

Orbital Inverse Faraday and Cotton-Mouton Effects in Hall Fluids

Gabriel Cardoso¹, Erlend Syljuåsen^{1,2} and Alexander V. Balatsky^{1,3}

¹*Nordita, Stockholm University, and KTH Royal Institute of Technology, Hannes Alfvéns väg 12, SE-106 91 Stockholm, Sweden*

²*Center for Quantum Spintronics, Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway*

³*Department of Physics, University of Connecticut, Storrs, Connecticut 06269, USA*



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We report two light-induced orbital magnetization effects in quantum Hall (QH) fluids, stemming from their transverse response. The first is a purely transverse contribution to the inverse Faraday effect (IFE), where circularly polarized light induces a dc magnetization by stirring the charged fluid. This contribution dominates the IFE in the QH regime. The second is the orbital inverse Cotton-Mouton effect (ICME), in which linearly polarized light generates a dc magnetization. Since the applied field in the ICME does not break time-reversal symmetry, the induced magnetization directly probes the chiral orbital response of the fluid at the driving frequency. We estimate that the resulting magnetization lies in the range of 0.5–10 Bohr magnetons per charge carrier in materials such as graphene and transition-metal dichalcogenides (TMDs) in the QH regime. Finally, we show that the induced magnetization is accompanied by a local correction to the static particle density, enabling optical quantum printing of density profiles into the QH fluid.

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Introduction—Recent progress in control of the dynamics of quantum states has been noted. The coupling of terahertz light to the low-energy degrees of freedom in materials enables the creation and control of quantum states on ultrafast timescales [1]. Applications of structured light can create, modify, or destroy magnetic [2–4], superconducting [5,6], and charge-density orders [7]. One can now use the light-matter coupling to realize *quantum printing*, i.e., to write information into the quantum state [8]. Examples are the transduction of quantum numbers into topological defects in superconductors [9,10] and magnetic materials [11].

An important example of quantum printing is the inverse Faraday effect (IFE) [12], where angular momentum is transferred from circularly polarized light to matter. Its signature is the generation of a net static (dc) magnetization,

$$\mathbf{M}_0 \propto \mathbf{E}_\omega \times \mathbf{E}_\omega^* \quad (1)$$

Here, $\mathbf{E}_\omega$ denotes the complex electric field of monochromatic light with frequency ω . Magnetization in IFE is nonzero only for a circularly (or elliptically) polarized light $\mathbf{E} \sim \hat{\mathbf{e}}_x + i\hat{\mathbf{e}}_y$, reflecting the handedness of light that is imprinted on the electron fluid. The IFE is also a prominent

example of rectification, where oscillating fields induce a static response. As was realized early [12,13], the IFE can be generated by various microscopic mechanisms. One example is the generation of spin polarization via spin-orbit coupling [14,15]. Alternatively, circularly polarized light can generate a magnetization by driving coherent orbital motion of charge carriers. The resulting circulating currents can generate a net static magnetization even in the absence of spin-orbit coupling (see Fig. 1). The orbital contribution was shown to be relevant in metals and superconductors [16,17], and the elegant description using hydrodynamics of a charged fluid was developed [18]. For the subsequent discussion we also recall the inverse Cotton-Mouton effect (ICME), that describes the appearance of rectified magnetization upon incidence of linearly polarized light, $\mathbf{M}_0 \propto \mathbf{E}_\omega \cdot \mathbf{E}_\omega^*$ [19,20].

In this Letter, we extend the hydrodynamics of electron fluids interacting with light to develop orbital magnetization effects in charged fluids with a transverse response. In particular, we find (i) a purely transverse contribution to the IFE. We find that the transverse (Hall) response of the fluid allows for IFE and the result depends on the Hall conductivity. Together with the longitudinal contribution, the net magnetization is

$$\mathbf{M}_0 = i \frac{|\sigma_\omega^L|^2 + |\sigma_\omega^H|^2}{e\omega\rho_0} \mathbf{E}_\omega \times \mathbf{E}_\omega^*, \quad (2)$$

where e is the charge of carriers in the fluid, ρ_0 is the particle number density, and $\sigma_\omega^{L(H)}$ is the longitudinal (Hall) conductivity, respectively. (ii) We also predict an orbital

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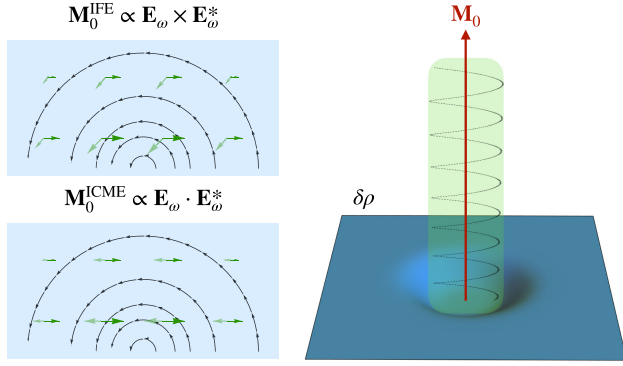


FIG. 1. Induction of a dc magnetization current $\mathbf{J}_0 = \nabla \times \mathbf{M}_0$ (black) by finite-frequency light fields $\mathbf{E}_\omega$ (green) in a Hall fluid. In the inverse Faraday effect (IFE), the orbital magnetization $\mathbf{M}_0^{\text{IFE}} \propto \mathbf{E}_\omega \times \mathbf{E}_\omega^*$ arises from stirring the fluid with circularly polarized light (top left). This effect can arise from purely transverse (Hall) response. In the inverse Cotton-Mouton effect (ICME), $\mathbf{M}_0^{\text{ICME}} \propto \mathbf{E}_\omega \cdot \mathbf{E}_\omega^*$ is generated by linearly polarized light (bottom left), revealing the fluid's intrinsic chiral orbital motion at the frequency of light. In the quantum Hall regime, the resulting dc magnetization is accompanied by a change of the local particle density $\delta\rho$ (right), enabling optical quantum printing of density patterns.

ICME, the generation of a static magnetization by linearly polarized light. In the orbital ICME the induced magnetization is given by

$$\mathbf{M}_0 = \frac{i}{e\omega\rho_0} \text{Tr}(\sigma_\omega^{2D} \times \sigma_\omega^{2D*}) \mathbf{E}_\omega \cdot \mathbf{E}_\omega^* \hat{\mathbf{k}}, \quad (3)$$

where $\hat{\mathbf{k}}$ is the direction of light propagation, σ_ω^{2D} denotes the optical conductivity tensor on the plane perpendicular to $\hat{\mathbf{k}}$, and $(\sigma_\omega^{2D} \times \sigma_\omega^{2D*})_{ij} = \epsilon^{kl} \sigma_{\omega,ki}^{2D} \sigma_{\omega,lj}^{2D*}$. This coefficient is only nonvanishing if the orbital motion of charges in the material breaks time-reversal symmetry (TRS), in contrast with the IFE (1), where TRS is broken by the circular polarization of light itself. In isotropic fluids it translates to a phase difference between the longitudinal and transverse ac responses (see Fig. 1). (iii) We estimate the strength of these effects in the quantum Hall (QH) fluid. In graphene and transition-metal dichalcogenides (TMDs), they lead to a significant local magnetization in the range of 0.5–10 Bohr magnetons per charge carrier, which can be experimentally detected via the light-induced anomalous Hall effect. (iv) We also find that in the QH fluid the induced magnetization is accompanied by a local change of the particle density. This is a modification of the Štředa formula, which relates the density to the magnetic flux, by a backreaction of the induced magnetization. As a result, light can be used to realize quantum Hall printing by optically writing density patterns into the fluid, as illustrated in Fig. 1. We now proceed to elaborate these claims.

Hydrodynamics—Our approach is based on the hydrodynamics of a charged fluid, similar to the original analysis by Pitaevskii [12] of the drift currents in plasmas. We focus on the high-frequency regime, where the oscillating electric field dominates the response to light [18,21]. In the THz-driven quantum Hall regime discussed later, one also has $\omega\tau \gg 1$, with τ the Drude lifetime. We assume that the electric current transported by the fluid is given by the constitutive relation

$$\mathbf{J} = e\rho\mathbf{v}, \quad (4)$$

where $\rho(t, \mathbf{x})$ and $\mathbf{v}(t, \mathbf{x})$ are the density and velocity fields of the fluid. Under an applied monochromatic high-frequency electromagnetic wave with electric field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r}) \cos(\omega t)$, the time dependence of the fluid properties can be expanded in terms of their harmonic components,

$$\rho(t, \mathbf{x}) = \rho_0 + \rho_\omega e^{i\omega t} + \rho_{-\omega} e^{-i\omega t} + \rho_{2\omega} e^{i2\omega t} + \dots, \quad (5)$$

and similarly for $\mathbf{v}$ and $\mathbf{J}$. The reality condition of these quantities implies that $\rho_{n\omega}^* = \rho_{-n\omega}$. It then follows from the constitutive relation (4) that

$$\mathbf{J}_0 = e(\rho_{-\omega}\mathbf{v}_\omega + \rho_\omega\mathbf{v}_{-\omega}) + \dots, \quad (6a)$$

$$\mathbf{J}_\omega = e\rho_0\mathbf{v}_\omega + \dots, \quad (6b)$$

where we assumed that the fluid is at rest on average, $\mathbf{v}_0 = 0$. The fluctuations of the density are fixed through the continuity equation

$$\partial_t(e\rho) + \nabla \cdot \mathbf{J} = 0, \quad (7)$$

from which we obtain

$$\rho_\omega = \frac{i}{e\omega} \nabla \cdot \mathbf{J}_\omega. \quad (8)$$

Substituting Eqs. (6b), (8) into Eq. (6a) yields the leading order contribution to the static current,

$$\mathbf{J}_0 = \frac{i}{e\omega\rho_0} [(\nabla \cdot \mathbf{J}_\omega)\mathbf{J}_{-\omega} - (\nabla \cdot \mathbf{J}_{-\omega})\mathbf{J}_\omega], \quad (9)$$

which can be expressed as

$$\mathbf{J}_0 = \nabla \times \mathbf{M}_0 + \mathbf{\Gamma}_0. \quad (10)$$

The static magnetization

$$\mathbf{M}_0 = \frac{i}{e\omega\rho_0} \mathbf{J}_\omega \times \mathbf{J}_\omega^* \quad (11)$$

arises by rectification of circulating currents $\mathbf{J}_\omega$ present in the lower-order response. The additional contribution

$$\mathbf{\Gamma}_0 = \text{Re} \left[\frac{i}{e\omega\rho_0} (\mathbf{J}_{-\omega} \cdot \nabla) \mathbf{J}_\omega \right], \quad (12)$$

leads to a net shift $\mathbf{v} \rightarrow \mathbf{v} + \mathbf{\Gamma}_0/(e\rho_0)$ of the fluid velocity, that determines the ponderomotive forces on the fluid [22].

Symmetry analysis—We now consider the particular case of magnetization generated along the direction of light propagation. Then, using $\mathbf{J}_\omega = \sigma_\omega \mathbf{E}_\omega$, Eq. (11) is expressed as

$$M_0 = \frac{i}{e\omega\rho_0} (\sigma_\omega^{2D} \times \sigma_\omega^{2D*})_{kl} E_\omega^k E_\omega^{l*}, \quad (13)$$

where we noted that, in this equation, only the 2×2 sub-matrix σ_ω^{2D} corresponding to the conductivity on the plane of incidence appears. Recall that a rank-two tensor can be decomposed into a scalar part, an antisymmetric part, and a quadrupolar (traceless symmetric) part. Applying this decomposition to the $\sigma \times \sigma^*$ matrix, we find that its fully antisymmetric part couples to $\mathbf{E} \times \mathbf{E}^*$, resulting in the generalized expression for the IFE,

$$\mathbf{M}_0 = \frac{i}{2e\omega\rho_0} \epsilon^{ij} (\sigma_\omega^{2D} \times \sigma_\omega^{2D*})_{ij} \mathbf{E}_\omega \times \mathbf{E}_\omega^*. \quad (14)$$

For an isotropic material, $(\sigma_\omega^{2D})_{ij} = \sigma_\omega^L \delta_{ij} + \sigma_\omega^H \epsilon_{ij}$, it simplifies to Eq. (2).

In the $\sigma_\omega^H \rightarrow 0$ case, Eq. (2) reduces to the usual formula for the IFE in a longitudinal fluid [18]. Conversely, in quantum Hall fluids it is the σ_ω^H contribution which dominates (see Table II). Note that, although the Hall conductivity is odd under time reversal, its absolute value squared, appearing in the coefficient of Eq. (2), is even.

Consider now the symmetric part of $\sigma_\omega^{2D} \times \sigma_\omega^{2D*}$. Taking the trace yields the ICME contribution (3). The ICME is most commonly studied in magnetic materials, where spin polarization is induced by light [19,20]. This is because linearly polarized light—unlike circularly polarized light—is invariant under TRS, and therefore cannot generate magnetization unless the material response itself breaks TRS. In our case, however, the fluid is not magnetically ordered and does not necessarily break TRS at the dc level. Instead, it exhibits chiral orbital motion at the frequency of light. This orbital chirality is encoded in the specific combination of components of the conductivity tensor appearing in the coefficient of Eq. (3). In the isotropic case, it simplifies to

$$M_0 = \frac{2 \text{Im}(\sigma_\omega^L \sigma_\omega^{H*})}{e\omega\rho_0} \mathbf{E}_\omega \cdot \mathbf{E}_\omega^*. \quad (15)$$

This shows that both longitudinal and transverse responses are required, along with a phase difference between them. The connection with the chiral orbital motion can be seen as follows: assume real σ_ω^H and imaginary σ_ω^L (as is the case

TABLE I. Rectified magnetization effects in an isotropic charged fluid. While in the IFE the applied field breaks TRS, in the ICME it is the chiral orbital response which breaks TRS. Both effects are observable in quantum Hall systems (see Table II).

Field component	Fluid response coefficients	Effect
$\mathbf{E}_\omega \times \mathbf{E}_\omega^*$ (TRS-odd)	$ \sigma_\omega^L ^2 + \sigma_\omega^H ^2$ (TRS-even)	IFE
$\mathbf{E}_\omega \cdot \mathbf{E}_\omega^*$ (TRS-even)	$\text{Im}(\sigma_\omega^L \sigma_\omega^{H*})$ (TRS-odd)	ICME

in the QH plateaus), and let $\mathbf{E} = E \cos(\omega t) \hat{\mathbf{e}}_x$. Then linear response gives the current

$$\mathbf{J} = E[i\sigma_\omega^L \sin(\omega t) \hat{\mathbf{e}}_x - \sigma_\omega^H \cos(\omega t) \hat{\mathbf{e}}_y], \quad (16)$$

which rotates with chirality determined by the sign of $\text{Im}(\sigma_\omega^L \sigma_\omega^{H*})$. According to Eq. (15), this also determines the sign of the static magnetization induced at the level of nonlinear response (compare with Fig. 1). We summarize the contrast between the symmetry properties of the IFE and the ICME in Table I.

Quantum Hall fluids—The QH system is a prime example of an isotropic fluid with large transverse response, and we find that it realizes both the transverse IFE and the ICME. The hydrodynamic description of QH states can be derived from the action principle [23,24]

$$S = - \int d^2x \left[\rho \left(\partial_t \theta + eA_t + \frac{m\mathbf{v}^2}{2} \right) + V(\rho) \right], \quad (17)$$

where e and m are the particles' charge and mass, respectively, and we assume the Landau level filling fraction ν to correspond to one of the integer or fractional QH plateaus. There are two fluctuating fields: the particle density ρ and the velocity potential θ , which parametrizes the local velocity of the fluid $\mathbf{v}(t, \mathbf{x})$ as

$$m\mathbf{v} = \nabla\theta + \frac{e}{c} \mathbf{A} + \frac{2\pi\hbar}{\nu} \nabla^\perp \left(\frac{\rho}{\nabla^2} \right). \quad (18)$$

Here we introduce convenient notations: $(\nabla^\perp)_i = \epsilon_{ij} \partial^j$, where ϵ_{ij} is the antisymmetric symbol, denotes the rotation of the gradient ∇ by $\pi/2$, and $(e\rho/\nabla^2)(\mathbf{x}) = \int d^2x' [\log|\mathbf{x} - \mathbf{x}'|/2\pi] e\rho(\mathbf{x}')$ denotes the 2D electrostatic potential generated by the charge distribution $e\rho$. In contrast to the approach to electron hydrodynamics based on the Navier Stokes equation [25,26], we focus on the particular hydrodynamics of the quantum Hall state, which can be understood as a superfluid with a particular equation of state (the QH constraint) [27]. The action (17) is derived from the Chern-Simons-Landau-Ginzburg theory of quantum Hall effect [28], and captures the universal transport features of both integer and fractional quantum Hall states at large scales. On the other hand, we note that this is a

semiclassical description, which does not explicitly include microscopic effects such as absorption, dephasing, and lattice defects, which may become relevant near resonance or for sharper probe fields.

Assuming a constant background magnetic field B_0 , the quantum Hall state corresponds to the homogeneous, static solution of the equations of motion. One finds that such a solution exists only for the specific value of density given by the Středa relation $\rho_0 = \nu(eB_0/2\pi\hbar)$. As we review in Supplemental Material [29] (see also Ref. [24]), adding a perturbation to the background field δA_μ and expanding the effective action to linear order around the static solution, one recovers the formulas for the ac conductivities at the center of the Hall plateau,

$$\sigma_\omega^H = \nu \frac{e^2}{h} \frac{\omega_0^2}{\omega_0^2 - \omega^2}, \quad \sigma_\omega^L = \nu \frac{e^2}{h} \frac{i\omega\omega_0}{\omega^2 - \omega_0^2}, \quad (19)$$

where $\omega_0 = (eB_0/m)$ is the cyclotron frequency. Importantly, the hydrodynamic action (17) should be seen as the leading part of a small-gradient expansion with respect to ω_0 . On the other hand, these formulas also take into account the typically very high mobility in QH samples, so that the mean free time is large compared to the period of driving, $\omega\tau \gg 1$. Thus in the collisionless, strong plateau quantization regime, the leading contribution to the IFE arises from the transverse component $\propto |\sigma_\omega^H|^2$ and is given by

$$\mathbf{M}_0 = \frac{i\nu^2 e^3}{h^2 \omega \rho_0} \mathbf{E}_\omega \times \mathbf{E}_\omega^*. \quad (20)$$

Similarly, the leading contribution to the ICME is

$$\mathbf{M}_0 = -\frac{\nu^2 e^3}{h^2 \omega_0 \rho_0} \mathbf{E}_\omega \cdot \mathbf{E}_\omega^* \hat{\mathbf{k}}. \quad (21)$$

Note that in this regime the IFE is larger than the ICME by a factor of ω_0/ω (see Table II). These formulas are confirmed by directly computing the nonlinear response from the effective action

$$S_{\text{eff}}[A] = -i \log \int \mathcal{D}\rho \mathcal{D}\theta e^{iS[\rho, \theta; A]}, \quad (22)$$

TABLE II. Material estimates for induced magnetization (in μ_B per carrier) in the quantum Hall regime at background $B_0 = 10$ T. The light field is at frequency $\omega = 1$ THz and intensity $|\mathbf{E}_\omega| = 10^5$ V/m.

	$\rho_0 (10^{11} \text{ cm}^{-2})$	m_*/m_e	$M_{\text{IFE}}(\mu_B \rho_0)$	$M_{\text{ICME}}(\mu_B \rho_0)$
MoS ₂	9.1	0.49	0.57	0.16
Graphene	2.0	0.007	10.1	0.04
GaAs	2.7	0.07	5.5	0.21

as we show in Supplemental Material [29]. We also investigate the opposite regime $(\omega/\omega_0) \gg 1$, where one can derive the effective Floquet Hamiltonian of the two-dimensional electron gas (2DEG) through a high-frequency expansion. Interestingly, we find that the result of this calculation is also matched by the general formulas (2), (15), when expanded for high frequencies, up to a dimensionless prefactor [29].

Material estimates—The QH effect can be realized in devices with optically accessible surfaces, allowing direct coupling of light to the 2DEG, notably in graphene [32,33] and TMDs [34]. THz light is ideal for probing the orbital IFE and ICME in the hydrodynamic regime, as it is generally below but comparable to the cyclotron frequency in these materials. Assuming a background field of $B_0 = 10$ T (which corresponds to $\omega_0 \approx 1.8$ THz for bare electrons), and applied light with frequency $\omega = 1$ THz and $|\mathbf{E}_\omega| = 10^5$ V/m, we find that the induced magnetization is in the range of 0.5–10 Bohr magneton per charge carrier (see Table II). Note that the induced orbital magnetization directly corresponds to a vortical current, which can be measured in terms of the light-induced anomalous Hall effect [35].

The induced magnetization can also be probed via Faraday and Kerr rotations, whose magnitude and sign depend on the orientation of the magnetization relative to the probe field. These measurements can be performed in a pump-probe setup that isolates the signal induced through the IFE or ICME from the large background field, typically at short pump-probe delays to suppress thermal effects. For the IFE, this corresponds to measuring the difference in Faraday or Kerr rotation between left- and right-handed circularly polarized pump pulses, whereas for the ICME the signal corresponds to the change in rotation relative to the unpumped state. Time-resolved measurements provide direct access to the rectified magnetization, with a low-frequency peak in the Fourier spectrum indicating the induced dc component. This method achieves microradian sensitivity for THz pulses [36]. In the visible-to-THz range, mono- or few-layer samples of the materials in Table II can exhibit very large effective Verdet constants, of order $107 \text{ deg T}^{-1} \text{ cm}^{-1}$ [37–40], corresponding to observable rotation angles in the μrad range.

Quantum Hall printing—Up to this point, we only used the transport properties of the quantum Hall fluid. In other words, Eqs. (2) and (15) apply even to classical transverse fluids. We turn now to a purely quantum feature of the QH state, namely, the Středa formula for the proportionality between the particle density and the magnetic flux. A natural question is whether the magnetization generated by the light through rectification effects feeds back into the Středa formula as an additional magnetic flux. To answer this question, we consider the time-averaged (dc) part of the density, determined from the effective action by $\rho = -(1/e)(\delta/\delta A_t)S_{\text{eff}}[A]$. As we show in Supplemental

Material [29], the nonlinear corrections are related to the magnetization $M = (\delta/\delta B)S_{\text{eff}}[A]$ by

$$\rho = \frac{\nu e}{2\pi\hbar}B_0 - \frac{1}{e\hbar^2\omega_0}\nabla^2 M, \quad (23)$$

where M denotes the total dc magnetization normal to the Hall plane, including both the IFE and ICME contributions. From the perspective of quantum printing [8], it implies that one can use light to write into the QH states by imprinting static density textures through the IFE and the ICME (see Fig. 1).

Note that the magnetization contribution in Eq. (23) depends on the sharpness of the applied field profile through the Laplacian operator, which is typically constrained by the diffraction limit. Estimating $\Delta M \approx -(\hbar^2/\lambda^2)M$, where λ is the wavelength of the incident light, we find that the correction to the density is

$$\delta\rho \approx \left(\frac{\ell_0}{\lambda}\right)^2 \frac{m_*}{m_e \mu_B} M, \quad (24)$$

where $\ell_0 = \sqrt{(\hbar/eB_0)}$ is the magnetic length in the external field B_0 . For THz light and $B_0 = 10$ T, the ratio (ℓ_0/λ) is of the order of 10^{-5} , so that induced density corrections are negligible. On the other hand, recent developments in scattering near-field optical microscopy (SNOM) have succeeded in resolving length scales comparable to the magnetic length in graphene quantum Hall samples [41]. This is the perfect candidate to observe the quantum Hall printing of density profiles, as the estimated density correction (24) can be of the order of $0.1\rho_0$ (see Table II). Importantly, it is not essential to generate circularly polarized near fields, since the ICME contribution which we introduced in this Letter also generates a sizable static magnetization.

Conclusions—We identified two distinct light-induced orbital magnetization effects in transverse (Hall) fluids. First, we showed that the IFE acquires an orbital contribution from the transverse response, which dominates in the QH regime. Second, we found that linearly polarized light can induce a dc magnetization via an orbital ICME, enabled by a phase difference between longitudinal and Hall conductivities. The resulting static magnetization reveals the chirality of the orbital ac response—even under time-reversal symmetric illumination. Third, we showed that this induced magnetization feeds back into the Středa relation, producing a static density shift. Together, these effects establish a mechanism for quantum Hall printing, in which light imprints density profiles into the electronic fluid.

We extended the hydrodynamic derivation of these effects to transverse fluids and developed an effective Floquet Hamiltonian approach in the complementary regime of high frequencies. We estimated the strength of

these effects in quantum Hall states in graphene and TMDs and found that these effects can generate local orbital magnetizations of 0.5–10 Bohr magnetons per carrier, which can be measured through the Faraday and Kerr rotations in pump-probe experiments. Our results point to new avenues for optical control and quantum printing in topological phases, and call for additional experimental exploration of near-field light-matter coupling in QH states, where our predicted density response is enhanced by the large field gradients. Finally, this last point motivates additional theoretical studies beyond the hydrodynamic gradient expansion. A microscopic formulation can address this limitation, as well as the role of magnetoplasmons associated with Landau-level transitions, the coupling to edge states, heating effects, and other dynamical processes that can be enhanced by the large momenta of near-field probes.

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Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

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