

Dual Spectroscopy of Quantum Simulated Fermi-Hubbard Systems

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Quantum gas microscopy with atoms in optical lattices provides remarkable insights into the real space properties of many-body systems, but does not directly reveal the nature of their fundamental excitation spectrum. Here, we demonstrate that radio-frequency spectroscopy can reveal the quasiparticle nature of doped quantum many-body systems, crucial for our understanding of, e.g., high-temperature superconductors. In particular, we showcase how the existence and energy of magnetic polaron quasiparticles in doped Fermi-Hubbard systems may be probed, revealed by hallmark peaks in the spectroscopic spectrum. In combination with fundamental dualities of the Fermi-Hubbard model, we describe how these findings may be tested using several experimental platforms.

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Quantum simulators using atoms in optical lattices provide a wealth of information regarding the spatial correlations of many-body systems at the level of single particles [1–4]. This outstanding success hinges on two core capabilities: (1) single-site resolution of the atoms and their internal state is achieved using quantum gas microscopy [5–9], and (2) the same state is faithfully reinitialized thousands of times [3]. One has observed antiferromagnetic correlations for the repulsive Fermi-Hubbard model in real space [10–15], and charge-density wave correlations for the associated attractive model [16,17]. Moreover, the magnetic frustration around dopants in repulsive [18–20] and attractive [17] Fermi-Hubbard models has been studied and argued to stem from quasiparticle states called magnetic polarons. The nonequilibrium dynamics of a hole released from a specific lattice site has also been observed [21] and successfully explained by a theory based on the formation of magnetic polarons [22]. So far, there is, however, no *direct* evidence for the existence of such magnetic polarons. In addition, it is debated theoretically if and when magnetic polarons form [23–28]. These questions have a direct impact on the nature of pairing in such systems at higher doping, including high-temperature superconductors [29], and having unambiguous evidence for the existence of magnetic polarons is paramount.

In continuous atomic gases, radio-frequency (rf) spectroscopy has proven to be a powerful probe for the properties of Fermi [30–35] and Bose polarons [36–41], i.e., quasiparticles of impurities in Fermi and Bose gases. In particular, the existence of these polarons is clearly confirmed by sharp spectral peaks at their energies. In a similar fashion, angle-resolved photoemission spectroscopy (ARPES) is a key technique for mapping out single particle excitations in condensed matter systems [42]. In optical lattices, ARPES techniques have explored excitations of the attractive Fermi-Hubbard model [43], and recently reached a level of accuracy that makes it possible to study magnon excitations in detail [44]. In such systems, quench spectroscopy also offers an interesting pathway for spectroscopy [45–52]. There is still, however, a dire need for exploring simple, yet powerful spectroscopic techniques for quantum simulated many-body systems in optical lattices.

Here, we present a careful analysis of how rf spectroscopy can be used to reveal the spectral properties of the fundamental states of the Fermi-Hubbard model. Performing microscopic calculations based on the self-consistent nonperturbative Born approximation, which is known to be quantitatively accurate even for strong interactions [53,54], we show that the presence of magnetic polarons close to half filling can be detected by the presence of a clear peak in ejection spectroscopy at a frequency determined by its energy [Figs. 1(a)–1(c) top to bottom], which is the hallmark feature of quasiparticles. We on the other hand find that injection spectroscopy [Figs. 1(a)–1(c) bottom to top] has crucial limitations for detecting delocalized quasiparticles. Additionally, because of the duality between repulsive

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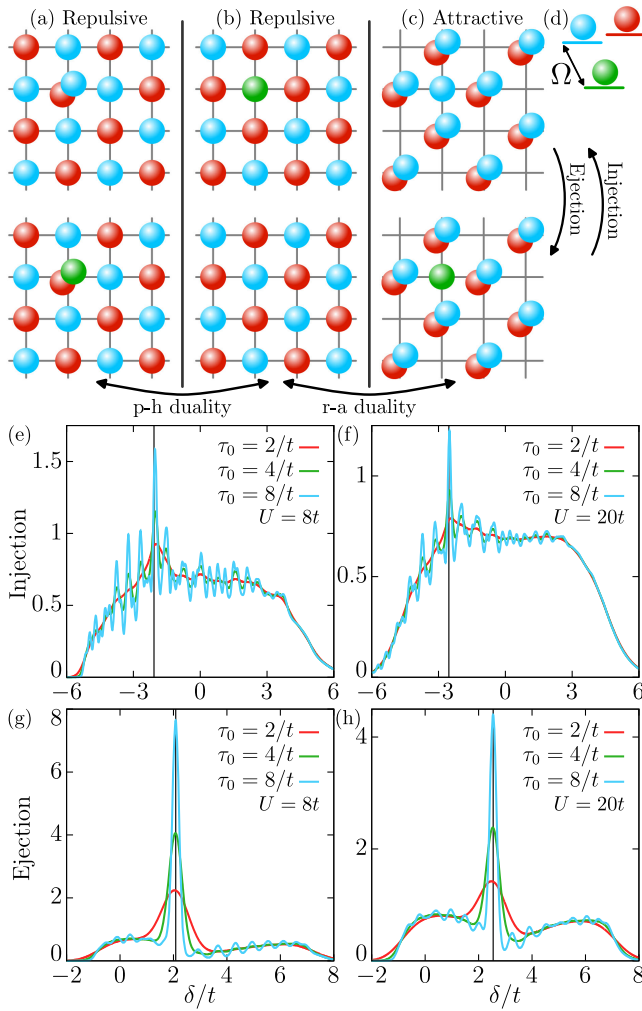


FIG. 1. Radio-frequency spectroscopy of Fermi-Hubbard systems (red spheres, spin- $\uparrow$; blue spheres, spin- $\downarrow$; green spheres, free $|f\rangle$ state). (a) Probing particle-doped system with repulsive interactions. (b) Probing hole-doped system with repulsive interactions. (c) Probing particle-doped system with attractive interactions. In ejection (injection) spectroscopy, the system starts out in one of the top (bottom) states and an rf transition (d) with Rabi-coupling Ω is driven. These systems are dual to each other (bidirectional arrows), up to the presence or absence of additional free particles (green). While we illustrate these processes as local flips, the probe naturally generates superpositions of dopants at any site. (e),(f) Normalized transferred atoms [Eq. (3)] in injection spectroscopy for indicated interaction strengths U and pulse lengths τ_0 , yielding a peak response at the quasiparticle energies (black vertical lines), but with broad spectra and poor signal to noise. (g),(h) Normalized transferred atoms [Eq. (3)] in ejection spectroscopy starting from the quasiparticle states instead singles out resonant responses at *minus* the quasiparticle energy (black vertical lines) and much better signal to noise.

and attractive Fermi-Hubbard models on bipartite lattices [55], we show that the desired evidence for magnetic polarons can be found both for repulsive and attractive interactions alike.

Model and dualities—Consider spin-1/2 fermions in a square lattice described by the Fermi-Hubbard model

$$\hat{H}_U = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \text{H.c.}) + U \sum_i \left(\hat{n}_{i\uparrow} - \frac{1}{2} \right) \left(\hat{n}_{i\downarrow} - \frac{1}{2} \right), \quad (1)$$

where $\hat{c}_{i\sigma}^\dagger$ creates a spin $\sigma = \uparrow, \downarrow$ particle at lattice site $\mathbf{i}$, $\hat{n}_{i\sigma} = \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma}$ is the local density operator for spin σ , t is the hopping amplitude, and U is the on-site interaction strength. In the above form, the Hamiltonian implicitly incorporates a chemical potential $\mu = U/2$, meaning that the ground state is at half filling, $\langle \hat{n}_i \rangle = \sum_\sigma \langle \hat{n}_{i\sigma} \rangle = 1$.

The Fermi-Hubbard model has two dualities. First, the particle-hole transformation, $\hat{\mathcal{P}}_{\text{ph}} \hat{c}_{i\sigma} \hat{\mathcal{P}}_{\text{ph}}^\dagger = \hat{c}_{i\sigma}^\dagger e^{i\mathbf{Q} \cdot \mathbf{i}}$, is a symmetry for any bipartite lattice, $\hat{\mathcal{P}}_{\text{ph}} \hat{H}_U \hat{\mathcal{P}}_{\text{ph}}^\dagger = \hat{H}_U$, where the phase factor $e^{i\mathbf{Q} \cdot \mathbf{i}}$ with $\mathbf{Q} = (\pi, \pi)$ for a square lattice (unit lattice constant) ensures an alternating sign on the two sublattices. Second, the Shiba transformation [55–57], $\hat{\mathcal{P}}_{\text{ra}} \hat{c}_{i\uparrow} \hat{\mathcal{P}}_{\text{ra}}^\dagger = -\hat{c}_{i\uparrow}$, $\hat{\mathcal{P}}_{\text{ra}} \hat{c}_{i\downarrow} \hat{\mathcal{P}}_{\text{ra}}^\dagger = e^{i\mathbf{Q} \cdot \mathbf{i}} \hat{c}_{i\downarrow}^\dagger$, maps the repulsive model to an attractive one, $\hat{\mathcal{P}}_{\text{ra}} \hat{H}_U \hat{\mathcal{P}}_{\text{ra}}^\dagger = \hat{H}_{-U}$. Crucially, these dualities take a given hole-doped eigenstate $|\Psi_n\rangle$ for repulsive interactions, and give dual eigenstates $\hat{\mathcal{P}}_{\text{ph}} |\Psi_n\rangle$ ($\hat{\mathcal{P}}_{\text{ra}} |\Psi_n\rangle$), corresponding to a doublon-doped repulsively interacting state (spin- $\downarrow$ doped attractively interacting state).

To understand how these dualities may be utilized for spectroscopy, we note that the antiferromagnetically ordered Néel state, $|\mathbf{N}\rangle = \prod_{\mathbf{i} \in A} \hat{c}_{i\uparrow}^\dagger \prod_{\mathbf{j} \in B} \hat{c}_{j\downarrow}^\dagger |0\rangle$, approximating the ground state of the repulsive Fermi-Hubbard model at half filling is dual to the charge-density wave (CDW) $\hat{\mathcal{P}}_{\text{ra}} |\mathbf{N}\rangle \propto \prod_{\mathbf{i} \in A} \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\downarrow}^\dagger |0\rangle = |\text{CDW}\rangle$, which approximates the ground state for attractive interactions [58]. These dualities, moreover, mean that the motion of a $\downarrow$ hole (one lacking spin $\downarrow$) just below half filling in a repulsively interacting Fermi-Hubbard system is dual to the motion of an additional spin-down (a doublon) just above half filling for attractive (repulsive) interactions, as illustrated in Fig. 2. These dualities are convenient because different experimental scenarios may favor either particle [18] or hole doping [21] and attractive or repulsive interactions, as have been explored in recent proposals to measure pairing correlations [59,60]. Here, we show how they are useful for an unambiguous spectral detection of magnetic polarons consisting of a hole or a doublon in an antiferromagnet (AFM), or equivalently a doublon in a CDW.

Radio-frequency spectroscopy—Inspired by the success of rf spectroscopy to probe quasiparticles in continuum gases [61,62], we now explore how it can be used to detect magnetic polarons in optical lattices. Consider a probe,

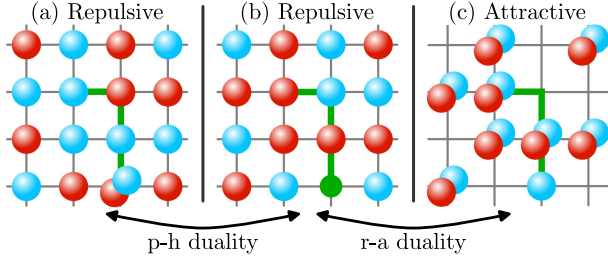


FIG. 2. Particle-hole duality between a doublon (a) and a hole (b) moving in a repulsively interacting Fermi-Hubbard system, and the repulsive-attractive duality with an additional spin $\downarrow$ moving in an attractively interacting system (c).

$$\hat{H}_{\text{rf}}(\tau) = \Omega(\tau) e^{-i\delta\tau} \sum_{\mathbf{k}} \hat{f}_{\mathbf{k}}^{\dagger} \hat{c}_{\mathbf{k}\downarrow} + \text{H.c.}, \quad (2)$$

transferring particles between the interacting $|\downarrow\rangle$ state and a third noninteracting (free) state, $|f\rangle$, at a detuning δ relative to the transition frequency between the $|f\rangle$ and $|\downarrow\rangle$ states in a vacuum. Here, $\hat{f}_{\mathbf{k}}$ removes a $|f\rangle$ particle with crystal momentum $\mathbf{k}$ and energy $\varepsilon_{\mathbf{k}}^f = -2t(\cos k_x + \cos k_y)$. The probe, hereby, creates or removes dopants in the system as illustrated in Fig. 1. We allow the Rabi frequency $\Omega(\tau)$ to depend on time τ to model a real experiment, focusing on Gaussian pulses with pulse length τ_0 , such that $\Omega(\tau) = \Omega_0/(2\pi)^{1/4} \exp[-\tau^2/(4\tau_0^2)]$. Since it is experimentally challenging to simultaneously trap the three internal states $|\uparrow\rangle$, $|\downarrow\rangle$, and $|f\rangle$, we assume in the following that the initial state, before the rf probe is applied, only contains $|\uparrow\rangle$ and $|\downarrow\rangle$ particles. Linear response then gives that the number of atoms in the $|f\rangle$ state after the rf pulse is [63]

$$N_f(\delta) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \Omega^2(\delta - \omega) A(\omega), \quad (3)$$

with the spectral function $A(\omega) = -2 \text{Im} G(\omega)$, and

$$\begin{aligned} G(\tau) &= -i\theta(\tau) \sum_{\mathbf{k}} \langle \Psi | \hat{c}_{\mathbf{k}\downarrow}^{\dagger}(\tau) \hat{f}_{\mathbf{k}}(\tau) \hat{f}_{\mathbf{k}}^{\dagger}(0) \hat{c}_{\mathbf{k}\downarrow}(0) | \Psi \rangle \\ &= -i\theta(\tau) \sum_{\mathbf{k}} e^{-i\varepsilon_{\mathbf{k}}^f \tau} \langle \Psi | \hat{c}_{\mathbf{k}\downarrow}^{\dagger}(\tau) \hat{c}_{\mathbf{k}\downarrow}(0) | \Psi \rangle \end{aligned} \quad (4)$$

the Green's function, where $\hat{a}(\tau) = \exp(i\hat{H}\tau)\hat{a}\exp(-i\hat{H}\tau)$ with $\hat{f}_{\mathbf{k}}(\tau) = e^{-i\varepsilon_{\mathbf{k}}^f \tau} \hat{f}_{\mathbf{k}}(0)$ for the free state. Here, $|\Psi\rangle$ is the initial state containing only $\sigma = \uparrow, \downarrow$ particles. For notational simplicity, $F(\omega)$ denotes the frequency-dependent Fourier transform of the time-dependent $F(\tau)$.

Low energy description—We now introduce a powerful theory for the low energy $\epsilon \ll U$ dynamics of holes close to half filling for strong repulsion $U/t \gg 1$. Integrating out doubly occupied states with high energy $\sim U$ yields the t - J model [65] with an AFM ground state at half filling due to the superexchange coupling $J = 4t^2/U$. Using a

Holstein-Primakoff transformation generalized to include the presence of a hole (slave fermion representation), combined with linear spin wave theory, in turn gives the effective Hamiltonian

$$\hat{H}_{\text{eff}} = \sum_{\mathbf{q}, \mathbf{k}} g(\mathbf{q}, \mathbf{k}) \left[\hat{h}_{\mathbf{q}+\mathbf{k}}^{\dagger} \hat{h}_{\mathbf{q}} \hat{b}_{-\mathbf{k}}^{\dagger} + \text{H.c.} \right] + \sum_{\mathbf{k}} \omega_{\mathbf{k}} \hat{b}_{\mathbf{k}}^{\dagger} \hat{b}_{\mathbf{k}} \quad (5)$$

describing the low energy dynamics of holes in the AFM [66–68]. Here, $\hat{h}_{\mathbf{k}}^{\dagger}$ and $\hat{b}_{\mathbf{k}}^{\dagger}$ create a spinless holon and a spin wave with crystal momentum $\mathbf{k}$; the latter with energy $\omega_{\mathbf{k}} = J\sqrt{1-\gamma_{\mathbf{k}}^2}/2$, where $\gamma_{\mathbf{k}} = (\cos k_x + \cos k_y)/2$. The spin waves are defined on top of the AFM ground state: $\hat{b}_{\mathbf{k}}|\text{AFM}\rangle = 0$. The Hamiltonian in Eq. (5) describes the dominant process, in which the hopping-induced scattering of a hole between crystal momenta $\mathbf{q}$ and $\mathbf{q} + \mathbf{k}$ leads to emission or absorption of a spin wave with momentum $-\mathbf{k}$ with amplitude $g(\mathbf{q}, \mathbf{k})$ [63]. While the effective Hamiltonian Eq. (5) is considerably simpler than Eq. (1), there is still no analytical solution available for the ground state in the presence of a single hole. On the other hand, the self-consistent Born approximation (SCBA) [53,67], in which a subset of so-called non-crossing Feynman diagrams for the holon Green's function $\mathcal{G}(\mathbf{k}, \tau) = -i\theta(\tau) \langle \text{AFM} | \hat{h}_{\mathbf{k}}(\tau) \hat{h}_{\mathbf{k}}^{\dagger} | \text{AFM} \rangle$ is retained, *quantitatively* describes the low energy states of the t - J model with one hole in an AFM background [54]. Indeed, it reproduces a ground state magnetic polaron at crystal momentum $(\pi/2, \pi/2)$ with energies in good agreement with exact diagonalization for small systems [53].

Injection spectroscopy—Consider first the case where the initial state is the ground state AFM at half filling, i.e., $|\Psi\rangle = |\text{AFM}\rangle$ in Eq. (4). The rf probe then creates a hole by flipping a spin $\downarrow$ to the free state. Since the hole can create a magnetic polaron by interacting with the AFM, it corresponds to so-called injection spectroscopy [61]. Moreover, the indicated dualities are only up to the presence or absence of the third noninteracting state.

We can now use $\hat{c}_{\mathbf{k}\downarrow} \simeq (\hat{h}_{-\mathbf{k}}^{\dagger} - \hat{h}_{-\mathbf{k}+\mathbf{Q}}^{\dagger})/2$ to express the Green's function in Eq. (4) with the initial state $|\Psi\rangle = |\text{AFM}\rangle$, in terms of the holon Green's function as $G(\tau) = \sum_{\mathbf{k}} \exp(-i\varepsilon_{\mathbf{k}}^f \tau) \mathcal{G}(\mathbf{k}, \tau)/2$. The lattice symmetry $\mathbf{Q} - \mathbf{p} \leftrightarrow \mathbf{p}$ has here been used to simplify the expression. Figures 1(e) and 1(f) show the resulting rf injection spectrum calculated using the SCBA for various probe lengths and two different values of U/t . The spectra are broad because the system starts out in an AFM state with localized particles. The rf probe, therefore, creates holes at all crystal momenta with equal amplitude. It follows that a full continuum of states given by $\delta = \varepsilon_{\mathbf{k}}^{\text{pol}} + \varepsilon_{\mathbf{k}}^f$, where $\varepsilon_{\mathbf{k}}^{\text{pol}}$ is the polaron dispersion, appears in the spectral response. There is, however, a weak resonant signal remaining at the ground state polaron energy $\varepsilon_{(\pi/2, \pi/2)}^{\text{pol}}$, where the density of

states is large. Nevertheless, these results demonstrate that injection spectroscopy suffers from detrimental broadening that obscures a clear spectral observation of magnetic polarons. A workaround is to postselect the momentum of the free state [69], which, however, complicates an already involved experimental sequence.

Ejection spectroscopy—To circumvent this broadening and avoid challenging postselection, we propose instead to use ejection spectroscopy. Here, one starts out in the state of interest, i.e., a system with a single (or low density of) dopant(s) forming magnetic polarons. It is not favorable, however, to probe a repulsively interacting hole-doped system because the initial hole is formed by a noninteracting $|f\rangle$ particle, requiring the simultaneous trapping and cooling of all three internal states as illustrated in Fig. 1(b). We instead consider the case where the initial state $|\Psi_p\rangle$ contains one doublon, i.e., one extra $\downarrow$ particle, forming a magnetic polaron with momentum $\mathbf{p}$ on top of the AFM (or CDW) background for repulsive and attractive interactions respectively; see Figs. 1(a) and 1(c). Using the particle-hole or repulsive-attractive dualities discussed above, the Green's function in Eq. (4) can be expressed as $G(\mathbf{p}, \tau) = -i\theta(\tau) \sum_{\mathbf{k}} \exp(-i\varepsilon_{\mathbf{k}}^f \tau) \langle \tilde{\Psi}_p | \hat{c}_{\mathbf{k}\downarrow}(\tau) \hat{c}_{\mathbf{k}\downarrow}^\dagger(0) | \tilde{\Psi}_p \rangle$, where the state $|\tilde{\Psi}_p\rangle = \hat{P}_{ph} |\Psi_p\rangle$ (or $\hat{P}_{ra} |\Psi_p\rangle$) contains one $\downarrow$ hole on top of an AFM. This explicitly shows that ejection spectroscopy starting from a doublon in an AFM (CDW) for repulsive (attractive) interactions is equivalent to ejection spectroscopy starting with a $\downarrow$ hole in an AFM for repulsive interaction as illustrated in Fig. 1.

In addition to its experimental importance, this duality is useful theoretically, as we can use the SCBA to calculate the wave function $|\tilde{\Psi}_p\rangle$ of the magnetic polaron formed by a hole in the AFM. Indeed, one can expand $|\tilde{\Psi}_p\rangle$ in terms containing an increasing number of spin waves created by the hole as

$$|\Psi_p^{\text{pol}}\rangle = \sqrt{Z_p} \left(\hat{h}_p^\dagger + \sum_{\mathbf{k}} \phi_{\mathbf{p},\mathbf{k}} \hat{b}_{-\mathbf{k}}^\dagger \hat{h}_{\mathbf{p}+\mathbf{k}}^\dagger + \dots \right) |\text{AFM}\rangle, \quad (6)$$

where Z_p is the quasiparticle residue of the magnetic polaron, $\phi_{\mathbf{p},\mathbf{k}} = g(\mathbf{p}, \mathbf{k}) \mathcal{G}(\mathbf{p} + \mathbf{k}, \varepsilon_p^{\text{pol}} - \omega_{\mathbf{k}})$ are the expansion parameters set by the SCBA [70,71], and ... denote terms with more spin waves. In this picture, the Green's function is expressed as $G(\mathbf{p}, \tau) = -i\theta(\tau) \sum_{\mathbf{k}} \exp(-i\varepsilon_{\mathbf{k}}^f \tau) \langle \Psi_p^{\text{pol}} | \hat{h}_{\mathbf{k}}^\dagger(\tau) \hat{h}_{\mathbf{k}}(0) | \Psi_p^{\text{pol}} \rangle$, describing the dominant response by which the hole is refilled, and ignoring doublon production separated by the large energy scale U [72]. To illustrate the main physics, we first approximate the polaron wave function with the dominant first term $|\tilde{\Psi}_p\rangle \sim \sqrt{Z_p} \hat{h}_p^\dagger |\text{AFM}\rangle$. This gives $G(\mathbf{p}, \tau) \sim Z_p \exp[i(\varepsilon_p^{\text{pol}} - \varepsilon_p^f)\tau]$. Using this in Eq. (3) then gives a sharp peak in the rf response for

$$\delta = \varepsilon_p^f - \varepsilon_p^{\text{pol}}. \quad (7)$$

The interpretation of Eq. (7) is straightforward: the rf probe has to provide energy to flip the magnetic polaron with energy $\varepsilon_p^{\text{pol}}$ into a free particle with energy ε_p^f . Hence, observing this sharp peak will confirm the presence of a magnetic polaron and it is a major advantage of using ejection spectroscopy as compared to injection spectroscopy. Using the SCBA, we can derive exact expressions for the expansion coefficients in Eq. (6) to all orders in the number of spin waves [70,71], resulting in

$$G(\mathbf{p}, \delta) = Z_p \left[\frac{1}{\delta + \varepsilon_p^{\text{pol}} - \varepsilon_p^f + i\eta} + \sum_{\mathbf{k}} \frac{[g(\mathbf{p}, \mathbf{k}) \mathcal{G}(\mathbf{p} + \mathbf{k}, \varepsilon_p^{\text{pol}} - \omega_{-\mathbf{k}})]^2}{\delta + \varepsilon_p^{\text{pol}} - \omega_{-\mathbf{k}} - \varepsilon_{\mathbf{p}+\mathbf{k}}^f + i\eta} + \dots \right], \quad (8)$$

with the positive infinitesimal $\eta = 0^+$. Besides the resonant peak given by Eqs. (7) and (8) shows that a continuum, $\delta = \varepsilon_p^f + \sum_{i=1}^n \varepsilon_{\mathbf{k}_i} - \varepsilon_p^{\text{pol}}$, is present in the rf response corresponding to the probe removing the magnetic polaron and creating n spin waves alongside the free particle. To circumvent having to calculate Eq. (8) order by order, we make use of the fact that the SCBA endows the wave function with a self-similar structure, which has previously been used to derive self-consistency equations for correlation functions [22,68]. Indeed, by introducing an auxiliary function $\Gamma(\mathbf{p}, \delta; \omega)$ that is the self-consistent solution to

$$\Gamma(\mathbf{p}, \delta; \omega) = \frac{1}{\delta + \omega - \varepsilon_p^f + i\eta} + \sum_{\mathbf{k}} [g(\mathbf{p}, \mathbf{k}) \mathcal{G}(\mathbf{p} + \mathbf{k}, \omega - \omega_{-\mathbf{k}})]^2 \times \Gamma(\mathbf{p} + \mathbf{k}, \delta; \omega - \omega_{-\mathbf{k}}), \quad (9)$$

we compute the full Green's function as $G(\mathbf{p}, \delta) = Z_p \Gamma(\mathbf{p}, \delta; \omega = \varepsilon_p^{\text{pol}})$ [63], allowing us to include an infinite number of spin-wave terms in Eq. (6).

The resulting ejection spectra [Figs. 1(g) and 1(h)] are plotted at the same probe lengths and values of U/t as the injection spectra in Figs. 1(e) and 1(f), assuming the initial state to be the ground state polaron with momentum $\mathbf{p} = (\pi/2, \pi/2)$. As predicted, the resonant signal given by Eq. (7) is much clearer than in the injection spectra. Unlike what is often the case in ARPES and rf spectroscopy, the many-body continuum forms both below and above this resonance. This is a result of three properties: (1) the free state is formed at the variable momentum $\mathbf{p} + \sum_{i=1}^n \mathbf{k}_i$, (2) the free state has a large energetic bandwidth of $8t$, and (3) the polaron ground state at $\mathbf{p} = (\pi/2, \pi/2)$ is in the

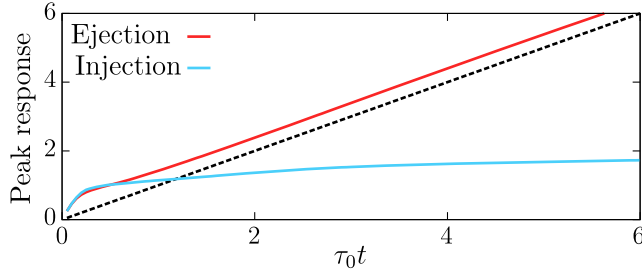


FIG. 3. Peak response for injection [units, $Nt/(\Omega_0^2\tau_0)$] (blue) and ejection spectroscopy [units, $t/(\Omega_0^2\tau_0)$] (red) vs pulse length, τ_0 , for $U = 8t$. While the former quickly saturates, the latter increases linearly (dashed black line).

middle of that bandwidth, $\epsilon_p^f = 0$. Figures 1(g) and 1(h) moreover show that the spectral weight of the continuum increases with U/t at the expense of the quasiparticle peak. This happens, because the system becomes more strongly interacting with increasing U/t , corresponding to decreasing J/t , so that the residue, Z_p , of the polaron decreases.

Finally, we make the principle advantage of ejection spectroscopy even more clear by calculating how the peak response, i.e., the height of the quasiparticle peak in the spectra, depends on the pulse length τ_0 (Fig. 3). This demonstrates that the ejection spectroscopy peak scales linearly $\sim Z_p\tau_0$ with pulse length, whereas the inferior injection peak response quickly saturates.

Experimental considerations—In addition to a finite pulse length, τ_0 , any rf experiment has frequency fluctuations. Writing the total frequency as $\delta + \Delta$, and assuming that the additional noise Δ follows a normal distribution around $\Delta = 0$ effectively leads to a shorter probing pulse, $\tilde{\tau}_0 = (\tau_0^{-2} + 4\sigma_\Delta^2)^{-1/2}$ [63]. Here, σ_Δ is the standard deviation in the probe frequency. In the absence of fluctuations, $\sigma_\Delta = 0$, Fig. 1 establishes that we need pulse lengths $> 1/t$ for a clear signal. Including fluctuations, $\sigma_\Delta \neq 0$, in turn means that we need $\tilde{\tau}_0 > 1/t$, or, put differently, for a certain pulse length τ_0 , the required precision on the probe frequency is $\sigma_\Delta \lesssim (t^2 - \tau_0^{-2})^{-1/2}/2$. More concretely, the attractive Fermi-Hubbard setup in Ref. [17] has a hopping amplitude of $t \simeq 2\pi \times 350$ Hz. Realistically probing the quasiparticle then requires $\tau_0 \gtrsim (2/t) \simeq 0.9$ ms and $\sigma_\Delta \lesssim 2\pi \times 150$ Hz. These explicit estimates show that our rf ejection spectroscopic scheme of the magnetic polaron can realistically be implemented in current experimental setups. The desired accuracy on the order of the hopping amplitude has indeed recently been achieved for probing dopant-dressed magnons in ferromagnetic backgrounds [44]. More generally, our proposed rf scheme requires three internal states of the same atomic species and to find a Feshbach resonance for which *only* the interaction between two states, $|\uparrow\rangle$ and $|\downarrow\rangle$, is significant. The ability to *freely* choose between attractively and repulsively interacting systems may, therefore, be instrumental for actual

implementations. An alternate setup could use *two* layers in which a second empty layer is used for detection and where the tunneling between the layers plays the role of the rf probe in Eq. (2) [69,73].

Conclusions—Using a nonperturbative theory for the Fermi-Hubbard model close to half filling, we systematically analyzed how its low energy spectrum can be explored using rf ejection spectroscopy with atoms in optical lattices. In particular, we demonstrated that a smoking gun detection of magnetic polarons is possible via the presence of a clear quasiparticle peak in the rf ejection spectrum. Moreover, we showed that the particle-hole and repulsive-attractive dualities of the Fermi-Hubbard model makes our scheme realistic for current quantum simulators using both repulsive [18,21] and attractive [74] interactions. Finally, we demonstrated how pulse lengths longer than the inverse hopping, $1/t$, and probing frequency accuracies on the order of 100 Hz are sufficient to clearly resolve the quasiparticle peaks, bringing the experimental realization within reach. Our probing scheme generally investigates transitions between states with N_d and $N_d - 1$ dopants, and, therefore, naturally extends to investigating many-body states in general such as, for example, bound magnetic polaron pairs.

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Data availability—The data that support the findings of this Letter are openly available [75].

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