

Sullivan Process near Threshold and the Pion Gravitational Form Factors

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We propose a novel method to experimentally access the gravitational form factors of the charged pion π^+ through the Sullivan process in electron-proton scattering. We demonstrate that the cross sections of J/ψ photoproduction and ϕ electroproduction near the respective thresholds are dominated by the gluon gravitational form factor of the pion to next-to-leading order in perturbative QCD. We predict cross sections for the Electron-Ion Collider and the Jefferson Lab experiments.

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Introduction—Recently, exclusive photoproduction of heavy vector mesons such as J/ψ and Υ near threshold has emerged as a promising tool to access the gluon gravitational form factors (GFFs) of the nucleon defined as the off-forward matrix elements of the QCD energy momentum tensor $\langle p'|T^{\mu\nu}|p\rangle$ [1–19]. (See [20–23] for other approaches to J/ψ photoproduction near threshold.) This is complementary to the “canonical” approach to extract the quark GFFs from the generalized parton distributions (GPDs) in deeply virtual Compton scattering [24–27]. The major challenge of GPD-based approaches is to isolate the contribution of the spin-2 energy momentum tensor from infinitely many leading twist operators with different spins. This problem can be mitigated in near-threshold productions characterized by large values of the skewness variable $\xi \sim \mathcal{O}(1)$. In this kinematical region, the sensitivity to the spin-2 component is significantly enhanced [7,8,11] compared to the situation in high energy processes with $\xi \ll 1$. This paves the way to efficiently access the nucleon’s gluon GFFs.

Importantly, one can realize the situation $\xi \sim \mathcal{O}(1)$ in the electroproduction of light vector mesons near threshold where the photon virtuality Q^2 serves as the hard scale for GPD factorization [7]. The case with ϕ production off the nucleon is studied in a separate publication [14], where it is demonstrated to next-to-leading order (NLO) in perturbative QCD that the cross section is dominated by the gluon and strangeness GFFs.

In this Letter, for the first time we explore the physics of near-threshold vector meson productions off the charged pion π^+ , with the goal of constraining the pion GFFs. The

GFFs of the pion are of special interest due to the unique nature of the pion as the Nambu-Goldstone bosons of spontaneously broken chiral symmetry. The soft pion theorem imposes various constraints on the GFFs and even GPDs [28–30] that could be experimentally tested. While there has been a recent surge of interest in computing the pion GFFs in various approaches [31–40], attempts to extract the pion GFFs from experimental data are quite limited [31]. We shall demonstrate that the new observables considered here have very good sensitivities to the pion GFFs, and make predictions for future experiments at Jefferson Laboratory (JLab) [41] and the Electron-Ion Collider (EIC) at Brookhaven National Laboratory [42].

Exclusive vector meson production in Sullivan process—The Sullivan process [43] collectively refers to a subset of electron-proton $e + p$ scattering events where the incoming proton transitions into a neutron by emitting a virtual pion $p(p) \rightarrow \pi^{+*}(p_\pi) + n(p_n)$ with virtuality $p_\pi^2 = (p - p_n)^2 = t < 0$; see Fig. 1. The pion then interacts with a virtual photon emitted from the electron

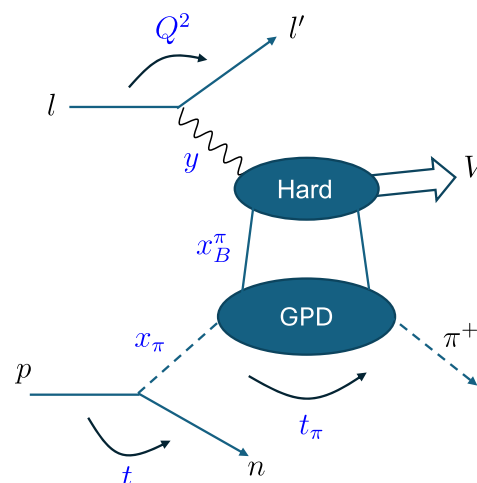


FIG. 1. Vector meson production in Sullivan process.

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$e(l) \rightarrow \gamma^*(q) + e(l')$ with virtuality $q^2 = (l - l')^2 = -Q^2 \leq 0$. Depending on subsequent $\gamma^* + \pi^{+*}$ reactions, different aspects of the pion structure can be studied [44]. The reaction $\gamma^* + \pi^{+*} \rightarrow \pi^+$ studied in the original paper [43] has become a cornerstone of modern experimental determinations of the pion electromagnetic form factor [45,46]. In the context of the pion GPDs, previous works focused on deeply virtual Compton scattering $\gamma^*(q) + \pi^{+*}(p_\pi) \rightarrow \gamma(q') + \pi^+(p'_\pi)$ at high center-of-mass energy $s_\pi = (p_\pi + q)^2 \sim Q^2$ [47,48]. There one encounters the usual complications such as the interference with the Bethe-Heitler process and the deconvolution problem of GPDs. In this work, we instead study two exclusive processes: (i) deeply virtual meson production $\gamma^*(q) + \pi^{+*}(p_\pi) \rightarrow V(q') + \pi^+(p'_\pi)$, where $Q^2 \gg \Lambda_{\text{QCD}}^2$ and V is a neutral, longitudinally polarized light vector meson such as $\rho^0(770)$, $\omega(783)$ or $\phi(1020)$; and (ii) photoproduction ($Q^2 = 0$) of $J/\psi(3097)$. The necessary hard scale for GPD factorization is provided by Q^2 in the former and the J/ψ mass in the latter. The novelty of our work is that we analyze these processes in the threshold region

$$s_\pi \gtrsim s_\pi^{\text{th}} \equiv (m_\pi + m_V)^2. \quad (1)$$

(See [49,50] for studies in the high energy limit.) Some caveats regarding the application of GPD factorization theorems to the threshold region have been discussed in [14]; see also [7,8]. In particular, ξ cannot be too close to 1, and the preferred region is $\xi = 0.4 \sim 0.6$ for the nucleon target. For the pion target, these caveats are less significant due to the light pion mass, and in fact even a lower region in ξ can be also studied, as we shall see. A more serious problem, which already arises in the context of form factor extractions (see, e.g., [46,51]), may be the fact that the incoming pion is virtual. However, as long as $p_\pi^2 = t$ is small, the impact on the pion GPDs is expected to be minor [44] and can be theoretically taken into account [52]. Assuming this to be the case, our goal is to assess to what extent the scattering amplitudes are sensitive to the pion GFFs.

Let us first consider deeply virtual meson production. We introduce the momentum fraction x_π of the proton carried by the pion and the Bjorken variable x_B^π for the $\gamma^* + \pi^{+*}$ subprocess,

$$x_\pi = \frac{p_\pi l}{p l}, \quad x_B^\pi = \frac{Q^2}{2p_\pi q}. \quad (2)$$

In the limit $Q^2 \rightarrow \infty$, the usual Bjorken variable x_B factorizes as [47]

$$x_B = \frac{Q^2}{2p q} \approx x_\pi x_B^\pi. \quad (3)$$

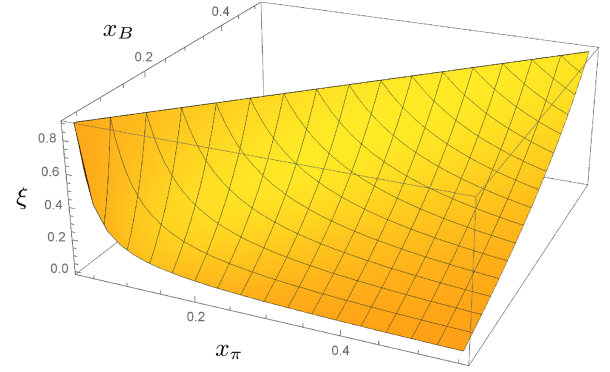


FIG. 2. Skewness ξ (5) in the (x_π, x_B) plane at $s_{ep} = 800 \text{ GeV}^2$, $Q^2 = 10 \text{ GeV}^2$, $t_\pi = -0.5 \text{ GeV}^2$.

At low energy, near the threshold $s_\pi^{\text{th}} \lesssim s_\pi \ll Q^2$, an additional condition $x_B^\pi \approx 1$ follows from the relation

$$s_\pi = Q^2 \left(\frac{1}{x_B^\pi} - 1 \right) + t. \quad (4)$$

Therefore, the threshold region sits along the diagonal line $x_B \approx x_\pi$ in the (x_π, x_B) plane. There the skewness variable is order unity

$$\xi \approx \frac{x_B^\pi}{2 - x_B^\pi \left(1 - \frac{t_\pi}{Q^2} \right)} \approx \frac{x_B}{2x_\pi - x_B \left(1 - \frac{t_\pi}{Q^2} \right)} \sim \mathcal{O}(1), \quad (5)$$

where we ignored $\mathcal{O}(1/Q^2)$ terms except for the dependence on the pion momentum transfer $t_\pi = (p_\pi - p'_\pi)^2$, which can become large in actual experiments. This is plotted in Fig. 2. The ranges of x_π , x_B depend on $s_{ep} = (p + l)^2$ and the upper cutoff $|t| < |t_{\text{max}}|$ [43,47]; see [53]. Working in the small- x_π region helps to avoid the Δ -resonance region $M_{\pi n}^2 = (p_n + p'_\pi)^2 \sim 1.5 \text{ GeV}^2$ [47].

In the case of J/ψ photoproduction $Q^2 = 0$, we remove the electron vertex and consider the reduced cross section $\gamma + p \rightarrow J/\psi + \pi^+ + n$. Again, ξ becomes order unity near the threshold $s_\pi \gtrsim (m_\pi + m_{J/\psi})^2 \approx 10.5 \text{ GeV}^2$. It is important to note that, due to the light pion mass, ξ can be tuned closer to unity for the pion target than for the nucleon target in both processes.

Threshold approximation—The basic ingredients of the QCD factorization approach are the convolution integrals of GPDs. To leading order they have the form

$$\mathcal{H}^a(\xi, t, \mu^2) = \int_{-1}^1 dx \frac{1}{\xi - x - i\epsilon} \left\{ \frac{1}{2} H^{q(+)}(x, \xi, t, \mu^2) + \frac{1}{x} H^q(x, \xi, t, \mu^2) \right\}, \quad (6)$$

where $q = u, d, s, \dots$ is the flavor label and $H^{q(+)}(x, \xi) = H^q(x, \xi) - H^q(-x, \xi)$ is the C -even part. μ^2 is the renormalization scale. Typically, at high energy $s_\pi \gg Q^2$, where $\xi \ll 1$, one tries to extract GPDs H from $\mathcal{H}$ by

deconvoluting the x integral. However, it has been originally observed in [7,8] and further refined in [11] that, when $\xi \sim \mathcal{O}(1)$ and for the gluon channel $a = g$, (6) can be well approximated by

$$\mathcal{H}_{\text{trunc}}^a(\xi, t, \mu^2) = \frac{2}{\xi^2} \frac{5}{4} [A^a(t, \mu^2) + \xi^2 D^a(t, \mu^2)], \quad (7)$$

where A, D are the gravitational form factors

$$A^a(t, \mu^2) + \xi^2 D^a(t, \mu^2) = \int_{-1}^1 dx \left\{ x H^a(x, \xi, t, \mu^2) + \frac{1}{2} H^a(x, \xi, t, \mu^2) \right\}. \quad (8)$$

Equation (7) is nothing but the leading order term in the conformal partial wave expansion [54,55] of (6). Drastic as it may seem, the validity of this “threshold approximation” for the nucleon target has been tested to next-to-leading order (NLO) in ϕ electroproduction [14] and J/ψ photoproduction [15]. The approximation works fine already for moderately large values $\xi \gtrsim 0.4$. Below we test this approach for the pion target.

For this purpose, we adopt the “algebraic” model of the pion GPDs $H_\pi^a(x, \xi, t_\pi, \mu^2)$ constructed in [30]. In Supplemental Material [53], we discuss another model to check the model dependence. A notable feature of this model is that it contains the so-called D term [56], which has support in the central region $|x| < \xi$,

$$H_\pi^a(x, \xi, t_\pi, \mu^2) \sim \theta(\xi - |x|) D_\pi^a(x/\xi, t_\pi, \mu^2). \quad (9)$$

Remarkably, in the chiral limit of QCD, the forward value $D_\pi^a(x/\xi, t_\pi = 0)$ is strongly constrained by the soft pion theorem for all parton species $a = q, g$ [29,30]. This in particular means that

$$D_\pi^q(0, \mu^2) = \int_{-1}^1 dz z D_\pi^q(z, 0, \mu^2) = -A_\pi^q(0, \mu^2), \quad (10)$$

and similarly for the gluon. The model also assumes isospin symmetry, which means $H_\pi^{u(+)}(x, \xi, t_\pi) = H_\pi^{d(+)}(x, \xi, t_\pi)$ exactly. In real QCD, there are deviations from these relations.

Using the algebraic model evolved to $\mu^2 = 10 \text{ GeV}^2$ and setting $t_\pi = 0$ for simplicity, we numerically evaluate (6) and compare the results with their truncated versions (7). This is plotted in Fig. 3 in terms of the ratio

$$R_f^a(\xi) = 1 - \frac{\sqrt{(\text{Re}\mathcal{H}_f^a)^2 + (\text{Im}\mathcal{H}_f^a)^2}}{\mathcal{H}_{f,\text{trunc}}^a}, \quad (11)$$

which is the measure of the truncation error. The index $f = \text{LO}, \text{NLO}$ denotes whether the leading order (LO) or next-to-leading order hard kernel has been used. We immediately notice that the approximation fails for u, d

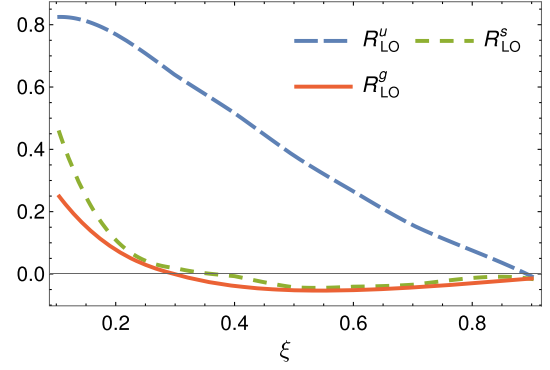


FIG. 3. Truncation error $R^{a=u,s,g}$ (11) of the leading order amplitude (6) at $\mu^2 = 10 \text{ GeV}^2$.

quarks, which are important for ρ^0, ω productions, unless one reaches $\xi \gtrsim 0.7$. The main reason is that the corresponding GPD is well supported in the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi region $\xi < |x| < 1$ and takes a sizable value at the diagonal point $x = \xi$, giving rise to a large imaginary part $\text{Im}\mathcal{H}_\pi^{u,d}(\xi) \propto H_\pi^{u,d}(\xi, \xi)$. This is related to the fact that u, d quarks constitute the valence quarks of the π^+ . In the asymptotic limit $\mu^2 \rightarrow \infty$, $H_\pi^{u,d}(\xi, \xi, \mu^2)$ should go to zero and the threshold approximation becomes exact. But this occurs at much higher scales than $\mu^2 = 10 \text{ GeV}^2$ in the present model.

On the other hand, the approximation works very well in the gluon and strangeness channels. The truncation error is less than 10% already for $\xi > 0.2$. In contrast to $H_\pi^{u,d}$, $H_\pi^{s,g}$ are dominantly supported in the central region $|x| < \xi$, leading to a small imaginary part in the amplitude $\text{Im}\mathcal{H}_\pi^{s,g}(\xi) \propto H_\pi^{s,g}(\xi, \xi)$. We therefore focus on ϕ electroproduction and J/ψ photoproduction in what follows and test the validity of the threshold approximation to NLO. For ϕ electroproduction, we use the full NLO scattering amplitude from [55,57,58], and its truncated version in [14]. In the J/ψ case, we use the full NLO result from [59] and its truncated version [15,53]. As for meson production mechanisms, we use the asymptotic distribution amplitude for ϕ production and the leading order non-relativistic QCD vertex for J/ψ production (see [14,15]). These approximations can be systematically improved in future.

The results for R_{NLO} are plotted in Fig. 4. Each plot shows R_{NLO} for different partonic channels $a = g, s, ps$, where ps means the “pure singlet” contribution (the sum of GPDs or GFFs over all flavors) that appears at NLO. The total NLO amplitude is indicated by the solid line. The threshold approximation is excellent $|R_{\text{NLO}}^\phi| < 0.05$ for ϕ electroproduction at $\mu^2 = 10 \text{ GeV}^2 = Q^2$ (upper plot). As for J/ψ photoproduction, the approximation is not as good as in the ϕ production case due to the partial cancellation between the quark and gluon terms. Still, it is decent $|R_{\text{NLO}}^{J/\psi}| < 0.15$ and increasingly gets better as $\xi \rightarrow 1$.

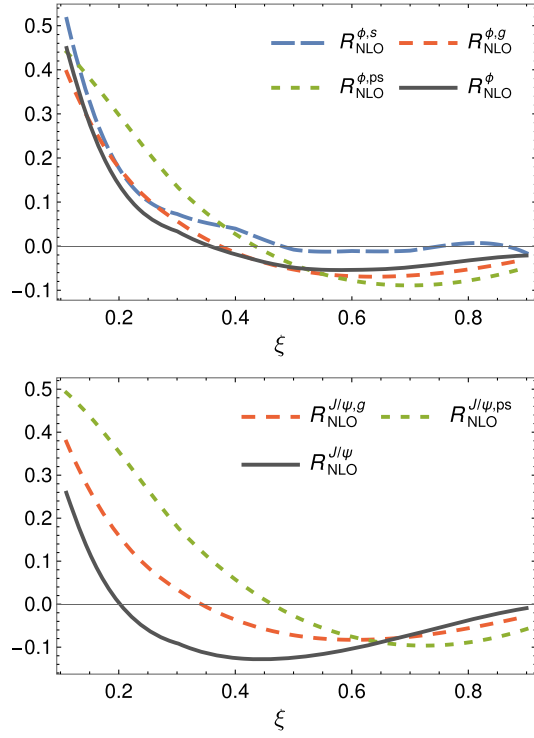


FIG. 4. Truncation error (11) in the full NLO amplitude at $\mu^2 = 10 \text{ GeV}^2$. Top: ϕ electroproduction. Bottom: J/ψ photoproduction.

Predictions at the EIC and JLab—Now that we have demonstrated the validity of the threshold approximation for ϕ electroproduction and J/ψ photoproduction, we make concrete NLO predictions for the JLab and EIC kinematics. The relevant cross section formulas are summarized in [53]. We assume the monopole and dipole form for the A , D form factors [32] (see also [33]),

$$A_\pi^a(t, \mu^2) = \frac{A_\pi^a(0, \mu^2)}{1 - t/m_A^2}, \quad D_\pi^a(t, \mu^2) = \frac{D_\pi^a(0, \mu^2)}{(1 - t/m_D^2)^2}. \quad (12)$$

We use $m_A = 1.6 \text{ GeV}$ and $m_D = 1.1 \text{ GeV}$ as in [14], but ultimately these parameters should be determined from future experimental data. The forward values $A_\pi^a(0, \mu^2)$ are constrained by the pion Parton Distribution Function. We use $A_\pi^u = A_\pi^d = 0.28$ (isospin symmetry), $A_\pi^s = 0.02$, and $A_\pi^g = 0.41$ at the reference scale $\mu^2 = 2 \text{ GeV}^2$ from the algebraic model and consider their one-loop evolution with $n_f = 4$ flavors. We impose the constraint $D_\pi^a(0, \mu^2) = -A_\pi^a(0, \mu^2)$ from the soft pion theorem, although possible deviations from this formula are interesting and can be implemented in a straightforward manner.

First consider ϕ electroproduction at the EIC with $s_{ep} = 800 \text{ GeV}^2$. We write the $e + p$ cross section as (not to be confused with the reduced $\gamma^* + p$ cross section $d\sigma_{T/L}^{\gamma^* p}$)

$$\frac{d\sigma^{ep}}{dx_B dQ^2 dt_\pi dx_\pi} \equiv \frac{d\sigma_T^{ep}}{dx_B dQ^2 dt_\pi dx_\pi} + \varepsilon \frac{d\sigma_L^{ep}}{dx_B dQ^2 dt_\pi dx_\pi}, \quad (13)$$

where the longitudinal or transverse photon flux ratio ε is

$$\varepsilon = \frac{1 - y_\pi}{1 - y_\pi + \frac{y_\pi^2}{2}}, \quad y_\pi = \frac{q p_\pi}{l p_\pi}. \quad (14)$$

In the present kinematics, y_π is approximately equal to the usual Deep Inelastic Scattering variable y [47],

$$y_\pi \approx y = \frac{q p}{l p} = \frac{Q^2}{(s_{ep} - m_N^2)x_B}, \quad (15)$$

where m_N is the nucleon mass. We focus on the longitudinal part $d\sigma_L^{ep}$, which admits QCD factorization [60]. Figure 5 shows $d\sigma_L^{ep}$ evaluated at a representative point $(x_\pi, x_B) = (0.3, 0.2)$ with two different values $Q^2 = 5 \text{ GeV}^2$ (top) and $Q^2 = 10 \text{ GeV}^2$ (bottom). The NLO cross section (blue band) is larger and has smaller scale uncertainties than the LO cross section (orange band). In these plots, $\xi > 0.5$, where the truncation error (see Fig. 4) is expected to be smaller than the scale uncertainties and possible higher twist effects of order t_π/Q^2 . The cross

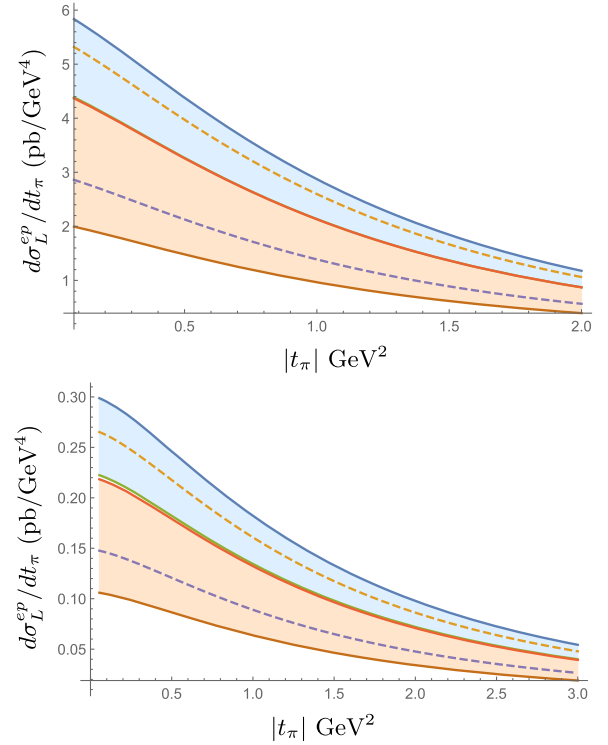


FIG. 5. Longitudinal part of the ep cross section (13) at $x_\pi = 0.3$, $x_B = 0.2$ as a function of $|t_\pi|$ at the EIC. Top: $Q^2 = 5 \text{ GeV}^2$, Bottom: $Q^2 = 10 \text{ GeV}^2$. Blue and orange band are the NLO and LO cross sections, respectively, varied within $Q/2 < \mu < 2Q$.

section is dominated by the gluon exchange, which is partly canceled by the NLO quark contribution. Because of the soft pion theorem $D_\pi^s(0) \approx -A_\pi^s(0)$, the large- D_s scenario for the nucleon considered in [7,14], is unlikely for the pion. The rather weak t_π dependence is due to the monopole ansatz (12), which can be experimentally tested.

The corresponding measurement at JLab will be more difficult, as the allowed ranges of x_π , x_B [53] are narrower at the current JLab energy $s_{ep} \approx 23 \text{ GeV}^2$. One is forced to go to lower regions in Q^2 at the risk of reducing the accuracy of perturbative predictions. The proposed 22 GeV upgrade ($s_{ep} \approx 42 \text{ GeV}^2$) [61], if realized, certainly improves the situation. The Electron-Ion Collider in China [62] offers another possibility. One advantage of low energy experiments is that it may be possible to perform the longitudinal or transverse (L/T) decomposition via the ‘‘Rosenbluth separation’’ method. Usually, one varies ε by changing the beam energy s_{ep} at fixed Q^2 , x_B [see (15)]. But in the Sullivan process, one may as well fix s_{ep} and vary x_B along the ‘‘degenerate’’ line $x_B^\pi \approx x_B/x_\pi = \text{const}$. This allows one to access the low- ε region critical for the method [63].

Next we consider J/ψ photoproduction at JLab. We compute the reduced $\gamma + p$ cross section

$$\frac{d\sigma^{\gamma p}}{dx_\pi dt_\pi}, \quad (16)$$

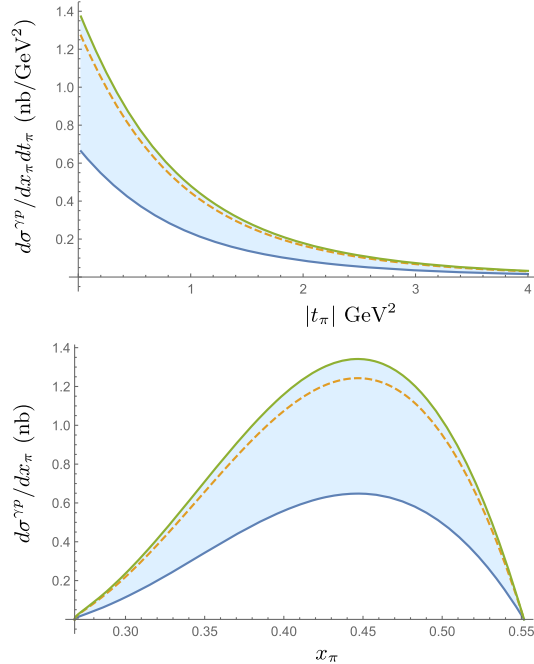


FIG. 6. Upper plot: J/ψ photoproduction cross section (16) at JLab with $s_{\gamma p} = 40 \text{ GeV}^2$ and $x_\pi = 0.45$. Lower plot: cross section integrated over t_π . The three curves correspond to $\mu = 2m_{J/\psi}$, $m_{J/\psi}$, and $m_{J/\psi}/2$ from top to bottom.

treating the photon energy $q^0 = E_\gamma$ as a fixed parameter. This can be controllably done as in the recent JLab measurements of J/ψ off the nucleon target [16–18]. However, the same is challenging for the pion target in the Sullivan process due to the lack of phase space. We thus assume the 22 GeV upgrade [61] and fix $s_{\gamma p} = (q + p)^2 = 40 \text{ GeV}^2$. In Fig. 6, we plot (16) at $x_\pi = 0.45$, corresponding to $\xi > 0.38$. The uncertainty band corresponds to the window $m_{J/\psi}/2 < \mu < 2m_{J/\psi}$. We observe that the region $\mu \sim m_{J/\psi}/2 \approx m_c$ suffers from a rather strong scale dependence that we attribute to the cancellation between the LO gluon and NLO quark contributions. This can be avoided by selecting $\mu > m_{J/\psi}$, where the NLO cross section shows a fairly stable plateau in μ [53]. The magnitude of the cross section is of the same order as in the nucleon target production [16–18] and falls slower with $|t_\pi|$. The lower plot is the t_π -integrated cross section, which clearly shows the rise of the cross section above the threshold $x_\pi > 0.27$. The upper limit $x_\pi \lesssim 0.55$ is an artificial cutoff imposed by the condition $|t| < |t_{\text{max}}| = 0.6 \text{ GeV}^2$ [47,64].

Conclusions—In this Letter we have proposed a novel method to experimentally constrain the gluon and strangeness gravitational form factors of the pion. We have demonstrated to NLO in QCD that ϕ electroproduction and J/ψ photoproduction near the threshold have strong sensitivities to the pion GFFs. Moreover, thanks to the light pion mass, one can explore larger ξ and lower $|t_\pi|$ regions than for the nucleon target, which makes theoretical treatments cleaner. The predicted NLO cross sections are well in the measurable range with the high luminosities of the EIC and JLab.

There are many directions for future exploration. First, it would be interesting to further investigate the sensitivity of near-threshold ρ^0 and ω productions to the up and down quark GFFs. The region $\xi > 0.7$ (see Fig. 3) is not inaccessible for the pion target, but there are practical issues such as model dependence and experimental feasibility. Second, while near-threshold photoproduction is challenging to measure at the EIC, electroproduction of J/ψ should be possible [5]. The calculation can be pushed to NLO using the result of [65]. One can also consider photoproduction in proton-nucleus ultraperipheral collisions [2]. The Sullivan process in these reactions has been studied in the high energy limit $s_\pi \rightarrow \infty$ [49,66] but not in the threshold region. Finally, the need for reconstructing the scattered particles e , V , π^+ , n poses significant experimental challenges. Careful investigations based on realistic experimental setups and event generator simulations (see, e.g., [14]) are crucial in this respect. We hope our work triggers future efforts in this direction.

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Data availability—No data were created or analyzed in this Letter.

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