

Next-to-Next-to-Leading-Order QCD Prediction for the Pion Form Factor

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We accomplish for the first time the two-loop computation of the leading-twist contribution to the pion form factor by employing the effective field theory formalism rigorously. The next-to-next-to-leading-order hard function is determined by evaluating the appropriate five-point QCD amplitude with the modern multiloop technique and by implementing the ultraviolet renormalization and infrared subtractions in the presence of evanescent operators. The yielding two-loop correction to this fundamental quantity turns out to be numerically significant at experimentally accessible momentum transfers. We further demonstrate that the newly computed two-loop QCD correction is highly beneficial for an improved determination of the leading-twist pion distribution amplitude.

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Introduction—It is generally accepted that the pion electromagnetic form factor (EMFF) is of paramount importance for probing the infrared structure of QCD scattering amplitudes of hard exclusive reactions and for unraveling the precise mechanisms that dictate the intricate nature of composite hadron systems. Advancing our understanding toward this gold-plated form factor is especially suitable for delivering deep and far-reaching insights into the delicate interplay between the emergent and Higgs mass generation mechanisms for such lightest pseudo-scalar mesons as the Nambu-Goldstone modes of QCD. Historically, the systematic analysis of the pion form factor played an indispensable role in establishing the perturbative factorization formalism for the entire domain of exclusive hadronic processes [1–7]. As a consequence, the dimensional-counting rules for the power-law behavior of numerous hard exclusive reactions on the basis of the parton

model [8,9] can be placed on a solid footing within this field-theoretic framework [10]. Additionally, the perturbative factorization technique has been extended to the first-principles calculations of the deeply virtual Compton scattering [11–14], the hard diffraction of light mesons at high energies [15–17], and the exclusive heavy-to-light B -meson decays [18–21]. Moreover, the timelike scalar-current pion form factors are intimately connected with the weak annihilation contributions to the hadronic $B \rightarrow \pi\pi$ decays [22–24].

Experimentally, the pion EMFF at low momentum transfers ($Q^2 \in [0.015, 0.253] \text{ GeV}^2$) has been measured from the elastic scattering of high-energy pions off the atomic electrons in a liquid-hydrogen target at Fermilab [25,26] and CERN [27,28]. The challenging measurement of the pion form factor at intermediate momentum transfers, up to $Q^2 = 10 \text{ GeV}^2$, has been further carried out at Cornell [29–31], DESY [32,33], and JLab [34–38], by exploiting the so-called Sullivan mechanism and by extrapolating the longitudinal cross section at negative values of the Mandelstam variable t to the pion pole at $t = m_\pi^2$ [39]. Extracting the charged pion form factor over a wide range of the moderate momentum transfers ($Q^2 \in [2.0, 6.0] \text{ GeV}^2$) can be also anticipated from the E12-06-101 experiment with the upgraded JLab accelerator [40]. Accessing the spacelike pion form factor at yet higher momentum transfers ($10 \text{ GeV}^2 \leq Q^2 \leq 40 \text{ GeV}^2$) with high precision will be one of the key targets for the forthcoming electron-ion collider (EIC) experiment at Brookhaven National Laboratory [41].

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According to the hard-collinear factorization theorem [1–6], the leading-power contribution to the pion EMFF in the large-momentum expansion can be expressed in terms of the short-distance coefficient and the twist-two light-cone distribution amplitude (LCDA) for the (anti)collinear pion state. The next-to-leading-order (NLO) QCD correction to the perturbatively calculable hard function had been determined with the diagrammatic factorization approach more than 40 years ago [42–46]. Including the complete NLO correction in the factorization analysis of the pion EMFF turns out to be extraordinarily advantageous to pin down the intrinsic uncertainty from varying the renormalization or factorization scale [47]. Accomplishing the next-to-next-to-leading-order (NNLO) computation of the short-distance coefficient is, therefore, in high demand for improving further the theory prediction of the charged pion form factor in order to match the ever-growing precision of the experimental measurements. It is our primary objective to fill such an important and long-standing gap in this Letter, by evaluating the appropriate QCD matrix element at $\mathcal{O}(\alpha_s^3)$ with the contemporary multiloop computational strategies and by carrying out the ultraviolet (UV) renormalization and infrared (IR) subtractions in a meticulous and factorization-compatible manner (see [48] for an earlier NNLO calculation of the pion EMFF with the covariant trace method which cannot be rigorously justified in the perturbative factorization framework [49–66]). Phenomenological implications of the determined two-loop correction to the spacelike pion EMFF at large momentum transfers will be then explored comprehensively.

General analysis—Applying the general decomposition for the hadronic matrix element of the quark electromagnetic current enables us to write down

$$\begin{aligned} \langle \pi^+(p') | j_\mu^{\text{em}}(0) | \pi^+(p) \rangle \\ = F_\pi(Q^2)(p + p')_\mu + \tilde{F}_\pi(Q^2)(p - p')_\mu, \end{aligned} \quad (1)$$

where $p(p')$ corresponds to the four-momentum carried by the initial (final) pion state and $Q^2 = -(p - p')^2$. Employing the vector-current conservation condition leads to $\tilde{F}_\pi(Q^2) = 0$, thus leaving us with a single form factor $F_\pi(Q^2)$. The quark electromagnetic current is given by

$$j_\mu^{\text{em}}(x) = \sum_q e_q \bar{q}(x) \gamma_\mu q(x), \quad (2)$$

with $e_u = 2/3$ and $e_d = -1/3$ for the up and down quarks, respectively. Introducing two lightlike vectors n_μ and $\bar{n}_\mu$ with $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$ allows for the decomposition $p_\mu = (n \cdot p/2)\bar{n}_\mu$ and $p'_\mu = (\bar{n} \cdot p'/2)n_\mu$ at leading power.

Applying the modern effective field theory technique [67,68] leads to the factorization formula for the pion form factor at leading power in the $\Lambda_{\text{QCD}}^2/Q^2$ expansion:

$$\begin{aligned} F_\pi(Q^2) = (e_u - e_d) \frac{4\pi\alpha_s(\nu)}{Q^2} f_\pi^2 \int dx \int dy T_1(x, y, Q^2, \nu, \mu) \\ \times \phi_\pi(x, \mu) \phi_\pi(y, \mu), \end{aligned} \quad (3)$$

with the pion decay constant $f_\pi = (130.2 \pm 1.2)$ MeV [69]. Apparently, the hard-scattering kernel T_1 depends on both the renormalization scale ν and the factorization scale μ . This short-distance coefficient function can be expanded perturbatively in terms of the strong coupling constant (similarly for any other QCD quantity)

$$T_1 = \sum_{\ell=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^\ell T_1^{(\ell)}. \quad (4)$$

The leading-twist pion distribution amplitude ϕ_π in the factorized expression (3) can be defined by the renormalized QCD matrix element on the light cone [70,71]:

$$\begin{aligned} \langle \pi^+(p') | \bar{u}(\tau\bar{n})[\tau\bar{n}, 0] \gamma_\mu \gamma_5 d(0) | 0 \rangle \\ = -i f_\pi p'_\mu \int_0^1 dx e^{ix\tau\bar{n} \cdot p'} \phi_\pi(x, \mu), \end{aligned} \quad (5)$$

where $[\tau\bar{n}, 0]$ is the finite-length collinear Wilson line. The one-loop renormalization-group (RG) equation for the light-ray operator on the left-hand side of (5) implies the conformal partial wave expansion of $\phi_\pi(x, \mu)$ in terms of the Gegenbauer polynomials with multiplicatively renormalizable coefficients [72]

$$\phi_\pi(x, \mu) = 6x(1-x) \sum_{m=0}^{\infty} a_m(\mu) C_m^{3/2}(2x-1). \quad (6)$$

The short-distance coefficient T_1 can be determined by investigating the five-point QCD matrix element below:

$$\Pi_\mu = \langle u(p'_1) \bar{d}(p'_2) | j_\mu^{\text{em}}(0) | u(p_1) \bar{d}(p_2) \rangle, \quad (7)$$

where the external momenta can be restricted to their leading components $p_1 = xp$, $p_2 = \bar{x}p$, $p'_1 = yp'$, and $p'_2 = \bar{y}p'$ with the “bar notation” $\bar{x} \equiv 1 - x$, $\bar{y} \equiv 1 - y$. We will perform the computation of the bare amplitude in dimensional regularization with $D = 4 - 2\epsilon$.

Next-to-next-to-leading-order QCD computation—We will dedicate this section to a brief technical description of the two-loop QCD calculation for Π_μ . We start with generating the NNLO Feynman diagrams with FeynArts [73] and, independently, by means of an in-house routine. Taking into account the observation that a subset of the two-loop diagrams yield the vanishing contribution due to the Furry theorem and/or the zero color (electric) charge factors, we eventually encounter 1066 nonvanishing diagrams of our interest. Two sample Feynman diagrams are depicted in Fig. 1 explicitly.

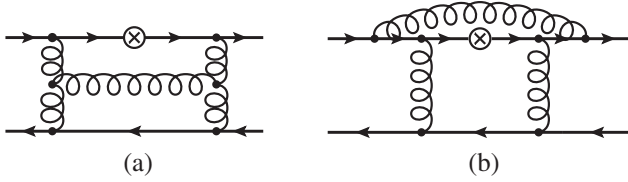


FIG. 1. Sample two-loop Feynman diagrams.

All tensor integrals can be expressed in terms of scalar integrals and algebraic tensorial structures using the Passarino-Veltman decomposition [74]. We then perform the Dirac and tensor reductions with in-house *Mathematica* routines, by applying the QCD equations of motion and on-shell conditions. The large amount of two-loop scalar integrals are further reduced to an irreducible set of master integrals with *Apart* [75] and *FIRE* [76]. In total, we obtain 57 two-loop master integrals. The evaluation of these master integrals can be accomplished in an analytic fashion by employing the method of differential equations [77,78]. In order to identify a suitable linear transformation such that the master integrals in the new basis satisfy a set of differential equations in an ϵ -factorized form [79], we take advantage of Lee's algorithm [80] as implemented in the program *Libra* [81]. The boundary conditions to the differential equations are determined by capitalizing on the package *AMFlow* [82] based upon [83,84]. The yielding results of the master integrals at three distinct kinematic points allow us to construct the boundary constants of the differential equations in terms of transcendental numbers using the PSLQ algorithm [85,86]. It is, thus, straightforward to express the master integrals through Goncharov polylogarithms [87,88] up to the required order in ϵ .

Ultraviolet renormalization and infrared subtractions—We proceed to deduce the master formula for T_1 by performing the UV renormalization and IR subtractions in the presence of evanescent operators [66,89–91]. To achieve this goal, we exploit the matching equation for Π_μ onto the effective matrix elements:

$$\Pi_\mu = (p + p')_\mu \left[(e_u - e_d) \frac{4\pi\alpha_s}{Q^4} \sum_k T_k \otimes \langle \mathcal{O}_k \rangle \right],$$

$$\mathcal{O}_k \in \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\}, \quad (8)$$

with the following choice of the collinear operator basis:

$$\begin{aligned} \mathcal{O}_1 &= [\bar{\chi}_u \not{\epsilon} \gamma_5 \chi_d] [\bar{\xi}_d \not{\epsilon} \gamma_5 \xi_u], \\ \mathcal{O}_2 &= [\bar{\chi}_u \gamma_\perp \alpha \xi_u] [\bar{\xi}_d \gamma_\perp \alpha \chi_d] - \frac{1}{4} \mathcal{O}_1, \\ \mathcal{O}_3 &= [\bar{\chi}_u \gamma_\perp \alpha \gamma_{\perp\mu_1} \gamma_{\perp\mu_2} \xi_u] [\bar{\xi}_d \gamma_\perp^\alpha \gamma_{\perp}^{\mu_2} \gamma_{\perp}^{\mu_1} \chi_d]. \end{aligned} \quad (9)$$

One can readily verify that $\mathcal{O}_1$ is the only physical operator in our problem and the other two operators are evanescent. To reduce our notation to the essentials, we strip off the

collinear Wilson lines and the position arguments of quark fields from $\mathcal{O}_k$. Following [49], we label the collinear and anticollinear effective fields moving into the directions of $\bar{n}$ and n by ξ and χ , respectively.

Now we organize the perturbative expansion of the renormalized QCD correlation function Π_μ in terms of the tree-level matrix elements of the effective operators:

$$\begin{aligned} \Pi_\mu &= (p + p')_\mu (e_u - e_d) \frac{(4\pi)^2}{Q^4} \sum_k \sum_{\ell=1,2,3} \\ &\times \left[\left(\frac{Z_\alpha \alpha_s}{4\pi} \right)^{\ell+1} A_k^{(\ell)} \otimes \langle \mathcal{O}_k \rangle^{(0)} \right], \end{aligned} \quad (10)$$

where Z_α denotes the renormalization constant of the strong coupling in the $\overline{\text{MS}}$ scheme [92–94]. The scalar quantities $A_k^{(\ell)}$ in (10) represent the bare ℓ -loop on-shell QCD amplitudes. In the same vein, the UV-renormalized matrix elements of $\mathcal{O}_k$ can be expanded according to

$$\langle \mathcal{O}_k \rangle = \sum_i \sum_{\ell=0}^{\infty} \left(\frac{\alpha_s}{4\pi} \right)^\ell Z_{ki}^{(\ell)} \otimes \langle \mathcal{O}_i \rangle^{(0)}. \quad (11)$$

Since the collinear fields for distinct directions already decouple at the hard scale, the renormalization constant $Z_{11}^{(\ell)}$ can be, therefore, determined from the Efremov-Radyushkin-Brodsky-Lepage kernel with the one- and two-loop results obtained in [2,4] and [95–99]. Substituting (4), (10), and (11) into the matching equation (8) leads to the master formulas for the hard function:

$$\begin{aligned} T_1^{(0)} &= A_1^{(0)}, \\ T_1^{(1)} &= A_1^{(1)} + Z_\alpha^{(1)} A_1^{(0)} - \sum_{k=1,2} Z_{k1}^{(1)} \otimes T_k^{(0)}, \\ T_1^{(2)} &= A_1^{(2)} + 2Z_\alpha^{(1)} A_1^{(1)} + Z_\alpha^{(2)} A_1^{(0)} \\ &\quad - \sum_{k=1,2,3} \sum_{\ell=1,2} Z_{k1}^{(\ell)} \otimes T_k^{(2-\ell)}. \end{aligned} \quad (12)$$

The renormalization constants for the evanescent operators are adjusted to ensure that the IR-finite matrix elements $\langle \mathcal{O}_{2,3} \rangle$ vanish [100–104]. Adopting the collinear operator basis (9) yields the evanescent-to-physical operator mixing, which is captured by the *finite* renormalization constant $Z_{21}^{(2)}$ at two-loop order. This justifies the essential role of introducing evanescent operators in constructing the factorization formulas [49–62]. It is perhaps worthwhile to mention that the yielding short-distance coefficient functions $T_1^{(\ell)}$ do not depend on the particular choice of the evanescent four-quark operators $\mathcal{O}_{2,3}$.

Inserting the two-loop expression of T_1 into the factorization formula (3) and then performing the twofold integration over $x \in (0, 1)$ and $y \in (0, 1)$ with the

Mathematica package PolyLogTools [105] in the asymptotic approximation

$$\begin{aligned}
 F_{\pi}^{\text{Asy}}(Q^2) = & (e_u - e_d) \frac{4\pi\alpha_s}{Q^2} 2f_{\pi}^2 \left\{ 1 + \left(\frac{\alpha_s}{4\pi} \right) \left[9 \ln \frac{\nu^2}{Q^2} + \frac{79}{3} \right] \right. \\
 & + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[81 \ln^2 \frac{\nu^2}{Q^2} + 538 \ln \frac{\nu^2}{Q^2} \right. \\
 & + \left(120 - \frac{560}{9} \zeta_2 - \frac{128}{9} \zeta_3 \right) \ln \frac{\mu^2}{Q^2} + \frac{13136}{9} \\
 & - \frac{1100}{9} \zeta_2 - \frac{1736}{3} \zeta_3 - 24 \zeta_4 + \frac{3280}{9} \zeta_5 \left. \right] \\
 & \left. + \mathcal{O}(\alpha_s^3) \right\}, \quad (13)
 \end{aligned}$$

where ζ_n represents the Riemann zeta function [106–108]. Including the higher conformal spin contributions from ϕ_{π} leads to the lengthy results for the nonasymptotic corrections to the pion EMFF, whose explicit expressions are collected in Supplemental Material [109]. We have verified that the thus achieved NNLO QCD computation of $F_{\pi}(Q^2)$ is truly independent of the renormalization or factorization scale at $\mathcal{O}(\alpha_s^3)$, by applying the two-loop evolution equation of $\phi_{\pi}(x, \mu)$ [95–99]. Adopting the RG formalism enables us to accomplish an all-order summation of the enhanced logarithms of $Q^2/\Lambda_{\text{QCD}}^2$ entering the factorized expression of $F_{\pi}(Q^2)$ in the next-to-next-to-leading-logarithmic (NNLL) approximation with the three-loop evolution of $\phi_{\pi}(x, \mu)$ [110,111].

Numerical analysis—We proceed by first discussing our choice for the phenomenological models of the twist-two pion LCDA in the hard-collinear factorization formula (3). The first model $\phi_{\pi}^{\text{Model I}}(x, \mu_0) = [\Gamma(2 + 2\alpha_{\pi})/\Gamma^2(1 + \alpha_{\pi})](x\bar{x})^{\alpha_{\pi}}$ is motivated from the anti-de Sitter–QCD correspondence [112] with $\alpha_{\pi}(\mu_0) = 0.585^{+0.061}_{-0.055}$ [113] determined by matching to the lattice result of $a_2(\mu_0) = 0.116^{+0.019}_{-0.020}$ at $\mu_0 = 2.0$ GeV [114]. Our second model $\{a_2, a_4, a_6, a_8\}(\mu_0) = \{0.181(32), 0.107(36), 0.073(50), 0.022(55)\}$ [115] is obtained from the comparison of the light-cone sum rule for the pion EMFF [116] with the experimental measurements (see also [117,118]). By contrast, the numerical intervals for the two lowest conformal coefficients in model III $\{a_2, a_4\}(\mu_0) = \{0.149^{+0.052}_{-0.043}, -0.096^{+0.063}_{-0.058}\}$ [119–121] are extracted from the method of QCD sum rules [122]. Furthermore, we employ an alternative model (hereafter labeled as model IV) with the first three Gegenbauer coefficients $\{a_2, a_4, a_6\}(\mu_0) = \{0.196(32), 0.085(26), 0.056(15)\}$ [123] determined from the lattice prediction for the shape of the pion LCDA with large momentum effective theory [124–126]. We will vary the renormalization scale of the strong coupling α_s in the interval $\nu^2 \in [Q^2/2, 2Q^2]$. The

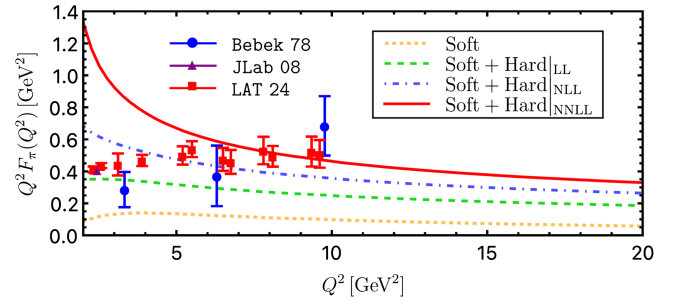


FIG. 2. Theory predictions of the pion EMFF from the soft Feynman mechanism [116] (yellow curve) and the hard-scattering contribution. We further collect the experimental data points (Bebek 78 [31] and JLab 08 [38]) and the recent lattice QCD results (LAT 24 [141]).

factorization scale μ will be taken as $\mu^2 = \langle x \rangle Q^2$ with $1/4 \leq \langle x \rangle \leq 3/4$ [127] (see also [47,57,128–134]).

We further display in Fig. 2 the obtained theory predictions for the leading-power contributions at leading-logarithmic (LL), NLL, and NNLL accuracy as well as the subleading power corrections arising from (I) an atypical “end-point” configuration of twist two in the one-loop approximation and (II) the higher-twist pion LCDAs up to the twist-six accuracy [116], by adopting model I of ϕ_{π} as our default choice. Apparently, the newly determined two-loop correction to the hard-scattering contribution can substantially enhance the NLL prediction of the pion EMFF: numerically at the level of (30–50)%. In analogy to the QCD anatomy of $B \rightarrow \gamma \ell \bar{\nu}_{\ell}$ [135–139], the soft nonfactorizable correction to the charged pion form factor can constantly shift the NNLL resummation improved leading-power contribution by approximately an amount of $\mathcal{O}(25\%)$ for $5.0 \text{ GeV}^2 \leq Q^2 \leq 20.0 \text{ GeV}^2$. We are then led to conclude that an ironclad and *fully analytical* extraction of the two-loop short-distance coefficient in the factorized expression (3) is vital for obtaining the robust and accurate theory prediction of $F_{\pi}(Q^2)$ [140].

We now turn to present in Fig. 3 our final theory predictions for the pion form factor with the four different

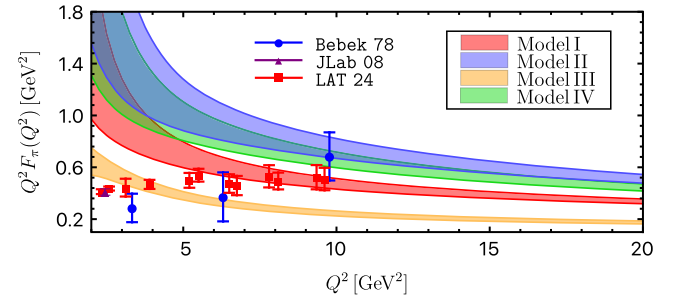


FIG. 3. Theory predictions of the pion EMFF with the four sample models of $\phi_{\pi}(x, \mu_0)$ obtained by adding together both the hard-scattering contribution and the subleading power correction. The uncertainties from varying the renormalization and factorization scales are indicated by the color bands.

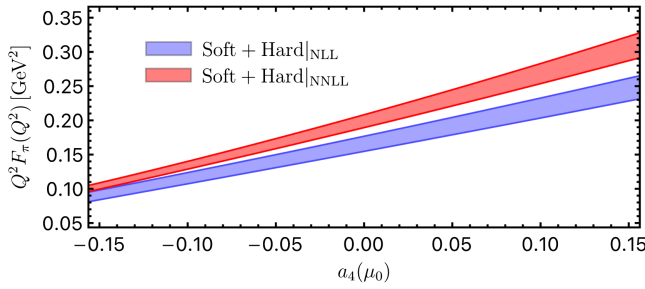


FIG. 4. Theory predictions for the dependence of the pion EMFF on the fourth conformal coefficient $a_4(\mu_0)$ at $Q^2 = 30.0 \text{ GeV}^2$, by taking advantage of the lattice determination of $a_2(\mu_0)$ [114] and by discarding the yet higher moments.

phenomenological models of the twist-two pion LCDA discussed above, confronting with the two experimental measurements Bebek 78 [31] and JLab 08 [38] at $Q^2 = \{2.45, 3.33, 6.30, 9.77\} \text{ GeV}^2$. It can then be observed that the yielding numerical prediction with the anti-de Sitter–QCD-inspired model fulfilling an additional constraint from the lattice determination of $a_2(\mu_0)$ [114] (i.e., our model I) provides us with the most optimized description of the experimental data points. In contrast with the NNLL QCD computation for the photon-pion transition form factor [59,143], our two-loop prediction of the pion EMFF approaches the desired scaling behavior in the formal $Q^2 \rightarrow \infty$ limit rather slowly [144].

We finally address the genuine benefit of the two-loop QCD calculation of the charged pion form factor on the model-independent extraction of the shape parameters dictating the twist-two pion LCDA. To this end, we proceed to explore the intrinsic sensitivity of $Q^2 F_\pi(Q^2)$ on the poorly constrained conformal coefficient $a_4(\mu_0)$. It is evident from Fig. 4 that including the newly obtained two-loop correction to the pion EMFF will indeed be advantageous to enhance the sensitivity of extracting the key shape parameter $a_4(\mu_0)$ in anticipation of the prospective precision EIC measurements [41].

Conclusions—In conclusion, we have endeavored to accomplish for the first time the rigorous two-loop computation of the pion EMFF analytically, by applying the modern effective field theory formalism. Crucially, we have demonstrated that the thus-determined NNLO QCD correction to the short-distance coefficient can bring about a sizable impact on the pion EMFF over a wide range of momentum transfers. In particular, the very inclusion of the two-loop correction allowed for an improved extraction of the shape parameters dictating the intricate profile of the pion LCDA. Extending and developing further our factorization prescription to the charged kaon form factor and to the exclusive $B_c \rightarrow \eta_c \ell \bar{\nu}_\ell$ form factors [148–151] will be beneficial for deepening our understanding toward the diverse facets of the strong interaction dynamics encoded in the hard-scattering processes.

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Data availability—The data supporting this study’s findings are available within the article.

- [1] G. P. Lepage and S. J. Brodsky, *Phys. Lett.* **87B**, 359 (1979).
- [2] G. P. Lepage and S. J. Brodsky, *Phys. Rev. D* **22**, 2157 (1980).
- [3] A. V. Efremov and A. V. Radyushkin, *Theor. Math. Phys.* **42**, 97 (1980).
- [4] A. V. Efremov and A. V. Radyushkin, *Phys. Lett.* **94B**, 245 (1980).
- [5] A. Duncan and A. H. Mueller, *Phys. Rev. D* **21**, 1636 (1980).
- [6] A. Duncan and A. H. Mueller, *Phys. Lett.* **90B**, 159 (1980).
- [7] S. J. Brodsky and G. P. Lepage, *Adv. Ser. Dir. High Energy Phys.* **5**, 93 (1989).
- [8] S. J. Brodsky and G. R. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973).
- [9] V. A. Matveev, R. M. Muradian, and A. N. Tavkhelidze, *Lett. Nuovo Cimento* **7**, 719 (1973).
- [10] F. Gross *et al.*, *Eur. Phys. J. C* **83**, 1125 (2023).
- [11] X.-D. Ji, *Phys. Rev. D* **55**, 7114 (1997).
- [12] A. V. Radyushkin, *Phys. Rev. D* **56**, 5524 (1997).
- [13] M. Diehl, T. Feldmann, R. Jakob, and P. Kroll, *Phys. Lett. B* **460**, 204 (1999).
- [14] M. Diehl, T. Feldmann, R. Jakob, and P. Kroll, *Eur. Phys. J. C* **8**, 409 (1999).
- [15] J. C. Collins, L. Frankfurt, and M. Strikman, *Phys. Rev. D* **56**, 2982 (1997).
- [16] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller, and M. Strikman, *Phys. Rev. D* **50**, 3134 (1994).
- [17] B. Pire, K. Semenov-Tian-Shansky, and L. Szymanowski, *Phys. Rep.* **940**, 1 (2021).
- [18] M. Beneke and T. Feldmann, *Nucl. Phys.* **B592**, 3 (2001).
- [19] M. Beneke and T. Feldmann, *Nucl. Phys.* **B685**, 249 (2004).

- [20] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, *Phys. Rev. D* **63**, 114020 (2001).
- [21] C. W. Bauer, D. Pirjol, and I. W. Stewart, *Phys. Rev. D* **67**, 071502(R) (2003).
- [22] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, *Nucl. Phys.* **B606**, 245 (2001).
- [23] M. Duraisamy and A. L. Kagan, *Eur. Phys. J. C* **70**, 921 (2010).
- [24] C.-D. Lü, Y.-L. Shen, C. Wang, and Y.-M. Wang, *Nucl. Phys.* **B990**, 116175 (2023).
- [25] E. B. Dally *et al.*, *Phys. Rev. D* **24**, 1718 (1981).
- [26] E. B. Dally *et al.*, *Phys. Rev. Lett.* **48**, 375 (1982).
- [27] S. R. Amendolia *et al.*, *Phys. Lett.* **146B**, 116 (1984).
- [28] S. R. Amendolia *et al.* (NA7 Collaboration), *Nucl. Phys.* **B277**, 168 (1986).
- [29] C. J. Bebek, C. N. Brown, M. Herzlinger, S. D. Holmes, C. A. Lichtenstein, F. M. Pipkin, S. Raither, and L. K. Sisterson, *Phys. Rev. D* **13**, 25 (1976).
- [30] C. J. Bebek *et al.*, *Phys. Rev. Lett.* **37**, 1326 (1976).
- [31] C. J. Bebek *et al.*, *Phys. Rev. D* **17**, 1693 (1978).
- [32] P. Brauel, T. Canzler, D. Cords, R. Felst, G. Grindhammer, M. Helm, W. D. Kollmann, H. Krehbiel, and M. Schadlich, *Z. Phys. C* **3**, 101 (1979).
- [33] H. Ackermann, T. Azemoon, W. Gabriel, H. D. Mertiens, H. D. Reich, G. Specht, F. Janata, and D. Schmidt, *Nucl. Phys.* **B137**, 294 (1978).
- [34] J. Volmer *et al.* (Jefferson Lab $F\pi$ Collaboration), *Phys. Rev. Lett.* **86**, 1713 (2001).
- [35] V. Tadevosyan *et al.* (Jefferson Lab $F\pi$ Collaboration), *Phys. Rev. C* **75**, 055205 (2007).
- [36] T. Horn *et al.* (Jefferson Lab $F\pi$ -2 Collaboration), *Phys. Rev. Lett.* **97**, 192001 (2006).
- [37] H. P. Blok *et al.* (Jefferson Lab Collaboration), *Phys. Rev. C* **78**, 045202 (2008).
- [38] G. M. Huber *et al.* (Jefferson Lab Collaboration), *Phys. Rev. C* **78**, 045203 (2008).
- [39] J. D. Sullivan, *Phys. Rev. D* **5**, 1732 (1972).
- [40] J. Arrington *et al.*, *Prog. Part. Nucl. Phys.* **127**, 103985 (2022).
- [41] R. Abdul Khalek *et al.*, *Nucl. Phys.* **A1026**, 122447 (2022).
- [42] R. D. Field, R. Gupta, S. Otto, and L. Chang, *Nucl. Phys.* **B186**, 429 (1981).
- [43] F. M. Dittes and A. V. Radyushkin, *Sov. J. Nucl. Phys.* **34**, 293 (1981).
- [44] M. H. Sarmadi, The asymptotic pion form factor to the leading order and beyond, Ph.D. thesis, University of Pittsburgh, 1982.
- [45] R. S. Khalmuradov and A. V. Radyushkin, *Sov. J. Nucl. Phys.* **42**, 289 (1985).
- [46] E. Braaten and S.-M. Tse, *Phys. Rev. D* **35**, 2255 (1987).
- [47] B. Melic, B. Nizic, and K. Passek, *Phys. Rev. D* **60**, 074004 (1999).
- [48] L.-B. Chen, W. Chen, F. Feng, and Y. Jia, *Phys. Rev. Lett.* **132**, 201901 (2024).
- [49] M. Beneke and S. Jager, *Nucl. Phys.* **B751**, 160 (2006).
- [50] M. Beneke and S. Jager, *Nucl. Phys.* **B768**, 51 (2007).
- [51] T. Becher and R. J. Hill, *J. High Energy Phys.* **10** (2004) 055.
- [52] R. J. Hill, T. Becher, S. J. Lee, and M. Neubert, *J. High Energy Phys.* **07** (2004) 081.
- [53] M. Beneke and D. Yang, *Nucl. Phys.* **B736**, 34 (2006).
- [54] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
- [55] A. J. Buras, M. Misiak, and J. Urban, *Nucl. Phys.* **B586**, 397 (2000).
- [56] Y.-M. Wang and Y.-L. Shen, *Nucl. Phys.* **B898**, 563 (2015).
- [57] Y.-M. Wang and Y.-L. Shen, *J. High Energy Phys.* **12** (2017) 037.
- [58] J. Gao, C.-D. Lü, Y.-L. Shen, Y.-M. Wang, and Y.-B. Wei, *Phys. Rev. D* **101**, 074035 (2020).
- [59] J. Gao, T. Huber, Y. Ji, and Y.-M. Wang, *Phys. Rev. Lett.* **128**, 062003 (2022).
- [60] Y.-K. Huang, B.-X. Shi, Y.-M. Wang, and X.-C. Zhao, *arXiv:2407.18724*.
- [61] H.-D. Li, C.-D. Lü, C. Wang, Y.-M. Wang, and Y.-B. Wei, *J. High Energy Phys.* **04** (2020) 023.
- [62] B.-Y. Cui, Y.-K. Huang, Y.-M. Wang, and X.-C. Zhao, *Phys. Rev. D* **108**, L071504 (2023).
- [63] J. Aebischer, A. J. Buras, and J. Kumar, *Phys. Rev. D* **107**, 075007 (2023).
- [64] M. I. Buchoff and M. Wagman, *Phys. Rev. D* **93**, 016005 (2016); **98**, 079901(E) (2018).
- [65] A. Buras, *Gauge Theory of Weak Decays* (Cambridge University Press, Cambridge, England, 2020).
- [66] J. C. Collins, *Renormalization*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2023), Vol. 26.
- [67] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, *Phys. Rev. D* **66**, 014017 (2002).
- [68] I. Z. Rothstein, *Phys. Rev. D* **70**, 054024 (2004).
- [69] Y. Aoki *et al.* (Flavour Lattice Averaging Group (FLAG)), *Eur. Phys. J. C* **82**, 869 (2022).
- [70] V. M. Braun and I. E. Filyanov, *Z. Phys. C* **44**, 157 (1989).
- [71] V. M. Braun and I. E. Filyanov, *Z. Phys. C* **48**, 239 (1990).
- [72] V. M. Braun, G. P. Korchemsky, and D. Müller, *Prog. Part. Nucl. Phys.* **51**, 311 (2003).
- [73] T. Hahn, *Comput. Phys. Commun.* **140**, 418 (2001).
- [74] G. Passarino and M. J. G. Veltman, *Nucl. Phys.* **B160**, 151 (1979).
- [75] F. Feng, *Comput. Phys. Commun.* **183**, 2158 (2012).
- [76] A. V. Smirnov, *J. High Energy Phys.* **10** (2008) 107.
- [77] A. V. Kotikov, *Phys. Lett. B* **254**, 158 (1991).
- [78] T. Gehrmann and E. Remiddi, *Nucl. Phys.* **B580**, 485 (2000).
- [79] J. M. Henn, *Phys. Rev. Lett.* **110**, 251601 (2013).
- [80] R. N. Lee, *J. High Energy Phys.* **04** (2015) 108.
- [81] R. N. Lee, *Comput. Phys. Commun.* **267**, 108058 (2021).
- [82] X. Liu and Y.-Q. Ma, *Comput. Phys. Commun.* **283**, 108565 (2023).
- [83] X. Liu, Y.-Q. Ma, and C.-Y. Wang, *Phys. Lett. B* **779**, 353 (2018).
- [84] X. Liu and Y.-Q. Ma, *Phys. Rev. D* **105**, L051503 (2022).
- [85] H. Ferguson, D. Bailey, and S. Arno, *Math. Comput.* **68**, 351 (1999).
- [86] H. R. Ferguson, D. H. Bailey, and P. Kutler, A polynomial time, numerically stable integer relation algorithm,

- Technical Report No. RNR-91-032, Supercomputing Research Center, 1998.
- [87] A. B. Goncharov, *Math. Res. Lett.* **5**, 497 (1998).
 - [88] A. B. Goncharov, [arXiv:math/0103059](https://arxiv.org/abs/math/0103059).
 - [89] P. Breitenlohner and D. Maison, *Commun. Math. Phys.* **52**, 11 (1977).
 - [90] G. Bonneau, *Nucl. Phys.* **B167**, 261 (1980).
 - [91] G. Bonneau, *Nucl. Phys.* **B171**, 477 (1980).
 - [92] F. Herzog, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt, *J. High Energy Phys.* **02** (2017) 090.
 - [93] T. Luthe, A. Maier, P. Marquard, and Y. Schroder, *J. High Energy Phys.* **10** (2017) 166.
 - [94] K. G. Chetyrkin, G. Falcioni, F. Herzog, and J. A. M. Vermaseren, *J. High Energy Phys.* **10** (2017) 179; **12** (2017) 006(A).
 - [95] M. H. Sarmadi, *Phys. Lett.* **143B**, 471 (1984).
 - [96] F. M. Dittes and A. V. Radyushkin, *Phys. Lett.* **134B**, 359 (1984).
 - [97] G. R. Katz, *Phys. Rev. D* **31**, 652 (1985).
 - [98] S. V. Mikhailov and A. V. Radyushkin, *Nucl. Phys.* **B254**, 89 (1985).
 - [99] A. V. Belitsky, D. Mueller, and A. Freund, *Phys. Lett. B* **461**, 270 (1999).
 - [100] M. J. Dugan and B. Grinstein, *Phys. Lett. B* **256**, 239 (1991).
 - [101] S. Herrlich and U. Nierste, *Nucl. Phys.* **B455**, 39 (1995).
 - [102] A. J. Buras and P. H. Weisz, *Nucl. Phys.* **B333**, 66 (1990).
 - [103] M. Jamin and A. Pich, *Nucl. Phys.* **B425**, 15 (1994).
 - [104] A. J. Buras, *Phys. Rep.* **1025**, 1 (2023).
 - [105] C. Duhr and F. Dulat, *J. High Energy Phys.* **08** (2019) 135.
 - [106] J. A. M. Vermaseren, *Int. J. Mod. Phys. A* **14**, 2037 (1999).
 - [107] J. Blumlein and S. Kurth, *Phys. Rev. D* **60**, 014018 (1999).
 - [108] V. A. Smirnov, *Analytic Tools for Feynman Integrals* (Springer, New York, 2012), Vol. 250.
 - [109] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/physrevd.103.034007> for the manifest expressions of the non-asymptotic corrections to the pion form factor at two loops.
 - [110] V. M. Braun, A. N. Manashov, S. Moch, and M. Strohmaier, *J. High Energy Phys.* **06** (2017) 037.
 - [111] M. Strohmaier, Conformal symmetry breaking and evolution equations in quantum chromodynamics, Ph.D. thesis, Regensburg University, 2018.
 - [112] S. J. Brodsky and G. F. de Teramond, *Phys. Rev. D* **77**, 056007 (2008).
 - [113] A. Khodjamirian, B. Melić, Y.-M. Wang, and Y.-B. Wei, *J. High Energy Phys.* **03** (2021) 016.
 - [114] G. S. Bali, V. M. Braun, S. Bürger, M. Göckeler, M. Gruber, F. Hutzler, P. Korcyl, A. Schäfer, A. Sternbeck, and P. Wein (RQCD Collaboration), *J. High Energy Phys.* **08** (2019) 065; **11** (2020) 037(A).
 - [115] S. Cheng, A. Khodjamirian, and A. V. Rusov, *Phys. Rev. D* **102**, 074022 (2020).
 - [116] V. M. Braun, A. Khodjamirian, and M. Maul, *Phys. Rev. D* **61**, 073004 (2000).
 - [117] J. Bijnens and A. Khodjamirian, *Eur. Phys. J. C* **26**, 67 (2002).
 - [118] A. Khodjamirian, T. Mannel, N. Offen, and Y. M. Wang, *Phys. Rev. D* **83**, 094031 (2011).
 - [119] A. P. Bakulev, S. V. Mikhailov, and N. G. Stefanis, *Phys. Lett. B* **508**, 279 (2001); **590**, 309(E) (2004).
 - [120] S. V. Mikhailov, A. V. Pimikov, and N. G. Stefanis, *Phys. Rev. D* **93**, 114018 (2016).
 - [121] N. G. Stefanis, *Phys. Rev. D* **102**, 034022 (2020).
 - [122] S. V. Mikhailov and A. V. Radyushkin, *Phys. Rev. D* **45**, 1754 (1992).
 - [123] I. Cloet, X. Gao, S. Mukherjee, S. Syritsyn, N. Karthik, P. Petreczky, R. Zhang, and Y. Zhao, *Phys. Rev. D* **110**, 114502 (2024).
 - [124] X. Ji, *Phys. Rev. Lett.* **110**, 262002 (2013).
 - [125] X. Ji, *Sci. China Phys. Mech. Astron.* **57**, 1407 (2014).
 - [126] X. Ji, Y. Liu, Y. S. Liu, J.-H. Zhang, and Y. Zhao, *Rev. Mod. Phys.* **93**, 035005 (2021).
 - [127] S. S. Agaev, V. M. Braun, N. Offen, and F. A. Porkert, *Phys. Rev. D* **83**, 054020 (2011).
 - [128] B. Melic, B. Nizic, and K. Passek, *Phys. Rev. D* **65**, 053020 (2002).
 - [129] N. G. Stefanis, *Eur. Phys. J. Direct* **1**, 7 (1999).
 - [130] M. Mojaza, S. J. Brodsky, and X.-G. Wu, *Phys. Rev. Lett.* **110**, 192001 (2013).
 - [131] X.-G. He, T. Li, X.-Q. Li, and Y.-M. Wang, *Phys. Rev. D* **74**, 034026 (2006).
 - [132] C.-D. Lu, Y.-M. Wang, H. Zou, A. Ali, and G. Kramer, *Phys. Rev. D* **80**, 034011 (2009).
 - [133] Y.-M. Wang and Y.-L. Shen, *J. High Energy Phys.* **02** (2016) 179.
 - [134] L. Di Giustino, S. J. Brodsky, P. G. Ratcliffe, X.-G. Wu, and S.-Q. Wang, *Prog. Part. Nucl. Phys.* **135**, 104092 (2024).
 - [135] V. M. Braun and A. Khodjamirian, *Phys. Lett. B* **718**, 1014 (2013).
 - [136] Y.-M. Wang, *J. High Energy Phys.* **09** (2016) 159.
 - [137] Y.-M. Wang and Y.-L. Shen, *J. High Energy Phys.* **05** (2018) 184.
 - [138] M. Beneke, V. M. Braun, Y. Ji, and Y.-B. Wei, *J. High Energy Phys.* **07** (2018) 154.
 - [139] A. Khodjamirian, B. Melić, and Y.-M. Wang, *Eur. Phys. J. Special Topics* **233**, 271 (2024).
 - [140] It remains interesting to observe that the very inclusion of the NNLO QCD correction to the pion EMFF turns out to be especially beneficial for better accommodating the benchmark lattice simulation results at intermediate momentum transfers [141]. Moreover, the obtained QCD predictions of the pion form factor at $Q^2 \geq 10.0 \text{ GeV}^2$ shown in Fig. 2 are also in nice agreement with the encouraging nonperturbative determinations using the dispersive matrix approach [142].
 - [141] H.-T. Ding, X. Gao, A. D. Hanlon, S. Mukherjee, P. Petreczky, Q. Shi, S. Syritsyn, R. Zhang, and Y. Zhao, *Phys. Rev. Lett.* **133**, 181902 (2024).
 - [142] S. Simula and L. Vittorio, *Phys. Rev. D* **108**, 094013 (2023).
 - [143] V. M. Braun, A. N. Manashov, S. Moch, and J. Schonleber, *Phys. Rev. D* **104**, 094007 (2021).
 - [144] This intriguing pattern can be attributed to the fact that the determined hierarchy between the subleading conformal spin effect and the counterpart asymptotic contribution grows steadily with the increasing loop order ℓ at realistic momentum transfers accessible in the current and

forthcoming experimental facilities, thus postponing the onset of the asymptotic regime to an enormously higher value of Q^2 (see [60,145–147] for further discussions in the context of the nucleon electromagnetic form factors).

- [145] V. M. Braun, A. Lenz, N. Mahnke, and E. Stein, *Phys. Rev. D* **65**, 074011 (2002).
- [146] V. M. Braun, A. Lenz, and M. Wittmann, *Phys. Rev. D* **73**, 094019 (2006).
- [147] I. V. Anikin, V. M. Braun, and N. Offen, *Phys. Rev. D* **88**, 114021 (2013).
- [148] P. Böer, QCD factorisation in exclusive semileptonic B decays: New applications and resummation of rapidity logarithms, Ph.D. thesis, Siegen University, 2018.
- [149] G. Bell and T. Feldmann, *Nucl. Phys. B, Proc. Suppl.* **164**, 189 (2007).
- [150] G. Bell, Higher order QCD corrections in exclusive charmless B decays, Ph.D. thesis, Munich University, 2006, G. Bell [arXiv:0705.3133](#).
- [151] P. Böer, G. Bell, T. Feldmann, D. Horstmann, and V. Shtabovenko, *Proc. Sci. RADCOR2023* (**2024**) 086 [[arXiv:2309.08410](#)].