

Core-Periphery Detection in Multilayer Networks

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Multilayer networks provide a powerful framework for modeling complex systems that capture different types of interactions between the same set of entities across multiple layers. Core-periphery detection involves partitioning the nodes of a network into core nodes, which are highly connected across the network, and peripheral nodes, which are densely connected to the core but sparsely connected among themselves. In this paper, we propose a new model of core-periphery structure in multilayer networks and a nonlinear spectral method that simultaneously detects the corresponding core and periphery structures of both nodes and layers in weighted and directed multilayer networks. Our method reveals novel structural insights in three empirical multilayer networks from distinct application areas: the citation network of complex network scientists, the European airlines transport network, and the world trade network.

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Introduction—The study of networks is a powerful approach to understanding complex systems across various disciplines, including physics, the natural sciences, economics, engineering, and sociology [1–4]. Multilayer network models offer the flexibility to represent different types of relationships between the same set of nodes across multiple layers [5,6]. In the early 2010s, several breakthroughs established the study of multilayer networks as a distinct subdiscipline within network science [7–11]. Since then, the generalization of concepts and techniques from single-layer networks to multilayer networks has become an active and evolving field of research.

A crucial aspect of analyzing complex networks involves examining structural properties such as community structure, metric structure, degree distributions, and centralities [3,4]. The core-periphery (also referred to as rich club or core-fringe) detection problem involves dividing the nodes of a network into two subsets: the core, which is well connected to the entire network, and the periphery, which is densely connected to the core but sparsely connected internally. Borgatti and Everett’s seminal work first formalized core-periphery structure, characterizing it as an ideal L-shape in the network’s adjacency matrix [12]. Since

then, numerous methods have been proposed for detecting core-periphery structures, and such structures have been identified in various types of networks, including physical, social, computer, biological, brain, and transport networks [13–23].

Core-periphery detection can be formulated as a combinatorial optimization problem, the exact solution of which becomes computationally infeasible for even moderately sized networks. To address this, standard methods often rely on a relaxed formulation where one computes a positive coreness vector $\mathbf{x} \in \mathbb{R}_{\geq 0}^n$, where larger values of x_i indicate a higher likelihood that node i belongs to the core. Many core-periphery detection methods exist for standard, single-layer networks and it was noted in [19,20] that a large number of available approaches, including, e.g., [12,15,17,24,25], can be formalized in terms of the following objective function:

$$f(\mathbf{x}) = \sum_{i,j=1}^n A_{ij} \kappa(x_i, x_j), \quad (1)$$

where $\mathbf{A} \in \mathbb{R}_{\geq 0}^{n \times n}$ is the adjacency matrix and $\kappa: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ is a suitable kernel function.

While well understood in the standard single-layer setting, core-periphery detection in multilayer networks remains a relatively underdeveloped area. Existing approaches, such as those based on degree counts, offer limited insights into the more intricate structures of multilayer systems [18]. A recent approach introduced in [22] proposes a nonlinear spectral method to optimize a multilayer version of (1), specifically for unweighted, undirected

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multiplex networks without interlayer edges. This method has demonstrated superior performance compared to degree-based approaches in several multiplex networks, including genetic, transportation, and social systems. However, extending this method to general multilayer networks that incorporate interlayer edges is nontrivial.

In this Letter, we introduce a model to detect and quantify core and periphery sets for both nodes and layers in general multilayer networks, accommodating arbitrary inter- and intralayer edges, potentially with weights and directions. Our model is based on the maximization of a generalized core-periphery kernel function, resulting in a challenging nonconvex and nonlinear optimization problem. Despite this, we show that the model can be applied in practical settings by proposing a tailored core-periphery detection algorithm that efficiently solves the optimization task and determines core and periphery sizes for both layers and nodes. We provide theoretical results demonstrating the convergence of the algorithm to either global or local optima. We validate the proposed method on three empirical networks representing collaboration, transportation, and commercial trade data, revealing novel structural insights.

Multilayer core-periphery model—Defining the notion of core and periphery in multilayer networks is a nontrivial task, as nodes may belong to the core in some layers while belonging to the periphery in others, or peripheral nodes may be weakly connected within layers but strongly connected across layers.

In this section, we propose a novel model for detecting core-periphery structures in general multilayer networks. The key idea is to simultaneously detect core structures among both nodes and layers by optimizing a coreness vector for each, using a provably convergent and efficiently implementable optimization procedure.

For the representation of multilayer networks with n nodes and L layers, we define a fourth-order adjacency tensor

$$\mathcal{A} \in \mathbb{R}_{\geq 0}^{n \times n \times L \times L}, \quad [\mathcal{A}_{ij}^{k\ell}]_{i,j=1,\dots,n}^{k,\ell=1,\dots,L},$$

which encodes possibly weighted and directed edges from node i in layer k to node j in layer ℓ .

Our goal is to optimize entry-wise positive node and layer coreness vectors, denoted by $\mathbf{x} \in \mathbb{R}_{>0}^n$ and $\mathbf{c} \in \mathbb{R}_{>0}^L$, respectively. High values in these vectors correspond to core nodes and layers, while low values correspond to peripheral ones. To prevent the blowup of the objective function (2), we impose the norm constraints $\|\mathbf{x}\|_p = \|\mathbf{c}\|_q = 1$ using the p norm and q norm, respectively. The choice of p and q does not affect the model, but will allow us to provide convergence guarantees of the proposed computational strategy.

We extend the objective function in (1) to multilayer networks by summing the entries of the adjacency tensor, weighted by a kernel function that incorporates both the

node and layer coreness vectors. As our kernel function κ , we select the product of two smoothed maximum kernels, which encourages at least one large node coreness score x_i , x_j and one large layer coreness score c_k, c_ℓ whenever the edge weight $\mathcal{A}_{ij}^{k\ell}$ is significant [19–22]. The objective function to maximize is given by

$$f_{\alpha,\beta}(\mathbf{x}, \mathbf{c}) = \sum_{i,j=1}^n \sum_{k,\ell=1}^L \mathcal{A}_{ij}^{k\ell} (x_i^\alpha + x_j^\alpha)^{1/\alpha} (c_k^\beta + c_\ell^\beta)^{1/\beta}. \quad (2)$$

Here, the scalar parameters $\alpha, \beta > 1$ control the smoothness of the kernel. In the limit $\alpha \rightarrow \infty$, the term $(x_i^\alpha + x_j^\alpha)^{1/\alpha}$ converges to $\max\{x_i, x_j\}$, yielding the standard maximum kernel. Notably, when $\alpha = \beta$, the roles of $\mathbf{x}$ and $\mathbf{c}$ are interchanged upon transforming the original network into its dual, as explored in Ref. [26]. The choice $\alpha \neq \beta$, however, allows for additional model flexibility.

To enforce the norm constraints, we normalize the objective function (2) by dividing by $\|\mathbf{x}\|_p \|\mathbf{c}\|_q$. The optimization problem is then solved by setting the gradients with respect to both $\mathbf{x}$ and $\mathbf{c}$ to zero, yielding the following fixed-point equations:

$$\mathbf{x} = J_{p^*}[\nabla_{\mathbf{x}} f_{\alpha,\beta}(\mathbf{x}, \mathbf{c})], \quad \mathbf{c} = J_{q^*}[\nabla_{\mathbf{c}} f_{\alpha,\beta}(\mathbf{x}, \mathbf{c})], \quad (3)$$

using similar arguments as in Refs. [19,22]. Here, $J_p(\mathbf{x}) = \|\mathbf{x}\|_p^{1-p} \mathbf{x}^{p-1}$ is the gradient of the p norm, and p^* is the Hölder conjugate of p , defined by $1/p + 1/p^* = 1$.

Our proposed method consists of alternating fixed-point iterations of the two equations in (3), which are summarized in Algorithm 1. This method can be efficiently implemented in a few lines of code. We refer to it as a nonlinear spectral method, as it can be interpreted as a power iteration for a generalized nonlinear eigenvalue problem [19,22].

The global convergence of Algorithm 1 is ensured by nonlinear and multilinear Perron-Frobenius theory [27–29]. Specifically, we define the following 2×2 coefficient matrix:

$$\mathbf{M} = \begin{bmatrix} \frac{2|\alpha-1|}{p-1} & \frac{2}{p-1} \\ \frac{2}{q-1} & \frac{2|\beta-1|}{q-1} \end{bmatrix}, \quad (4)$$

and its spectral radius $\rho(\mathbf{M})$. We can then state the following convergence result:

Theorem 1—For parameters $\alpha, \beta, p, q > 1$ such that

$$\rho(\mathbf{M}) < 1,$$

Algorithm 1 converges to the global maximum of $f_{\alpha,\beta}$ defined in (2), for any initial vectors $\mathbf{x}_0 \in \mathbb{R}_{>0}^n$ and $\mathbf{c}_0 \in \mathbb{R}_{>0}^L$, with a linear rate of convergence.

Algorithm 1. Nonlinear spectral method for core-periphery detection in multilayer networks.

Input: $\mathcal{A} \in \mathbb{R}_{\geq 0}^{n \times n \times L \times L}$, Adjacency tensor.
 $\mathbf{x}_0 \in \mathbb{R}_{\geq 0}^n$, Initial node coreness vector.
 $\mathbf{c}_0 \in \mathbb{R}_{\geq 0}^L$, Initial layer coreness vector.
Parameters: $\alpha, \beta, p, q > 1$, $\text{tol} \in \mathbb{R}_{> 0}$, $\text{maxIter} \in \mathbb{N}$.

- 1: $\mathbf{x}_0 = \mathbf{x}_0 / \|\mathbf{x}_0\|_{p/(p-1)}$
- 2: $\mathbf{c}_0 = \mathbf{c}_0 / \|\mathbf{c}_0\|_{q/(q-1)}$
- 3: **for** $k = 1 : \text{maxIter}$ **do**
- 4: $\tilde{\mathbf{x}} = \nabla_{\mathbf{x}} f_{\alpha, \beta}(\mathbf{x}_{k-1}, \mathbf{c}_{k-1})$
- 5: $\mathbf{x}_k = (\tilde{\mathbf{x}} / \|\tilde{\mathbf{x}}\|_{p/(p-1)})^{1/(p-1)}$
- 6: $\tilde{\mathbf{c}} = \nabla_{\mathbf{c}} f_{\alpha, \beta}(\mathbf{x}_{k-1}, \mathbf{c}_{k-1})$
- 7: $\mathbf{c}_k = (\tilde{\mathbf{c}} / \|\tilde{\mathbf{c}}\|_{q/(q-1)})^{1/(q-1)}$
- 8: **if** $\|\mathbf{x}_k - \mathbf{x}_{k-1}\| < \text{tol}$ **and** $\|\mathbf{c}_k - \mathbf{c}_{k-1}\| < \text{tol}$ **then**
- 9: **break**
- 10: **end if**
- 11: **end for**

Output: $\mathbf{x} \in \mathcal{S}_p^+$, Optimized node coreness vector.
 $\mathbf{c} \in \mathcal{S}_q^+$, Optimized layer coreness vector.

Proof—The proof is provided in Sec. VI of Supplemental Material [30]. ■

It has been observed that for single-layer and multiplex networks, selecting relatively small values of p and q may yield core-periphery structures closer to the ideal L-shape [22]. As such values do not necessarily satisfy the assumption of Theorem 1, the global convergence of Algorithm 1 cannot be guaranteed in general. In this situation, however, convergence to a local optimum is still ensured by a previous result [22] (Thm. 3.2).

Determining core sizes—Once the node and layer coreness vectors $\mathbf{x}$ and $\mathbf{c}$ have been optimized, the next challenge is determining where to draw the boundary between core and periphery.

In single-layer networks, core-periphery structures are often visualized by permuting the rows and columns of the adjacency matrix $\mathbf{A} \in \mathbb{R}_{\geq 0}^{n \times n}$ in descending order of the nodes' coreness scores. A recent method proposes identifying the core size by sweeping over the permuted adjacency matrix and comparing it to Borgatti and Everett's idealized L-shape model for different core sizes [21].

We extend this approach to multilayer networks by generalizing the single-layer L-shape model. The multilayer network is represented by flattening the adjacency tensor into the supra-adjacency matrix, a block matrix of size $L \times L$, where each block is of size $n \times n$. Block (k, ℓ) represents edges between nodes in layer k and layer ℓ [5, 7–9]. To facilitate comparison, we normalize the supra-adjacency matrix by dividing it by its largest entry, allowing edge weights of 1 to represent present edges, 0 missing edges, and weights in $(0, 1)$ partially present edges.

We propose a two-level permutation of the supra-adjacency matrix. First, the block rows and columns are

permuted using the layer coreness vector $\mathbf{c}$, while each block is further permuted according to the node coreness vector $\mathbf{x}$, as done in the single-layer case. A strong multilayer core-periphery structure is characterized by dense blocks forming an L-shape at the block level, with each block itself also exhibiting an inner L-shape. Figure 1 provides an illustrative example.

To identify the core size in single-layer networks, the sweeping procedure evaluates n binary vectors $\bar{\mathbf{x}} \in \{0, 1\}^n$, where each binary vector $s = 1, \dots, n$ contains nonzeros in the first s entries [21]. Given the permuted binary adjacency matrix $\mathbf{A}$, the objective function

$$\sum_{i,j=1}^n \left(\frac{A_{ij}}{n_1} \max\{\bar{x}_i, \bar{x}_j\} + \frac{1 - A_{ij}}{n_2} (1 - \max\{\bar{x}_i, \bar{x}_j\}) \right), \quad (5)$$

is evaluated for each $\bar{\mathbf{x}}$, where $n_1 = \sum_{i,j=1}^n A_{ij}$ is the total number of present edges, and $n_2 = n^2 - n_1$ is the number of absent edges. Equation (5) compares the adjacency matrix with an ideal L-shape model for core size s , with present edges in the first s rows and columns, and absent edges in the bottom-right $(n - s) \times (n - s)$ block. The normalization factors n_1 and n_2 ensure that both contributions are weighted according to the overall structure of the network. The core size s^* is selected as the value of s that maximizes (5) [21, 22].

For multilayer networks, we aim to detect both the node core size s_{node}^* and the layer core size s_{layer}^* via separate sweeping procedures. To determine the node core size, we compute the similarity of each block in the permuted supra-adjacency matrix to an ideal L-shape using the layer coreness values $\gamma_{k\ell} = \max\{c_k, c_\ell\}$ as weights. Using the same binary vectors $\bar{\mathbf{x}} \in \{0, 1\}^n$ as in the single-layer case, the objective function is

$$\sum_{k, \ell=1}^L \gamma_{k\ell} \sum_{i, j=1}^n \left(\frac{\mathcal{A}_{ij}^{k\ell}}{n_1^{(k\ell)}} \bar{x}_{ij} + \frac{1 - \mathcal{A}_{ij}^{k\ell}}{n_2^{(k\ell)}} (1 - \bar{x}_{ij}) \right), \quad (6)$$

where $n_1^{(k\ell)} = \sum_{i, j=1}^n \mathcal{A}_{ij}^{k\ell}$, $n_2^{(k\ell)} = n^2 - n_1^{(k\ell)}$, and $\bar{x}_{ij} = \max\{\bar{x}_i, \bar{x}_j\}$. The node core size s_{node}^* is chosen as the number of nonzero entries in $\bar{\mathbf{x}}$ that maximizes (6).

For the layer core size, we perform a similar sweep over binary layer coreness vectors $\bar{\mathbf{c}} \in \{0, 1\}^L$ and define the corresponding objective function as

$$\sum_{i, j=1}^n \chi_{ij} \sum_{k, \ell=1}^L \left(\frac{\mathcal{A}_{ij}^{k\ell}}{n_1^{(ij)}} \bar{c}_{k\ell} + \frac{1 - \mathcal{A}_{ij}^{k\ell}}{n_2^{(ij)}} (1 - \bar{c}_{k\ell}) \right), \quad (7)$$

where $n_1^{(ij)} = \sum_{k, \ell=1}^L \mathcal{A}_{ij}^{k\ell}$, $n_2^{(ij)} = L^2 - n_1^{(ij)}$, $\chi_{ij} = \max\{x_i, x_j\}$, and $\bar{c}_{k\ell} = \max\{\bar{c}_k, \bar{c}_\ell\}$. This respects the symmetry between nodes and layers imposed by (2).

Normalizing (6) by $(1/\sum_{k,\ell=1}^L \gamma_{k\ell})$ and (7) by $(1/\sum_{i,j=1}^n \chi_{ij})$ ensures that the objective function values lie in the range $[-1, 1]$, where larger values indicate stronger multilayer core-periphery structures, with configurations closer to the ideal two-level L-shape. Note that the described procedure relies on the node and layer coreness rankings generated by Algorithm 1 that are used to obtain the permuted supra-adjacency matrix. Details on the efficient numerical evaluation of (6) and (7) are provided in Sec. V of the Supplemental Material [30].

Numerical results on empirical multilayer networks—We test Algorithm 1 on three real-world multilayer networks: the citation network of complex network scientists, the European airlines transport network, and the world trade network (in End Matter). In addition, we report results on synthetic networks in Sec. I of the Supplemental Material [30]. In all experiments, we use the parameters $\alpha = \beta = 10$ [32], $\text{tol} = 10^{-8}$, $\text{maxIter} = 200$, with initial vectors $\mathbf{x}_0 = \mathbf{1} \in \mathbb{R}^n$ and $\mathbf{c}_0 = \mathbf{1} \in \mathbb{R}^L$, where $\mathbf{1}$ denotes the vector of all ones. A JULIA implementation is publicly available [33], and all the network data is publicly available [34].

Citation multilayer network of complex network scientists: We constructed the citation network of complex network scientists using publicly available data from OpenAlex [35]. We collected data on all recorded complex network science publications by filtering for the level-2 concept [36] “complex network,” resulting in 38 346 works published before January 1, 2024. After author disambiguation to handle hyphenated names, we identified $n = 53\,423$ unique authors, representing the nodes in the multilayer network. The layers are instead formed by the level-0 concepts in the OpenAlex topics hierarchy. We detected $L = 19$ level-0 concepts, including “computer science,” “mathematics,” and “physics.”

Edges originate from one complex network science paper citing another. Directed edges were inserted from all citing authors and disciplines associated with the citing paper toward all cited authors and disciplines of the cited paper, resulting in 1.55×10^7 nonzero entries in the

adjacency tensor $\mathcal{A}$. We weighted citations such that the total edge weight of one paper citing another is 1, with each citation weight divided by the product of the number of citing authors, citing disciplines, cited authors, and cited disciplines. Additional results, including analyses on “unweighted” multilayer networks (which omit the citation weighting), are provided in Sec. II of the Supplemental Material [30]. Both weighting approaches yield sparse networks with $\mathcal{O}(nL)$ edges.

Figure 1 shows the original as well as the permuted supra-adjacency matrices of the weighted and directed OpenAlex citation network. Both the outer and the inner permutation, i.e., the permutation of and within blocks shows a strong concentration of edges with large weights in the upper left corner of supra-adjacency matrix and all individual blocks, respectively. Using $\alpha = \beta = 10$ in Algorithm 1, the parameters $p = q = 22$ satisfy the assumptions of Theorem 1, ensuring convergence to the global optimum. With $p = q = 2$, convergence is only guaranteed to a local optimum, yet the intra-block reorderings exhibit strong L-shapes. Both parameter settings detect $s_{\text{layer}}^* = 2$ disciplines, computer science and mathematics, as the layer core. For $p = q = 22$, $s_{\text{node}}^* = 4058$ authors belong to the node core, whereas for $p = q = 2$, $s_{\text{node}}^* = 2566$. Despite the large problem size of $n = 53\,423$ and $L = 19$, Algorithm 1 required only 135 sec on a standard laptop, completing in 26 iterations for $p = q = 22$.

We also constructed a sequence of adjacency tensors for publications up to the years 2000 to 2023. Figure 2 shows the evolution of the top 10 author coreness rankings over this period. After strong reorderings following several landmark publications in the early 2000s, the top 5 core authors remain constant from year 2006 onward.

European airlines multilayer network: We analyze the largest connected component of the European Airlines (EUAir) network as of 2012 [37]. This multiplex network consists of $n = 417$ airports and $L = 37$ airlines. Intralayer edges are undirected and unweighted, indicating the existence of flight connections between airports for a given airline. Interlayer edges follow the approach in Ref. [38],

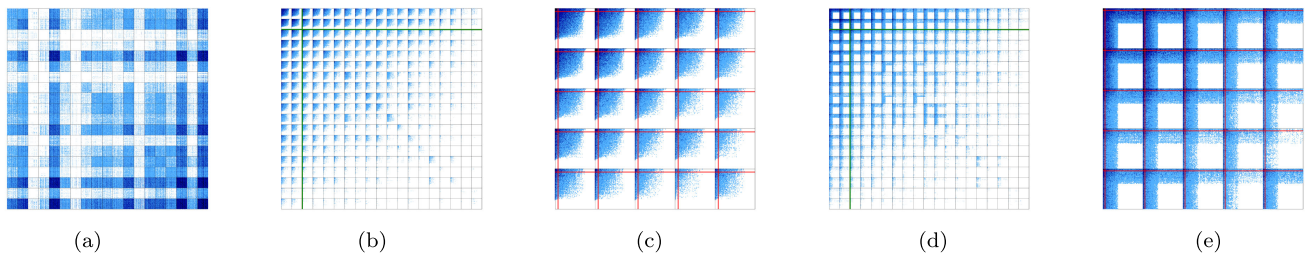


FIG. 1. Supra-adjacency matrix plots for the weighted OpenAlex multilayer citation network of complex network scientists of the year 2023. Dark fonts indicate large edge weights. Green lines in panels (b) and (d) indicate the layer core sizes $s_{\text{layer}}^* = 2$ and red lines in panels (c) and (e) indicate the node core sizes $s_{\text{node}}^* = 4058$ and $s_{\text{node}}^* = 2566$, respectively, in each block. (a) Original supra-adjacency. (b) Full permuted supra-adjacency for $p = q = 22$. (c) Upper left 5×5 blocks of permuted supra-adjacency for $p = q = 22$. (d) Full permuted supra-adjacency for $p = q = 2$. (e) Upper left 5×5 blocks of permuted supra-adjacency for $p = q = 2$.

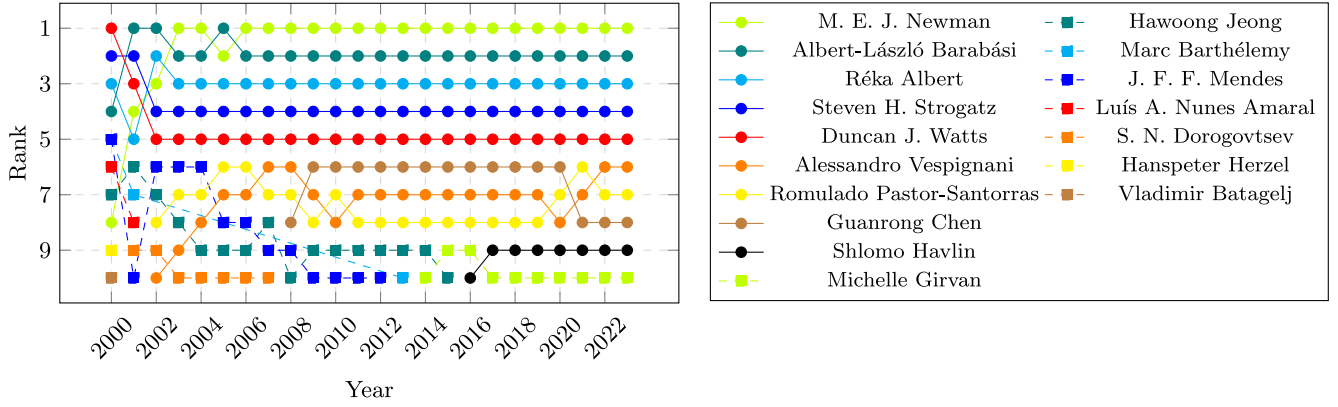


FIG. 2. Top 10 authors by node coreness score in the weighted OpenAlex citation multilayer network over the years 2000 to 2023 for the parameters $p = q = 22$.

where nodes (airports) in different layers (airlines) are connected if the airport is served by both airlines, allowing passengers to switch between airlines.

We ran Algorithm 1 with $p = q = 22$, detecting $s_{\text{node}}^* = 57$ core airports (see Table III in the Supplemental Material [30]) and $s_{\text{layer}}^* = 4$ core airlines: Lufthansa, easyJet, Ryanair, and Air Berlin. Figure 3 illustrates the core airports alongside intralayer connections with aggregated edge weights greater than 1.

An interesting observation is that Ryanair, ranked highly by several centrality measures [39], is the third-highest layer by coreness score for $p = q = 22$, but drops to rank 18 out of $L = 37$ layers for $p = q = 2$. This reflects Ryanair's tendency to serve peripheral airports, despite having numerous connections.

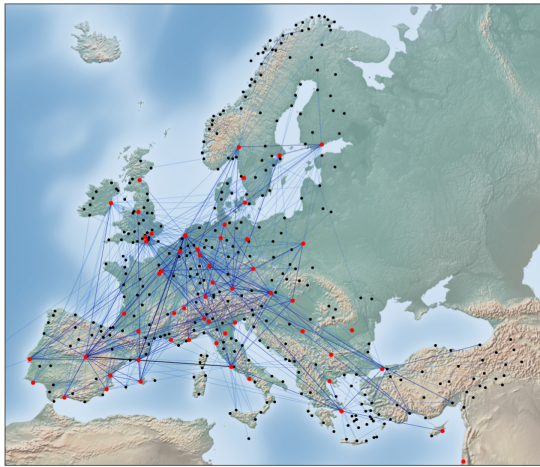


FIG. 3. Plot of the aggregated European airlines network. Core airports are marked red while periphery airports are marked black. Only edges with aggregated intralayer edge weight larger than 1 are recorded. Dark fonts indicate large edge weights. The figure was created with matplotlib's basemap library.

Conclusion—In this work, we proposed a new model of core-periphery structure in general multilayer networks along with a provably convergent nonlinear spectral method for detecting the modeled cores and peripheries. Our method simultaneously optimizes both node and layer coreness vector, allowing for a comprehensive understanding of core-periphery dynamics across multiple layers. We also introduced a procedure for determining the sizes of node and layer cores and applied our framework to three real-world multilayer networks from diverse application domains, uncovering novel structural insights into citation, transportation, and world trade networks.

Data availability—The data that support the findings of this Letter are openly available [34].

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End Matter

World trade multilayer network—We consider the world trade network, recorded in the World Input Output Database (WIOD) [40,41] for the years 2000 to 2014.

Nodes correspond to $n = 56$ industries, and layers correspond to $L = 43$ countries. A separate multilayer network was constructed for each year. Directed and

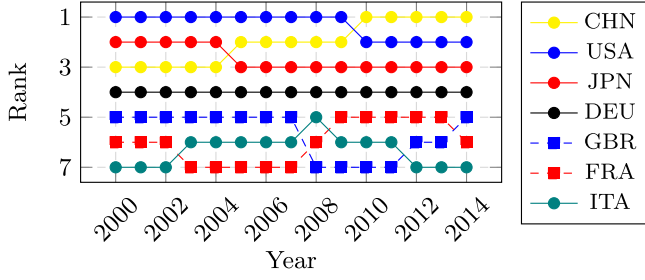


FIG. 4. Rankings of country cores by layer coreness scores over the years 2000 to 2014 in the WIOD world trade multilayer network.

weighted edges represent trade volumes between industries in different countries, measured in millions of USD. Although the network is not fully connected, it is dense with $\mathcal{O}(n^2 L^2)$ edges, but the edge weights vary widely.

Figures 4 and 5 illustrate the evolution of country and industry cores from 2000 to 2014, showing their coreness score rankings. Figure 4 shows a consistent country core of $s_{\text{layer}}^* = 7$ countries over all years, with notable shifts in rankings, such as China's rise to the top position. In contrast, Fig. 5 shows that the industry core size s_{node}^* varies between 9 and 16, with a sharp drop from 16 in 2007 to 10 in 2008, reflecting the global financial crisis.

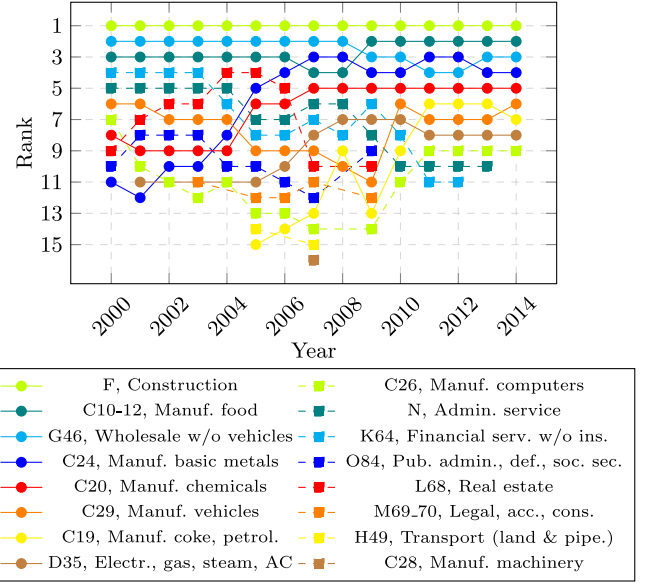


FIG. 5. Rankings of industry cores by node coreness scores over the years 2000 to 2014 in the WIOD world trade multilayer network.

Noteworthy trends include the increasing coreness of the “manufacturing of basic metals” industry and the decreasing coreness of “financial services” over the years.