

Shake before Use: Universal Enhancement of Quantum Thermometry by Unitary Driving

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(Received 10 December 2025; accepted 23 April 2026; published 23 June 2026)

Quantum thermometry aims at determining temperature with ultimate precision in the quantum regime. Standard equilibrium approaches, limited by the quantum Fisher information given by static energy fluctuations, lose sensitivity outside a fixed temperature window. Nonequilibrium strategies have therefore been recently proposed to overcome these limits, but their advantages are typically model dependent or tailored for a specific purpose. This Letter establishes a general, model-independent result showing that any temperature-dependent unitary driving applied to a thermalized probe enhances its quantum Fisher information with respect to its equilibrium value. Such information gain is expressed analytically through a positive semidefinite kernel of information currents that quantify the flow of statistical distinguishability. Our results, together with an analysis of the relation between information gain and control cost, are benchmarked on a driven spin-1/2 thermometer, furthermore showing that resonant modulations remarkably restore the quadratic-in-time scaling of the Fisher information and allow the sensitivity peak to be shifted across arbitrary temperature ranges.

DOI: [10.1103/z6rz-d9sq](https://doi.org/10.1103/z6rz-d9sq)

Introduction—Unlike standard observables in quantum mechanics, temperature is notoriously not associated with a Hermitian operator and must be instead indirectly inferred [1–4]. In equilibrium quantum thermometry, a probe, hereby referred to as thermometer, is allowed to thermalize with a bath, and the temperature is extracted from the statistics of energy measurements. According to quantum-estimation theory [5–7], the ultimate precision for any temperature estimation protocol is fixed by the quantum Fisher information (QFI) through the Cramér-Rao bound [8,9]. For thermal states the QFI equals the thermometer’s energy variance, and is thus proportional to its heat capacity [3,10]. Yet equilibrium thermometry is fundamentally limited. Because Gibbs states commute with their Hamiltonian, the QFI coincides with the classical Fisher information of the thermal population distribution, with no contributions from coherence. Moreover, as one reaches equilibrium, the QFI saturates and does not increase with the interrogation time. Last but not least, high sensitivity can be crucially achieved only within a fixed and non-tunable temperature window, dictated by the probe energy

spectrum, and vanishes both in the low- and high-temperature limits [11,12]. In practice, this hinders experimental control over the operating range of any equilibrium thermometer.

These limitations have motivated the development of alternative, nonequilibrium approaches, such as interferometric [13–16] and control-assisted [17–19] schemes, where external drivings [20,21] or transient dynamics [22–25] are exploited to enhance temperature sensitivity. Despite substantial progress, existing results remain largely model dependent, and a unifying principle quantifying the fundamental role of unitary driving in quantum thermometry has insofar remained elusive. Here we establish a model-independent result: any unitary perturbation applied to a probe initially in thermal equilibrium cannot degrade its temperature sensitivity, and in fact enhances it under mild and physically reasonable assumptions. In particular, our main result is an analytical expression for the time-dependent QFI for a generic unitary driving, in the decomposed form

$$\mathcal{F}_t^\beta = \mathcal{F}_{\pi_0}^\beta + \mathcal{I}_t^\beta, \quad \mathcal{I}_t^\beta \geq 0, \quad (1)$$

where $\mathcal{F}_{\pi_0}^\beta$ is the equilibrium value and $\mathcal{I}_t^\beta$ quantifies the non-negative contribution arising from the unitary driving. A general conclusion following from Eq. (1) is that temperature-dependent unitary drivings implemented on thermalized probes also enable the shift-and-reshape of the

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QFI profile across temperatures, effectively relocating the thermometer's high-sensitivity region to any desired range. This result holds for arbitrary finite-dimensional probes initially in a full-rank equilibrium state. Extensions to more general regimes (e.g., infinite-dimensional systems, rank-deficient or nonequilibrium initial states), although possible, lie beyond the scope of this Letter. Although exemplified in the context of thermometry, our result extends directly to quantum estimation theory at large, providing a benchmark for precision enhancement in driven quantum systems.

Equilibrium thermometry—In order to determine the temperature $(\beta^*)^{-1}$ (in units where $k_B = 1$) of a target sample, acting as a thermal bath, the minimal strategy is to infer it from a probe (i.e. the thermometer) that has interacted with it until thermalization has occurred. Owing to the so-called “zeroth law” [26,27] of thermodynamics, such a thermalization process in fact generally encodes the temperature β of a generic sample into the probe, leading to the definition of a smooth one-parameter statistical manifold $\{\pi_0(\beta)\}_\beta$, with $\pi_0(\beta) = e^{-\beta H_0}/Z_0$ being the Gibbs state of the probe with respect to its bare Hamiltonian H_0 . Within local quantum estimation theory, the maximum achievable precision in inferring the true value of the parameter $\beta = \beta^*$ for any unbiased estimator $\hat{\beta}$ based on n independent measurements is set by the quantum Cramér-Rao bound

$$\text{Var}[\hat{\beta}] \geq \frac{1}{n\mathcal{F}_{\pi_0}^{\beta^*}}, \quad (2)$$

where $\mathcal{F}_{\pi_0}^{\beta^*} = \mathcal{F}_{\pi_0}^\beta|_{\beta=\beta^*}$ is the QFI, which represents the maximum of the classical Fisher information optimized over all positive operator-valued measures, and quantifies the infinitesimal distinguishability between states in the statistical model. For an arbitrary smooth family of states $\sigma(\beta)$, the QFI can be explicitly expressed as

$$\mathcal{F}_\sigma^\beta = \text{Tr}[(\partial_\beta \sigma) L_\sigma^\beta] = \text{Tr}[\sigma (L_\sigma^\beta)^2], \quad (3)$$

where the symmetric logarithmic derivative (SLD) [8] L_σ^β is the Hermitian operator satisfying the Lyapunov-type equation

$$\partial_\beta \sigma = \mathcal{J}_\sigma(L_\sigma^\beta), \quad \mathcal{J}_\sigma(X) := \frac{1}{2}\{\sigma, X\}. \quad (4)$$

The Bures-Jordan multiplication superoperator $\mathcal{J}_\sigma$ [28,29] induces a metric on the quantum statistical manifold parametrized by β through the inner product

$$\langle\langle A, B \rangle\rangle_\sigma = \text{Tr}[A^\dagger \mathcal{J}_\sigma(B)]. \quad (5)$$

Accordingly, the SLD L_σ^β is the metric dual [30–32] of the tangent vector $\partial_\beta \sigma$, i.e., $L_\sigma^\beta = \mathcal{J}_\sigma^{-1}(\partial_\beta \sigma)$, where, for

full-rank σ , an explicit representation of the inverse Bures superoperator is $\mathcal{J}_\sigma^{-1}(X) = 2 \int_0^\infty ds e^{-s\sigma} X e^{-s\sigma}$. At thermal equilibrium, $[\pi_0, H_0] = 0$ implies

$$L_{\pi_0}^\beta = -(H_0 - \text{Tr}[H_0 \pi_0]), \quad (6)$$

$$\mathcal{F}_{\pi_0}^\beta = \text{Var}_{\pi_0}[H_0], \quad (7)$$

which is proportional to the heat capacity of the probe up to a β^2 factor [33]. Because Eq. (2), with $\mathcal{F}_{\pi_0}^{\beta^*} = \mathcal{F}_{\pi_0}^\beta|_{\beta=\beta^*}$, is saturated by energy measurements, equilibrium thermometry is effectively classical and the achievable precision is fixed by static energy fluctuations. As the probe approaches π_0 , the QFI asymptotically saturates to Eq. (7), independently of the interrogation time t , in contrast with unitary metrological protocols where phase accumulation yields the characteristic quadratic scaling $\mathcal{F}_\sigma^\beta \propto t^2$ [34]. Moreover, the sensitivity window of equilibrium thermometry is static and determined by the spectrum of H_0 : for gapped systems the QFI is exponentially suppressed in both the $\beta \rightarrow 0$ (maximally mixed) and $\beta \rightarrow \infty$ (ground-state) limits, whereas for gapless ones it decays algebraically [3,33]. These intrinsic limitations have motivated the exploration of nonequilibrium and control-assisted thermometric strategies, in which external drivings, engineered couplings [18,35,36], or memory effects [24,25,37] are exploited to activate coherence and modify the scaling and range of the QFI. A paradigmatic instance is that of periodically driven thermometers, where modulation of the probe spectrum can alter the low-temperature scaling of the achievable precision [21]. While such schemes demonstrate that control can outperform static thermometry, they remain tied to specific dynamics and platforms: a general, model-independent understanding of how driving can reshape the fundamental bounds on thermal sensitivity is still lacking.

Driving a thermometer—To single out the role of unitary control in determining temperature sensitivity, we consider the minimal scenario in which the probe, having first thermalized with the sample, is then subjected to an external perturbation mediated by a classical field, as depicted in Fig. 1. We stress that, at this stage, any interaction between probe and sample has been switched off: the probe is fully isolated and the sample is excluded from the subsequent dynamics. In this regime the dynamics is unitary, consistent with the standard formulation of quantum optimal control theory. The evolution is described by the propagator $U_{t,\beta}$, generated by the time-dependent Hamiltonian

$$H(t, \beta) = H_0 + \lambda(t, \beta)V, \quad (8)$$

where in general $[H_0, V] \neq 0$ and $\lambda(t, \beta)$ is the driving profile, smooth in β . The explicit temperature dependence of the drive encompasses the widest class of perturbations one can

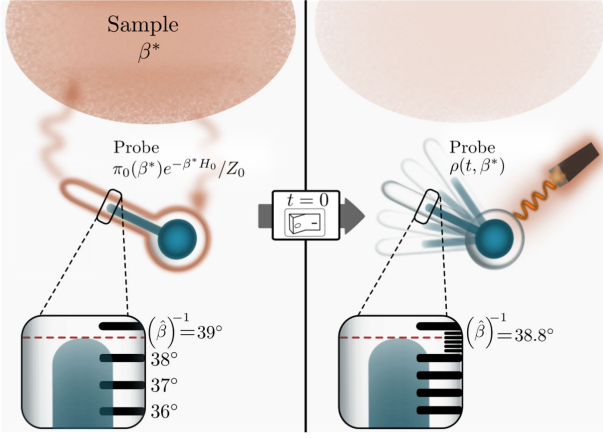


FIG. 1. Schematic illustration of the protocol. In the initial stage (left), the probe is allowed to thermalize with the sample, encoding the unknown inverse temperature β^* into the Gibbs state $\pi_0(\beta^*)$, and yielding an estimation precision bounded by the equilibrium quantum Cramér-Rao bound in Eq. (2). At $t = 0$ the driving is switched on, perturbing the now isolated probe. The resulting QFI of the nonequilibrium state $\rho(t, \beta^*)$ evolves according to Eq. (1), increasing whenever the drive depends nontrivially on temperature and thereby enhancing the attainable precision in the estimation (here illustrated as the acquisition of an additional significant digit in the estimator $\hat{\beta}$).

consider. From a thermodynamical perspective, the control field $\lambda(t, \beta)$ may arise within an open quantum systems framework when the probe is coupled to an auxiliary work reservoir, itself in equilibrium at the same temperature as the sample [38,39] (for further details, see Sec. 2 of Ref. [40]). In this regime, such a reservoir behaves in fact as an ideal work source with negligibly small fluctuations, unitarily exchanging energy with the probe and thus providing a deterministic, nondissipative drive: tracing out its degrees of freedom yields an effective Hamiltonian correction on the probe whose amplitude inherits its β dependence directly from the reservoir's thermal state. Operationally, one may tune the driving profile in Eq. (8) adaptively, consistently with a local estimation framework: First of all, a preliminary coarse-grained measurement of the equilibrium probe provides a prior estimate $\beta_0 \approx \beta^*$ via Eq. (7), which is then used as a working point [50] to shift the operating region of the control field and maximize the sensitivity in a neighborhood of β_0 . In the following, β_0, s_β are to be interpreted as fixed design parameters. For concreteness, in the rest of the Letter we consider drivings of the form

$$\lambda(t, \beta) = \lambda_0 G_{\beta_0, s_\beta}(\beta) f(t), \quad (9)$$

where λ_0 is the driving strength, $f(t)$ its temporal modulation, and $G_{\beta_0, s_\beta}(\beta) = \exp[-(\beta - \beta_0)^2 / (2s_\beta^2)] \equiv G(\beta)$ a dimensionless Gaussian envelope centered around the initial estimate β_0 , with standard deviation s_β . We remark that the above choice, Eq. (9), is purely illustrative (one could consider any

physically motivated smooth temperature dependence) and does not affect the generality of the subsequent results. To assess how such drivings modify thermal sensitivity, we analyze the corresponding family of evolved states $\rho(t) = U_{t, \beta} \pi_0(\beta) U_{t, \beta}^\dagger$. Exploiting the unitary covariance of $\mathcal{J}_\sigma$ and its inverse, the corresponding SLD can be expressed as

$$L_\rho^\beta = U_{t, \beta} (L_{\pi_0}^\beta + \delta L_t^\beta) U_{t, \beta}^\dagger, \quad (10)$$

with $L_{\pi_0}^\beta$ as in Eq. (6), while

$$\delta L_t^\beta = \int_0^t ds \partial_\beta \lambda(s) J_V(s) \quad (11)$$

captures the nonequilibrium contribution induced by the temperature dependence of the driving. The operator

$$J_V(s) = -\frac{i}{\hbar} \mathcal{J}_{\pi_0}^{-1}([V_H(s), \pi_0]), \quad (12)$$

here introduced, which we call information current, quantifies the instantaneous flow of statistical distinguishability generated by the perturbation $V_H(s)$ (in the Heisenberg picture) with respect to the equilibrium reference state π_0 . It vanishes whenever $[V_H(s), \pi_0] = 0$, i.e., when no quantum friction is generated through the driving, since the latter leaves the eigenbasis of π_0 invariant, acting only on its eigenvalues [51–55]. Hence, a nonzero $J_V(s)$ signals that the control field injects coherences in the eigenbasis of π_0 : the ensuing enhancement of distinguishability thus stems from a genuinely quantum mechanism, as further discussed in Sec. 4 of Ref. [40]. Similarly, when $\partial_\beta \lambda = 0$, i.e., when the driving is not sensitive to temperature, one recovers $L_{\rho(t)}^\beta = L_{\pi_0}^\beta$. This identifies a genuine no-go condition, as it excludes any unitary control that does not explicitly depend on β from generating additional information about temperature, regardless of its time dependence or strength. Geometrically, this reflects the fact that a parameter-independent unitary is an isometry of the Bures metric, rotating the probe state within the statistical manifold without changing its sensitivity to temperature. This conclusion is fully consistent with the quantum data-processing inequality [56], which forbids any Fisher information gain under parameter-independent completely positive trace-preserving maps, including unitaries.

Using Eq. (3) and noticing that $\text{Tr}[\pi_0 \delta L_t^\beta L_{\pi_0}^\beta] = 0$, allows us to derive [40] our main result Eq. (1), which quantifies the QFI increment due to a generic unitary driving with

$$\begin{aligned} \mathcal{I}_t^\beta &= \text{Tr}[\pi_0(\beta) (\delta L_t^\beta)^2] \\ &= \int_0^t \int_0^t \partial_\beta \lambda(s, \beta) \partial_\beta \lambda(u, \beta) K(s, u, \beta) ds du, \end{aligned} \quad (13)$$

which is manifestly non-negative, and where we have introduced the kernel

$$K(s, u, \beta) = \text{Tr}[\pi_0(\beta)J_V(s)J_V(u)] \quad (14)$$

which encodes the two-time correlation function of the information current. We remark once again that the QFI must be evaluated at $\beta = \beta^*$ and that, if the perturbation commutes with the initial Hamiltonian, i.e., $[H_0, V] = 0$, then the improvement vanishes and no precision enhancement is expected.

Interestingly, by interpreting the external driving as part of a measurement scheme, our results can be recast within the framework of quantum parameter estimation with parameter-dependent measurement apparatuses, put forward in Ref. [57], where the standard Cramér-Rao bound may in principle be surpassed. In the present Letter, however, we focus explicitly on the modification of the QFI induced by the β -dependent dynamics, without optimizing the drive. In this sense, Eq. (1) holds true for any arbitrary choice of driving provided it depends nontrivially on temperature.

The positivity of Eq. (13) thus provides a measure of how temperature-dependent drivings amplify thermal sensitivity. At short times, regularity of the kernel guarantees that $\mathcal{I}_t^\beta \sim t^2$, reproducing the standard scaling of the QFI in closed evolutions where distinguishability grows linearly in time at the level of the SLD. Importantly, the same quadratic scaling can reemerge asymptotically when the driving preserves constructive correlations in the information current, such as for constant (in time) controls or for periodic modulations resonant with the Bohr frequencies of H_0 . This guarantees the scaling of the QFI with t to be quantumlike, i.e., quadratic, at all times [34]. Since the kernel depends on the β derivative of the control profile $\lambda(t, \beta)$, shaping this dependence provides a direct way of shifting the sensitivity window of the probe. In this sense, the driving acts as an operational handle on the probe sensitivity landscape, and can naturally accommodate feedback-assisted strategies in which $\lambda(t, \beta)$ is updated according to intermediate estimates of β , keeping the probe dynamically locked to the region of maximal information gain. To benchmark our results, we analyze a single spin-1/2 probe as a minimal reference model in which all relevant mechanisms can be made explicit.

Single spin probe—In this minimal setting, the dynamics is governed by a driven two-level Hamiltonian,

$$H(t, \beta) = \frac{\Omega}{2}\sigma_z + \lambda(t, \beta)\sigma_x, \quad (15)$$

where Ω (in units of $\hbar = 1$) denotes the characteristic frequency of the system, while $\lambda(t, \beta)$ is chosen according to Eq. (9), with periodic temporal modulation $f(t) = \cos(\omega_d t + \phi)$, driving frequency ω_d , and phase ϕ . In the

absence of control, the probe is allowed to reach thermal equilibrium, characterized by the baseline Fisher information

$$\mathcal{F}_{\pi_0}^\beta = \left(\frac{\Omega}{2}\right)^2 \text{sech}^2\left(\frac{\beta\Omega}{2}\right). \quad (16)$$

Switching on the drive turns on the information current, leading to the emergence of a nontrivial $K(s, u, \beta)$.

The time integral governing $\mathcal{I}_t^\beta$ contains harmonic components oscillating at $\Omega \pm \omega_d$, with the detuning $\Omega - \omega_d$ determining whether correlations in the information current interfere constructively or destructively over time. Figure 2 illustrates this effect for three representative driving frequencies: below, on, and above resonance. On resonance, i.e., $\omega_d \simeq \Omega$, and in a weak-field approximation, the increment $\mathcal{I}_t^\beta$ can be shown explicitly (see Sec. 3 of Ref. [40]) to achieve quadratic scaling for long times,

$$\mathcal{I}_t^\beta \approx (\lambda_0 G'(\beta))^2 \alpha(\beta) t^2, \quad (17)$$

with $G'(\beta) = \partial_\beta G(\beta)$ and $\alpha(\beta)$ being a time-independent quantity dictated only by the equilibrium state of the thermometer, confirming that resonance restores the fully unitary t^2 scaling discussed above. For detuned drivings ($\omega_d \neq \Omega$), instead, the oscillatory kernel induces dephasing that suppresses the long-time growth and saturates the information gain. In Fig. 3, we fix $\omega_d \simeq \Omega$ and vary the center and standard deviation of the Gaussian temperature envelope, showing how the QFI can be reshaped and shifted across distinct temperature regions. This demonstrates that the probe's sensitivity window can be tailored by tuning the functional dependence of the control amplitude on β .

Finally, we investigate the energetic cost associated with the external driving that is responsible for the predicted information gain. To this end, inspired by shortcuts-to-adiabaticity and counterdiabatic-driving methods [59,60], we introduce the following cost functional to quantify the accumulated driving effort:

$$\mathcal{C}_2(t, \beta) := \int_0^t ds |\lambda(s, \beta)|^2 \|V\|_2^2, \quad (18)$$

with $\|V\|_2$ denoting the Frobenius norm. This allows us to further consider a dimensionless ratio measuring the information gain per unit of energetic cost

$$\mathcal{R}_2(t, \beta) := \frac{\mathcal{I}_t^\beta}{\mathcal{C}_2(t, \beta)}. \quad (19)$$

In the time-asymptotic limit of our model, we find that $\mathcal{R}_2(t, \beta) \sim t$ in the resonant regime, while it vanishes away from resonance (see Sec. 5 of Ref. [40] for further details).

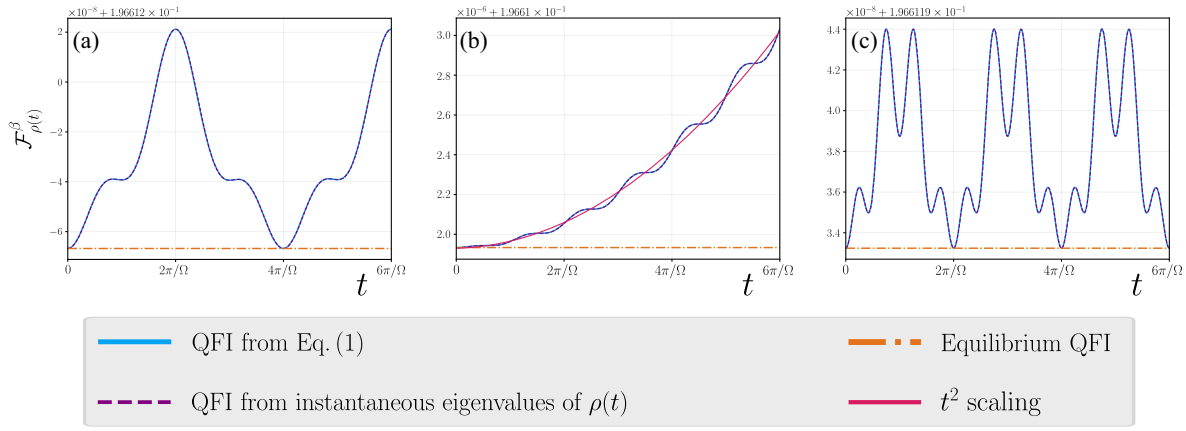


FIG. 2. Time evolution of the QFI for a single-spin probe under a temperature-dependent transverse driving. Time t is measured in units of the spin characteristic period $2\pi/\Omega$, with $\Omega = 1$. Solid and dashed lines denote, respectively, the QFI as given by Eq. (1) and by the standard expression in terms of the instantaneous eigenvalues of $\rho(t)$ [58], showing perfect agreement. The dash-dotted line marks the equilibrium QFI benchmark. The three panels illustrate the transition from bounded dynamics to resonant quadratic scaling, identifying the optimal driving frequency. The Gaussian temperature control profile is chosen with a standard deviation saturating the equilibrium quantum Cramér-Rao bound, as if an optimal equilibrium estimation had been performed beforehand, and is randomly centered within the interval $\max(0, \beta^* \pm 1/\sqrt{\mathcal{F}_{\pi_0}^{\beta^*}})$. For all panels $\phi = 0$ and $\lambda_0 = 0.1$. (a) Off-resonant regime with $\omega_d = 0.5\Omega$, where the QFI exhibits bounded oscillations. (b) Resonant regime with $\omega_d = \Omega$, where the QFI displays an almost monotonic t^2 growth, signalling a build-up of sensitivity. (c) High-frequency regime with $\omega_d = 2\Omega$, where one observes once again a bounded, oscillatory response, albeit at a faster characteristic timescale.

This clearly emphasizes the additional efficacy of driving the spin-1/2 probe on resonance, since a linearly growing energetic cost $\mathcal{C}_2(t, \beta) \sim t$ would allow for a quadratically increasing information gain $\mathcal{I}_t^\beta \sim t^2$.

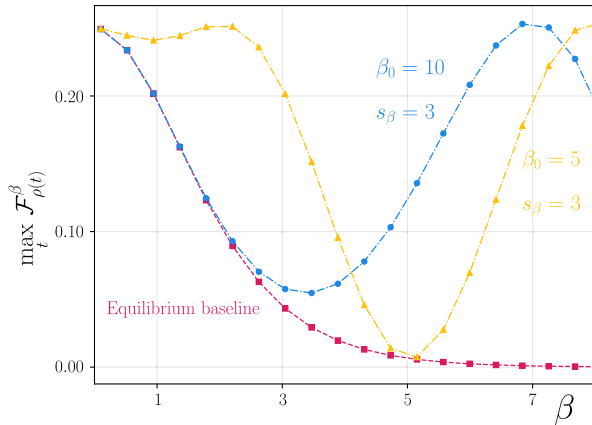


FIG. 3. Behavior of the maximum QFI at a fixed evolution time $t = 12s$ in the resonant regime, as a function of the inverse temperature β . The magenta dashed line (with squares) represents the equilibrium baseline $\mathcal{F}_{\pi_0}^\beta$, while the blue circles and yellow triangles show the maxima for the driven case, for $G(\beta)$ with $(\beta_0, s_\beta) = (10, 3)$ and $(5, 3)$, respectively. By tuning the mean and standard deviation of the Gaussian envelope, the sensitivity window can be shifted to different temperature ranges. The resulting QFI profiles display two symmetric lobes around β_0 , reflecting the underlying Gaussian modulation of the driving amplitude, as exemplified by the yellow curve.

Conclusion—In this Letter we have derived a general analytic expression for the temperature sensitivity, i.e., its quantum Fisher information, ensuing from any temperature-dependent unitary driving acting on an initially thermalized probe. Our results show that implementing such a perturbation increases temperature precision, quantified by a non-negative correction to the equilibrium heat capacity of the thermometer expressed in terms of a kernel of information-geometric currents that captures the unitary buildup of temperature information along the drive. We then showcase our findings on a driven spin-1/2 model, remarkably recovering a t^2 scaling of the QFI in the resonant-driving regime.

Furthermore, we explicitly demonstrate that the sensitivity window of the qubit probe can be tuned across the whole temperature range by suitably acting on the driving parameters. We want to emphasize that the framework and the results here explicitly obtained within the context of quantum thermometry can be straightforwardly applied to study the enhancement in the estimation of other parameters. Finally, it is worth mentioning that the predicted sensitivity increase and tunability can be directly applicable to current experimental setups for quantum thermometry such as bichromatically driven trapped ions [61] or microwave-driven dispersive circuit QED [62–64]. A natural next step will be to allow for the control field to act already during thermalization, or to consider nonunitary drivings; in this case, the noise introduced by dissipative channels may in general compete with the information gain, reducing the maximum achievable sensitivity [56,65].

Our results therefore pave the way for future studies characterizing the general trade-off between unitary control and dissipation in thermal sensitivity enhancements.

Acknowledgments—E. T. acknowledges support from the PNRR MUR Project No. PE0000023-NQSTI. L. M. acknowledges support from the PRIN MUR Project No. 2022RATBS4. C. M. acknowledges support from the National Research Centre for HPC, Big Data and Quantum Computing, PNRR MUR Project No. CN0000013-ICSC. M. G. A. P. acknowledges partial support by Khalifa University of Science and Technology through the Project No. C2PS-8474000137. G. G. kindly acknowledges support from the Ministero dell’Università e della Ricerca (MUR) under the “Rita Levi-Montalcini” grant and to INFN.

Data availability—The underlying code used for data generation and analysis in this study is openly available on GitHub at [66].

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