Information-Theoretic Derivation of Energy, Speed Bounds, and Quantum Theory

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We provide a derivation of quantum theory in which the existence of an energy observable that generates the reversible dynamics follows directly from information-theoretic principles. Our first principle is that every reversible dynamics is implementable through a sequence of fast collisions with an array of identically prepared systems. Combined with four additional principles, known as causality, classical decomposability, purity preservation, and strong symmetry, this collision model pins down the quantum framework, sets up a one-to-one correspondence between energy observables and generators of the dynamics, and provides an information-theoretic derivation of the Mandelstam-Tamm bound on the speed of quantum evolutions.

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Introduction—Information-theoretic notions play an important role in modern physics, from thermodynamics to quantum field theory and gravity. With the advent of quantum information and computation, new links between information-theoretic primitives and physical laws have been uncovered, suggesting that information could be the key to understand the counterintuitive laws of quantum mechanics [1,2]. Over the past two decades, this research program blossomed into a series of reconstructions of the quantum framework from information-theoretic principles [3–10].

A limitation of most quantum reconstructions, however, is that they do not provide direct insights into the dynamics of quantum systems. Crucially, they do not provide an information-theoretic characterization of the notion of energy, and an explanation of its dual role of observable quantity and generator of the dynamics. For long time, this observable-generator duality has been known to be a key feature from which much of the algebraic structure of quantum theory can be derived [11]. More recently, the observable-generator duality was assumed as an axiom in a quantum reconstruction [8], or as a requirement for post-quantum dynamics [12–14]. And yet, little is known about the origin of the duality. How is "energy" emerging from "information," and why does it drive the evolution of quantum systems?

Here, we provide a reconstruction of quantum theory in which the key features of quantum dynamics are reconstructed directly from the principles, without invoking the machinery of Hilbert spaces. Our first principle is that every reversible dynamics should be implementable by a sequence of fast collisions with an array of identically prepared systems, as illustrated in Fig. 1. In this model, we explore the idea that dynamics arises from *informational nonequilibrium*, the condition that the system's state deviates from the state of the systems it collides with. We then combine this idea with four principles that have featured in previous quantum reconstructions: causality [4,15], classical decomposability [8], strong symmetry [8],

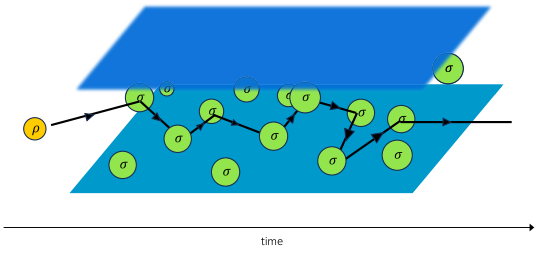


FIG. 1. Collision model for reversible dynamics. A target system in an initial state ρ undergoes a sequence of interactions with a set of systems, independently and identically prepared in the reference state σ. Here we require that every reversible dynamics of the target system can be obtained by varying the state σ, in the limit of instantaneous and infinitely frequent interactions.

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and purity preservation [16–18]. We show that, together, the above principles pin down the framework of finite-dimensional quantum theory and, at the same time, provide direct access to the key features of quantum dynamics, including in particular the existence of an energy observable in one-to-one correspondence with the generator of the dynamics. We then show that the variance of the energy observable yields a lower bound on the speed of state evolution, thereby providing a purely information-theoretic derivation of the Mandelstam-Tamm bound in quantum theory [19]. Our results establish a direct link between information-theoretic principles and dynamical notions such as energy and speed.

Collisional models in general probabilistic theories—A general class of physical theories, including quantum theory and potential candidates of postquantum theories, can be formulated as general probabilistic theories [3,20–24]. More specifically, here we adopt the framework of operational probabilistic theories (OPTs) [4,15,23,25], describing a set of physical systems, closed under composition, a set of transformations thereof, closed under parallel and sequential composition.

Let us start with a brief overview of the OPT framework (a more in-depth presentation can be found in Supplemental Material [26] and in the original papers). Mathematically, the structure of an OPT is underpinned by the graphical language of symmetric monoidal categories [27–32]. For a pair of systems A and B, $\mathrm{A}\otimes\mathrm{B}$ denotes the composite system consisting of subsystems A and B. The trivial system I, representing degrees of freedom that are not relevant in the OPT, satisfies the relation $\mathrm{A}\otimes\mathrm{I}=\mathrm{I}\otimes\mathrm{A}=\mathrm{A}$ for every system A.

The set of physical transformations with input system A and output system A' is denoted by $\mathsf{Transf}(\mathrm{A}\to\mathrm{A}')$. For general systems $\mathrm{A},\mathrm{A}',\mathrm{B},\mathrm{B}'$, two transformations $\mathcal{T}\in\mathsf{Transf}(\mathrm{A}\to\mathrm{A}')$ and $\mathcal{S}\in\mathsf{Transf}(\mathrm{B}\to\mathrm{B}')$ can be performed in parallel, giving rise to the transformation $\mathcal{T}\otimes\mathcal{S}\in\mathsf{Transf}(\mathrm{A}\otimes\mathrm{B}\to\mathrm{A}'\otimes\mathrm{B}')$. If $\mathrm{A}'=\mathrm{B}$, the transformations $\mathcal{T}$ and $\mathcal{S}$ can also be performed in a sequence, giving rise to the transformation $\mathcal{S}\circ\mathcal{T}\in\mathsf{Transf}(\mathrm{A}\to\mathrm{B}')$.

For every system A, there exists an identity transformation, denoted by $\mathcal{I}_{\mathrm{A}}\in\mathsf{Transf}(\mathrm{A}\to\mathrm{A})$, which leaves the system unchanged. A transformation $\mathcal{T}\in\mathsf{Transf}(\mathrm{A}\to\mathrm{B})$ is called reversible if there exist another transformation $\mathcal{S}\in\mathsf{Transf}(\mathrm{B}\to\mathrm{A})$ such that $\mathcal{S}\circ\mathcal{T}=\mathcal{I}_{\mathrm{A}}$ and $\mathcal{T}\circ\mathcal{S}=\mathcal{I}_{\mathrm{B}}$. The reversible transformations in $\mathsf{Transf}(\mathrm{A}\to\mathrm{A})$ form a group, hereafter denoted by G_{A}. In quantum theory, reversible transformations are associated to unitary operators $U\colon\mathcal{H}_{\mathrm{A}}\to\mathcal{H}_{\mathrm{A}}$, where $\mathcal{H}_{\mathrm{A}}$ is the Hilbert space associated to system A, while general transformations in $\mathsf{Transf}(\mathrm{A}\to\mathrm{B})$ are described by quantum operations [68–70], that is, completely positive, trace nonincreasing linear maps transforming linear operators on $\mathcal{H}_{\mathrm{A}}$ into linear operators on $\mathcal{H}_{\mathrm{B}}$.

In quantum theory, the possible experiments are described by *quantum instruments* [71], that is, collections of quantum operations that sum up to trace-preserving maps. In a general OPT, an N-outcome experiment transforming an input system A into an output system B is described by a collection of transformations $(\mathcal{T}_i)_{i=1}^{N}$, with the transformation $\mathcal{T}_i\in\mathsf{Transf}(\mathrm{A}\to\mathrm{B})$ corresponding to the ith outcome. The OPT specifies the set of all possible experiments, with the constraint that such a set must be closed under parallel composition, sequential composition, and coarse graining (see Supplemental Material [26] for more details).

Experiments with a single outcome ($N=1$) are associated to a single transformation, called *deterministic*. The set of deterministic transformations with input A and output B is denoted by $\mathsf{Transf}_1(\mathrm{A}\to\mathrm{B})$. Transformations from the trivial system to system A are called *states*, while transformations from system A to the trivial system are called *effects*. We will use the notation $\mathsf{St}(\mathrm{A})\coloneqq\mathsf{Transf}(\mathrm{I}\to\mathrm{A})$ and $\mathsf{Eff}(\mathrm{A})\coloneqq\mathsf{Transf}(\mathrm{A}\to\mathrm{I})$ for the sets of states and effects, and $\mathsf{St}_1(\mathrm{A})\coloneqq\mathsf{Transf}_1(\mathrm{I}\to\mathrm{A})$ and $\mathsf{Eff}_1(\mathrm{A})\coloneqq\mathsf{Transf}_1(\mathrm{A}\to\mathrm{I})$ for the subsets of deterministic states and deterministic effects, respectively.

OPTs associate transformations with trivial input and trivial output ($\mathrm{A}=\mathrm{B}=\mathrm{I}$) to probabilities. In particular, the combination of a state $\rho\in\mathsf{St}(\mathrm{A})$ with an effect $e\in\mathsf{Eff}(\mathrm{A})$ yields a probability $e\circ\rho$. As a consequence, states can be represented as functions on the set of effects, and vice versa. In turn, this fact implies that states and effects can be embedded into vector spaces over the real field, hereafter denoted by $\mathsf{St}_{\mathbb{R}}(\mathrm{A})$ and $\mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$, respectively. More generally, each set of physical transformations $\mathsf{Transf}(\mathrm{A}\to\mathrm{B})$ can be embedded in a real vector space, hereafter denoted by $\mathsf{Transf}_{\mathbb{R}}(\mathrm{A}\to\mathrm{B})$. In the following we will make the standard assumption that the vector spaces $\mathsf{Transf}_{\mathbb{R}}(\mathrm{A}\to\mathrm{B})$ are finite dimensional for all systems A and B, and that the sets $\mathsf{Transf}(\mathrm{A}\to\mathrm{B})$ are compact convex sets. Under this assumption, the group of reversible transformations G_{A} is a compact Lie group and admits a faithful matrix representation for every system A (see Supplemental Material [26] for a proof).

We now formulate a definition of collision model in general probabilistic theories. Collision models have been extensively studied in the theory of quantum open systems [33–39], with applications to nonequilibrium quantum thermodynamics [72,73] and quantum machine learning [40]. In a collision model, a target system evolves through a sequence of pairwise interactions with an array of independent and identically prepared systems, as depicted in Fig. 1. We assume that the interaction is the same at all steps of the sequence, and gives rise to a reversible joint evolution, specified by a one-parameter subgroup $(\mathcal{S}_t)_{t\in\mathbb{R}}$ of the group $\mathsf{G}_{\mathrm{A}\otimes\mathrm{A}'}$, where A is the target system and A' is the system it collides with. As a result of the interaction, the target system undergoes the transformation

$$\mathcal{C}_{\tau,\sigma} := (\mathcal{I}_A \otimes u_{\mathrm{A}'}) \circ \mathcal{S}_\tau \circ (\mathcal{I}_A \otimes \sigma), \tag{1}$$

where τ is the interaction time, $\sigma \in \mathsf{St}_1(\mathrm{A}')$ is the initial state of system A′, and $u_{\mathrm{A}'} \in \mathsf{Eff}_1(\mathrm{A}')$ is a deterministic effect, representing the process of discarding system A′ after the interaction. Hereafter, we will call σ the *reference state* of the collisional model.

We now consider the effective transformation resulting from an asymptotically long sequence of collisions, taking place in a finite time t. We will assume that (i) the number of collisions grows linearly with t, as $n = \lfloor t/\tau \rfloor$, and (ii) the collisions are instantaneous ($\tau \to 0$). In this setting, the effective evolution of system A over time t is given by $\mathcal{U}_{t,\sigma} := \lim_{\tau\to 0} (\mathcal{C}_{\tau,\sigma})^{\lfloor t/\tau \rfloor}$. We name $(\mathcal{U}_{t,\sigma})_{t\in\mathbb{R}}$ the *collisional dynamics* generated by σ. In Supplemental Material we prove the following:

Theorem 1—For every reference state σ, the collisional dynamics $(\mathcal{U}_{t,\sigma})_{t\in\mathbb{R}}$ is a one-parameter Lie group, of the form $\mathcal{U}_{t,\sigma} = e^{G_\sigma t}$, with $G_\sigma := (\mathcal{I}_{\mathrm{A}} \otimes u_{\mathrm{A}'}) \left(\mathrm{d}\mathcal{S}_\tau/\mathrm{d}\tau\right)\Big|_{\tau=0} (\mathcal{I}_A \otimes \sigma)$.

Here, the derivative $\mathrm{d}\mathcal{S}_\tau/\mathrm{d}\tau$ is well-defined because $(\mathcal{S}_\tau)_{\tau\in\mathbb{R}}$ is a one-parameter subgroup of a compact Lie group with a faithful matrix representation, and therefore it is also a Lie group [41,74].

An important consequence of Theorem 1 is that the transformation $\mathcal{U}_{t,\sigma}$ is physically reversible: for example, it can be reversed by replacing $\mathcal{S}_\tau$ with its inverse $\mathcal{S}_{-\tau}$ in Eq. (1), while keeping the state σ fixed. Note that the map $G^{(\mathrm{A})}\colon \sigma \mapsto G_\sigma$ transforms states of system A into elements of the Lie algebra $\mathfrak{g}_{\mathrm{A}}$ associated to the Lie group G_{A}. Physically, the elements of $\mathfrak{g}_{\mathrm{A}}$ represent the generators of the possible dynamics of system A.

Informational equilibrium—We now consider *symmetric* collision models, for which $\mathrm{A} = \mathrm{A}'$. In these models, we say that the system and its environment are at *informational equilibrium* if they are in the same state ($\rho = \sigma$). The model is *stationary at equilibrium* if the condition

$$\mathcal{S}_\tau(\rho \otimes \rho) = \rho \otimes \rho \quad \forall\ \rho \in \mathsf{St}_1(\mathrm{A}), \quad \forall\ \tau \in \mathbb{R} \tag{2}$$

holds. This condition expresses the idea that dynamics only takes place when the system and its environment are in different states. A straightforward consequence of stationarity is the following:

Lemma 1—If a symmetric collision model is stationary at equilibrium, then the reference state σ is invariant under the collisional dynamics $(\mathcal{U}_{t,\sigma})_{t\in\mathbb{R}}$.

In the rest of this Letter, we explore the idea that every dynamics can be generated by a symmetric collisional model that is stationary at equilibrium. Specifically, we say that a collision model is *universal* if every reversible transformation $\mathcal{U} \in \mathsf{G}_{\mathrm{A}}$ is of the form $\mathcal{U} = \mathcal{U}_{t,\sigma}$ for some suitable time t and some suitable state $\sigma \in \mathsf{St}_1(\mathrm{A})$. Our first principle, called *dynamics from informational nonequilibrium (DIN)*, is that there exists a symmetric collision model that is both stationary and universal.

In Supplemental Material [26], we show that quantum theory on complex Hilbert spaces satisfies DIN, and we explicitly provide a symmetric, stationary, and universal collision model. In contrast, we show that for quantum theory on real Hilbert spaces, every stationary collision model is trivial (i.e., it only implements the identity transformation), and therefore fails to be universal.

An important consequence of DIN is the following theorem, proven in Supplemental Material [26]:

Theorem 2—In a theory satisfying DIN, the map $G^{(\mathrm{A})}\colon \mathsf{St}_1(\mathrm{A}) \to \mathfrak{g}_{\mathrm{A}}, \sigma \mapsto G_\sigma$ is injective if and only if system A has a unique invariant state.

Theorem 2 shows that a duality between states and generators of the reversible dynamics is equivalent to a simple property, the uniqueness of the invariant state (UIS). UIS is a well-studied property, which is satisfied by quantum theory [4,15,18] and plays a key role in the foundations of thermodynamics [75].

We conclude this section with an interesting aside. For systems with a unique invariant state χ_{A}, the evolution $\mathcal{U}_{t,\sigma}$ can be inverted by replacing the state σ and the time t with a new state $\sigma_{\mathrm{inv}} := (\chi_{\mathrm{A}} - p_\sigma \sigma)/(1 - p_\sigma)$ and a new time $t_{\mathrm{inv}} := t(1 - p_\sigma)/p_\sigma$, where p_σ is the maximum probability of the state σ in a convex decomposition of χ_{A} (see Supplemental Material [26] for details). It is interesting to observe that t_{inv} is generally different from t, thereby implying that, if our collision model is taken to be fundamental, dynamical evolutions, while in principle reversible, have a privileged direction that is physically easier to implement, in the sense that it takes a shorter evolution time. For systems with high-dimensional state space, the inversion time can be much larger than the time taken by the original evolution, thereby implying a form of physical irreversibility. For example, quantum dynamics generated by pure states σ have an inversion time $t_{\mathrm{inv}} = t(d-1)$, where d is the system's dimension.

Duality between observables and generators—In Supplemental Material [26], we show that the state-generator duality set up by Theorem 2 can be turned into an observable-generator duality whenever the theory satisfies an additional property known as strong self-duality [42,43], i.e., whenever the cones spanned by the sets $\mathsf{St}(\mathrm{A})$ and $\mathsf{Eff}(\mathrm{A})$ are isomorphic and the pairing defined by $\langle e, \rho\rangle := e \circ \rho$, $\forall\ \rho \in \mathsf{St}(\mathrm{A})$, $\forall\ e \in \mathsf{Eff}(\mathrm{A})$ can be expressed as a Euclidean inner product.

Theorem 3 (Observable-generator duality)—In a theory satisfying DIN, UIS, and strong self-duality, the elements of the Lie algebra $\mathfrak{g}_{\mathrm{A}}$ are in one-to-one linear correspondence with the elements of the subspace $\mathcal{O}_{\mathrm{A}} := \{H \in \mathsf{Eff}_{\mathbb{R}}(\mathrm{A}) | H \circ \chi_{\mathrm{A}} = 0\}$. Moreover, every element $H \in \mathcal{O}_{\mathrm{A}}$ is invariant under the dynamics generated by the corresponding element $G_H \in \mathfrak{g}_{\mathrm{A}}$.

Theorem 3 implies that every generator of the reversible dynamics is canonically associated to an *energy observable* $H \in \mathcal{O}_A$, whose expectation value can be estimated by performing suitable measurements on the system (see Supplemental Material [26] for more details). In summary, Theorem 3 provides a derivation of the notion of energy from the idea that dynamics arises from collisions out of informational equilibrium, combined with the properties of strong self-duality and uniqueness of the invariant state. In the following, we will replace these two properties with more basic requirements.

Derivation of quantum theory—Finite-dimensional quantum theory can be uniquely characterized by DIN, plus four additional assumptions that have appeared in other quantum reconstructions. Here we review these assumptions briefly, leaving a detailed discussion to Supplemental Material [26].

The first assumption is *causality (C)* [4,15], expressing the fact that the outcomes of present experiments should be independent of the setting of future experiments. Mathematically, this assumption is equivalent to the statement that every system has one and only one deterministic effect. The second assumption is *classical decomposability (CD)* [8], namely, the requirement that every state can be prepared as a random mixture of perfectly distinguishable pure states. This assumption provides a starting point for the quantification of information in terms of entropies [18,44,45]. The third assumption is *purity preservation (PP)* [18], namely, the requirement that composing two pure transformations in parallel or in sequence gives rise to another pure transformation (informally, a pure transformation is one that cannot be viewed as the coarse-graining of a set of transformations; see Supplemental Material [26] for more details). The fourth assumption is *strong symmetry (SS)* [8], the condition that every two sets of perfectly distinguishable pure states can be reversibly converted into one another if they have the same cardinality. SS guarantees that two sets of states that can perfectly encode the same amount of classical information are equivalent.

In Supplemental Material [26], we prove that the combination of C, CD, PP, and SS implies that the states of the theory form an irreducible Euclidean Jordan algebra [46,47]. This result implies in particular that every theory satisfying C, CD, PP, and SS also satisfies strong self-duality and UIS. We then combine this fact with Theorem 3, showing that DIN, UIS, and strong self-duality imply the observable-generator duality. Finally, we leverage a result of Barnum, Müller, and Ududec [8], who showed that irreducible Jordan algebras plus observable-generator duality imply quantum theory on complex Hilbert spaces. In summary, DIN, C, CD, PP, and SS imply the standard Hilbert space framework.

Speed bound—The above derivation of the quantum framework can be used to reconstruct properties of quantum dynamics directly from the principles, thereby establishing a closer connection between information-theoretic and dynamical features. For example, in Supplemental Material [26] we use the principles to show that every dynamics has a maximal set of perfectly distinguishable stationary states $\{\psi_i\}_{i=1}^d$, corresponding to energy eigenstates in the Hilbert space framework. In addition, the energy observable associated to the dynamics can be decomposed as $H = \sum_{i=1}^d E_i e_{\psi_i}$, where $(e_{\psi_i})_{i=1}^d$ are the effects associated to the states $(\psi_i)_{i=1}^d$ by strong self-duality, and $(E_i)_{i=1}^d$ are real numbers. The effects $(e_{\psi_i})_{i=1}^d$ form a measurement, and the expectation value of the energy observable on a given state ρ can be computed as $\langle H \rangle_\rho := \sum_i E_i \langle e_{\psi_i}, \rho \rangle$. This expression suggests to interpret $(e_{\psi_i})_{i=1}^d$ as the *canonical energy measurement* and the numbers $(E_i)_{i=1}^d$ as the possible values of the energy, distributed with probability $p_\rho(E_i) = \langle e_{\psi_i}, \rho \rangle$, $\forall\, i \in \{1, \ldots, d\}$.

We now show that the principles yield an upper bound on the speed of state changes in terms of the variance of the energy observable H, providing an information-theoretic derivation of the celebrated Mandelstam-Tamm bound [19]. To this purpose, we introduce a notion of speed in general probabilistic theories:

Definition 1 (Speed of state change)—Let $(\mathcal{U}_t)_{t \in \mathbb{R}}$ be a one-parameter subgroup of G_A, representing a possible dynamics of system A, let $\{\rho_t = \mathcal{U}_t(\rho) | t \in \mathbb{R}\}$ be the trajectory of an initial state ρ, and let t_0 and $t_1 \geq t_0$ be two moments of time. The average speed of state change from time t_0 to time t_1 is defined as

$$v_\rho(t_0, t_1) := \frac{\|\rho_{t_1} - \rho_{t_0}\|}{t_1 - t_0}, \tag{3}$$

where $\|\cdot\|$ is an arbitrary norm on $\mathsf{St}_{\mathbb{R}}(A)$. Similarly, the instantaneous speed at time t is defined as $v_\rho(t) := \lim_{\delta t \to 0} v_\rho(t, t + \delta t)$.

In the following we will take the norm $\|\cdot\|$ to be the Euclidean norm induced by self-duality, namely, $\left\| \sum_j c_j \rho_j \right\| := \sqrt{\sum_{j,k} c_j c_k e_{\rho_j}(\rho_k)}$ for every linear combination of states (ρ_j) with real coefficients (c_j). With this choice of norm, we show that the instantaneous speed at time t is $v_\rho(t) = \|G(\rho)\|$ for every reversible dynamics with time-independent generator G [26]. Using this fact, we obtain the following:

Theorem 4 (Speed limit)—DIN, C, CD, PP, and SS imply that the time Δt taken by a system to transition from an initial state ρ_{t_0} to a final state ρ_{t_1} through a reversible dynamics satisfies the bound

$$\Delta t \geq \frac{D(\rho_{t_0}, \rho_{t_1})}{\Delta H}, \tag{4}$$

where $D(\rho,\rho') \coloneqq \|\rho-\rho'\|/\sqrt{2}$ is the normalized Euclidean distance between two states ρ and ρ', while ΔH is the standard deviation of the canonical energy measurement when performed on the state ρ.

The proof is provided in Supplemental Material [26]. For two perfectly distinguishable pure states, the bound becomes $\Delta t \geq 1/\Delta E$, which in the case of quantum theory coincides with the Mandelstam-Tamm bound up to a dimensional factor $h/4$, where h is Planck's constant [19]. Hence, Eq. (4) can be regarded as an alternative, purely information-theoretic derivation of the Mandelstam-Tamm bound.

Conclusions—In this Letter we explored the idea that reversible dynamics arises from informational nonequilibrium in a collisional model. In this setting, we provided an information-theoretic derivation of the notion of energy, and of its dual role of observable and generator of the dynamics. Our derivation can be extended to a full reconstruction of quantum theory, in which the principles give direct access to the key aspects of quantum dynamics, such as the Mandelstam-Tamm speed limit.

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