

Emergence of Navier-Stokes Hydrodynamics in Chaotic Quantum Circuits

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We construct an ensemble of two-dimensional nonintegrable quantum circuits that are chaotic but have a conserved particle current, and thus a finite Drude weight. The long-wavelength hydrodynamics of such systems is given by the incompressible Navier-Stokes equations. By analyzing circuit-to-circuit fluctuations in the ensemble we argue that these are negligible, so the circuit-averaged value of transport coefficients like the viscosity is also (in the long-time limit) the value in a typical circuit. The circuit-averaged transport coefficients can be mapped onto a classical irreversible Markov process. Therefore, remarkably, our construction allows us to efficiently compute the viscosity of a family of strongly interacting chaotic two-dimensional quantum systems.

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Introduction—The long-wavelength, late-time dynamics of generic many-body systems is governed by hydrodynamics. One can argue based on general principles based on chaos or scrambling that hydrodynamics must eventually emerge. However, microscopic derivations of hydrodynamic transport coefficients, or even the timescales on which hydrodynamic behavior sets in, starting from unitary dynamics or even classical dynamics of an isolated many-body system, remain intractable in general [1]. It was realized only recently that the emergence of hydrodynamics can be derived microscopically in ensembles of random circuits [2–5]. Random-circuit methods are inherently defined on the lattice, with the circuit randomness strongly breaking translation invariance. Therefore, momentum relaxes rapidly, and the resulting hydrodynamics is therefore generally diffusive, or subdiffusive if kinetic constraints or quenched randomness are present [6–16].

The richer phenomenology of Navier-Stokes hydrodynamics, which has stimulated a great deal of experimental and theoretical work in the context of low-temperature transport in graphene and other ultraclean metals [17,18], might appear to require momentum conservation and thus not be tractable using random-circuit techniques (or even, more generally, using lattice models with finite local Hilbert-space dimension).

In this work, we construct a family of two-dimensional random unitary circuits, which we dub Frisch-Hasslacher-

Pomeau (FHP) random unitary circuits, inspired by lattice gas automata which exhibit incompressible Navier-Stokes hydrodynamics [19,20]. Although the presence of the lattice breaks translation symmetry and momentum conservation, we show that the resulting hydrodynamics still exhibits an emergent momentum conservation law. The behavior we find bears close similarities to emergent Navier-Stokes hydrodynamics in systems with polygonal Fermi surfaces [21–23]. The dynamics averaged over the ensemble of circuits is described by a classical Markov chain, and the averaged correlation functions are easy to compute classically. Furthermore, there have been numerous works on quantum lattice Boltzmann methods which effectively simulate classical Markov processes capturing fluid behavior using quantum circuits [24]. However, our interest is in the dynamics of individual circuits. One of our main technical contributions is to compute the circuit-to-circuit fluctuations of transport coefficients and show that these are subleading (and quantitatively negligible) at late times. Relying on this “self-averaging” result, we use the ensemble averaged dynamics to study transport coefficients in a typical circuit. We indeed find that the model exhibits damped sound modes as well as a diffusive mode with a finite dc shear viscosity (up to logarithmic corrections [25]). Our construction has two particularly notable features. First, it provides a class of interacting nonintegrable two-dimensional lattice systems that still possess a nonzero Drude weight. Though this arises from current conservation, to the best of our knowledge the only known systems with current conservation have integrable dynamics. Second, it allows us to extract transport coefficients—including the viscosity of a strongly interacting two-dimensional quantum system—through efficient classical simulations.

FHP circuit rules—The system resides on a two-dimensional triangular lattice where each site, $\vec{x}$, hosts a Hilbert

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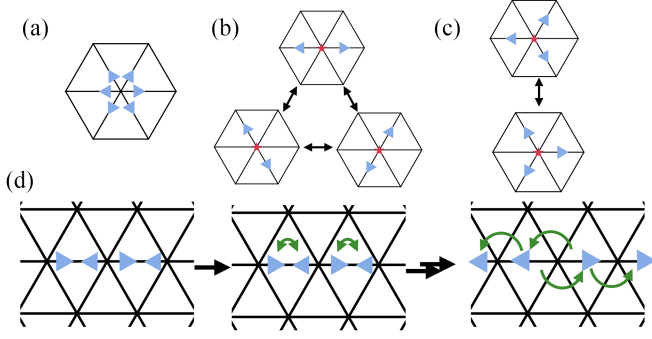


FIG. 1. Hilbert space and circuit rules. (a) Degrees of freedom, represented as outwardly pointing arrows, reside on links incident to a site denoted by the red star. (b),(c) Two- and three-body configurations, respectively, that the collision unitary has nonzero matrix elements. (d) Propagation step for the automaton achieved by two layers of SWAP gates. Particles moving to the left (right) are indicated with a (left) right blue arrow here for clarity. The first layer interchanges degrees of freedom associated with different nearest neighbor sites but reside on the same link. The second layer interchanges degrees of freedom associated with the same site but on different links along the same reflection axis of the hexagon.

space, $\mathcal{H}_{\vec{x}}$, comprised of degrees of freedom living on links incident to site $\vec{x}$. More precisely, $\mathcal{H}_{\vec{x}} = \bigotimes_{\ell} \mathcal{H}_{\vec{x}}^{\ell}$ where $\ell = 1, \dots, 6$ labels the six links incident to a given site $\vec{x}$ and

$$\mathcal{H}_{\vec{x}}^{\ell} = \text{span}\{|\nearrow\rangle, |\nwarrow\rangle\} \otimes \mathbb{C}^d. \quad (1)$$

This means each incident link to site $\vec{x}$ hosts a particle (represented as an arrow moving away from site $\vec{x}$ as shown in Fig. 1) and a d -dimensional ancillary degree of freedom which gives analytical control in the large- d limit when computing circuit to circuit fluctuations.

The unitary operator generating the evolution $\mathcal{U}$ is split up into two parts: a collision step, whose unitary is denoted by $\mathcal{U}_C$, followed by a propagation step whose unitary is denoted by $\mathcal{U}_P$, i.e., $\mathcal{U} = \mathcal{U}_P \mathcal{U}_C$. The collision unitary operator $\mathcal{U}_C$ is comprised of unitary gates that act on each site at position $\vec{r}$, i.e., $\mathcal{U}_C = \prod_{\vec{r}} U_{C,\vec{r}}$. Each gate $U_{C,\vec{r}}$ has the following form:

$$U_{C,\vec{r}} = P_2 V_{2,\vec{r}} P_2 + P_3 V_{3,\vec{r}} P_3 + \sum_k e^{i\phi_{k,\vec{r}}} P_k, \quad (2)$$

where the P_a project onto the separately conserved sectors. The sectors with no collisions, collectively $\mathcal{H}_{\perp}$, are enumerated by an index k , i.e., $\mathcal{H}_{\perp} = \bigoplus_k \mathcal{H}_{\perp,k}$. Since these sectors are one dimensional, the Haar random unitary simply gives each of them a random phase $e^{i\phi_k}$. For the sectors with two- and three-body collisions, we have the projectors P_a given by

$$P_2 = \left| \begin{array}{c} \nearrow \nwarrow \\ \nwarrow \nearrow \end{array} \right\rangle \left\langle \begin{array}{c} \nearrow \nwarrow \\ \nwarrow \nearrow \end{array} \right| + \left| \begin{array}{c} \nwarrow \nearrow \\ \nearrow \nwarrow \end{array} \right\rangle \left\langle \begin{array}{c} \nwarrow \nearrow \\ \nearrow \nwarrow \end{array} \right| + \left| \begin{array}{c} \nwarrow \nearrow \\ \nwarrow \nearrow \end{array} \right\rangle \left\langle \begin{array}{c} \nwarrow \nearrow \\ \nwarrow \nearrow \end{array} \right| + \left| \begin{array}{c} \nearrow \nwarrow \\ \nearrow \nwarrow \end{array} \right\rangle \left\langle \begin{array}{c} \nearrow \nwarrow \\ \nearrow \nwarrow \end{array} \right| \quad (3)$$

$$P_3 = \left| \begin{array}{c} \nearrow \nwarrow \nwarrow \\ \nwarrow \nearrow \nearrow \end{array} \right\rangle \left\langle \begin{array}{c} \nearrow \nwarrow \nwarrow \\ \nwarrow \nearrow \nearrow \end{array} \right| + \left| \begin{array}{c} \nwarrow \nearrow \nearrow \\ \nearrow \nwarrow \nwarrow \end{array} \right\rangle \left\langle \begin{array}{c} \nwarrow \nearrow \nearrow \\ \nearrow \nwarrow \nwarrow \end{array} \right| + \left| \begin{array}{c} \nwarrow \nearrow \nwarrow \\ \nearrow \nwarrow \nearrow \end{array} \right\rangle \left\langle \begin{array}{c} \nwarrow \nearrow \nwarrow \\ \nearrow \nwarrow \nearrow \end{array} \right| + \left| \begin{array}{c} \nearrow \nwarrow \nearrow \\ \nwarrow \nearrow \nwarrow \end{array} \right\rangle \left\langle \begin{array}{c} \nearrow \nwarrow \nearrow \\ \nwarrow \nearrow \nwarrow \end{array} \right|. \quad (4)$$

Furthermore, $V_{2,\vec{r}}$ and $V_{3,\vec{r}}$ are $3d^6 \times 3d^6$ and $2d^6 \times 2d^6$ Haar random unitaries, respectively. Intuitively, $U_{C,\vec{r}}$ updates collisions between particles only for the corresponding two-body and three-body configurations shown in Fig. 1. We note that the $U_{C,\vec{r}}$ commute with one another since (1) degrees of freedom with arrows pointing outward from a site $\vec{r}$ are not shared with another site; (2) a collision gate at site $\vec{r}$ acts as the identity on degrees of freedom with arrows pointing outward on a different site $\vec{r}'$.

The propagation step is a deterministic step that is enacted by the unitary operator $\mathcal{U}_P = \prod_{\ell=1}^3 U_{P,\ell}^{\text{inter}} U_{P,\ell}^{\text{intra}}$ where $U_{P,\ell}^{\text{inter}}$ and $U_{P,\ell}^{\text{intra}}$ are given by

$$U_{P,\ell}^{\text{intra}} = \prod_{\vec{x}} \text{SWAP}_{(\vec{x}-\hat{b}_{\ell},\ell+3),(\vec{x},\ell)}, \quad (5)$$

$$U_{P,\ell}^{\text{inter}} = \prod_{\vec{x}} \text{SWAP}_{(\vec{x},i),(\vec{x},\ell+3)}, \quad (6)$$

where $\text{SWAP}_{(\vec{x},\ell),(\vec{y},\ell')}$ denotes the usual SWAP operator which interchanges degrees of freedom on link ℓ associated with $\vec{x}$ and with degrees of freedom on link ℓ' associated with $\vec{y}$. The unit vector $\hat{b}_{\ell}$ is given by $\hat{b}_{\ell} = [\cos(2\pi\ell/6), \sin(2\pi\ell/6)]$ and $\ell+3$ implicitly should be evaluated modulo six.

From the expression one can see that $\mathcal{U}_P$ is a brickwork circuit of SWAP gates acting on the three different reflection axes of the hexagonal cell. The first layer of the SWAP circuit swaps particles residing on the same link and the second layer of the SWAP circuit swaps particles associated to the same site but on different nearest neighbor links (along one of the reflection axes of the hexagonal cell) as shown in Fig. 1. This procedure ensures that particles will move in the direction their arrow points to a link one lattice spacing away.

Hydrodynamics—With the dynamics of the model specified, we now identify the conservation laws of the model and determine the hydrodynamics up to the diffusive scale of an individual circuit. Any given circuit realization conserves the following operators:

$$N = \sum_{\vec{x}} \mathbf{n}_{\vec{x}} = \sum_{\vec{x}} \sum_{\ell=1}^6 n_{\vec{x}}^{\ell}, \quad (7)$$

$$\vec{P} = \sum_{\vec{x}} \vec{\mathbf{p}}_{\vec{x}} = \sum_{\vec{x}} \sum_{\ell=1}^6 \hat{b}_{\ell} n_{\vec{x}}^{\ell}, \quad (8)$$

where n_x^ℓ corresponds to the occupation number on the link ℓ of site $\vec{x}$. The first conservation law reflects total particle number conservation while the second reflects a type of “momentum” conservation—although this operator is not associated with the generator of translations. In this context, momentum conservation refers to the fact that the vector sum of the particles’ direction of motion is conserved. One can see that this is indeed the case since collision unitary gates only cause transitions between states whose vector sum is zero and the propagation step does not change the direction in which particles travel.

The momentum operator $\vec{p}_x$ also coincides exactly with the particle current: this current cannot relax, corresponding to dissipationless, ballistic particle transport. Denoting the expectation values of the local conserved quantities $n(\vec{x}, t) = \langle \vec{n}_x \rangle$ and $n(\vec{x}, t) \vec{v}(\vec{x}, t) = \langle \vec{p}_x \rangle$, with $\vec{v}(\vec{x}, t) \equiv \langle \vec{p}_x \rangle / \langle \vec{n}_x \rangle = \sum_{\ell=1}^6 \hat{b}_\ell \langle n_x^\ell \rangle / \sum_{\ell=1}^6 \langle n_x^\ell \rangle$, the long time dynamics of a typical FHP circuit are expected to be governed by the following hydrodynamics equations:

$$\begin{aligned} \partial_t n + \partial_i (n v_i) &= 0, \\ \partial_t (n v_i) + \partial_j (\pi_{ij}) &= 0. \end{aligned} \quad (9)$$

The second equation corresponds to momentum conservation and the expectation value of the stress tensor $\pi_{ij}(\vec{x}) = \sum_{\ell=1}^6 \hat{b}_\ell^i \hat{b}_\ell^j n_x^\ell$, with $i, j = x, y$ denoting the x and y components, can be expressed as a gradient expansion up to the diffusive scale $\langle \pi_{ij} \rangle = P \delta_{ij} + A_{ijkl} c^2 n g(n) v_k v_l - (\nu - \tilde{\nu}) A_{ijkl} \partial_k (n v_l)$, with $A_{ijkl} = \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \delta_{ij} \delta_{kl}$, $P = c^2 n$, $c = 1/\sqrt{2}$, $g(n) = (3 - n/6 - n)$, and ν denotes the kinematic viscosity and $\tilde{\nu} = 1/8$ [26].

These are not quite the Navier-Stokes equations due to effects from the lattice: (1) the factor $g(n)$ which breaks Galilean invariance [20], (2) the tensor A_{ijkl} signifying that the dynamics is invariant under the group of symmetries of a regular hexagon. The latter feature has been noted in other works exploring hydrodynamics of hexagonal Fermi surfaces [21]. We note that the A_{ijkl} is invariant under the full rotation group [20] and so the above equations still describe an isotropic fluid. Additionally, one can show that when the average density is approximately constant, i.e., $n \simeq n_0$, one recovers the incompressible Navier-Stokes equations [20].

To characterize the hydrodynamics of an individual circuit we will examine linear response coefficients. For the above hydrodynamics one finds two damped sound modes and one diffusive mode characterized by the speed of sound, c , and shear viscosity η . In the next section, we will study sample-to-sample fluctuations by studying fluctuations of these transport coefficients. In particular, we will show that the speed of sound is fixed to the same value for every circuit realization.

Fluctuations in transport coefficients—As mentioned in the previous section there are two linear response coefficients which we can diagnose transport with. We begin by

discussing the fluctuations in the speed of sound, c . Owing to the conservation of momentum, $\vec{P}$, sound modes appear at the Euler scale for hydrodynamics. Since all transport coefficients at the Euler scale are fixed by thermodynamics [27], so is the speed of sound, in particular $c = \sqrt{(\partial P / \partial n)}$ [28,29]. Thus the speed of sound only depends on thermodynamic expectation values, and does not depend on details of the random Haar gates: it is fixed to $c = 1/\sqrt{2}$ for all circuit realizations.

For fluctuations of the shear viscosity, we know that the associated current, i.e., the momentum stress tensor, is not conserved and so the shear viscosity is generally not set by thermodynamics so we must compute fluctuations from circuit to circuit via other means. In general sample-to-sample fluctuations are generically encoded in nonlinear observables, i.e., observables which are nonlinear functions of the density matrix. In this work we will compute the variance from circuit to circuit of the ac shear viscosity, $\text{Var}[\eta(\omega)]$. For each circuit realization, the ac shear viscosity can be related to autocorrelation functions of the momentum stress tensor via the Kubo formula [30],

$$\eta(\omega) = \frac{1}{2} C(0) + \sum_{t=1}^{\infty} e^{i\omega t} C(t), \quad (10)$$

where $C(t) = (1/V) \langle \Pi_{xy}(t) \Pi_{xy}(0) \rangle$ with V the volume of the system and $\Pi_{ij} = \sum_{\vec{x}} \pi_{ij}(\vec{x})$. In this case $\langle A \rangle \equiv (1/Z) \text{tr}(A e^{-\mu N})$ with μ denoting the chemical potential.

Evaluating the average of this quantity over circuits involves computing the Haar average of $\mathcal{U} \otimes \mathcal{U}^*$, whereas computing the variance over circuit realizations requires averaging the two-copy replicated unitary, $(\mathcal{U} \otimes \mathcal{U}^*)^{\otimes 2}$. Using standard tools developed for one-dimension quantum circuits [31–38], we find that this averaging procedure maps the single copy Haar average onto a classical irreversible Markov process, while the two copy average maps onto a statistical mechanics model with permutation degrees of freedom $\sigma \in S_2$ corresponding pairings of the replica living on the vertices and charge degrees of freedom on the edges [26].

In the limit of infinite ancilla dimension, $d \rightarrow \infty$, the average of the two-copy replica unitary corresponds to two decoupled copies of $\mathbb{E}[\mathcal{U} \otimes \mathcal{U}^*]$, i.e., two decoupled classical Markov processes [5,26]. Generalizing the one-dimensional results of Ref. [5], we find that at large but finite d , one can systematically account for corrections and consequently show that leading order correction corresponds to a new effective Markov process which couples the two single-copy Markov processes (the ancillae in this limit exhibit no dynamics and the Markov process generates dynamics for the qubit degrees of freedom) [26].

This allows us to study the variance of the viscosity over quantum circuits through efficient numerical simulations—presented in Fig. 2. In Fig. 2, the variance of the ac shear

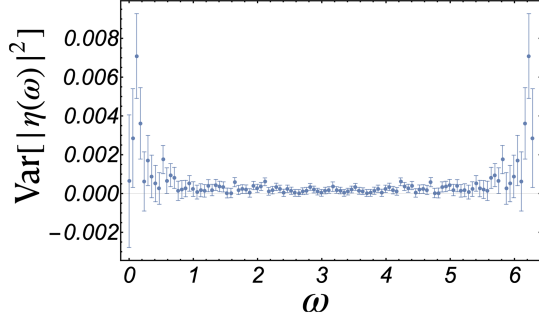


FIG. 2. Sample-to-sample fluctuations of shear viscosity. The fluctuations of the ac shear viscosity evaluated using the effective Markov process from the large- d limit of the statistical mechanics model. This was evaluated for a system with 216×216 sites at a time $T = 108$. Data are averaged over 8.9×10^6 realizations and $\mu = 0.5$.

viscosity is computed at a finite time $T = L/2$ where L is the linear size of the system with the ancilla dimension, $d = 2$. In principle, fluctuations are expected to be larger in this case but strikingly we observe a quantitative variance for the given amount of sampling we were able to achieve. Since the variance of the ac shear viscosity appears to show subleading fluctuations, this indicates that a typical circuit will also be characterized by the ac shear viscosity of the ensemble average with corrections that are vanishingly small. Given that the speed of sound and ac shear viscosity correspond to their ensemble averaged values, we can now study the hydrodynamics of a typical circuit via its ensemble averaged dynamics.

Hydrodynamic transport—We illustrate this mapping by computing the unequal time density correlation functions as well as momentum density correlation functions of typical circuits by evaluating their ensemble average. These quantities are linear functions of the density matrix, since they are of the form $\text{tr}[O(t)O'(0)\rho]$, and hence the ensemble averaged dynamics can be described by a classical Markov process [2–4,6,11]. More precisely,

$$\mathbb{E}\{\text{tr}[O(t)O'(0)\rho]\} = \text{tr}(\mathcal{T}^t[O(0)]O'(0)\rho) \quad (11)$$

where $\mathcal{T}$ is a superoperator (transfer matrix), which is positive semidefinite and is Markovian, i.e., $\sum_i \mathcal{T}_{ij} = 1$. Importantly, this Markov process is irreversible because of the propagation step of the model. We note that in this case the ancillae decouple from the ensemble averaged dynamics and thus only the qubits have a nontrivial evolution even without the $d \rightarrow \infty$ limit [26].

For the FHP random unitary circuit, one can show the corresponding Markov process is a variant of the well-known FHP lattice gas automaton [19,20]. Like the random unitary circuit, the transfer matrix of this FHP variant generates stochastic dynamics consisting of a random collision step, denoted by $\mathcal{T}_C$, followed by a deterministic

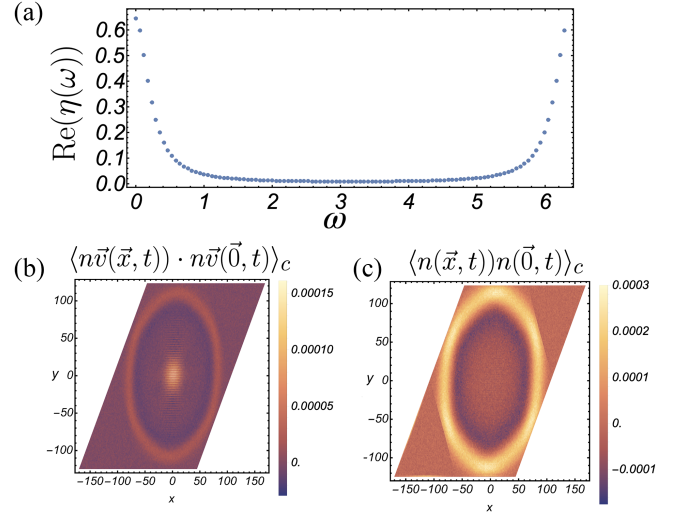


FIG. 3. Ensemble averaged structure factors and shear viscosity. (a) The real part of the ac shear viscosity. (b) The momentum density structure factor features a ballistic portion corresponding to the sound modes and a heat mode located at the center. (c) One sees that the particle density structure factor also features a clear ballistic behavior indicative of sound modes. The above data were evaluated at $T = 108$ for a system size of 216×216 and the data are averaged over 8.9×10^6 realizations and $\mu = 0.5$.

propagation step in which particles move one lattice spacing in the direction their arrow points, $\mathcal{T}_P$. So the full transfer matrix is given by $\mathcal{T} = \mathcal{T}_P \mathcal{T}_C$. Like the random unitary circuit, the collision portion has a gate structure, i.e., $\mathcal{T}_C = \prod_{\vec{x}} \mathcal{T}_{C,\vec{x}}$. The collision gates for the model are then given by

$$\mathcal{T}_{C,\vec{r}} = \frac{1}{3} \mathcal{P}_2 + \frac{1}{2} \mathcal{P}_3 + \mathcal{P}_\perp, \quad (12)$$

where $\mathcal{P}_2$ projects onto the symmetrized state $\frac{1}{3}(|\diamond\rangle + |\diamond\rangle + |\diamond\rangle)$, $\mathcal{P}_3$ projects onto the state $\frac{1}{2}(|\diamond\rangle + |\diamond\rangle)$, and $\mathcal{P}_\perp$ projects onto the space orthogonal to the two and three-particle collision spaces [here we have denoted the classical states for a particle configuration Γ by $|\Gamma\rangle$]. This model has equiprobable transition to any state within a given collision subspace and all other states are unchanged. We note that the present stochastic evolution differs from the original FHP model since collision updates have a probability to not reconfigure particles in the two-body or three-body collision subspaces—such a modification will only change transport coefficients but not the hydrodynamic behavior.

Using the transfer matrix, we numerically obtain the structure factors for the particle and momentum density, $\langle n(\vec{x}, t)n(\vec{0}, 0) \rangle_c$ and $\langle (n\vec{v})(\vec{x}, t) \cdot (n\vec{v})(\vec{0}, 0) \rangle_c$ as well as the real part of the ac shear viscosity. Our results are shown in Fig. 3 and we indeed observe the presence of two

damped sound modes and a heat mode, as expected from standard linearized hydrodynamics.

Discussion—In this Letter, we constructed an ensemble of random quantum circuits whose hydrodynamics corresponds to the incompressible Navier-Stokes equations. We characterized sample-to-sample fluctuations of transport coefficients such as the ac shear viscosity by mapping fluctuations onto an effective classical statistical mechanics model. Our results are consistent with fluctuations being subleading, indicating that a typical circuit behaves as its ensemble average and furthermore a typical circuit has a shear viscosity that is effectively the same as the ensemble average.

Since sample-to-sample fluctuations of transport coefficients are quantitatively very small, we are able to study the hydrodynamics of a typical circuit via its ensemble averaged dynamics. We show that the ensemble averaged dynamics corresponds to a variant of a well-known classical Markov process known as the FHP model, and numerically verify that the system does host two sound modes and a heat mode. The presence of sound modes places these random circuits as a rare example of a lattice nonintegrable system with—albeit fine-tuned—ballistic transport and an exact Drude weight.

Strikingly, the hydrodynamics of the coupled model features an additional conservation law coming from the presence of anticommuting charges in the dynamics [39]. Although the effects of the anticommuting charges do not appear to affect typical thermal states [40], it would be interesting to see how the presence of these new conservation laws changes the approach towards the ensemble average.

We also remark that the Navier-Stokes hydrodynamics in two dimensions is not stable to stochastic noise which results in logarithmic corrections to the hydrodynamics [25]. This has been observed in the original FHP model by studying the finite size scaling of the dc shear viscosity [40]. In future work, it would be interesting to see if such effects have any consequence on quantum fluctuations.

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