t) \langle x_0(t) \rangle + p_{\text{in}}(t) \langle p_0(t) \rangle]. \quad (\text{A3})$$

The second term accounts for the energetic cost of the parametric pump, which is obtained as [72]

$$E_{\text{pump}}(t) = \int_0^t dt' P_{\text{pump}}(t'), \quad (\text{A4})$$

with

$$P_{\text{pump}}(t) = \omega_C \tilde{h}(t) \sinh r(t) \sum_{j=-N}^{N-1} [\langle x_j(t) x_{j+1}(t) \rangle - \langle p_j(t) p_{j+1}(t) \rangle]. \quad (\text{A5})$$

Indeed, this contribution vanishes in the absence of the parametric pump, namely, for a squeezing parameter $r = 0$.

The overall charging efficiency is then defined as

$$\eta(t) = \frac{E_{\text{bat}}(t)}{E_{\text{cost}}(t)} = \frac{E_L(t) + E_R(t)}{E_{\text{pulse}}(t) + E_{\text{pump}}(t)}. \quad (\text{A6})$$

Equivalently, one may introduce the individual charging efficiencies $\eta_\alpha(t) = E_\alpha(t)/E_{\text{cost}}(t)$ for $\alpha = L, R$, so that $\eta(t) = \eta_L(t) + \eta_R(t)$. As shown in Fig. 2(e), when the setup is kept on continuously, the efficiency initially increases since the amplified, ordered fluctuations are transported to the chain ends and loaded into the batteries. It subsequently decreases after $t = t^*$ because of the continued chain-battery energy exchange together with the accumulation of energetic cost. We therefore choose t^* , the time at which $\eta(t)$ is maximal, as the operational switch-off time. The finite-time protocol is implemented by setting $h = \Delta = g = 0$ at $t = t^*$.

The average charging power shown in Fig. 2(f) is defined as the energy gained by the batteries per unit charging time,

$$P(t) = \frac{E_{\text{bat}}(t) - E_{\text{bat}}(0)}{t}. \quad (\text{A7})$$

Correspondingly, decomposing the stored energy into coherent and fluctuation contributions, $E_{\text{bat}} = E_{\text{bat}}^{\text{coh}} + E_{\text{bat}}^{\text{fluc}}$, gives $P(t) = P^{\text{coh}}(t) + P^{\text{fluc}}(t)$ with

$$P^{\text{coh(fluc)}}(t) = \left[E_{\text{bat}}^{\text{coh(fluc)}}(t) - E_{\text{bat}}^{\text{coh(fluc)}}(0) \right] / t.$$

Ergotropy versus intrinsic energy fluctuation—To better understand the origin of the nearly unity worklike SNR of our protocol, in Fig. 5 we replot the data of Fig. 3 in terms of the ergotropy $\mathcal{W}(t^*)$ versus the battery-energy fluctuation $\sigma_E(t^*)$, both evaluated at the optimal charging time t^* . The different data points along each curve are obtained by varying the chain half-length N from 2 to 6 (as in Fig. 3), while each curve corresponds to a fixed bath thermal occupancy n_{th} , taken to be uniform across the chain.

For all considered values of n_{th} , the data points lie almost on the same straight line. This nearly linear coscaling shows that increasing the chain length enhances the extractable work and the intrinsic energy fluctuation in a correlated manner. For the smallest chain sizes, particularly $N = 2$, the data retain a visible dependence on the bath thermal occupancy n_{th} . As N increases, however, the curves rapidly converge, indicating that the influence of the thermal occupancy becomes progressively negligible for the larger chains considered here. As a result, their ratio $\text{SNR}^{\text{work}}(t^*) = \mathcal{W}(t^*)/\sigma_E(t^*)$ remains close to unity and becomes insensitive to n_{th} , consistent with the saturation behavior in Fig. 3.

Relation to other nonreciprocal and chiral charging schemes—We finally distinguish the present mechanism from previous quantum-battery proposals based on nonreciprocal or chiral energy transfer. In the reservoir-engineered nonreciprocal charging scheme [28], directionality is created by balancing the coherent and dissipative charger–battery couplings so as to suppress

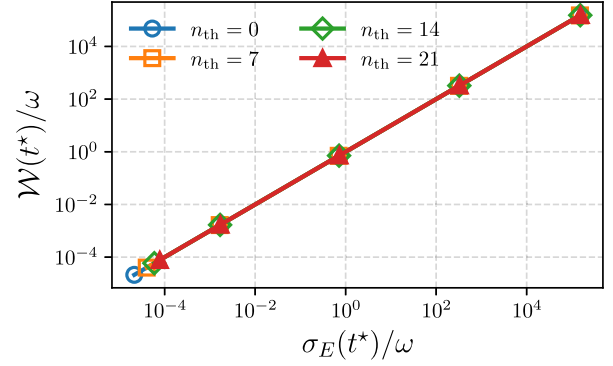


FIG. 5. Ergotropy $\mathcal{W}(t^*)$ as a function of the battery-energy fluctuation $\sigma_E(t^*)$, evaluated at the optimal charging time t^* . Each curve is obtained by varying the chain half-length N from 2 to 6 at fixed bath thermal occupancy n_{th} , as in Fig. 3. The data lie almost on the same straight line, showing that $\mathcal{W}(t^*)$ and $\sigma_E(t^*)$ grow in a correlated manner as N increases. All other parameters are the same as in Fig. 2.

energy backflow. In the recently proposed chiral waveguide implementation [86], chirality instead originates from spin-momentum locking, which produces asymmetric couplings to oppositely propagating waveguide modes and thereby enables nonreciprocal energy transfer. Moreover, a recent giant-atom scheme [16] exploits self-interference between spatially separated coupling points to suppress dissipative loss while preserving coherent energy exchange, with related configurations also enabling chiral long-range transfer. These mechanisms primarily control the directionality or loss of coherently supplied energy, without introducing parametric amplification.

In our BKC scheme, by contrast, chirality arises from the *parametric pairing* and is intrinsically *quadrature dependent*: the x and p sectors realize effective Hatano-Nelson chains with opposite directional amplification. Importantly, the active pairing process also gives rise to chiral amplification and squeezing, thereby converting passive fluctuations into ordered, non-passive energy. Therefore, the key resource here is the active chiral squeezing and fluctuation-to-work conversion, rather than the directional redistribution or protection of coherent energy alone.