

Cosmological Magnetic Fields from Ultralight Dark Matter

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We propose a mechanism for the generation of magnetic fields on cosmological scales that is operative after recombination. An essential ingredient is an instability (of parametric resonance type) of the electromagnetic field driven by an oscillating pseudoscalar dark-matter field, ϕ , that is coupled to the electromagnetic field tensor via a $\phi F \wedge F$ term in the Lagrangian of axion electrodynamics. We find that magnetic fields larger than the observational lower bounds can be generated soon after recombination on scales of 1 Mpc.

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Introduction—There is increasing evidence (see, e.g., Refs. [1–4] for reviews on cosmological magnetic fields) for the existence of magnetic fields on cosmological scales. Since such fields are present in voids, it is natural to assume that they are primordial in origin. In particular, there is a lower bound [5] on the amplitude of intergalactic magnetic fields from the nonobservation of γ -ray cascade emission,

$$B(r) > 10^{-17} \text{ Gauss}, \quad (1)$$

on length scales, r , larger than 10^{-1} Mpc [6]. The lower bound on the magnetic field increases on smaller scales, but on such scales the origin of the magnetic fields may be astrophysical. We will not consider them in this Letter.

The origin of cosmological magnetic fields remains a mystery. One popular mechanism for the generation of such fields relies on processes breaking the scale invariance in the electromagnetic sector that occur during a primordial period of cosmological inflation (see, e.g., Refs. [9,10]). They may occur if the inflaton, ϕ_I , is a pseudoscalar field coupling to the photon field via a $\phi_I F \wedge F$ term (see Refs. [11–17]) [18]. In the presence of such terms in the Lagrangian violating scale invariance it is also possible that magnetic fields are generated in cosmological phase transitions [25]. A key challenge for the viability of this particular mechanism is to show that magnetic fields are

generated on sufficiently large scales, because the typical length scale on which the magnetic field is produced initially is smaller than the Hubble radius at the time of the phase transition. The fact that, in magnetohydrodynamics, an inverse energy cascade (flow of energy from shorter to longer wavelength modes) has been established may help the argument. However, in both mechanisms, one faces the problem that the amplitude of the magnetic field decreases between the time of the phase transition and the present.

In this Letter we propose a mechanism for the generation of magnetic fields on cosmological scales that becomes relevant after the time of recombination. Our mechanism links magnetogenesis to the dark-matter mystery. We assume that dark matter is “wavelike” (see Refs. [26,27] for reviews of wave dark matter) and is described by a pseudoscalar axion field ϕ that couples to electromagnetism via the standard $\phi F \wedge F$ term. We assume that dark matter is created via a standard misalignment mechanism [28] and, hence, that the axion field ϕ starts to oscillate coherently in our Hubble patch once the Hubble rate H has fallen below the dark-matter mass, m . The oscillations of the field ϕ induce a “tachyonic” resonance in the electromagnetic field, a phenomenon previously explored in different contexts, such as [32] (axion monodromy inflation), [33] (inflationary magnetogenesis), [34] [graviton-induced axion-like particle (ALP) decay], and [35] (graviton to photon conversion). In our scenario, magnetic fields are generated at rather late times, namely after the time of recombination (see also the parametric resonance instability to matter generation from an oscillating inflaton field at the end of the period of inflation [36,37]).

Magnetic field generation from an oscillating ultralight dark-matter field—As stated, we assume that dark matter comes from a pseudoscalar, ultralight dark-matter field ϕ with a potential, V , that is quadratic in ϕ near its minimum,

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$$V(\phi) \simeq \frac{1}{2} m^2 \phi^2. \quad (2)$$

Considerations of structure formation imply a lower bound [38] on m of the order of $m > 10^{-20}$ eV (for lower values of m , structures on scales probed by Lyman α emitters would be suppressed). Assuming dark matter to be wavelike yields an upper bound on m , namely $m < 10$ eV (see, e.g., Ref. [39]). The field quanta of ϕ can be identified with axion or axionlike particles constituting dark matter [40]. Axion fields are ubiquitous in string theory, and it is not hard to argue for the existence of fields with ultralight masses (see, e.g., Ref. [41] for a recent construction).

We will introduce a dimensionless measure of the dark-matter mass via

$$m \equiv m_{20} 10^{-20} \text{ eV}. \quad (3)$$

A homogeneous coherent scalar-field configuration will remain frozen by Hubble damping until the time when $H \sim m$, where H is the Hubble expansion rate. Making use of the Friedmann equation we find that this is the case up to the time when the temperature T of the Universe drops below a value of roughly

$$T \sim m_{20}^{1/2} 10^4 T_{\text{eq}}, \quad (4)$$

where T_{eq} is the temperature at the time of matter-radiation equality (this equation is obtained by setting the Hubble expansion rate H equal to the mass m , and using the first Friedmann equation to relate H to the temperature T , taking into account the fact that before the time of equal matter and radiation the energy density is given by T^4). For our mechanism to be relevant for an explanation of the observed cosmic magnetic fields, we must require the coherence of the ϕ configuration to persist until the time of recombination. This implies that one must assume the mass m to be very small, since, in this case, the field ϕ starts to oscillate at a rather late time and, as a consequence, there is no time for its coherence to decay too early.

Axion fields typically interact with the electromagnetic field as described by a Lagrangian,

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} g_{\phi\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} - J^\mu A_\mu, \quad (5)$$

where $F_{\mu\nu}$ is the field strength tensor of the vector potential A_μ , $\tilde{F}_{\mu\nu}$ is its dual, J^μ is the current due to matter, and the coupling constant $g_{\phi\gamma}$ has inverse mass units. Using the homogeneous Maxwell equations to eliminate the electric field, $\vec{E}$, and assuming that ϕ only depends on physical time t , one finds that the Lagrangian in (5) yields the following equation for the magnetic induction, $\vec{B}$:

$$-\Delta \vec{B} + \ddot{\vec{B}} + [3H + \sigma] \dot{\vec{B}} + \left[\frac{3}{2} \dot{H} + \left(\frac{3}{2} H \right)^2 + \frac{3}{2} H \sigma \right] \vec{B} - g_{\phi\gamma} \dot{\phi} \vec{\nabla} \wedge \vec{B} = 0, \quad (6)$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the background spacetime, σ is the electric conductivity, and the overdot indicates a derivative with respect to physical time t , while the field equation for ϕ , with $V(\phi)$ as in (2), is given by

$$\square \phi = -g_{\phi\gamma} \vec{E} \cdot \vec{B} - m^2 \phi. \quad (7)$$

From the nonobservation of events caused by interactions of dark matter with photons there are mass-dependent upper limits on the coupling constant $g_{\phi\gamma}$. A conservative upper bound is (see, e.g., Ref. [42] for a review and [43] for a website with updated bounds)

$$\tilde{g}_{\phi\gamma} \ll 1, \quad (8)$$

where

$$g_{\phi\gamma} \equiv \tilde{g}_{\phi\gamma} 10^{-10} \text{ GeV}^{-1}. \quad (9)$$

The interaction term in (5) violates CP symmetry and affects the two photon polarization states differently. Thus, it may lead to the generation of a helical magnetic field.

Assuming that the axion field ϕ only depends on time, and setting the conductivity σ to zero, the field equations (6) yield the following equations of motion for the amplitudes, $\mathcal{A}_\pm$, of the two helicity modes of the electromagnetic vector potential in Fourier space (see Ref. [3] for a detailed discussion):

$$(\partial_\eta^2 + k^2 \pm k g_{\phi\gamma} a(\eta) \dot{\phi}) \mathcal{A}_\pm = 0, \quad (10)$$

where η is conformal time. Note that, in an expanding background, k is the comoving wave number. Equations (6) and (10) are valid in the absence of an electromagnetic plasma, i.e., only during a period when photons propagate without scattering. Thus, they are applicable after recombination [44]. We assume that the initial values of $\mathcal{A}_\pm(k)$ begin in their vacuum state with amplitude $1/\sqrt{2k}$.

From (10) we conclude that, in the presence of a dynamical axion field, there is a tachyonic instability in the evolution of $\mathcal{A}_\pm$. Depending on the sign of $\dot{\phi}$ this instability affects different polarization modes. Under our assumptions, ϕ is oscillating in time, and in this case during half of the oscillation period one polarization state is unstable, while during the other half of the period the other state is unstable. Since a varies in time, the Floquet exponent describing the exponential instability varies in time, and thus, when integrated over time, an asymmetry

between the two polarization states arises, resulting in a helical magnetic field.

The instability described here affects only long wavelength modes. For an oscillating ϕ background, $\dot{\phi} \sim \phi_0 m$, with ϕ_0 the initial amplitude of oscillation at recombination, the critical comoving wave number, k_c , above which the instability shuts off is given by [for larger values of k the k^2 term in (10) is always larger than the oscillating term and there is no instability]

$$k_c(\eta) \simeq g_{\phi\gamma} m \phi_0 a(\eta). \quad (11)$$

Note that the instability shifts to higher values of k as time increases. Since the instability sets in at the time of recombination and proceeds rapidly, we focus on this particular time.

We assume that the field ϕ accounts for most of the dark matter in the Universe. At the time of recombination $m\phi_0$ must then be given by

$$m\phi_0 \sim T_{\text{rec}}^2, \quad (12)$$

where $T_{\text{rec}} \sim 10^{-1}$ eV is the temperature of the Universe at the time of recombination (at the time of recombination the energy density T_{rec}^4 of radiation is close to the dark-matter density. Since the density in ϕ is proportional to $m^2\phi^2$ and we are assuming that ϕ dominates the dark matter, this equation follows immediately.). Hence [inserting (12) into (11)], the comoving wavelength, λ_c , below which the instability is shut off is given by

$$\lambda_c \sim \tilde{g}_{\phi\gamma}^{-1} 10^{-6} \text{ Mpc}. \quad (13)$$

Next, we provide an order of magnitude estimate of the magnetic field generated according to our mechanism. The instability in the evolution of the electromagnetic field manifests itself in an exponential growth of $\mathcal{A}_{\pm}(k)$ with the Floquet exponent [the Floquet exponent describes the exponential growth of the amplitude during the time interval of tachyonic resonance, $\mathcal{A}_{\pm}(k) \sim e^{\mu_k t}$, and can be determined by inserting this ansatz into the equation of motion, neglecting the k^2 term] μ_k , given by

$$\mu_k = (g_{\phi\gamma} a m \phi(t) k)^{1/2}, \quad (14)$$

where t is physical time, and $\phi(t)$ here denotes the amplitude of the envelope of the oscillating scalar field, which is equal to ϕ_0 at the time of recombination. Note that the Floquet exponent for the resonance increases with k until $k = k_c$. Since the phase space of Fourier modes scales as k^3 , most of the dark-matter energy flows into modes with $k \sim k_c$. Evaluated at this value [i.e., inserting the value of k_c from (11) into (15)], we obtain

$$\mu_{k_c} = g_{\phi\gamma} a m \phi(t). \quad (15)$$

The expansion of space is negligible, provided that $\mu_k(t) > H(t)$ (since in this case the energy transfer is exponential and takes place within a Hubble time H^{-1}). Taking into account the decay of the amplitude of ϕ , namely $\phi(t) \sim T^{3/2}$, where T is the temperature at time t , and making use of the Friedmann equations to determine $H(t)$, and inserting the scaling $a(t) = (t/t_0)^{2/3}$ of the scale factor during the matter dominated epoch (where t_0 is the present time), the efficiency criterion at time t becomes

$$g_{\phi\gamma} m \phi_0 > m_{pl}^{-1} T_{\text{rec}}^2 \frac{T}{T_0}, \quad (16)$$

where T_0 is the current temperature of the cosmic microwave background (CMB). With relation (12) we obtain the condition

$$\tilde{g}_{\phi\gamma} > 10^{-9} \frac{T}{T_0}, \quad (17)$$

for effective resonance (we remind the reader that by “effective resonance” we mean the requirement that the timescale of exponential increase in the gauge field amplitude is shorter than the Hubble timescale). Note that since μ decreases in time more slowly than H , it is possible that the efficiency condition is not satisfied at $T = T_{\text{rec}}$, but becomes satisfied later on.

The resonant production of photons drains energy from the axion condensate and induces a decrease in the amplitude of this condensate. In addition, the inhomogeneities in the gauge field produced by the resonance will backreact both on the condensate and on the gauge field itself and will shut off the resonance, in a similar way to how backreaction effects shut down the parametric resonance instability of the preheating phase of inflation (see, e.g., Ref. [45]) for a review). We will not study the details of this process here, but simply assume that the tachyonic instability shuts off once a fraction $\mathcal{F} \ll 1$ of the dark-matter density has been drained. Since we are considering the case of effective resonance when the timescale of the instability is less than the Hubble time, the shutoff of the resonance is expected to happen within a couple of Hubble times after recombination.

Since a fraction $\mathcal{F}$ of the dark-matter energy density at recombination (which is of the order T_{rec}^4) is transformed into electromagnetic energy peaked at the frequency k_c , the power spectrum, $P_B(k_c)$ of the magnetic field (which is the mean-square value of B in a sphere of radius k^{-1}) at the critical scale k_c is given by

$$P_B(k_c) \sim \mathcal{F} T_{\text{rec}}^4, \quad (18)$$

which (inserting the value of T_{rec} and remembering that $1 \text{ Gauss} \simeq 2 \times 10^{-20} \text{ GeV}^2$ in the natural units that we are using) corresponds to a characteristic position space value of B on the scale k_c of

$$B(k_c) \sim \mathcal{F}^{1/2} \text{Gauss.} \quad (19)$$

Note that this is the amplitude at $T = T_{\text{rec}}$, the time when the instability takes place. Afterward, the amplitude of B will decay as $a(t)^{-2}$.

The scaling law of the magnetic field on larger length scales follows the one of the magnetic field in a phase transition, which scales with wave number on length scales larger than the characteristic wavelength of the transition. It is thus given by (see Ref. [3] for a review)

$$B(k) \simeq \left(\frac{k}{k_c}\right)^n B(k_c), \quad (20)$$

where $n = 3/2$ for the helical magnetic field component, and $n = 1$ for the nonhelical component. Evaluating the result for $n = 3/2$, making use of the field amplitude (19) at the critical scale and the value (11) for the critical wave number, and taking into account that after creation, the magnetic field amplitude decays as $a(t)^{-2}$, the magnetic field on 1 Mpc scale ($k = k_1 \simeq 10^{-38}$ GeV) at the present time is found to be given by

$$B(k_1) \sim \tilde{g}_{\phi\gamma}^{-3/2} \mathcal{F}^{1/2} 10^{-15} \text{Gauss.} \quad (21)$$

We conclude that the mechanism described here can give rise to the growth of magnetic fields on megaparsec scales sufficiently large to be compatible with observational bounds. Note that the scaling of our final result, $\propto \tilde{g}_{\phi\gamma}^{-3/2}$, follows from the fact that k_c scales as $\tilde{g}_{\phi\gamma}$. This scaling relation only holds down to a value of $\tilde{g}_{\phi\gamma}$ when (17) ceases to be satisfied.

If the e-folding timescale μ_k^{-1} of the resonance is small compared to the oscillation timescale m^{-1} of the condensate, then the energy transfer takes place within less than an oscillation time and only one helicity component of the magnetic field will be produced. On the other hand, if $\mu_k^{-1} > m^{-1}$, then the resonance extends over multiple oscillation times and both helicity modes are produced. Since the amplitude of the condensate decreases and hence μ_k decreases, there will be a small remaining net helicity in the generated magnetic field. Using the expression for μ_k it follows that if $\tilde{g}_{\phi\gamma} m_{20}^{-1} \ll 1$, then the resonance extends over more than an oscillation timescale. Since $\tilde{g}_{\phi\gamma} \ll 1$ and $m_{20} > 1$ from observational constraints, we expect the resonance to extend over many oscillation time intervals.

Conclusions and discussion—In this Letter we have proposed a plausible mechanism for the generation of magnetic fields on cosmological scales. Our mechanism works at late times—specifically after the time of recombination. It is based on the tachyonic instability of infrared modes of the electromagnetic field, which sets in if a coherently oscillating pseudoscalar field ϕ is coupled to electromagnetism through a $\phi F \wedge F$ term in the Lagrangian. This term is typical for an axion field ϕ , which we have assumed to have an ultralight mass.

A key point of our proposal is that it provides a link between dark matter and cosmological magnetic fields. A mechanism similar to ours has been used recently [46] to provide sufficient Lyman-Werner radiation to allow direct collapse black hole formation from energy density fluctuations in the standard cosmological scenario.

We have assumed that ϕ makes up all of the dark matter. If it were to make up only a fraction f of the dark matter, our mechanism would still operate, but it would be less efficient. In particular, the critical wave number k_c and the Floquet exponent would be reduced by a factor of $f^{1/2}$. We have also assumed that no magnetic fields on cosmological scales are present at the time of recombination. If evidence for such field were found in the CMB, this would be problematic for the specific mechanism we have studied here, but it does not preclude the possibility that a coherently oscillating axion field might contribute to magnetic field production before the time of recombination. A study of this possibility would, however, be more involved.

Note added—It has been brought to our attention that primordial magnetic field generation produced during the formation of a large-scale structure has been explored assuming that dark energy is a pseudoscalar field that couples to electromagnetism [47]. There has also been a study [48] of axion dark matter producing large-scale magnetic fields via its interaction with baryons and vorticity generation. This mechanism is very different from the one we are proposing (and it also produces much weaker magnetic fields). In the study of [49], a coherently oscillating ALP field induces late time magnetogenesis via a coupling to a dark photon field, which induces a tachyonic instability. In terms of making use of a tachyonic instability there is a close connection between our Letter and that of [49]. However, our mechanism makes use of a direct channel of producing magnetic fields. It also produces magnetic fields of larger amplitude and on larger scales.

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Data availability—No data were created or analyzed in this study.

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End Matter

Equation of motion for the magnetic field—The Lagrangian for the electromagnetic U(1) gauge field coupled to an axion field is

$$\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} g_{\phi\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} - J^\mu A_\mu, \quad (\text{A1})$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor of the gauge field A_μ and $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ is the dual of $F_{\mu\nu}$. $J^\mu = (\rho, \vec{J})$ is the four-current, where $\vec{J}$ is the electric current density, and ρ is the charge density. In the expanding Universe, $F_{\mu\nu}$ is related to the electric and magnetic fields as

$$F_{\mu\nu} = a^2(\eta) \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}. \quad (\text{A2})$$

By varying the Lagrangian with respect to A_μ , we can obtain the equation of motion for the electromagnetic field,

$$\partial_\nu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\nu A_\mu)} \right) - \frac{\partial \mathcal{L}}{\partial A_\mu} = 0 \quad (\text{A3})$$

$$\Rightarrow \partial_\nu F^{\mu\nu} = J^\mu + g_{\phi\gamma} \dot{\phi} \tilde{F}^{\mu 0}, \quad (\text{A4})$$

which corresponds to the Maxwell equations with an extra term due to the axion field [the rhs of Eq. (A8)],

$$\nabla \cdot \vec{E} = \rho, \quad (\text{A5})$$

$$\nabla \cdot \vec{B} = 0, \quad (\text{A6})$$

$$\vec{\nabla} \wedge \vec{E} + \dot{\vec{B}} + \frac{3}{2} H \vec{B} = 0, \quad (\text{A7})$$

$$\vec{\nabla} \wedge \vec{B} - \dot{\vec{E}} - \frac{3}{2} H \vec{E} = \vec{J} + g_{\phi\gamma} \dot{\phi} \vec{B}, \quad (\text{A8})$$

where $\rho = 0$ and $\vec{J} = \sigma \vec{E}$ in the background. As we are interested in the magnetic field, we can derive its equation of motion by taking the curl of (A8) and using (A7) to eliminate $\vec{E}$, and thus we have

$$\begin{aligned}
 & -\Delta\vec{B} + \left(\partial_0 + \frac{3}{2}H + \sigma\right) \left(\ddot{\vec{B}} + \frac{3}{2}H\vec{B}\right) - g_{\phi\gamma}\dot{\phi}\vec{\nabla} \wedge \vec{B} = 0 \\
 \Rightarrow & -\Delta\vec{B} + \ddot{\vec{B}} + [3H + \sigma]\dot{\vec{B}} + \left[\frac{3}{2}\dot{H} + \left(\frac{3}{2}H\right)^2 + \frac{3}{2}H\sigma\right]\vec{B} \\
 & - g_{\phi\gamma}\dot{\phi}\vec{\nabla} \wedge \vec{B} = 0. \tag{A9}
 \end{aligned}$$

The equation of motion for the axion field is obtained by varying $\mathcal{L}$ with respect to ϕ ,

$$\partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0, \tag{A10}$$

where $[\partial \mathcal{L} / \partial(\partial_\nu \phi)] = \frac{1}{2} [\partial(\partial^\mu \phi \partial_\mu \phi) / \partial(\partial_\nu \phi)] = \partial^\nu \phi$, so the equation of motion becomes

$$\Rightarrow \partial^\mu \partial_\mu \phi + \frac{\partial V(\phi)}{\partial \phi} - g_{\phi\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} = 0. \tag{A11}$$

In the last term $F_{\mu\nu} \tilde{F}^{\mu\nu} = -4\vec{E} \cdot \vec{B}$. Therefore, (A11) reproduces (7) for $V(\phi) \simeq \frac{1}{2} m^2 \phi^2$.