

Fluid Flow and Spatiotemporal Chaos in Chemically Active Emulsions

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We study phase-separating fluid mixtures as they demix in the presence of chemical reactions that maintain them away from thermodynamic equilibrium. We show that in such chemically active emulsions the interplay of chemical reactions, phase separation, and hydrodynamics effects complex self-organization and pattern formation that can give rise to spatiotemporal chaos. This chaotic dynamics, unlike in classical turbulence, is not due to fluid inertia—we analyze the system in the Stokes flow regime—and it is different from the *turbulence* of active nematics at low Reynolds number, for our fluid mixtures lack any orientational order. To explore the generic features of nonlinear dynamics in our system, we derive amplitude equations which we find to be identical to those obtained for Rayleigh-Benard convection with mean flow and stress-free conditions at the top and bottom plates. Chemically active emulsions possessing no internal order, we thus establish, can exhibit chaoticity that is driven by interfacial stresses in the fluid mixture.

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Active emulsions are phase-separating mixtures that are maintained away from thermodynamic equilibrium by energy input at small scales [1,2]. Unlike passive emulsions [3,4] which typically reach a thermodynamically equilibrated macrophase-separated state, active emulsions can display intriguing nonequilibrium dynamics such as suppression and even reversal of the ripening process [5,6]. Active emulsions also provide a general framework to study the principles of self-organization of multicomponent fluids inside living cells. Cells need to organize complex biochemical reactions within their confines [2,7,8] and do so by creating distinct chemical environments in the form of compartments. Some of these compartments, such as the mitochondria—“the powerhouse of the cell” [9], have membranes, while others, like centrosomes which control shape, division and motility of the cells [10], do not. Why then the compartments without membranes just not mix with their surroundings? This is because many membraneless organelles are, in fact, liquid droplets formed via phase separation inside cells [7,11]. Their liquidlike properties such as viscosity and surface tension have been measured

experimentally in several cases [12,13]. In contrast to droplets in passive emulsions though, droplets inside cells are present in a nonequilibrium environment where chemical reactions can actively regulate their size and composition.

The observation of liquid-liquid phase separation in biology provides a key reason to study the coupled physics of chemical reactions and phase separation far from thermodynamic equilibrium. This physics is also of significance beyond biology, for example, in self-assembly of polymer mixtures and reaction-driven phase transformations in electrochemistry [14–17]. Often in studying these chemically active emulsions, hydrodynamic flows have not been considered, or in other words, the viscosity of the phase-separating mixture taken infinite [18]. This begs the question: How does the viscosity of a chemically active phase-separating mixture affect its dynamics? After all, we know that in passive phase separation, viscosity alters the kinetics of the separation process and the morphology of its transient structures [3,19]. The motivation behind the present article is to explore the importance of fluid mechanics in phase-separating mixtures with out-of-equilibrium chemical reactions.

We study an incompressible binary fluid mixture with components *A* and *B* which undergo reversible chemical reactions $A \rightleftharpoons B$ with constant forward and backward reaction rates Γ_f and Γ_b , respectively. Having constant reaction rates in a phase-separating system is inconsistent with the thermodynamic condition of detailed balance [20–22] and implies an active system (see [23]). An important remark is

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already in place here. When fluid flows are neglected, as they are in many previous studies on active emulsions, the coarse-grained model of the binary active emulsion that we have just described is equivalent to the model of spinodal decomposition of diblock copolymers, which is an equilibrium problem [24–26]. This mapping onto an equilibrium system breaks down when fluid flows are accounted for, thereby highlighting the flows’ importance in effectuating the non-equilibrium dynamics.

In this Letter, we highlight the most intriguing aspects of our minimal system, a binary mixture with the simplest of chemical reactions. We show that active chemical reactions drive persistent flows. When these flows are weak, e.g. for low reactions rates and high viscosities, the system reaches a steady state that is either stationary or traveling with a constant drift velocity [27]. When the flows are strong, the system exhibits chaotic dynamics. This chaotic dynamics is novel in that it is neither due to fluid inertia nor like the turbulence of active nematics: we analyze the system in the Stokes flow regime and contra nematics, our system lacks any orientational order and is a *scalar* mixture.

Minimal model of an active emulsion with flows—The dynamics of passive phase separation of an incompressible binary fluid is governed by model H of Hohenberg and Halperin [1,28]. The model comprises of the Navier-Stokes equations for fluid momentum and mass conservation, and an equation for conserved order parameter ϕ . We choose ϕ as the local molecular composition of the mixture: $\phi \equiv \langle n_A - n_B \rangle / \langle n_A + n_B \rangle$, where $n_{A,B}$ represents the concentration of molecules of A,B [1]. We describe the phases using the Ginzburg-Landau free-energy [1,4] $F[\phi] \equiv \int (V(\phi) + \kappa(\nabla\phi)^2/2) dV$ with $V(\phi) \equiv -a\phi^2/2 + b\phi^4/4$, which has a double-well structure with minima at $\pm\phi_b$, where $\phi_b \equiv \sqrt{a/b}$, and a , b , and κ are positive constants. The equations that we study are

$$\frac{\partial\phi}{\partial t} + \mathbf{v} \cdot \nabla\phi = M\nabla^2\mu - R(\phi - \phi_s), \quad (1)$$

$$-\nabla p + \eta\nabla^2\mathbf{v} = \phi\nabla\mu \quad \text{with} \quad \nabla \cdot \mathbf{v} = 0, \quad (2)$$

where, expecting small inertial forces compared to viscous forces at the scale of biological cells, we have considered only the Stokes equations. The fluid components A and B here have, for simplicity, the same mass density ρ and viscosity η but an interfacial tension $\gamma \equiv \sqrt{8\kappa a^3/9b^2}$ between them. The field ϕ is convected by the flow velocity $\mathbf{v}$, and p is the Lagrange multiplier imposing incompressibility of the flow field, $\nabla \cdot \mathbf{v} = 0$. The chemical potential is defined as $\mu \equiv \delta F/\delta\phi$ and the mobility M is chosen to be constant. The force density $-\phi\nabla\mu$ equals the divergence of the equilibrium stress stemming from interfacial tension between the two fluids, in accordance with the Gibbs-Duhem relation. The chemical reactions make the system active and drive it towards a spatially

averaged value of $\langle\phi\rangle = \phi_s$, where $\phi_s \equiv (\Gamma_b - \Gamma_f)/R$ and $R \equiv \Gamma_f + \Gamma_b$ [29].

It is instructive to work with dimensionless quantities: we therefore scale lengths with $\ell \equiv w/\sqrt{2}$, where $w \equiv \sqrt{2\kappa/a}$ characterizes the interface width, time with $\tau \equiv w^2/D$ where $D \equiv 2aM$ is the effective diffusion constant at late times [4], and ϕ by ϕ_b to obtain

$$\frac{\partial\phi}{\partial t} + \mathbf{v} \cdot \nabla\phi = \nabla^2(-\phi + \phi^3 - \nabla^2\phi) - \mathcal{R}(\phi - m), \quad (3)$$

$$-\nabla p + \nabla^2\mathbf{v} = \mathcal{H}\phi\nabla(-\phi + \phi^3 - \nabla^2\phi), \quad (4)$$

along with $\nabla \cdot \mathbf{v} = 0$, where the variables now and henceforth represent scaled and dimensionless quantities. The dimensionless parameters $\mathcal{R} \equiv R\tau$, $\mathcal{H} \equiv 3\gamma\tau/2\eta w$, and $m \equiv \phi_s/\phi_b$ represent the strength of activity, of hydrodynamic flows, and of asymmetry in the composition, respectively, and control the dynamics of the system. We solve the equations numerically using the Fourier pseudo-spectral method in two spatial dimensions with a square box of side $256\sqrt{2}$ and periodic boundary conditions. The number of grid points in each spatial direction is initially 512, in other words two points per interface width, and 1024 at late times, including when quantifying chaos. The time integration is performed using the Runge-Kutta (2, 3, 3) implicit-explicit scheme [30]. As we have a cubic nonlinearity in the equations, we use 1/2-dealiasing (truncation) scheme to eliminate the aliasing error [31,32]. We initialize the simulations with a fully mixed state of $\phi = m$ plus a small random noise to destabilize it.

Results and discussion—Even as phase separation tends to demix the components, the chemical reactions mix them. As we stated above, the active emulsion model of (1) and (2) but without hydrodynamic flows, i.e., $\mathbf{v} = \mathbf{0}$, is equivalent to an equilibrium model for diblock copolymers with competing local and nonlocal interactions [25]. This system will eventually reach an equilibrium state. In two spatial dimensions, the global free-energy minima of the hydrodynamics-absent system can be (i) a homogeneous state with uniform concentration $\phi = m$, (ii) periodic stripe patterns, and (iii) periodic patterns with hexagonal symmetry [33]. Typically, small values of $|m|$ lead to stripes, large values close to the value $|m| = 1$ lead to uniform states. Hexagonal patterns are stable for intermediate values of m [33]. The characteristic size of the patterns is inversely proportional to the reaction rate [20,34,35]. When hydrodynamic flows are present, the active emulsion model can no longer be mapped onto the equilibrium problem due to the different definition of chemical potential in the active emulsion and diblock-copolymer models. Since the gradient of chemical potential enters the Stokes equation, the fluid flows in the two systems will be different (see [36,37] for the equilibrium model with hydrodynamics; cf. [38,39]).

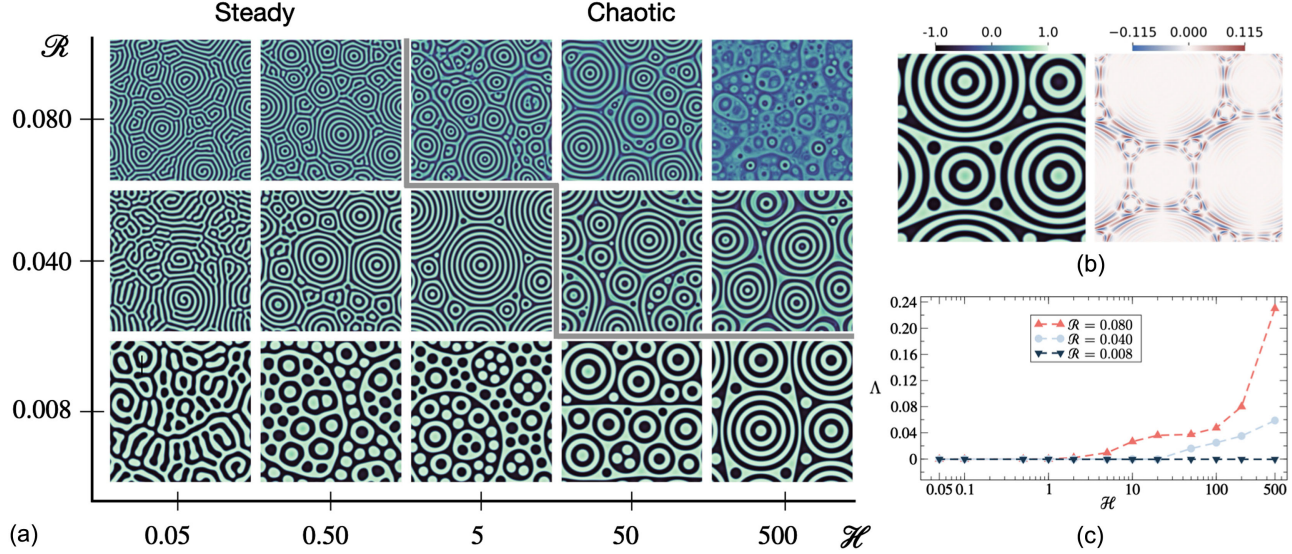


FIG. 1. For symmetric mixtures. (a) Patterns in ϕ after long times of $\mathcal{O}(10^5)$ for some values of $\mathcal{H}$ and $\mathcal{R}$. The gray line separates the steady patterns from chaotic ones. Color bar same as in (b). (b) A steady ϕ pattern (left) and the underlying vorticity (right) for $\mathcal{R} = 0.008$ and $\mathcal{H} = 200$. (c) Plot showing values of the maximal Lyapunov exponent for different $\mathcal{H}$ and $\mathcal{R}$. Contrary to the dynamics of Swift-Hohenberg equation with mean flow [41,42]—a system related to Rayleigh-Benard convection, we find the spiral defects to be stable whereas it is the target patterns that populate the chaotic state.

In order to explore the effect of fluid flows, we consider a wide range of viscosity values such that $\mathcal{H}$ ranges from 0.05 to 500, three different values for the strength of reactions $\mathcal{R}$, namely, 0.008, 0.04, 0.08, and two values of m , 0 and -0.2 corresponding to 50:50 and 40:60 component ratio, respectively (see End Matter for an estimate of $\mathcal{H}$ and $\mathcal{R}$ in physical systems.)

The presence of fluid flows brings in fundamentally new dynamics. For low values of $\mathcal{H}$ and $\mathcal{R}$, the system reaches a steady state with patterns whose morphology is dependent on these values. For symmetric mixtures ($m = 0$), in addition to the lamellae and stripes of the no-hydrodynamic case we now also find spiral and target patterns [see Fig. 1(a)], as was previously reported in [29,40] for chemically reactive binary mixtures. Fluid flows underpin these steady patterns, as can be seen in Fig. 1(b). If the hydrodynamics were to be turned off, the patterns would become unstable and decay towards the stripe pattern, which is the global energy minimizer in the absence of hydrodynamics in symmetric mixtures. Importantly, however, not every trajectory of the system leads to a steady state. At high $\mathcal{H}$ and $\mathcal{R}$, the patterns show chaotic dynamics, which we ascertain by calculating the maximal Lyapunov exponent Λ (see End Matter for the procedure and for the statistics of the kinetic energy). Positive values of Λ indicate unpredictability and in Fig. 1(c) we find these for high values of $\mathcal{H}$ and $\mathcal{R}$.

While the ensuing fluid flows are stronger in symmetric mixtures owing to the continuous nature of the phase domains [3], we also observe chaotic dynamics for asymmetric mixtures. In Fig. 2, we highlight the dynamics of a

40:60 mixture. Here, we initially see double phase separation [43] and shell formation in a few drops. These shells then grow adding layers onto themselves from the material of the surrounding drops, collide, collapse and reform, and so the process continues. Conversely, in the absence of hydrodynamics, the pattern would be one of arrested drops, ideally on a hexagonal lattice [33].

With the aim of understanding the generic features of the dynamics of our model, we derive amplitude equations which describe the effects of nonlinearities at large scales and long times on some simple steady-state patterns. In particular, we focus on stripes and hexagonal spots that appear in the absence of hydrodynamics as noted before,

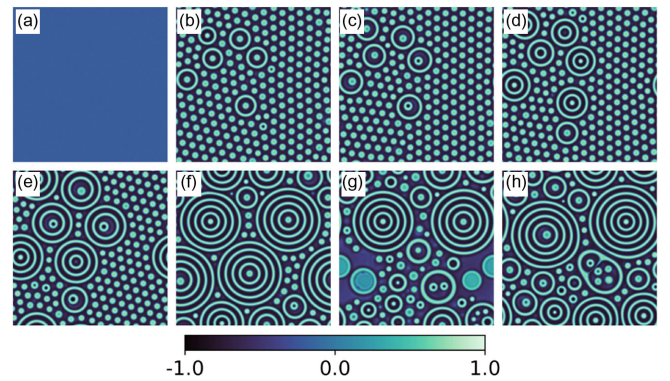


FIG. 2. Enroute to spatiotemporal chaos in a 40:60 mixture for $\mathcal{H} = 500$ and $\mathcal{R} = 0.04$. Snapshots of the ϕ field at (a) $t = 0$, (b) $t = 1250$, (c) $t = 2500$, (d) $t = 5000$, (e) $t = 10000$, (f) $t = 19500$, (g) $t = 19750$, (h) $t = 25000$. The Lyapunov exponent $\Lambda = 0.021$ at late times of $\mathcal{O}(10^6)$.

and show how hydrodynamics can affect the evolution of these near the threshold of pattern-forming instability.

We study the system near the threshold of instability which occurs when the reaction-diffusion length scale (dimensional) $\xi \equiv (D/R)^{1/2}$ becomes similar to the length scale (dimensional) w characterizing the interface width of the phase-separating domains. We proceed by defining $u \equiv \phi - m$ in (3) and (4), and use the stream function ψ to write $\mathbf{v}(x, y) = \nabla\psi \times \mathbf{e}_z$, exploiting the two-dimensional spatial nature of the problem.

We first discuss the linear stability of the uniform state, $u = 0$. For a perturbation of the form $u = \hat{u} \exp(i\mathbf{k} \cdot \mathbf{x} + \lambda t)$, the dispersion relation is

$$\lambda(k) = (1-3m^2)k^2 - k^4 - \mathcal{R}. \quad (5)$$

The wave vector $\mathbf{k}_c$ for which $d\lambda/dk = 0$, and therefore $k_c^2 = (1-3m^2)/2$, gives the fastest growing mode provided that $|m| < 1/\sqrt{3}$. The instability of this mode occurs at $\mathcal{R} = \mathcal{R}_c$ where $\mathcal{R}_c = k_c^4$, so that $\lambda = -(k^2 - k_c^2)^2 + \mathcal{R}_c - \mathcal{R}$. We study the dynamics near this bifurcation point, i.e., when $\mathcal{R}_c - \mathcal{R} = r\epsilon^2$, where ϵ is $\mathcal{O}(1)$ and r is the control parameter [44,45].

We then proceed systematically to derive the amplitude equations [44,46]. Looking first at the slow dynamics of stripe patterns aligned perpendicular to, say the x axis, we write

$$\begin{aligned} u(x, y, t) &= \epsilon A(\epsilon x, \epsilon^{1/2}y, \epsilon^2 t) e^{ik_c x} + \text{c.c.} + \mathcal{O}(\epsilon^2), \\ \psi(x, y, t) &= \epsilon^{3/2} B(\epsilon x, \epsilon^{1/2}y, \epsilon^2 t) + \mathcal{O}(\epsilon^{5/2}), \end{aligned} \quad (6)$$

where $A(X, Y, T)$ is complex and $B(X, Y, T)$ is real and $X = \epsilon x$, $Y = \sqrt{\epsilon}y$, $T = \epsilon^2 t$ are the slow scales. The dynamics of the amplitude A at the lowest order in ϵ is then given by

$$\begin{aligned} \frac{\partial A}{\partial T} &= rA - 3A|A|^2 k_c^2 + 4k_c^2 \left(\frac{\partial}{\partial X} - \frac{i}{2k_c} \frac{\partial^2}{\partial Y^2} \right)^2 A - ik_c \frac{\partial B}{\partial Y} A, \\ \frac{\partial^4 B}{\partial Y^4} &= 2k_c^2 \mathcal{H} \left[\bar{A} \left(\frac{\partial}{\partial X} - \frac{i}{2k_c} \frac{\partial^2}{\partial Y^2} \right) A \right] + \text{c.c.}, \end{aligned} \quad (7)$$

where $\bar{A}$ is complex conjugate of A . These equations (7) are identical in form to those obtained for the stability of rolls at low Prandtl number Rayleigh-Benard convection with stress-free boundary conditions [44,47].

Similarly, for hexagon patterns at leading order we write $u(\mathbf{x}, t) = \sum_{n=1}^3 \epsilon A_n(\epsilon \mathbf{x}, \epsilon^2 t) e^{i\mathbf{k}_n \cdot \mathbf{x}} + \text{c.c.}$ with $\psi(\mathbf{x}, t) = \epsilon B(\epsilon \mathbf{x}, \epsilon^2 t)$ where $\mathbf{k}_1 = k_c \mathbf{e}_x$, $\mathbf{k}_2 = k_c(-\mathbf{e}_x + \sqrt{3}\mathbf{e}_y)/2$ and $\mathbf{k}_3 = k_c(-\mathbf{e}_x - \sqrt{3}\mathbf{e}_y)/2$. For a systematic derivation now, however, we must substitute $3m = \epsilon s$ and $\mathcal{H} = \epsilon h$, where s and h are constants [48,49]. The resulting amplitude equations are

$$\begin{aligned} \frac{\partial A_n}{\partial T} &= rA_n + 4(\mathbf{k}_n \cdot \nabla)^2 A_n - 2sk_c^2 [\bar{A}_{n-1} \bar{A}_{n+1}] \\ &\quad - 3k_c^2 [|A_n|^2 + 2|A_{n+1}|^2 + 2|A_{n-1}|^2] A_n - iA_n \boldsymbol{\tau}_n \cdot \nabla B, \\ \nabla^4 B &= \sum_{l=1}^3 2h[(\boldsymbol{\tau}_l \cdot \nabla)(\mathbf{k}_l \cdot \nabla)|A_l|^2], \end{aligned} \quad (8)$$

where n is cyclically permuted. ∇ acts on (X, Y) , and $\boldsymbol{\tau}_l$ are obtained from $\mathbf{k}_l$ by an anticlockwise rotation of $\pi/2$. Again, Eq. (8) are of the same form as those obtained for the evolution of hexagon patterns in non-Boussinesq Rayleigh-Benard convection with mean flow [49].

The coupled amplitude equations signify that slow spatial modulations of concentration patterns can drive fluid flow which, due to the fluid's incompressibility, creates a long-range coupling between different regions of the pattern [50]. These amplitude equations, unlike the Newell-Whitehead-Segel equation that governs the dynamics in the absence of hydrodynamic flows [45,51,52], do not have a Lyapunov functional and therefore can admit oscillatory solutions [44]. The identical nature of Ginzburg-Landau type equations in Rayleigh-Benard and chemically active emulsions suggests that the latter also offer rich dynamical behavior, and this behavior can be further investigated through insights developed for the well-studied former problem.

Conclusion and outlook—We studied binary fluid phase separation in the presence of active chemical reactions, and found that the reactions drive persistent fluid flows. When these flows are weak, the system reaches a steady state; when strong, spatiotemporal chaos emerges. By deriving the amplitude equations, we showed that the weakly non-linear dynamics of our system is related to Rayleigh-Benard convection. Beyond the narrow confines set by the amplitude equations [50], however, the dynamics of the two systems differs. In the numerical simulations of the full governing equations, we did not observe the typical spiral defect chaos of the convection problem [41,42,53]: instead the chaotic states were dominated by target patterns. We note that similar patterns and an unsteady dynamics were reported for phase-field crystal model (a conserved version of the Swift-Hohenberg equation) with mean flow [54,55], where unlike in our system a neutrally stable Goldstone mode is present (see [56]). Binary fluid mixtures de-mixing under gravity and a continuously ramped temperature that creates a buoyancy effect have also shown oscillatory dynamics [57]. The similarities and differences of these systems inspire our future work.

It is important that we contrast our system with others that are similar. Previously, in [58,59] convective Cahn-Hilliard models of the form $\phi_t + \mathbf{v} \cdot \nabla \phi = \nabla^2 \mu$ were considered, where the velocity $\mathbf{v}$ was assumed to be $G\nabla \phi$ with the constant G representing the intensity of a driving force. The authors of those studies found that while for weak driving, $G \rightarrow 0$, the system showed coarsening

dynamics, as $G \rightarrow \infty$ chaotic structures typical of Kuramoto-Sivashinsky equation emerged. The convected Cahn-Hilliard models are, in fact, based on model B of Hohenberg and Halperin [28] with an additional flux term due to an external field such as gravity, magnetic or electric field [60,61]. In the present Letter, we do not assume a form of velocity nor do we include any external field but instead solve the fully coupled Stokes-Cahn-Hilliard equations with chemical reactions—between the phase-separating constituents themselves—which generate spontaneous flows by internal activity. Chemically driven hydrodynamic flows can also originate in non-phase-separating systems, for example, in the canonical Belousov-Zhabotinskii reaction, due to local changes in the properties of the system, e.g., viscosity, density, and surface tension [62–67]. Any oscillatory dynamics observed in these systems, however, is typically either due to the complex reactions in the system or because of the already present classical hydrodynamic instabilities. Only recently, it was shown that the simple chemical reaction $A + B \rightarrow C$ when coupled to the Navier-Stokes equations can generate oscillations in the system through buoyancy- and surface-tension-driven forces [68]. In contrast, in our phase-separating system, reactions constantly drive fluid flows not through buoyancy or solutal Marangoni stresses but Laplace pressure gradients via the $\phi \nabla \mu$ term in the Stokes equation [1]. These flows can lead to chaotic dynamics even in Stokes flow, without the presence of nonlinear inertial terms. Additionally, our model is still different, both in spirit and form, to the active phase-separating model Active Model H that presents a continuum theory for self-propelled particles [69]. In that model, although activity manifests itself at interfaces, as in our model, it is added phenomenologically through terms that break time-reversal symmetry in chemical potential and fluid stresses and are different to those that emerge here due to chemical activity.

In the end, it is worth emphasizing that we have studied only a minimal model of active emulsions. We did not consider fluid inertia along with any differences in the density and viscosity of the components, all of which can generate richer dynamics [57,70–72]. We ignored the Langevin noise of the original model H [1,28], when noise in active emulsions may play an important role. Moreover, many real systems are multicomponent systems which behave fundamentally differently from binary mixtures [73]. While biological emulsions are often rheologically complex [12,74], showing properties such as viscoelasticity, which brings in a new time scale to the problem [75], as a first step, we restricted our analysis to only Newtonian fluids. Finally, we studied active emulsions in two spatial dimensions but their dynamics may differ considerably in three dimensions due to a larger variety of possible equilibrium structures of equation (3) without hydrodynamics, e.g., gyroids, Fddd, BCC [37], and owing to the presence of the Rayleigh instability of tubes, which is absent in two dimensions [76].

We believe that this Letter will stimulate further exploration of the hydrodynamics of active emulsions.

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Data availability—The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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- [1] M. E. Cates and E. Tjhung, Theories of binary fluid mixtures: From phase-separation kinetics to active emulsions, *J. Fluid Mech.* **836**, P1 (2018).
 - [2] C. A. Weber, D. Zwicker, F. Jülicher, and C. F. Lee, Physics of active emulsions, *Rep. Prog. Phys.* **82**, 064601 (2019).
 - [3] E. D. Siggia, Late stages of spinodal decomposition in binary mixtures, *Phys. Rev. A* **20**, 595 (1979).
 - [4] A. J. Bray, Theory of phase-ordering kinetics, *Adv. Phys.* **51**, 481 (2002).
 - [5] D. Zwicker, A. A. Hyman, and F. Jülicher, Suppression of Ostwald ripening in active emulsions, *Phys. Rev. E* **92**, 012317 (2015).
 - [6] E. Tjhung, C. Nardini, and M. E. Cates, Cluster phases and bubbly phase separation in active fluids: Reversal of the Ostwald process, *Phys. Rev. X* **8**, 031080 (2018).
 - [7] A. A. Hyman, C. A. Weber, and F. Jülicher, Liquid-liquid phase separation in biology, *Annu. Rev. Cell Dev. Biol.* **30**, 39 (2014).
 - [8] J. Berry, C. P. Brangwynne, and M. Haataja, Physical principles of intracellular organization via active and passive phase transitions, *Rep. Prog. Phys.* **81**, 046601 (2018).
 - [9] P. Siekevitz, Powerhouse of the cell, *Sci. Am.* **197**, No. 1, 131 (1957).
 - [10] D. M. Glover, C. Gonzalez, and J. W. Raff, The centrosome, *Sci. Am.* **268**, No. 6, 62 (1993).
 - [11] C. P. Brangwynne, C. R. Eckmann, D. S. Courson, A. Rybarska, C. Hoege, J. Gharakhani, F. Jülicher, and A. A. Hyman, Germline p granules are liquid droplets that localize by controlled dissolution/condensation, *Science* **324**, 1729 (2009).
 - [12] L. Jawerth, E. Fischer-Friedrich, S. Saha, J. Wang, T. Franzmann, X. Zhang, J. Sachweh, M. Ruer, M. Ijavi, S. Saha, J. Mahamid, A. A. Hyman, and F. Jülicher, Protein condensates as aging Maxwell fluids, *Science* **370**, 1317 (2020).
 - [13] H. Wang, F. M. Kelley, D. Milovanovic, B. S. Schuster, and Z. Shi, Surface tension and viscosity of protein condensates quantified by micropipette aspiration, *Biophys. Rep.* **1**, 100011 (2021).
 - [14] A. C. Balazs, Modeling self-assembly and phase behavior in complex mixtures, *Annu. Rev. Phys. Chem.* **58**, 211 (2007).
 - [15] M. Z. Bazant, Theory of chemical kinetics and charge transfer based on nonequilibrium thermodynamics, *Acc. Chem. Res.* **46**, 1144 (2013).

- [16] M. Z. Bazant, Thermodynamic stability of driven open systems and control of phase separation by electro-autocatalysis, *Faraday Discuss.* **199**, 423 (2017).
- [17] Q. Tran-Cong-Miyata and H. Nakanishi, Phase separation of polymer mixtures driven by photochemical reactions: Current status and perspectives, *Polym. Int.* **66**, 213 (2017).
- [18] D. Zwicker, The intertwined physics of active chemical reactions and phase separation, *Curr. Opin. Colloid Interface Sci.* **61**, 101606 (2022).
- [19] C. Datt, S. P. Thampi, and R. Govindarajan, Morphological evolution of domains in spinodal decomposition, *Phys. Rev. E* **91**, 010101(R) (2015).
- [20] S. C. Glotzer, D. Stauffer, and N. Jan, Monte Carlo simulations of phase separation in chemically reactive binary mixtures, *Phys. Rev. Lett.* **72**, 4109 (1994).
- [21] R. Lefever, D. Carati, and N. Hassani, Comment on “Monte Carlo simulations of phase separation in chemically reactive binary mixtures,” *Phys. Rev. Lett.* **75**, 1674 (1995).
- [22] S. C. Glotzer, D. Stauffer, and N. Jan, Glotzer, Stauffer, and Jan reply, *Phys. Rev. Lett.* **75**, 1675 (1995).
- [23] M. te Vrugt, B. Liebchen, and M. E. Cates, What exactly is ‘active matter’?, [arXiv:2507.21621](https://arxiv.org/abs/2507.21621).
- [24] L. Leibler, Theory of microphase separation in block copolymers, *Macromolecules* **13**, 1602 (1980).
- [25] S. C. Glotzer and A. Coniglio, Self-consistent solution of phase separation with competing interactions, *Phys. Rev. E* **50**, 4241 (1994).
- [26] C. Sagui and R. C. Desai, Kinetics of phase separation in two-dimensional systems with competing interactions, *Phys. Rev. E* **49**, 2225 (1994).
- [27] R. E. Goldstein, G. H. Gunaratne, L. Gil, and P. Coulet, Hydrodynamic and interfacial patterns with broken space-time symmetry, *Phys. Rev. A* **43**, 6700 (1991).
- [28] P. C. Hohenberg and B. I. Halperin, Theory of dynamic critical phenomena, *Rev. Mod. Phys.* **49**, 435 (1977).
- [29] Y. Huo, X. Jiang, H. Zhang, and Y. Yang, Hydrodynamic effects on phase separation of binary mixtures with reversible chemical reaction, *J. Chem. Phys.* **118**, 9830 (2003).
- [30] U. M. Ascher, S. J. Ruuth, and R. J. Spiteri, Implicit-explicit Runge-Kutta methods for time-dependent partial differential equations, *Appl. Numer. Math.* **25**, 151 (1997).
- [31] N. B. Padhan, K. V. Kiran, and R. Pandit, Novel turbulence and coarsening arrest in active-scalar fluids, *Soft Matter* **20**, 3620 (2024).
- [32] S. Derevyanko, The $(n + 1)/2$ rule for dealiasing in the split-step Fourier methods for n -wave interactions, *IEEE Photonics Technol. Lett.* **20**, 1929 (2008).
- [33] R. Choksi, M. Maras, and J. F. Williams, 2D phase diagram for minimizers of a Cahn–Hilliard functional with long-range interactions, *SIAM J. Appl. Dyn. Syst.* **10**, 1344 (2011).
- [34] S. C. Glotzer, E. A. Di Marzio, and M. Muthukumar, Reaction-controlled morphology of phase-separating mixtures, *Phys. Rev. Lett.* **74**, 2034 (1995).
- [35] F. Liu and N. Goldenfeld, Dynamics of phase separation in block copolymer melts, *Phys. Rev. A* **39**, 4805 (1989).
- [36] Y. Shiwa, Hydrodynamic interactions in microphase separation of block copolymer films: Stability and spirals, *Phys. Rev. E* **61**, 2924 (2000).
- [37] T. Ohta, Morphological transitions in microphase-separated diblock copolymers, in *Kinetics of Phase Transitions* (CRC Press, Boca Raton, 2009).
- [38] G. Gonnella, E. Orlandini, and J. M. Yeomans, Spinodal decomposition to a lamellar phase: Effects of hydrodynamic flow, *Phys. Rev. Lett.* **78**, 1695 (1997).
- [39] A. Xu, G. Gonnella, A. Lamura, G. Amati, and F. Massaioli, Scaling and hydrodynamic effects in lamellar ordering, *Europhys. Lett.* **71**, 651 (2005).
- [40] K. Furtado and J. M. Yeomans, Lattice Boltzmann simulations of phase separation in chemically reactive binary fluids, *Phys. Rev. E* **73**, 066124 (2006).
- [41] A. Karimi, Z.-F. Huang, and M. R. Paul, Exploring spiral defect chaos in generalized Swift-Hohenberg models with mean flow, *Phys. Rev. E* **84**, 046215 (2011).
- [42] K.-H. Chiam, M. R. Paul, M. C. Cross, and H. S. Greenside, Mean flow and spiral defect chaos in Rayleigh–Bénard convection, *Phys. Rev. E* **67**, 056206 (2003).
- [43] H. Tanaka, Hydrodynamic interface quench effects on spinodal decomposition for symmetric binary fluid mixtures, *Phys. Rev. E* **51**, 1313 (1995).
- [44] R. B. Hoyle, *Pattern Formation: An Introduction to Methods* (Cambridge University Press, Cambridge, England, 2006).
- [45] Y. Shiwa, The amplitude and phase-diffusion equations for lamellar patterns in block copolymers, *Phys. Lett. A* **228**, 279 (1997).
- [46] G. H. Gunaratne, Q. Ouyang, and H. L. Swinney, Pattern formation in the presence of symmetries, *Phys. Rev. E* **50**, 2802 (1994).
- [47] E. D. Siggia and A. Zippelius, Pattern selection in Rayleigh–Bénard convection near threshold, *Phys. Rev. Lett.* **47**, 835 (1981).
- [48] P. Matthews, Hexagonal patterns in finite domains, *Physica (Amsterdam)* **116D**, 81 (1998).
- [49] Y.-N. Young and H. Riecke, Mean flow in hexagonal convection: Stability and nonlinear dynamics, *Physica D (Amsterdam)* **163**, 166 (2002).
- [50] M. Cross and H. Greenside, *Pattern Formation and Dynamics in Nonequilibrium Systems* (Cambridge University Press, Cambridge, England, 2009).
- [51] A. C. Newell and J. A. Whitehead, Finite bandwidth, finite amplitude convection, *J. Fluid Mech.* **38**, 279 (1969).
- [52] L. A. Segel, Distant side-walls cause slow amplitude modulation of cellular convection, *J. Fluid Mech.* **38**, 203 (1969).
- [53] E. Bodenschatz, W. Pesch, and G. Ahlers, Recent developments in Rayleigh–Bénard convection, *Annu. Rev. Fluid Mech.* **32**, 709 (2000).
- [54] Y. Shiwa, Hydrodynamic coarsening in striped pattern formation with a conservation law, *Phys. Rev. E* **72**, 016204 (2005).
- [55] Y. Shiwa, Spirals, hexagons, and all that through hydrodynamic coupling with a zero mode, *AIP Conf. Proc.* **832**, 205 (2006).
- [56] H. Ohnogi and Y. Shiwa, Nucleation, growth, and coarsening of crystalline domains in order-order transitions between lamellar and hexagonal phases, *Phys. Rev. E* **84**, 011611 (2011).
- [57] T. C. B. McLeish, M. E. Cates, J. S. Higgins, P. D. Olmsted, M. E. Cates, J. Vollmer, A. Wagner, and D. Vollmer, Phase

- separation in binary fluid mixtures with continuously ramped temperature, *Phil. Trans. R. Soc. A* **361**, 793 (2003).
- [58] A. A. Golovin, A. A. Nepomnyashchy, S. H. Davis, and M. A. Zaks, Convective Cahn-Hilliard models: From coarsening to roughening, *Phys. Rev. Lett.* **86**, 1550 (2001).
- [59] A. A. Golovin and L. M. Pismen, Dynamic phase separation: From coarsening to turbulence via structure formation, *Chaos* **14**, 845 (2004).
- [60] K.-T. Leung, Theory on morphological instability in driven systems, *J. Stat. Phys.* **61**, 345 (1990).
- [61] C. Yeung, T. Rogers, A. Hernandez-Machado, and D. Jasnow, Phase separation dynamics in driven diffusive systems, *J. Stat. Phys.* **66**, 1071 (1992).
- [62] H. Miike, S. C. Müller, and B. Hess, Oscillatory deformation of chemical waves induced by surface flow, *Phys. Rev. Lett.* **61**, 2109 (1988).
- [63] H. Miike, S. C. Müller, and B. Hess, Hydrodynamic flows traveling with chemical waves, *Phys. Lett. A* **141**, 25 (1989).
- [64] M. Diewald and H. R. Brand, Chemically driven convection can stabilize turing patterns, *Phys. Rev. E* **51**, R5200 (1995).
- [65] H. Kitahata, R. Aihara, N. Magome, and K. Yoshikawa, Convective and periodic motion driven by a chemical wave, *J. Chem. Phys.* **116**, 5666 (2002).
- [66] A. De Wit, Chemo-hydrodynamic patterns and instabilities, *Annu. Rev. Fluid Mech.* **52**, 531 (2020).
- [67] A. Huffman and H. Shum, Boundary-bound reactions: Pattern formation with and without hydrodynamics, *Phys. Rev. E* **108**, 055103 (2023).
- [68] M. A. Budroni, V. Upadhyay, and L. Rongy, Making a simple $A + B \rightarrow C$ reaction oscillate by coupling to hydrodynamic effect, *Phys. Rev. Lett.* **122**, 244502 (2019).
- [69] A. Tiribocchi, R. Wittkowski, D. Marenduzzo, and M. E. Cates, Active model H: Scalar active matter in a momentum-conserving fluid, *Phys. Rev. Lett.* **115**, 188302 (2015).
- [70] J. Lowengrub and L. Truskinovsky, Quasi-incompressible Cahn-Hilliard fluids and topological transitions, *Proc. R. Soc. A* **454**, 2617 (1998).
- [71] A. Onuki, Domain growth and rheology in phase-separating binary mixtures with viscosity difference, *Europhys. Lett.* **28**, 175 (1994).
- [72] Y. Huo, H. Zhang, and Y. Yang, Effects of reversible chemical reaction on morphology and domain growth of phase separating binary mixtures with viscosity difference, *Macromol. Theory Simul.* **13**, 280 (2004).
- [73] J. Bauermann, S. Laha, P. M. McCall, F. Jülicher, and C. A. Weber, Chemical kinetics and mass action in coexisting phases, *J. Am. Chem. Soc.* **144**, 19294 (2022).
- [74] H. Tanaka, Viscoelastic phase separation in biological cells, *Commun. Phys.* **5**, 167 (2022).
- [75] H. Tanaka, Viscoelastic phase separation, *J. Phys. Condens. Matter* **12**, R207 (2000).
- [76] M. San Miguel, M. Grant, and J. D. Gunton, Phase separation in two-dimensional binary fluids, *Phys. Rev. A* **31**, 1001 (1985).
- [77] D. Zwicker, R. Seyboldt, C. A. Weber, A. A. Hyman, and F. Jülicher, Growth and division of active droplets provides a model for protocells, *Nat. Phys.* **13**, 408 (2017).
- [78] A. Martínez-Calvo, J. Zhou, Y. Zhang, and N. S. Wingreen, Sticky enzymes: Increased metabolic efficiency via substrate-dependent enzyme clustering, *PRX Life* **3**, 033011 (2025).
- [79] B. Sorkin and N. S. Wingreen, Accelerated Ostwald ripening by chemical activity, *J. Am. Chem. Soc.* **147**, 44673 (2025).
- [80] H. L. Snyder, P. Meakin, and S. Reich, Dynamical aspects of phase separation in polymer blends, *Macromolecules* **16**, 757 (1983).
- [81] T. Hashimoto, M. Itakura, and H. Hasegawa, Late stage spinodal decomposition of a binary polymer mixture. I. Critical test of dynamical scaling on scattering function, *J. Chem. Phys.* **85**, 6118 (1986).
- [82] T. Hashimoto, M. Itakura, and N. Shimidzu, Late stage spinodal decomposition of a binary polymer mixture. II. Scaling analyses on $Q_m(\tau)$ and $I_m(\tau)$, *J. Chem. Phys.* **85**, 6773 (1986).
- [83] T. Ohta, O. Urakawa, and Q. Tran-Cong, Phase separation of binary polymer blends driven by photoisomerization: An example for a wavelength-selection process in polymers, *Macromolecules* **31**, 6845 (1998).
- [84] J. K. Kim and H. W. Son, The rheological properties of polystyrene/poly(vinylmethylether) blend near the critical region and in the homogenous region, *Polymer* **40**, 6789 (1999).
- [85] G. Benettin, L. Galgani, and J.-M. Strelcyn, Kolmogorov entropy and numerical experiments, *Phys. Rev. A* **14**, 2338 (1976).
- [86] S. Mukherjee, R. K. Singh, M. James, and S. S. Ray, Intermittency, fluctuations and maximal chaos in an emergent universal state of active turbulence, *Nat. Phys.* **19**, 891 (2023).
- [87] R. Alert, J. Casademunt, and J.-F. Joanny, Active turbulence, *Annu. Rev. Condens. Matter Phys.* **13**, 143 (2022).
- [88] P. Davidson, *Turbulence: An Introduction for Scientists and Engineers* (Oxford University Press, Oxford, 2015).
- [89] V. M. Kendon, M. E. Cates, I. Pagonabarraga, J.-C. Desplat, and P. Bladon, Inertial effects in three-dimensional spinodal decomposition of a symmetric binary fluid mixture: A lattice Boltzmann study, *J. Fluid Mech.* **440**, 147 (2001).

End Matter

On the values of the parameters $\mathcal{R}$ and $\mathcal{H}$ —We first note that our motivation to study a minimal model is to reveal generic principles that govern a broader class of systems. Active emulsions, for example, biomolecular condensates and polymer blends driven by chemical

reactions, can be rheologically complex, where the Newtonian fluid description of our model is insufficient, with reactions that may not follow first-order kinetics, unlike in our minimal model. Measuring physical quantities such as interface width and diffusion in such

systems can also be challenging. The following values of $\mathcal{R}$ and $\mathcal{H}$ are, therefore, only an order-of-magnitude estimate.

For biomolecular condensates, the interface width $w \sim 10$ nm and protein diffusivities $D \sim 10^{-13}$ – 10^{-10} m²/s. With reaction rates $R \sim 10^{-3}$ – 10^{-1} s⁻¹, one obtains $\mathcal{R} \sim 10^{-9}$ – 10^{-4} (see [77–79] and references therein). The condensates' viscosity is $\sim 10^{-1}$ – 10^3 Pa s and surface tension $\gamma \sim 10^{-4}$ – 10^{-1} mN m⁻¹ [13], which gives $\mathcal{H} \sim 10^{-8}$ – 10^2 .

The phase separation of polystyrene (PS) and poly (vinyl methyl ether) (PVME) driven by photoisomerization reaction may also be described as an active emulsion. For shallow temperature jumps about the critical point, $w \sim 10$ – 100 nm with $D \sim 10^{-17}$ – 10^{-15} m²/s, and τ is reported $\sim 10^0$ – 10^2 s [80–82], cf. [75]. A photoisomerization reaction rate of $R \sim 10^{-3}$ s⁻¹ [83] then gives $\mathcal{R} \sim 10^{-3}$ – 10^{-1} . The interfacial tension between PS and PVME for shallow jumps is $\sim 10^0$ – 10^1 mN m⁻¹ [80] and viscosity of the homogeneous solution—an approximation for shallow jumps—is $\sim 10^3$ – 10^5 Pa s [84]. These values lead to $\mathcal{H} \sim 10^0$ – 10^4 . Note that the variation in the magnitude of these physical quantities arises from their composition and temperature dependence.

In light of the above estimates, we vary $\mathcal{R}$ as 0.008, 0.004, and 0.08, and $\mathcal{H}$ between 0.05 to 500 in our simulations.

Calculating the Lyapunov exponent—We use the procedure described in [85] to calculate the Lyapunov exponent. We begin the procedure after pattern statistics have fully developed, and denote with $\phi(0)$ the state of the concentration field at the procedure's start; $\phi(t)$ then represents the state after time t , $\phi(t) = \mathcal{Q}'[\phi(0)]$, where $\mathcal{Q}'$ is the time- t map due to the dynamics of the system. Next, to the initial state $\phi(0)$, we add a random perturbation $\delta\phi(0)$ with $\|\delta\phi(0)\| = \sigma\|\phi(0)\|$, where σ is a small real positive value and $\|\cdot\|$ represents the L_2 norm. We then determine $\phi(t)$ and $\phi(t) + \delta\phi(t)$ in parallel in time. After every time span of duration τ_Λ , we rescale the perturbation such that $\|\delta\phi(n\tau_\Lambda)\| = \sigma\|\phi(n\tau_\Lambda)\|$, where $n = 1, 2, 3, \dots$. The maximal Lyapunov exponent is then estimated as

$$\Lambda \approx \frac{1}{N_\Lambda \tau_\Lambda} \sum_{n=1}^{N_\Lambda} \ln \frac{\|\mathcal{Q}^{\tau_\Lambda}[\phi(n\tau_\Lambda) + \delta\phi(n\tau_\Lambda)] - \mathcal{Q}^{\tau_\Lambda}[\phi(n\tau_\Lambda)]\|}{\|\delta\phi(n\tau_\Lambda)\|}. \quad (\text{B1})$$

We have used $\tau_\Lambda = 0.25$ and $\sigma = 10^{-4}$ and verified that these parameters had no significant bearing on the results by increasing and decreasing σ by an order of magnitude, and halving and doubling the value of τ_Λ . N_Λ was chosen as 2.4×10^5 and in Fig. 1(c), we plot the average values and error bars of Λ for the last 1.2×10^5 rescalings. We did not see any long time trends in the values of the

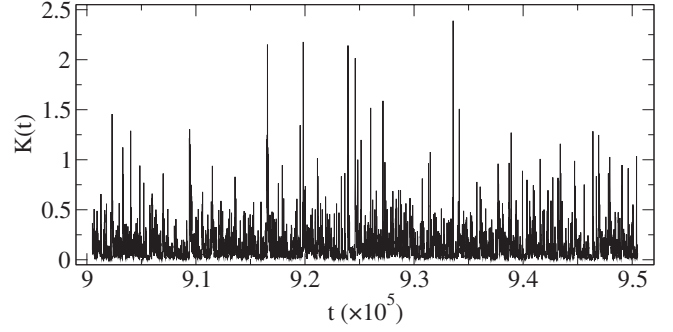


FIG. 3. Spatially averaged kinetic energy as a function of time, after long times, for a symmetric mixture with $\mathcal{H} = 50$ and $\mathcal{R} = 0.080$. The large variation suggests intermittency. $K(t) \equiv 0.5\langle|\mathbf{v}(\mathbf{x})|^2\rangle$ where angular brackets denote spatial average. Quantities in both the x axis and the y axis have been made dimensionless.

exponent, and the error bars in the figure are smaller than the symbol size, indicating convergence of values.

On intermittency and energy spectrum—We see signs of intermittency in the chaotic dynamics of our system. The unpredictability in the trajectory of the system, quantified by the Lyapunov exponent, arises as the target patterns collide and burst (see Fig. 2); this happens inhomogeneously in space and time, and gives rise to large fluctuations in the observables in the system: Fig. 3 shows the variations in the spatially averaged kinetic energy for a chaotic symmetric mixture. We have, however, not attempted to quantify the intermittent behavior (cf. [86]); this will be an important next step. For completeness, we report in Fig. 4 the kinetic energy

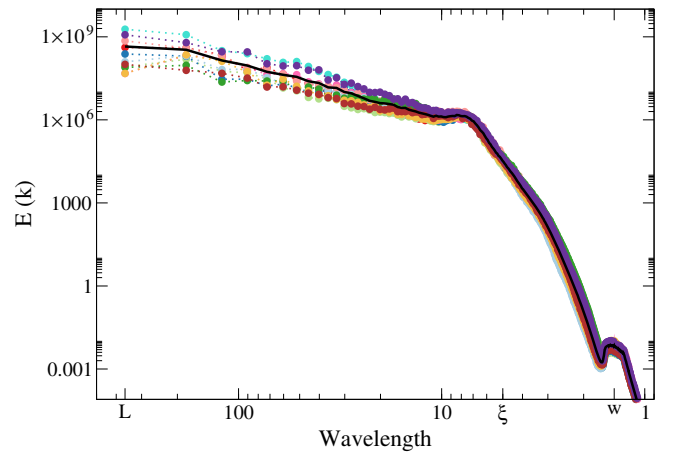


FIG. 4. Kinetic energy spectrum for a symmetric mixture with $\mathcal{H} = 50$ and $\mathcal{R} = 0.080$. The unit for the y axis is arbitrary while wavelengths on the x axis have been made dimensionless. L indicates the simulation box size. The dots, differently colored, correspond to eleven time snapshots taken at equal intervals from the time period corresponding to that in Fig. 3. The solid black line represents the mean. $E(k) \equiv k\langle|\mathbf{v}(\mathbf{k})|^2\rangle$, where angular brackets denote average over mode $|\mathbf{k}| \equiv k$ in spectral space [89].

spectrum for the same system; here, it will be instructive to evaluate the energy and enstrophy transfer across scales and compare them with other instances of active turbulence [87]. It will also be interesting to study the chaotic dynamics in three spatial dimensions, where topological structures distinct from the two-dimensional

case may emerge, and see how the statistical properties and scaling laws depend on spatial dimensions, although we note that our model, which describes the dynamics in the Stokes regimes, lacks the advective nonlinearity of classical turbulence which differs markedly in two and three spatial dimensions [88].