

Isotensor $\pi\pi\pi$ Scattering with a ρ -Resonant Subsystem from QCDRaúl A. Briceño^{1,2,*}, Maxwell T. Hansen^{3,†}, Andrew W. Jackura^{4,‡},
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This Letter presents a lattice quantum chromodynamics (QCD) determination of $\pi\pi\pi$ scattering amplitudes for the isospin-2 channel with angular momentum and parity $J^P = 1^+$. The calculation is performed using unphysically heavy light-quark masses, corresponding to a pion mass of $m_\pi \approx 400$ MeV, for which the ρ meson manifests as a narrow resonance. The analysis employs a previously developed formalism that nonperturbatively relates the finite-volume spectra of two- and three-pion systems to their infinite-volume scattering amplitudes. We combine earlier lattice QCD results for $\pi\pi$ systems with isospins 1 and 2 with new determinations of the isospin-2 three-pion finite-volume spectrum, obtained for three cubic, periodic lattice volumes with box length ranging from $\approx 4/m_\pi$ to $6/m_\pi$. These combined data constrain the low-lying partial waves of the infinite-volume three-pion K matrix, from which we solve a set of coupled integral equations to extract the corresponding scattering amplitudes.

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Introduction—Although quantum chromodynamics (QCD) is formulated in terms of quarks and gluons, confinement prevents experiments from observing these constituents directly. One instead detects composite hadrons, such as pions, protons and neutrons, and the properties of the underlying theory are inferred from their phenomenology. Precise, first-principles predictions of multi-hadron amplitudes are thus essential to advancing our understanding of nuclear and particle physics.

As an example, low-energy nuclear physics would benefit immensely by applying QCD determinations of multinucleon amplitudes to reduce uncertainties in the

two-neutron force (currently $\sim 10\%$) and even larger uncertainties in three-body forces, which currently hinder the precise determination of nuclear properties [1]. Various precision tests of the standard model, such as neutrino-nucleus scattering [2], double-beta decay [3], and multi-hadron weak decays [4–14], are similarly limited by poor knowledge of multihadron dynamics and would thus also clearly profit from reliable *ab initio* determinations.

Hadron spectroscopy, currently experiencing a remarkably rich period of discoveries, is another area where a deeper understanding of such observables would have a significant impact. A recent example of such a discovery is the doubly charmed tetraquark candidate, T_{cc} , observed to decay as $T_{cc} \rightarrow DD^* \rightarrow DD\pi$ [15]. Understanding the true nature of such states from first principles requires precise three-hadron decay amplitudes. [16] More generally, because most hadrons are unstable resonances that decay to two or more lighter hadrons [26], such amplitudes are essential in rigorously defining and extracting their properties.

To enhance the potential across all these fields, a concerted lattice QCD effort has been initiated to systematically determine scattering amplitudes from QCD. This is challenged by the fact that numerical lattice QCD is defined

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in a finite, discretized Euclidean spacetime volume where scattering observables are not directly accessible. Indirect constraints can be obtained from a correspondence between the finite-volume spectrum and the desired amplitudes. For two-particle systems, this correspondence is well understood [17–19,27–55] and the determination of low-lying two-hadron scattering amplitudes is now a mature aspect of this program, even in kinematic regimes where multiple two-hadron channels are open [20–22,56–112].

Given this success, increased attention has been directed towards advancing the determination of scattering amplitudes involving three hadrons. Further motivation comes from the fact that scattering amplitudes involving more than two hadrons generally become more relevant for calculations with close-to-physical-mass light quarks, since three-hadron states generically must be incorporated to rigorously treat center-of-momentum energies above the lowest lying three-hadron threshold. In the three-particle sector, the correspondence between the finite-volume spectrum and scattering amplitudes is known for a more limited, though ever growing, set of systems [113–148]. To date, most systematic constraints of three-hadron scattering dynamics have been for weakly repulsive mesonic systems [149–154]. Three notable exceptions are lattice QCD studies constraining the $\pi\pi\pi$ scattering amplitudes in the $a_1(1260)$ [155], $\omega(782)$ [156] and $\pi(1300)$ [157] channels. For recent reviews on the general approach, we point the reader to Refs. [158–160].

The calculation presented here advances the goal of reliably predicting multi-hadron amplitudes by presenting the first determination of a $\pi\pi\pi \rightarrow \pi\pi\pi$ scattering amplitude with coupled (resonant and non-resonant) two-pion subchannels. In particular, we study the coupled $(\pi\pi)_{I_2=1}\pi$ and $(\pi\pi)_{I_2=2}\pi$ channels, where I_2 is the isospin of the two-pion subsystem, within the total isospin $I_3 = 2$ (isotensor) channel with $J^P = 1^+$, where J and P are total angular momentum and parity. This calculation is performed using a single set of unphysically heavy light-quark masses, corresponding to a pion mass of $m_\pi \approx 400$ MeV [161], with the strange quark having an approximately physical mass. For these values of the quark masses, the $(\pi\pi)_{I_2=1}$ channel couples strongly to the unstable ρ resonance, which is narrow with a width of $\Gamma \approx 10$ MeV [56,162]. In contrast to Refs. [155–157], this Letter relies on QCD, unitarity, and analyticity constraints, together with a set of parametrizations intended to estimate the systematic uncertainties associated with the unknown true functional forms of the amplitudes. In particular, we do not impose constraints from chiral effective theory or use quark-mass-dependent parametrizations. As we discuss in the outlook, it is not clear whether the former are applicable for this system, with our value of the light-quark mass corresponding to $m_\pi \approx 400$ MeV.

In the remainder of this Letter, we describe the key elements of the calculation. We begin with a determination

of the finite-volume spectrum, followed by analysis of the spectrum to obtain an infinite-volume scheme-dependent K matrix using the formalism derived and implemented in Refs. [114,139,164]. This is subsequently used to determine physical observables (scattering amplitudes and intensities) by solving integral equations derived in Refs. [115,139,165] using advances described in Refs. [166–169]. We close by discussing conclusions that can be drawn from the analysis. More details of our methods and their implementation are given in Supplemental Material [170].

Spectrum determination—The spectra of three-hadron systems are substantially more challenging to determine than for two-hadron systems. As with previous two-hadron calculations, we use a state-of-the-art method where a large set of two-point correlation functions involving appropriate interpolating operators is computed in each symmetry channel. Then the spectrum is determined by solving a generalized eigenvalue problem [171–173]—our implementation is described in Ref. [174]. Here we use operators with $\rho\pi$ -like and $\pi\pi\pi$ -like structures [79,152,161], up to 6 or 7 of each depending on the lattice volume. More details of the spectrum determination and operator constructions are given in the Supplemental Material [170]. To evaluate the correlation functions, one must compute all possible Wick contractions of the quark fields, including those involving quark-antiquark annihilation. We do this efficiently using the distillation method [175].

The calculations are performed on the anisotropic set-up detailed in Ref. [176], with three cubic lattices differing only in the spatial extents L corresponding to $m_\pi L \approx 3.8, 4.8, 5.7$ and temporal extents T , given by $m_\pi T \approx 8.8, 17.8, 8.8$, respectively. The temporal lattice spacing is $a_t = 0.0348(3)$ fm, where the omega baryon mass [177] is used to set the scale, and the anisotropy (ratio of spatial and temporal lattice spacings) is $\xi = a_s/a_t = 3.444(6)$ [161]. Other details of the lattices, including masses of relevant mesons, are given in Table I. From these, we note that there are two additional relevant thresholds between the

TABLE I. Details of the lattice ensembles used in this Letter. Top: lattice geometry, number of gauge field configurations (N_{cfgs}), number of time sources per configuration (N_{tsrc}), and number of distillation vectors (N_v). Bottom: relevant stable meson masses in units of the temporal lattice spacing [59,79,161].

$(L/a_s)^3 \times (T/a_t)$	N_{cfgs}	N_{tsrc}	N_v
$16^3 \times 128$	479	4	64
$20^3 \times 256$	286	4	128
$24^3 \times 128$	553	1	160
$a_t m_\pi$			0.06906(13)
$a_t m_K$			0.09698(9)
$a_t m_\omega$			0.15541(29)

three- and five-pion thresholds: $E^{\text{thr}}(K\bar{K}\pi)/m_\pi = 3.81$ and $E^{\text{thr}}(\omega\pi\pi)/m_\pi = 4.25$. For the lower of these two, we have additionally evaluated correlators with $K\bar{K}\pi$ operators to confirm that the effect of these is negligible (more quantitatively the shift in central values is much less than the statistical uncertainty) in the energy region we are considering. This is consistent with the findings of Ref. [61]. Here it is also relevant to note that the lowest lying noninteracting energies in the neglected channels are significantly above the indicated thresholds, as two of the hadrons must carry back-to-back momentum to achieve the quantum numbers of interest.

A technically distinct but related question is the effect of neglecting these channels in the implementation of the three-particle formalism used to extract the amplitudes. If the coupling between the channel of interest and the neglected channels is sufficiently weak, then the associated effects will be subdominant to the statistical uncertainties reported for our final results. The observation that adding the corresponding operators shifts the finite-volume energies by amounts well below their statistical uncertainties makes it plausible that this hierarchy of uncertainties is preserved in the final amplitudes. In addition, the fact that the fits applied in the next section successfully describe the data using a formalism that neglects the additional channels gives further support to this conclusion. Still, future work is needed to address this point more directly.

The reduced symmetry of the finite-volume discretized system, as compared to an infinite-volume continuum, means that angular momentum is no longer a good quantum number and states are instead labeled by irreps, Λ , of the cubic group or a subgroup. To avoid complications in the subsequent analysis, we restrict ourselves to systems with total momentum zero in this first study. This ensures that parity, P , is a good quantum number, reducing the number of partial waves that can mix. For $J^P = 1^+$, the relevant lattice irrep is then $\Lambda^P = T_1^+$, which also includes contributions from $J^P = 3^+$ and higher J . At low energies, we expect this system to be saturated by the $J = 1$ $\pi\pi\pi$ system since the other contributions are kinematically suppressed at low energies by additional barrier factors [168].

In Fig. 1, we show the four lowest energy levels obtained in the T_1^+ irrep for each of the three volumes. In addition to presenting the lattice QCD spectra, we show various volume-dependent energy curves predicted by our formalism as described in the caption.

We note that the level counting is consistent with the expectations from a theory of noninteracting pions and rho mesons. The ground state corresponds to a rho and a pion, each at rest, while the first two excited states correspond to the same configuration with one unit of back-to-back momentum. The third excited state corresponds to three pions, two of which carry one unit of back-to-back momentum. We represent this in Fig. 1 by plotting in gray the

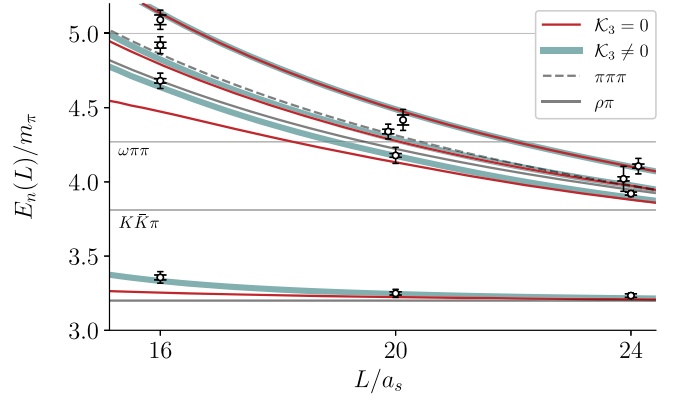


FIG. 1. Energies in the isospin-2 three-pion channel plotted as a function of the box size L . The black points represent lattice QCD-computed energies; inner error bars indicate statistical uncertainties, while outer error bars also include systematic uncertainties as discussed in the Supplemental Material [170]. The red curves are predictions of the finite-volume quantization condition with a vanishing three-body K matrix $\mathcal{K}_3 = 0$, using two-pion K matrices from a previous analysis of the two-pion spectrum, while the blue curves show results incorporating a best-fit $\mathcal{K}_3$ as detailed in the text. The horizontal light-gray lines indicate neglected three-hadron thresholds, while the other solid and dashed gray lines correspond to the hypothetical spectrum of noninteracting hadrons, as explained in the text. The first excited gray curve has a twofold multiplicity.

appropriate functions, given by summing individual particle energies for the relevant momenta. In addition to the pion mass, here we require a choice for our hypothetical noninteracting ρ . We set its mass to $2.184m_\pi$, which is the central value of the Breit-Wigner mass, determined from an analysis of the two-particle spectrum of this system in Ref. [56]. We now go beyond this simple counting to a detailed analysis of the interacting theory implied by this spectrum.

Spectrum analysis—All formalisms available to date that connect finite-volume spectra to infinite-volume three-hadron scattering, first relate the spectrum to intermediate functions that describe the short-distance dynamics, denoted in the formalism that we use here as K matrices. These are known to have exponentially suppressed volume effects that we neglect, and are real and smooth in the kinematic region considered.

The relation in the $I_3 = 2$ channel can be written as a determinant condition over all degrees of freedom that are not fixed by the system's energy [139]

$$\det_{l_2 k, S, m} [\mathbf{F}_3^{-1}(E_n, L|\boldsymbol{\eta}_2) + \mathbf{K}_3(E_n|\boldsymbol{\eta}_3)] = 0. \quad (1)$$

Before giving some details of this equation, we explain the essential idea. For a given $\mathbf{K}_3$ and for given two-pion dynamics (encoded by $\boldsymbol{\eta}_2$, a vector of two-pion scattering parameters), solutions of Eq. (1) predict the finite-volume spectrum $\{E_n\}$. We fix $\boldsymbol{\eta}_2$ from previous calculations

[56,161] and parametrize $\mathbf{K}_3$ with parameters η_3 to predict $\{E_n(L|\eta_2, \eta_3)\}$. We then form a chi-squared with these predictions and the lattice-QCD-determined energies and vary η_3 to minimize, thereby determining the best description.

Both matrices in Eq. (1) are indexed by $I_2 k S m$ for the initial state and a second primed set for the final state. $\mathbf{K}_3$ is the three-body K matrix, and $\mathbf{F}_3$ is a finite-volume matrix that also depends on two-pion interactions. Here S, m , and I_2 denote, respectively, the angular momentum, z component of angular momentum, and isospin of one of the pairs in the initial state, while k denotes the momentum of the third particle. We refer to the two pions labeled by S, m , and I_2 as the dipion, and the third as the spectator.

In the Supplemental Material [170], we provide a detailed description of the definitions of $\mathbf{K}_3$ and $\mathbf{F}_3$ for this channel. As explained in Refs. [115,139], $\mathbf{K}_3$ can be converted to a function of three incoming and three outgoing momenta by contracting the S, m indices with spherical harmonics. An important freedom in the definition of $\mathbf{K}_3$ determines whether or not the resulting function is invariant under simultaneous exchange of pion momentum and flavor labels. This choice is referred to as the symmetric or asymmetric formalism [165], though we emphasize it only enters the intermediate scheme dependent quantity and either must lead to a final amplitude that is consistent within uncertainties. Here we use the asymmetric formalism. As was shown in Ref. [168], this naturally accommodates an arbitrarily large number of parametrizations and thus provides a better handle on systematic errors.

As briefly indicated above, a key input in the definition of $\mathbf{F}_3$ are the two-body K matrices for the two dipion channels ($I_2 = 1, 2$), which have been previously constrained in Refs. [56,161]. For the kinematic region considered, these K matrices, denoted $\mathcal{K}_2$ are dominated by the $S = 1, 0$ partial waves, respectively [178].

Given $\mathbf{F}_3$ and $\mathbf{K}_3$, we subduce to the T_1^+ irreducible representation of the cubic group, following Refs. [127,164] for $\mathbf{F}_3$ and extending the procedure here for the asymmetric $\mathbf{K}_3$. The T_1^+ spectrum is expected to be well described by the $J = 1$ three-pion system at low energies, and we restrict our analysis to channels with $J^P = 1^+$ and $\ell \leq 2$, namely, $^3S_1$, $^3D_1$, and $^1P_1$, where we are using the spectroscopic notation $^{2S+1}\ell_J$ with ℓ being the angular momentum between the dipion and spectator pion. See Fig. 2 for a pictorial representation of the various angular momenta. For these channels, the dipion will have $I_2 = 1$ in the $^3S_1$ and $^3D_1$ and $I_2 = 2$ for the $^1P_1$.

The strategy for parametrizing the three-pion K matrix in this basis follows Ref. [168]. We consider 10 parametrizations, fit to the lattice QCD spectra of Fig. 1, and perform a model average using the Akaike information criterion (AIC) [179,180]. Figure 3(a) shows the results for the so-called reduced K matrix, $\tilde{\mathcal{K}}_3$, as a function of the total energy E , for

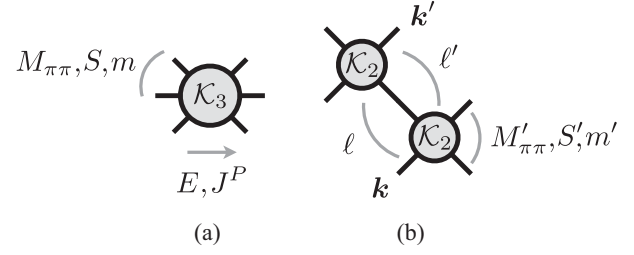


FIG. 2. Representative diagrams contained in the nonperturbative scattering amplitude, where the black lines represent pions. We show the two leading contributions: (a) the three-body K matrix constrained in this Letter, and (b) the one-pion exchange diagram responsible for additional kinematic singularities in the amplitude. Kinematic quantities and angular momenta are defined in the text.

all initial state channels going to the final state $^3S_1$. The reduced K matrix, introduced in Ref. [168], is a matrix in channel space and is defined by the decomposition $\mathbf{K}_3 \equiv \mathbf{h} \cdot \tilde{\mathcal{K}}_3 \cdot \mathbf{h}^\top$ where $\mathbf{h}$ are known matrices which convert from channel to $I_2 k S m$ space. We find that after propagating systematic errors, all of the components of the K matrix are consistent with zero. The best fit for the K matrix is the first listed in the Supplemental Material [170]. It is reached by setting the $^3S_1 \rightarrow ^3S_1$ component to a floating constant, while all other components of $\tilde{\mathcal{K}}_3$ are set to 0.

Amplitude analysis—Having constrained the K matrix, we focus on the determination of the physical observables, namely, scattering amplitudes and intensities. The K matrix is generally a scheme-dependent quantity that can be mapped to physical amplitudes via integral equations. [181] These were first derived in Ref. [115] for three scalar particles. Using the work of Ref. [139], one can immediately write the case for generic $\pi\pi\pi$ systems, and Refs. [167,168] show how these can be projected to arbitrary angular momentum and parity, yielding the specific form we use here.

This framework first yields unsymmetrized $J^P = 1^+$ amplitudes, denoted by $\mathcal{M}_3^{(a,b)}$. As for $\tilde{\mathcal{K}}_3$ this refers to the fact that, when this intermediate quantity is written as a function of pion momenta and flavor labels, it is not invariant under their simultaneous interchange (as any physical quantity must be). This issue is corrected in a final symmetrization step described below.

Each amplitude is a function of total three-pion energy E and initial and final dipion invariant masses $M_{\pi\pi}$ and $M'_{\pi\pi}$. [182] The indices $a, b \in \{1, 2, 3\}$ indicate one of the three pions. Schematically, the integral equations depend on two inputs, the two- and three-body K matrices:

$$\mathcal{M}_3^{(a,b)} = \mathcal{M}_3^{(a,b)}[\mathcal{K}_2, \mathcal{K}_3]. \quad (2)$$

The driving terms of these integral equations are $\mathcal{K}_3$ and the one-pion exchange (OPE) diagram, shown schematically in

Fig. 2. As with the spectrum, we use the previously constrained elastic $\pi\pi$ $\mathcal{K}_2$ for isospin-1 and 2 from Refs. [56,161] as an input. The partial wave $\mathcal{K}_3$ used in the integral equations is related to $\mathbf{K}_3$ by usual partial wave projection [168], see the Supplemental Material [170] for further discussion and its relation to $\tilde{\mathcal{K}}_3$.

We solve the system of integral equations numerically for the different components of $\mathcal{M}_3^{(a,b)}$ at a fixed E . Results are shown in Fig. 3(b) for the $\{^3S_1, ^3D_1, ^1P_1\} \rightarrow ^3S_1$ amplitudes as a function of the initial dipion invariant mass, $M_{\pi\pi}$, for fixed $E = 3.4m_\pi$ and a final state $M'_{\pi\pi} = 2.1m_\pi$. The 1σ band is generated by propagating the weighted uncertainties from the 10 fits. For comparison, we also show the amplitudes in the limit that $\mathcal{K}_3 = 0$. As can be seen, depending on the channel and kinematics, for a particular fit, there might be statistically significant deviations between the $\mathcal{K}_3 = 0$ solution and the full amplitude. [183] However, when we propagate the error associated with the choice of the parametrization, we find that the $\mathcal{K}_3 = 0$ solution and the full amplitude are consistent within uncertainties. Notice the $^3S_1$ and $^3D_1$ channels are dominated by the presence of the ρ resonance, with its peak at $M_{\pi\pi} \approx 2.2m_\pi$, while the $^1P_1$ channel is dominated by the logarithmic singularity at $M_{\pi\pi} \approx 2.1m_\pi$ associated with one-pion exchange (OPE).

Finally, we form the fully symmetric amplitude by summing over all spectators choices from the results of Eq. (2) using the procedure presented in Ref. [169],

$$\mathcal{M}_{3,I_2\lambda',I_2\lambda} = \sum_{a,b} \mathcal{R}_{I_2\lambda'}^{(a)} \cdot \mathcal{M}_3^{(a,b)} \cdot \mathcal{R}_{I_2\lambda}^{(b)\top}. \quad (3)$$

Here λ, λ' are helicity quantum numbers formed by linear combinations of the participating channels, and $\mathcal{R}_{I_2\lambda}^{(a)}$ are recoupling matrices in isospin and partial waves from the a th spectator to the desired quantum numbers. Their explicit expression is given in Ref. [169].

Reference [169] also provided a suitable definition for intensities for three-body systems with arbitrary angular momentum and isospin,

$$\mathcal{I} = \sum_{I_2, I_2'} \sum_{\lambda, \lambda'} |\mathcal{M}_{3,I_2\lambda',I_2\lambda}|^2. \quad (4)$$

In Fig. 4 we show Dalitz plots of the intensities for $E/m_\pi = 3.3, 3.4, 3.5$. We choose the initial state kinematic configuration to mirror those of the final state. The most striking features are dense bands due to the ρ resonance at $M_{\pi\pi} \approx 2.2m_\pi$. Because the amplitude is symmetrized, one sees reflections of the ρ bands which constructively interfere in the kinematic region where two bands overlap. Note that the logarithmic singularity that is visible in Fig. 3 is unresolvable in the intensities. This is due in part to the fact that the intensity is dominated by the $^3S_1$ amplitude, which is itself dominated by the ρ peak, and for the matrix

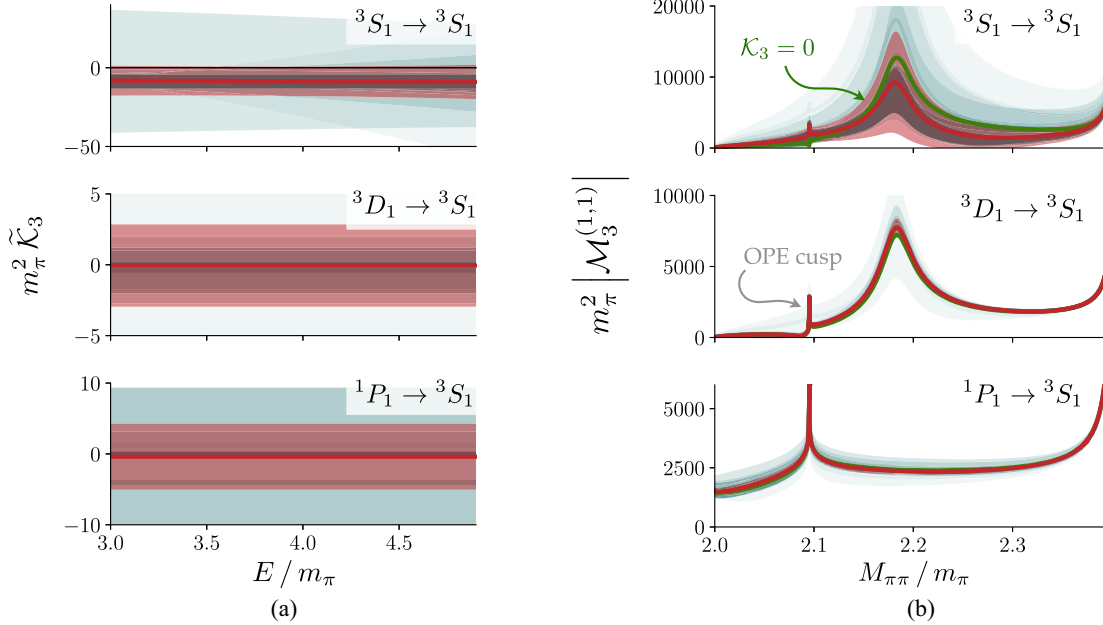


FIG. 3. (a) The light blue bands show fit results for a given parametrization of the reduced K matrix as a function of total energy E in the channels $\{^3S_1, ^3D_1, ^1P_1\} \rightarrow ^3S_1$. Bands represent 1σ uncertainty of a given fit, and the opacity of the band is directly proportional to the likelihood of the fit, as given by the AIC described in the text, with the darker bands being preferred. The red is the weighted average of the different fits, with the band including the propagated uncertainty. (b) Shown are the $\{^3S_1, ^3D_1, ^1P_1\} \rightarrow ^3S_1$ unsymmetrized amplitudes as a function of the incoming dipion invariant mass for a fixed $E = 3.4m_\pi$ and outgoing state defined by $M'_{\pi\pi} = 2.1m_\pi$. The red band includes the error propagation of the K matrices shown in (a). The green curve corresponds to amplitudes for $\mathcal{K}_3 = 0$. The cusp near $2.1m_\pi$ is due to physical OPE.

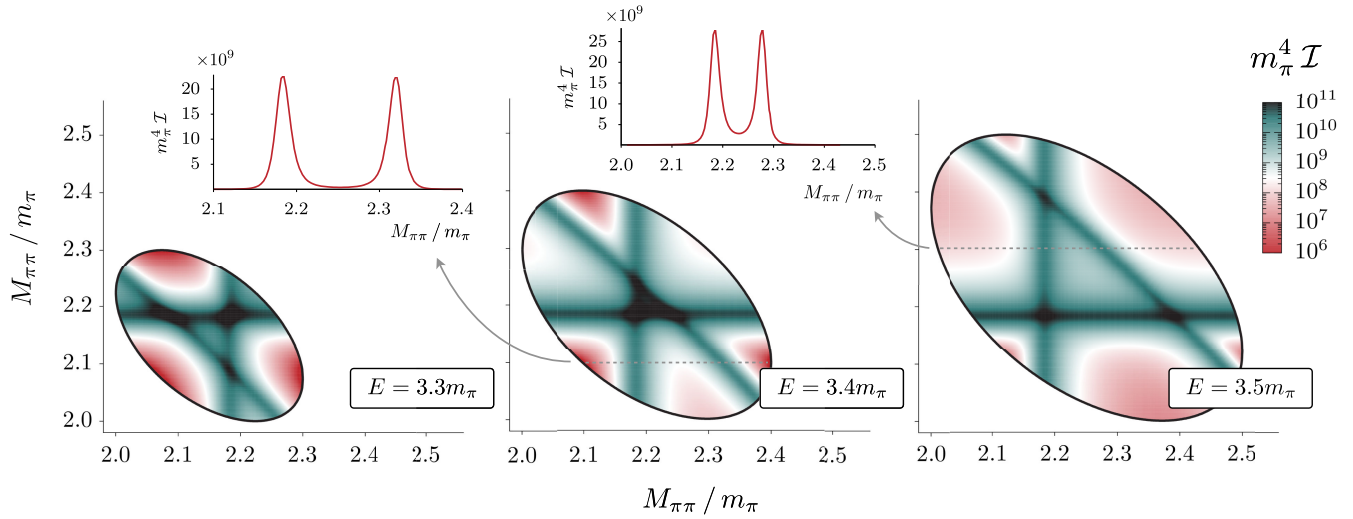


FIG. 4. Dalitz plots of the intensity distributions in Eq. (4) for the $J^P = 1^+$ amplitude for $E/m_\pi = \{3.3, 3.4, 3.5\}$. The axes represent the invariant masses corresponding to two different choices for the dipion within a given state. Initial and final state kinematics are fixed to be the same. Amplitudes are determined from the best-fitted values of the K matrix described in the text. The insets show slices of the distribution for fixed $(M_{\pi\pi}, E) = (2.1m_\pi, 3.4m_\pi)$ and $(2.3m_\pi, 3.5m_\pi)$.

sizes for which we were able to solve the integral equations, the $S = 1$ softened OPE singularity is too narrow to be resolved [167]. See the Supplemental Material [170] for a detailed discussion of this point and other aspects of the amplitude analysis.

Outlook—In this Letter, we have presented the first determination of the full kinematic dependence of a three-hadron scattering amplitude for a coupled-channel system with a resonant subsystem. Achieving this result required the development of a general, flexible formal and computational machinery, which can now be applied to the full set of $\pi\pi\pi$ channels of immediate physical interest, including those with three-pion resonances.

One promising future direction of this program is to map out the pion-mass dependence of such scattering amplitudes from very high pion masses down to the physical point. This will provide new insight into the analytic properties and potential universality across different channels. Together with the previously published $\rho\pi$ scattering amplitudes at $m_\pi \sim 700$ MeV [163], this Letter at $m_\pi \sim 400$ MeV provides a second value with which to begin such an investigation. As one pushes to lighter values of m_π , chiral perturbation theory (χ PT) can be compared to the data, at least for near-threshold kinematics, to more fully understand this dependence as well as to cross-check the lattice calculation. A relevant formal contribution in this direction was made by Ref. [184], which derived the leading-order χ PT expressions for the symmetric $\mathcal{K}_3$ at threshold across all $\pi\pi\pi$ isospins. Given the presence of the narrow, near-threshold ρ in the present Letter, we do not expect those results to directly apply here. In the context of two-pion scattering, unitarized extensions of χ PT [185–188] have proven valuable for studying the ρ

resonance in lattice QCD [62,189], and similar methods could be explored for three-pion systems in future work. We also note that χ PT has been successfully applied for maximal-isospin three-hadron systems in Refs. [154,190].

An important conclusion of this calculation is that varying the parametrizations of the three-body K matrix gives an essential handle on the systematic uncertainty that arises from not knowing the full analytic form. In other words, using an insufficient set of parametrizations risks underestimating uncertainties. [191] We find that the asymmetric formalism makes the allowed forms of $\mathcal{K}_3$ more transparent, and thereby offers the flexibility needed for robust uncertainty quantification.

The present calculation applies the formalism first derived in Ref. [129], initiated in Refs. [114,115], and later generalized to $2 \rightarrow 3$ processes [116] as well as systems with nondegenerate particles and spin [23,141–145]. Together with recent advances connecting K matrices to amplitudes [166–169], these developments will enable an ambitious program of applications, including $\omega(782) \rightarrow 3\pi$, $a_1(1260) \rightarrow \rho\pi \rightarrow 3\pi$, the Roper resonance $N(1440) \rightarrow N\pi\pi$, and fully coupled three-hadron systems such as $\Sigma\pi\pi$ and $K\bar{K}N$ in the hyperon sector. Such calculations will provide novel model-independent links between QCD’s quark-gluon dynamics and complex low-energy phenomena, laying the groundwork for a systematic, first-principles knowledge of the multihadron landscape.

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The software codes Chroma [194], QUDA [195,196], QUDA-MG [197], QPhiX [198], MG_PROTO [199], QOPQDP [200,201], and Redstar [202] were used.

Data availability—The data are not publicly available. The data are available from the authors upon reasonable request.

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