

Duality between Correlation Functions and Wilson Loops in Gauge Theory from Effective Field Theory

Hao Chen^{1,*}, Pier Francesco Monni^{2,†}, Zhaoyan Pang^{3,‡}, Gherardo Vita^{2,§}, and Hua Xing Zhu^{3,4,||}

¹*Center for Theoretical Physics—A Leinweber Institute, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

²*CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*

³*School of Physics, Peking University, Beijing 100871, China*

⁴*Center for High Energy Physics, Peking University, Beijing 100871, China*



(Received 22 October 2025; revised 11 March 2026; accepted 21 April 2026; published 15 June 2026)

In this Letter, we initiate a systematic study of the n -point correlation functions (CFs) in gauge theories in the sequential light-cone (SLC) limit. Focusing on QCD, we formulate a factorization theorem for the CF of four vector currents in this limit using tools from soft-collinear effective field theory (SCET). This result unveils a duality between CF and Wilson loops in nonconformal field theories, according to which the singular structure of the CF is described by a null polygonal Wilson loop dressed by universal jet functions and current form factors, directly related to well-known ingredients in collider physics. We employ this new factorization theorem to obtain the singular terms of the four-point CF in QCD in the SLC limit up to three loops. This constitutes the first determination of terms beyond one loop, and in particular, determines, for the first time, conformal-symmetry-breaking terms induced by the nonvanishing β function of QCD. As a stringent check, we verify that our result satisfies the anomalous conformal Ward identities and that its leading transcendental term reproduces the known correlation function in $\mathcal{N} = 4$ SYM to three loops. Our results are easily generalized to the multipoint case, offering a powerful tool for future applications of SCET to the study of the fundamental structure of gauge theories.

DOI: [10.1103/y5rf-ssw3](https://doi.org/10.1103/y5rf-ssw3)

Introduction—Correlation functions (CFs) are fundamental quantities in quantum field theory (QFT). They provide critical insight into the structural and dynamical properties of the underlying theory across high-energy physics, condensed matter systems, and cosmology. In the context of collider physics, correlation functions play a key role due to their relation of scattering amplitudes and cross sections [1–3] as well as in the description of an important class of collider observable, that of collider correlators, that admit a representation in terms of these objects [4–6]. Besides being fascinating from a formal point of view, this representation is a powerful tool to obtain precise theory predictions for these observables by completely avoiding the complication of IR divergences cancellation in the more standard amplitude approach. This

critically provides an alternative perspective on the factorization and renormalization group (RG) structure of collider observables, complementary to the standard diagrammatic and effective field theory approaches, which opens new angles for the study of collider physics with CFs [4–17].

Although extensively studied in formal QFT, there are only a few concrete calculations of higher-point CFs of local operators in QCD [18]. The nonconformal nature of QCD induces a more complicated kinematic dependence in local CFs, and hence, makes the calculations beyond one loop prohibitively difficult. Given the importance of CFs in the study of QCD, it is paramount to explore alternative methods for the quantitative analysis of these quantities. In this Letter, we initiate a study of higher-point local CFs beyond one loop in QCD by focusing on a special kinematic regime: the sequential light-cone (SLC) limit. This limit has a rich structure and encodes several interesting theoretical features. In $\mathcal{N} = 4$ SYM, local CFs in the SLC limit are proportional to polygonal lightlike Wilson loops [19], which in turn, are interestingly dual to scattering amplitudes [20–29]. There has also been interest in the geometric structure of such objects in terms of the correlahedron and positive geometry [30–34]. They have also been systematically studied in conformal field theories via the light-cone bootstrap [35–44] and integrability-based method [45,46], which revealed many analytic features of

*Contact author: hao_chen@mit.edu

†Contact author: pier.monni@cern.ch

‡Contact author: zypang@stu.pku.edu.cn

§Contact author: gherardo.vita@cern.ch

||Contact author: zhuhx@pku.edu.cn

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large spin physics in these theories. Furthermore, in the context of collider physics, the SLC limit of four-point CFs is dual to the di-jet (back-to-back) limit of the aforementioned collider correlators [9,15]. Remarkably, we find that, much like in conformal field theories, multipoint correlation functions in QCD are proportional to closed lightlike Wilson loops in the SLC limit. The details of this connection are unveiled by means of a novel factorization formula that describes the CFs in this limit and can be used to make quantitative predictions of their perturbative structure. To our knowledge, this is the first proof of such duality in any nonconformal gauge theory, and it offers a powerful tool for the study of these key objects in an array of applications to collider physics and formal QFT aspects.

We consider the correlation function of four operators,

$$G^{\mu\nu\rho\sigma}(\{x\}) = \langle j^\mu(x_1) j^\nu(x_2) j^\rho(x_3) j^\sigma(x_4) \rangle, \quad (1)$$

where $j^\mu(x_i) = \bar{\psi}(x_i) \gamma^\mu \psi(x_i)$ are electromagnetic currents evaluated at points x_i^μ such that $y_i^2 \equiv (x_{i+1} - x_i)^2 < 0$ are spacelike separated (with $x_5^\mu \equiv x_1^\mu$), and $\langle \cdot \rangle$ denotes a Wightman expectation value in the interacting vacuum. The region in which some or all of the $y_i^2 > 0$ is accessible through analytic continuation. Note that the CF in Eq. (1) has a corresponding collider correlator, the charge-charge correlator (QQC) [4–7,18], which measures correlations between electromagnetic charges in electron-positron annihilation. A recent analysis [17] established that the QQC is infrared and collinear safe in the back-to-back limit and derived a corresponding factorization formula, enabling its perturbative resummation to N⁴LL accuracy.

We are interested in the behavior of Eq. (1) in the SLC limit, which is where the x_i^μ are distinct and separated by a null distance, i.e., $y_i^2 \rightarrow 0$. In this limit, G exhibits pole singularities in y_i^2 related to the propagation of particles along the y_i^μ directions. At lowest perturbative order, one finds

$$G_{\text{tree}}^{\mu\nu\rho\sigma}(\{x\}) \sim \frac{\text{tr}[\gamma^\mu \not{y}_1 \gamma^\nu \not{y}_2 \gamma^\rho \not{y}_3 \gamma^\sigma \not{y}_4]}{(y_1^2)^2 (y_2^2)^2 (y_3^2)^2 (y_4^2)^2} + \dots, \quad (2)$$

where $y_i^\mu = x_{i+1}^\mu - x_i^\mu$, and we keep only the connected, leading-power term in this limit and neglect less singular Wick contractions in the calculation of Eq. (1), which would enter at subleading power in the SLC limit. This situation is depicted schematically in Fig. 1. To study Eq. (1) at higher loops, we define the quantity

$$R = \lim_{y_i^2 \rightarrow 0} G^{\mu\nu\rho\sigma}(\{x\}) / G_{\text{tree}}^{\mu\nu\rho\sigma}(\{x\}), \quad (3)$$

which packages the radiative corrections and contains logarithmic light-cone divergences at each order in perturbation theory. These light-cone divergences manifest themselves as logarithms of the conformal cross ratios

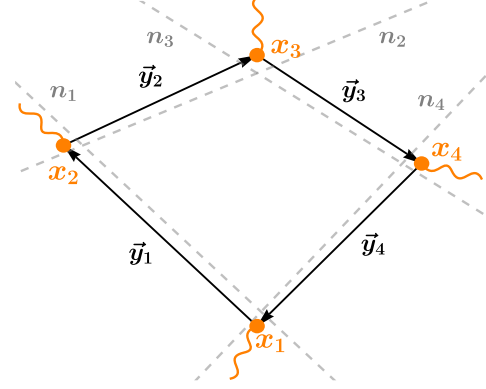


FIG. 1. Graphical depiction of $G(\{x\})$ in Eq. (1) in the sequential light-cone limit. The dashed gray lines identify the light-cone directions approached by y_i^μ .

$u = y_1^2 y_3^2 / x_{13}^2 x_{24}^2$ and $v = y_4^2 y_2^2 / x_{13}^2 x_{24}^2$, where $x_{13}^2 = (y_2^\mu + y_1^\mu)^2 < 0, x_{24}^2 = (y_3^\mu + y_2^\mu)^2 < 0$, which become small in the SLC limit. In the following, we will show that such logarithmic singularities are related to the infrared structure of classes of collider observables, and they exhibit a remarkably simple factorizing structure at all perturbative orders.

Factorization in the sequential light-cone limit—The starting point to build a factorization theorem for R is the identification of the power counting that defines the SLC limit. Translational invariance forces R to depend on differences in the vertices coordinates, in terms of which the SLC limit can be expressed as

$$|y_i^2| \ll |(y_i + y_{i+1})^2| \sim |x_{13}^2| \sim |x_{24}^2| \sim L^2, \quad (4)$$

where L^2 is the scale of the large separations x_{13}^2, x_{24}^2 . Note that we assume that $x_{13}^2 \sim x_{24}^2$, so our discussion will be valid in this geometric configuration, and that further modes may contribute in configurations where the ratio x_{13}^2 / x_{24}^2 (or its inverse) becomes small and commensurate with y_i^2 . The physical picture is given by four wide angle, lightlike directions n_i^μ that the separations y_i^μ are approaching in the SLC limit. To formalize the limit, we define the four lightlike vectors $\{n_i^\mu\}_{i=1}^4$ as

$$n_i^\mu \equiv \left(\frac{y_i^0}{|y_i^0|}, \frac{\vec{y}_i}{|\vec{y}_i|} \right), \quad n_i^2 = 0. \quad (5)$$

This allows us to define the standard light-cone decomposition for the y_i^μ vectors,

$$y_i^\mu = y_i^+ \frac{n_{i-1}^\mu}{n_i \cdot n_{i-1}} + y_i^- \frac{n_i^\mu}{n_i \cdot n_{i-1}} + y_{i,\perp}^\mu \equiv (y_i^+, y_i^-, y_{i,\perp}), \quad (6)$$

with $y_i^2 = 2y_i^+ y_i^- / (n_i \cdot n_{i-1}) + y_{i,\perp}^2, y_{i,\perp}^2 < 0$. Equation (4) implies the following hierarchy between the components of y_i :

In the end, we arrive at the following remarkably simple factorization theorem for R at leading power in the SLC limit:

$$R = \mathcal{W}(\{x\}) \prod_{i=1}^4 \int_0^\infty d\omega_i \mathcal{C}_i(\omega_i \omega_{i+1}) \mathcal{J}_{n_i} \left(\frac{\omega_i}{y_i^-} \right) e^{-\omega_i \frac{y_i^2}{2y_i^-}}. \quad (12)$$

Equation (12) reveals that the CF in the SLC limit can be simply described by the product of a polygonal Wilson loop and convolutions of form factors with jet functions. A pictorial representation of the factorization theorem in Eq. (12) is provided in Fig. 3. Importantly, each of the elements of the factorization formula in Eq. (12) is a well-known object in collider physics, entering the theoretical description of the infrared structure of collider observables such as event shapes. An important consequence of this observation is that both the finite remainder of the form factor $\mathcal{C}_i$ and the jet function $\mathcal{J}_{n_i}$ can be extracted at high-loop orders from the literature [54,56]. On the other hand, the full, finite remainder of the Wilson loop $\mathcal{W}(\{x\})$ in QCD was previously unknown beyond one loop [21,57], and we compute it to two loops in this Letter (all results are provided in Ref. [58]). The ingredients $\{\mathcal{W}, \mathcal{C}_i, \mathcal{J}_{n_i}\}$ of Eq. (12) also depend on a common renormalization scale, μ , that has been omitted in Eq. (12) to simplify the notation. This dependence is governed by the following linear RG equations that can be used to resum the large logarithmic corrections in R :

$$\begin{aligned} \frac{d \ln \mathcal{W}(\{x\})}{d \ln \mu} &= -2\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{x_{13}^2 x_{24}^2 \mu^4}{16} + \gamma^W(\alpha_s), \\ \frac{d \ln \mathcal{J}_{n_i}(\omega_i/y_i^-)}{d \ln \mu} &= -2\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{2\omega_i}{y_i^- \mu^2} + \gamma^J(\alpha_s), \\ \frac{d \ln \mathcal{C}_i(\omega_i \omega_{i+1})}{d \ln \mu} &= \Gamma_{\text{cusp}}(\alpha_s) \ln \frac{2\omega_i \omega_{i+1}}{\mu^2 n_i \cdot n_{i+1}} + \gamma^C(\alpha_s), \end{aligned} \quad (13)$$

where $x_{13}^2 = 2y_1^- y_2^- / n_1 \cdot n_4 = 2y_3^- y_4^- / n_2 \cdot n_3$ and $x_{24}^2 = 2y_1^- y_4^- / n_3 \cdot n_4 = 2y_2^- y_3^- / n_1 \cdot n_2$ up to power corrections in λ . The μ dependence cancels order-by-order in the rhs of Eq. (12) such that R is RG invariant, leading to the following consistency relation for the noncusp anomalous dimensions:

$$\gamma^{W_n}(\alpha_s) = -n\gamma^J(\alpha_s) - n\gamma^C(\alpha_s) = -\frac{n}{2}\gamma^{\text{thr}}(\alpha_s), \quad (14)$$

where $\gamma^{\text{thr}}(\alpha_s)$ is the soft anomalous dimension for threshold resummation that is currently known to four loops [59–61], analogously to γ^J [60,62] and γ^C [63]. Our derivation of the factorization formula for the CF in the SLC limit can be easily extended to the n -point case. This simply entails evaluating the Wilson loop $\mathcal{W}$ on the n -gon Γ , defined by the n lightlike directions of the problem, and extending the product in Eq. (12) up to n . The resulting equation for the n -point CF constitutes the generalization of the results of Ref. [19] to nonconformal gauge theories such as QCD. In this case, our result reveals that even without the presence of a conformal symmetry, the CF is dual to a polygonal Wilson loop. However, the proportionality factor has a more complex structure than in the conformal case, and it is entirely captured by our factorization formula.

Predictions for $\mathcal{N} = 4$ SYM and QCD—Equation (12) can be used to calculate the CF in the SLC limit in different QFTs. We examine the four-point case in QCD, for which we extract the three-loop form factor and jet function from Refs. [54,56]. The four-point two-loop Wilson loop is computed for the first time in this Letter [58], and we predict its three-loop structure up to terms that are regular in the SLC limit using the RG equations [Eq. (13)]. We work in conventional dimensional regularization scheme in $D = 4 - 2\epsilon$ dimensions, which allows us to verify the cancellation of all ϵ poles and the consistency of the (local) renormalization of the ingredients of Eq. (12). We write the result as an expansion in the strong coupling α_s as $R = 1 + \sum_n [\alpha_s^n(\mu)/(4\pi)^n] R_n$. The one-loop result is given by

$$R_1 = C_F(-4\zeta_2 - 2L_u L_v + L_{uv}), \quad (15)$$

where we have introduced the abbreviations $L_u = \ln u$, $L_v = \ln v$, and $L_{uv} = \ln(uv)$. The one-loop result of Eq. (15) can also be extracted by taking the SLC limit of the full one-loop result of Ref. [18].

Our factorization theorem, Eq. (12), allows us to calculate the new two-loop result as well as all the logarithmic terms at three loops (while the calculation of regular terms in the SLC limit requires the knowledge of the four-point Wilson loop at three loops, which is currently unknown). These read

$$\begin{aligned} R_2 &= C_F^2 \left[-28\zeta_2 - 48\zeta_3 + 116\zeta_4 + L_u(8 - 2L_{uv})L_v + 2L_u^2 L_v^2 + \left(4\zeta_2 - \frac{7}{2}\right)L_{uv}^2 + \left(4\zeta_2 - 24\zeta_3 + \frac{5}{2}\right)L_{uv} \right] \\ &+ C_F C_A \left[\frac{68\zeta_2}{9} + \frac{28\zeta_3}{3} - 16\zeta_4 + L_u L_v \left(4\zeta_2 - \frac{11L_{uv}}{6} - \frac{235}{18}\right) + \frac{11L_{uv}^2}{12} + \left(-\frac{112\zeta_2}{3} + 12\zeta_3 + \frac{221}{18}\right)L_{uv} \right] \\ &+ C_F n_f \left[-\frac{8\zeta_2}{9} + \frac{8\zeta_3}{3} + L_u \left(\frac{L_{uv}}{3} + \frac{17}{9}\right)L_v + \left(\frac{16\zeta_2}{3} - \frac{13}{9}\right)L_{uv} - \frac{L_{uv}^2}{6} \right] + 16 \left(C_F^2 - \frac{1}{2} C_F C_A \right) \\ &+ \frac{1}{2} \beta_0 R_1 \ln \frac{x_{13}^2 x_{24}^2 \mu^4}{16} + C_F \beta_0 \left[r_2(L_{uv}, x) + r_2\left(L_{uv}, \frac{1}{x}\right) + \frac{L_{13}^2}{4} + \frac{L_{24}^2}{4} + \ln(x)L_{13}L_{24} - \frac{1}{2}(L_v L_{13}^2 + L_u L_{24}^2) \right], \end{aligned} \quad (16)$$

$$\begin{aligned}
 R_3 = & C_A C_F^2 \left(-\frac{4}{9} \pi^4 L_u^2 - \frac{2}{45} \pi^4 L_u L_v - \frac{2}{3} \pi^2 L_u^2 L_v^2 - \frac{56}{3} \pi^2 L_u \zeta_3 - 24 L_u^2 L_v \zeta_3 - 120 L_u \zeta_5 \right) \\
 & + C_F^2 C_F \left(-\frac{11}{45} \pi^4 L_u L_v - 40 L_u \zeta_5 \right) \\
 & + C_F^3 \left(-\frac{4}{9} \pi^4 L_u^2 - \frac{26}{15} \pi^4 L_u L_v - \frac{4}{3} \pi^2 L_u^3 L_v - \frac{2}{3} \pi^2 L_u^2 L_v^2 - \frac{2}{3} L_u^3 L_v^3 + 32 \pi^2 L_u \zeta_3 - \frac{16}{3} L_u^3 \zeta_3 + 48 L_u^2 L_v \zeta_3 + 240 L_u \zeta_5 \right) \\
 & + [u \leftrightarrow v] + \dots
 \end{aligned} \tag{17}$$

where we defined $\beta_0 = 11C_A/3 - 2n_f/3$, $x = x_{13}^2/x_{24}^2$, $L_{13} = \ln(y_1^2/y_3^2)$, $L_{24} = \ln(y_2^2/y_4^2)$ and

$$r_2(L_{uv}, x) = 2\text{Li}_3(-x) - 2\ln(x)\text{Li}_2(-x) - \frac{1}{4}L_{uv}\ln^2(x) - \frac{1}{2}(6\zeta_2 + \ln^2(x))\ln\frac{(1+x)^2}{x} + \frac{1}{4}\ln^2(x). \tag{18}$$

Checks and discussion—A first consistency check of the validity of the factorization theorem in Eq. (12) is the complete cancellation of poles in our perturbative results. This check is far from trivial, as the cancellation happens after combining all the bare ingredients via the convolution structure dictated by Eq. (12). Moreover, both the hard Wilson coefficient and jet functions are fixed and taken from independent results, while the soft function directly follows from the calculation of the closed Wilson loop. On a related note, we have also independently checked that, after individually renormalizing the objects in the factorization theorem, the scale dependence of the renormalized objects cancels when combined according to Eq. (12). In other words, this implies that the factorization theorem holds both at the bare and at the renormalized level, further strengthening its validity. As a consistency check of our results, we verify that our expressions for the CF satisfy the anomalous conformal Ward identities given in Ref. [64], expanded in the sequential lightlike limit

$$K^\mu \langle j_1 \dots j_4 \rangle = -\beta(g) \int d^4x 2x^\mu \langle \frac{\partial}{\partial g} \mathcal{L}(x) j_1 \dots j_4 \rangle, \tag{19}$$

where $j_i \equiv j_{\mu_i}(x_i)$, $\beta(g)$ is the QCD beta function, $\mathcal{L}$ is the QCD Lagrangian, and K^μ is the special conformal transformation

$$K^\mu = \sum_{i=1}^4 2x_i^\mu (3 + x_i \cdot \partial_{x_i}) - 2\Sigma_\nu^\mu x_i^\nu - x_i^2 \partial_{x_i}^\mu. \tag{20}$$

We have verified with an explicit calculation that the action of K^μ on our two-loop prediction for the correlation function agrees with the rhs of Eq. (19) that we computed following Ref. [18] by evaluating the position-space integrals with a Lagrangian insertion. As a further check, we use the principle of leading transcendentality to obtain the two- and three-loop results for $\mathcal{N} = 4$ SYM by performing the replacements $C_F \rightarrow N_c$ and $C_A \rightarrow N_c$, keeping only the leading transcendental terms in the QCD expressions given above. For instance, for the three-loop correction, we find

$$\begin{aligned}
 R_3^{\mathcal{N}=4} = & -N_c^3 \left[\frac{2}{3} L_u^3 L_v^3 + \frac{4}{3} \pi^2 L_u^2 L_v^2 + \frac{4}{3} \pi^2 L_u^3 L_v \right. \\
 & + \frac{16}{3} L_u^3 \zeta_3 - 24 L_u^2 L_v \zeta_3 + \frac{8}{9} \pi^4 L_u^2 + \frac{91}{45} \pi^4 L_u L_v \\
 & \left. - \frac{40}{3} \pi^2 L_u \zeta_3 - 80 L_u \zeta_5 \right] + [u \leftrightarrow v],
 \end{aligned} \tag{21}$$

which is in full agreement with the direct calculation performed in Ref. [65]. This provides a nontrivial check of our procedure.

Conclusions—In this Letter, we have initiated a systematic study of the sequential light-cone limit of multipoint correlation functions in QCD. We have shown that this limit admits an effective-field-theory description owing to the large hierarchy of scales in the problem. We showed how SCET can be used to derive a factorization formula for the multipoint CFs that generalizes the duality between these objects and Wilson loops, previously observed for conformal field theories, to nonconformal gauge theories such as QCD. This result provides a powerful tool to predict the perturbative structure of these key objects in different QFTs, which we exploit to derive the first prediction for the full singular behavior of the correlator of four electromagnetic currents in the SLC limit at two loops in QCD and of its log-enhanced terms at three loops.

This Letter opens several interesting avenues that are worth exploring in the future. A first line of research concerns the study of the multipoint case, which will require the calculation of the finite terms of the n -gon Wilson loop with more than four cusped vertices. Secondly, it will be interesting to apply the techniques developed in this Letter to study other correlation functions, for instance, correlators of the electromagnetic current and the energy-momentum tensor, which enter the description of energy correlators at collider experiments. A third, related direction concerns the use of the results presented in this Letter to predict the structure of collider correlators, such as the back-to-back limit of the charge-charge correlation [17]. This, crucially, requires understanding the details of the detector limit and light transform of the renormalized CF, which is still an open problem in nonconformal theories.

The above explorations will greatly deepen our knowledge of CFs in QCD and their connection with scattering observables, offering a novel angle to approach the derivation of precise theoretical predictions at collider experiments.

Acknowledgments—We thank A. Zhiboedov for valuable discussions and comments on the draft and D. Chicherin for helpful discussions and correspondence. We also thank the participants of Resummation, Amplitudes, and Subtractions 2024 and 2025 at CERN and the Energy Operators in Particle Physics, Quantum Field Theory and Gravity at Simons Center for Geometry and Physics for useful discussion. P. M. and G. V. wish to thank the Center for High Energy Physics of Peking University for hospitality while this work was carried out. H. C. is supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics under Grant Contract No. DESC0011090. Z. Y. P. and H. X. Z. are funded by the National Natural Science Foundation of China under Contract No. 12425505. The work of P. M. and G. V. is funded by the European Union (ERC, Grant Agreement No. 101044599).

Views and opinions expressed are those of the authors only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

Data availability—The data that support the findings of this article are openly available [58].

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