

Temporally Localized Quantum Operations on Continuous-Wave Thermal Light

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Previous work showed that thermal light with a blackbody spectrum cannot be decomposed into a mixture of independent localized pulses. However, we find that in the weak-source limit and under the assumption of a flat spectrum, the first nonvacuum term in the state expansion does form a mixture of such pulses. This decomposition is essential for quantum-enhanced astronomical interferometry, which typically operates on localized pulses even though stellar light is inherently continuous-wave. We present a quantum derivation of the van Cittert–Zernike theorem that incorporates finite bandwidth, thereby justifying the operations on localized pulses while processing continuous-wave thermal light. For general spectra in the weak-source limit, we establish a criterion under which correlations between pulses can be safely neglected. When this criterion is not met, we provide a corrected strategy that accurately accounts for both the spectral profile and the detector-defined pulse shape.

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Introduction—Thermal light, such as sunlight, is ubiquitous in nature and features in some protocols of quantum optics. Here, we use the term “thermal light” to refer to radiation emitted by a system in thermal equilibrium at a given temperature and exhibiting thermal photon statistics [1]. Recent proposals have explored the use of distributed entanglement for astronomical interferometry to image stellar thermal sources [2], which is an important application of quantum networks [3–6]. Subsequent work has investigated various enhancements to such schemes, using quantum memories to reduce the entanglement requirements [7,8], employing nonlinear controlled quantum gates to improve performance [9], and analyzing the entanglement cost of different protocols [10]. Other studies have demonstrated the advantage of using multiple entangled states without the need for quantum repeaters [11], proposed continuous-variable versions of entanglement-

assisted interferometry [12–14], and established fundamental limits along with optimal protocols under superselection rule constraints [15]. It is commonly assumed that stellar light is in a thermal state upon detection on Earth’s surface. In such schemes, it is common to interfere stellar light with terrestrial photons, the latter typically prepared in localized temporal modes. This raises a subtle issue: when a localized terrestrial photon is interfered with continuous-wave stellar light, how should one account for the mismatch in mode structure, and what are the consequences?

This question is related to the decomposition of thermal light into localized pulses, which has been discussed in a series of papers [16–18]. These works show that thermal light with a blackbody spectrum cannot be decomposed into a mixture of independent localized pulses. We will refer to any optical state associated with a localized temporal mode as a pulse and use the terms “temporal mode” and “pulse” interchangeably throughout this Letter. References [16,17] demonstrate the impossibility of such a decomposition by showing that it leads to contradictions in the first- and second-order correlation functions. Attempts to resolve these contradictions using unphysical constructs, such as a trace-improper density matrix, still fail to reproduce both correlation functions accurately. These results call into question the validity of typical analyses in quantum-enhanced astronomical interferometry that apply quantum operations independently to each localized temporal mode of thermal light without accounting for intertemporal mode correlations [2,7–15]. In this Letter, we examine whether and when these temporal correlations have any observable consequences for such protocols.

We begin by considering a simple one-dimensional model with thermal light propagating along a single spatial

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dimension, e.g., in optical fiber, and focusing on specific cases different from the blackbody spectrum studied in Refs. [16–18]. Specifically, we show that when the spectrum of thermal light is flat, the light can indeed be decomposed into a direct product of independent localized pulses. We show that this decomposition gives the correct first- and second-order correlation functions. We further find that in the weak strength limit, the leading nonvacuum term is a mixture of independent localized pulses. We also observe that the trace-improper density matrix constructed in Refs. [16,17], which yields the correct first-order correlation function, naturally emerges in a similar form as the leading nonvacuum term in the weak-strength limit. The additional terms in the decomposition ensure proper normalization of the density matrix, making the overall state physical.

We then analyze the impact of temporal mode correlations on quantum-enhanced astronomical interferometry [2,7–15]. We present a complete derivation of the van Cittert–Zernike theorem within the quantum description of multimode thermal light with finite spectral bandwidth, followed by a discussion of how intermode correlations influence the measurement process in such schemes. For thermal light with a flat spectrum, operations can be performed in any set of localized temporal modes without the need to account for correlations between different modes. Furthermore, we consider the weak-source limit with a general spectrum and establish a criterion under which correlations between localized temporal modes can be neglected. When this criterion is not satisfied, we provide a strategy that accounts for these correlations for arbitrary spectral shapes and temporal mode profiles, tailored for experimental convenience. This addresses some of the naivety present in typical single-mode quantum optics analyses applied to quantum-enhanced astronomical interferometry. This framework also allows us to investigate the rich interplay among key properties of thermal light, including spatial correlations between the two telescopes, temporal correlations between localized modes induced by a nonflat spectrum, and the quantum statistical features of thermal light such as the Hanbury Brown–Twiss (HBT) effect [19–21].

Preliminary—We first review the theoretical framework needed for the decomposition of a thermal state in the frequency domain into localized modes in the spatial domain. For simplicity, we first consider the one-dimensional case such that the wave vector k is scalar. We consider thermal light with a continuous spectrum consisting of $N \rightarrow \infty$ frequency modes $a_{m=1,2,\dots,N}$, and denote the wave vector corresponding to the m th mode by k_m . After it has passed through a bandpass filter, which is commonly employed in astronomical interferometry [22], the light can be described as

$$\rho = \int \frac{d^{2N}\vec{\alpha}}{\det(\pi\Gamma)} F(\vec{\alpha}) |\vec{\alpha}\rangle \langle \vec{\alpha}|, \quad F(\vec{\alpha}) = \exp(-\vec{\alpha}^\dagger \Gamma^{-1} \vec{\alpha}), \quad \Gamma_{mn} = \delta_{mn} n_m, \quad (1)$$

where $|\vec{\alpha}\rangle = \exp(\sum_m \alpha_m a_m^\dagger - \alpha_m^* a_m) |0\rangle$ is a multimode coherent state spanning all frequency modes. In this P representation of the state [1], Γ is the diagonal matrix of mean thermal photon numbers, with n_m denoting the mean photon number of the m th mode. We assume $n_m \neq 0$ only when $k_m \in [k_0 - \Delta k/2, k_0 + \Delta k/2]$, corresponding to the photons transmitted by the bandpass filter centered at k_0 with bandwidth Δk . We will see that the spectrum described by n_m is important for the decomposition of thermal light. Using a construction similar to Refs. [18,23,24], we define the localized temporal modes as $c_s = \sum_{m=1}^N O_{sm} a_m$, for $s = 1, 2, \dots, N$, where each index s labels a temporal mode c_s . The matrix O can be chosen such that the resulting modes c_s are both orthonormal and spatially localized. This allows us to write Eq. (1) as

$$\rho = \int \frac{\prod_{s=1}^N d^2\gamma_s}{\det(\pi\Lambda)} F(\vec{\gamma}) |\vec{\gamma}\rangle \langle \vec{\gamma}|, \quad F(\vec{\gamma}) = \exp(-\vec{\gamma}^\dagger \Lambda^{-1} \vec{\gamma}), \quad \Lambda = (O^{-1})^\dagger \Gamma O^{-1}, \quad (2)$$

where $|\vec{\gamma}\rangle = \exp(\sum_s \gamma_s c_s^\dagger - \gamma_s^* c_s) |0\rangle$. This is the decomposition derived in Ref. [18]. For more details on the preliminary, see Sec. I of Supplemental Material (SM) [25].

Decomposition of a thermal state into localized pulses—We now discuss the general decomposition of states in the one-dimensional case and address the relation of our discussion with the previous work of Refs. [16–18]. We first show that, in general, thermal light cannot be decomposed into independent localized pulses. We construct a set of orthonormal functions $\{\omega_s(z)\}$ to serve as the pulse shapes, or equivalently their Fourier transforms $\tilde{\omega}_s(k)$, which must also form an orthonormal set,

$$\int_{k_0-\Delta k/2}^{k_0+\Delta k/2} dk \tilde{\omega}_{s_1}^*(k) \tilde{\omega}_{s_2}(k) = \delta_{s_1 s_2}. \quad (3)$$

As an example, the wave function for a localized mode could be $\omega_s(z) \propto e^{ik_0(z-x_s)} \text{sinc}[\Delta k(z-x_s)/2]$, where z denotes the spatial coordinate along the propagation direction. The pulses are separated by $2\pi/\Delta k$, and each has a spatial width on the order of Δk^{-1} , determined by the k -space bandwidth Δk . For more details on these calculations, see Sec. I of SM [25]. If we calculate the first-order correlation function between the two localized spatial modes a_{s_1} and a_{s_2} ,

$$G^{(1)}(s_1, s_2) = \int_{k_0-\Delta k/2}^{k_0+\Delta k/2} dk \tilde{\omega}_{s_1}^*(k) \tilde{\omega}_{s_2}(k) n(k) = 0, \quad (4)$$

where $s_1 \neq s_2$. Satisfying both Eqs. (3) and (4) is equivalent to constructing a set of functions that are orthogonal with respect to two different weight functions, which is generally not possible for an arbitrary spectrum $n(k)$, where we use $n_m = n(k_m)$ for notational convenience. In this Letter,

we refer to a nonvanishing correlation function between different temporal modes as the presence of temporal-mode correlations. The modes are considered independent when no such correlations are present.

We now consider several special cases where such a decomposition either exists exactly or holds approximately. In studies of quantum-enhanced astronomical imaging at optical wavelengths [2,7,8], it is typically appropriate to assume the weak-source limit, due to the extremely low mean photon number received in each localized temporal mode. In this weak limit, the decomposition of thermal light with a general spectrum, as given in Eq. (6), can be expanded as follows:

$$\rho \approx |0\rangle\langle 0| + \sum_{s,s'=1}^N \Lambda_{s's} |s'\rangle\langle s| + \cdots, \quad (5)$$

where $|s\rangle = c_s^\dagger |0\rangle$ and we assume weak-source limit $|\lambda_{\max}(\Lambda)| \rightarrow 0$. Note that, in general, Λ is not diagonal, so neighboring modes c_s and c_{s+1} may exhibit correlations. This means for the case of a general spectrum, even in the weak limit, we cannot directly assume the state is a mixture of independent localized pulses.

Consider the decomposition of thermal light with a flat spectrum by choosing $n_m = n$ for $\forall m$. This choice results in $\Lambda_{ss'} = \delta_{ss'} \Lambda_{ss}$, which means the different c_s modes are independent of each other and Eq. (2) is simplified as

$$\rho = \otimes_s \rho'_s, \quad \rho'_s = \int \frac{d^2\gamma_s}{\pi n} \exp(-|\gamma_s|^2/n) |\gamma_s\rangle\langle \gamma_s|. \quad (6)$$

Note that each localized mode c_s is in a thermal state ρ'_s and all modes c_s are independent of each other.

By assuming both a flat spectrum and the weak-source limit, we find the decomposition

$$\rho \approx |0\rangle\langle 0| + \epsilon \sum_{s=1}^N |s\rangle\langle s| + \cdots, \quad (7)$$

where $\Lambda_{ss} = \epsilon$ and the $O(\epsilon)$ term represents the first nonvacuum contribution and corresponds to a mixture of independent localized pulses, where all pulses are in the vacuum state except for one that contains a single photon. We do not know which pulse contains the single photon, as reflected by the mixed state. This is a very common intuitive picture used in analyses of astronomical interferometry enhanced by quantum technologies [2,7–15]. We validate our decomposition by showing it reproduces the expected thermal-state correlation functions without requiring an unphysical trace-improper density matrix whose trace exceeds one, unlike previous discussions [16,17], and naturally maintains trace normalization as the volume increases. For further discussion of the correlation functions, their relation to previous works, and specific cases of

deviation from a flat spectrum, please refer to Sec. II of SM [25].

Astronomical interferometric imaging of a thermal source with a general spectrum—We now consider astronomical interferometry as in Ref. [2], which typically interferes continuous-wave stellar light with a terrestrial photon prepared in a localized temporal mode. As shown earlier, a flat spectrum allows thermal light to be decomposed into a product of independent, localized pulses in orthonormal temporal modes, and the interference between terrestrial and stellar light can then be safely treated as independent across modes. However, with a nonflat spectrum, ignoring intermode correlations can introduce inaccuracies. We now establish criteria under which such correlations can be neglected and present a corrected estimation procedure when the criteria is not satisfied. These corrections can be applied in postprocessing, enabling flexible mode choices convenient for experimental implementation while maintaining accuracy.

To quantify the influence of a general spectrum of thermal light on interferometric imaging, we rederive the van Cittert–Zernike theorem within a multimode quantum optics framework. We start from the state of a general incoherent source [1],

$$\rho = \int d^{2MW} \vec{\alpha} \Phi(\vec{\alpha}) |\vec{\alpha}\rangle\langle \vec{\alpha}|, \quad \Phi(\vec{\alpha}) = \frac{\exp(-\vec{\alpha}^\dagger \Gamma^{-1} \vec{\alpha})}{\det(\pi \Gamma)}, \quad (8)$$

where $\Gamma_{i_1 m_1, i_2 m_2} = \delta_{i_1 i_2} \delta_{m_1 m_2} n_{i_1 m_1}$, the multimode coherent state $|\vec{\alpha}\rangle = \otimes_{i=1}^W \otimes_{m=1}^M \exp(\alpha_{im} \hat{a}_{i,k_m}^\dagger - \alpha_{im}^* \hat{a}_{i,k_m}) |0\rangle$ is used as a basis to represent the thermal state, the index $i = 1, 2, \dots, W$ labels spatial points on the source, and we consider the continuous spatial intensity distribution in the limit $W \rightarrow \infty$. The index $m = 1, 2, \dots, M$ labels frequencies k_m , with the continuous frequency spectrum recovered in the limit $M \rightarrow \infty$. The quantity n_{im} denotes the mean photon number emitted at frequency k_m from the i th spatial point on the source. For simplicity, we model the source as one-dimensional by neglecting variations along the y axis. Thus, the spatial coordinates of the i th source point are (x_i, d) , where x_i is its position along the x axis and d is the distance between the source and the telescope array along the z axis.

We consider the case of two telescopes. Given that the astronomical source is in the far-field limit relative to the telescopes on Earth, we assume the incoming light propagates exclusively along the z axis near the telescopes. Let the two lenses be located at positions $(u_0, 0)$ and $(u_1, 0)$. Under this assumption, the source annihilation operators $\hat{a}_{i,k_m}$ evolve as $\hat{a}_{i,k_m} \rightarrow \int dz \sqrt{\eta} e^{i\phi(k_m, u_1, z, x_i)} \hat{b}_{z0} + \int dz \sqrt{\eta} e^{i\phi(k_m, u_2, z, x_i)} \hat{b}_{z1} + \hat{v}_{i,k_m}$, where $\hat{v}_{i,k_m}$ describes the environmental modes and η quantifies the transmission efficiency from the source to the spatial modes near the telescopes. The operators $\hat{b}_{z0}$ and $\hat{b}_{z1}$ correspond to the

spatial modes immediately above the first and second telescopes, located at positions (u_0, z) and (u_1, z) , respectively. The accumulated propagation phase is given by $\phi(k, u, z, x) = k\sqrt{(x-u)^2 + (d-z)^2}$. With this setup, we can now determine the quantum state of the light field near the two telescopes. In the weak-source limit, we expand the total state into Fock states in each localized temporal mode,

$$\rho = |0\rangle\langle 0| + \sum_{s,s'=1}^N \sum_{p,q \in \{0,1\}} \vec{\omega}_{s'p}^\dagger \Lambda \vec{\omega}_{sq} |s', p\rangle\langle s, q| + \dots$$

$$\Lambda_{lt,pq} = \sum_{i=1}^W \sum_{m=1}^M \eta n_{im} e^{i[\phi(k_m, u_p, z_q, x_i) - \phi(k_m, u_l, z_t, x_i)]} \quad (9)$$

where $|s, p\rangle = \hat{c}_{sp}^\dagger |0\rangle$, $\hat{c}_{sp}^\dagger = \sum_{n=1}^N \omega_s(z_n) b_{zn,p}^\dagger |0\rangle$, we define the vector $\vec{\omega}_{sq}$ such that the double sum over spatial coordinates is expressed as matrix multiplication $\sum_{n,m} \omega_{s'}^*(z_n) \Lambda_{pn,qm} \omega_s(z_m) = \vec{\omega}_{s'p}^\dagger \Lambda \vec{\omega}_{sq}$, $\omega_s(z)$ is the s th localized temporal mode, we denote the position of the n th point near the two telescopes as $(u_{0,1}, z_n)$, and we take the limit $N \rightarrow \infty$ to describe a continuous light field. Note that, in general, nearby temporal modes exhibit correlations $\vec{\omega}_{s+1,p}^\dagger \Lambda \vec{\omega}_{sq} \neq 0$, so we cannot claim that the state is a mixture of independent localized pulses.

We now investigate the impact of a nonflat spectrum on the estimation results. For the measurement applied to the s th temporal mode, the state is projected onto $|00\rangle_s$ when no photon is detected, and onto the state $(|01\rangle_s \pm e^{i\theta}|10\rangle_s)/\sqrt{2}$ when a single photon is detected. Here, $|01\rangle_s = \sum_{n=1}^N \omega_s(z_n) \hat{b}_{0z_n}^\dagger |0\rangle$ and $|10\rangle_s = \sum_{n=1}^N \omega_s(z_n) \hat{b}_{1z_n}^\dagger |0\rangle$. Suppose we perform this measurement successively across N temporal modes in time and detect a total of Q photons. Let $\vec{s} = [s_1, s_2, \dots, s_Q]$ denote the indices of the temporal modes in which the photons are detected. We define $\vec{\delta} = \{e^{i\theta}, -e^{i\theta}\}^{\otimes Q}$ to label the possible outcomes of detecting Q photons, which projects the state onto

$$|\vec{s}, \vec{\delta}\rangle = \sum_{\vec{p} \in \{0,1\}^{\otimes Q}} \left(\prod_{i=1}^Q \delta_i^{p_i} \right) |\vec{s}, \vec{p}\rangle / 2^{Q/2},$$

$$P(\vec{s}, \vec{\delta}) = \sum_{\vec{p}, \vec{q}} \sum_{\mathbb{P}} \left[\prod_{i=1}^Q \frac{1}{2} \delta_i^{p_i+q_i} \vec{\omega}_{s_i p_i}^\dagger \Lambda \vec{\omega}_{s_i q_i} \right], \quad (10)$$

where $|\vec{s}, \vec{p}\rangle = \bigotimes_{i=1}^Q (\sum_{n=1}^N \omega_{s_i}(z_n) b_{pn,z_n}^\dagger) |0\rangle$, $\sum_{\mathbb{P}}$ means the sum over permutations.

The imaging task aims to estimate the intensity distribution n_{im} encoded in Λ , by inferring it from the empirical distribution obtained by sampling the true probability distribution in Eq. (10). Correlations between different temporal modes, i.e., $\vec{\omega}_{s'p}^\dagger \Lambda \vec{\omega}_{sq} \neq 0$ for $s \neq s'$, can influence the probability $P(\vec{s}, \vec{\delta})$ and thus affect astronomical

imaging. However, unlike Eq. (9), which involves all modes, $P(\vec{s}, \vec{\delta})$ depends only on correlations between modes where photons are actually detected. For light with bandwidth Δk , these correlations decay over a time-scale $\sim \Delta k^{-1}$; when detection events are separated by $\Delta t \gg \Delta k^{-1}$, such correlations can be neglected. This intuition is formalized below to establish the criteria under which intermode correlations can be safely ignored.

Proposition 1: Assuming $n_{im} = I_i f_m$, where f_m is the normalized spectrum satisfying $\sum_m f_m = 1$, implying that the relative intensity I_i across spatial points is independent of frequency—reasonable within a narrow bandwidth $\Delta k \ll k_0$ —and $\omega_r(z) = \omega(z - z_r)$, and further approximating the phase $\phi(k, u, z, x)$ by its value at $k = k_0$, thereby neglecting its frequency dependence, we compute the deviation between the actual probability distribution $P(\vec{s}, \vec{\delta})$ —which includes correlations between different temporal modes under a general spectrum—and the distribution $P_0(\vec{s}, \vec{\delta})$ assuming uncorrelated temporal modes,

$$|P(\vec{s}, \vec{\delta}) - P_0(\vec{s}, \vec{\delta})| = (2\epsilon)^Q \sum_{\mathbb{P} \neq I} \prod_{i=1}^Q \xi(x_{s_i} - x_{s_{\mathbb{P}_i}}), \quad (11)$$

where $\sum_{\mathbb{P} \neq I}$ means the sum over permutations except the identity, $\epsilon = \sum_{i=1}^W \eta I_i$, and the function quantifying correlations $\xi(x_{s_1} - x_{s_2}) = \Omega_{s_1 s_2} / \Omega_{ss}$, we find $\xi(\Delta x) \rightarrow 0$ as $|\Delta x| \gg 2\|P(k)\|_\infty + \Delta k \|P'(k)\|_\infty$, $P(k) = f(k) |\tilde{\omega}(k)|^2 / (\int_{k_0-\Delta k/2}^{k_0+\Delta k/2} dk f(k) |\tilde{\omega}(k)|^2)$, $P_0(\vec{s}, \vec{\delta}) = \prod_{i=1}^Q (\frac{1}{2} \sum_{p,q \in \{0,1\}} \delta_i^{p+q} \vec{\omega}_{s_i p}^\dagger \Lambda \vec{\omega}_{s_i q})$, $\Omega_{s_1 s_2} = \sum_{m=1}^M f_m \tilde{\omega}_{s_1}^*(k_m) \tilde{\omega}_{s_2}(k_m)$, $P'(k) = dP(k)/dk$.

The factor $\Omega_{s_1 s_2} \propto \vec{\omega}_{s_1 p}^\dagger \Lambda \vec{\omega}_{s_2 q}$ captures the effect of correlations between different temporal modes. Clearly, when $f_m = f$ is constant for all m , we have $\Omega_{s_1 s_2} = \delta_{s_1 s_2} f$, and no intermode correlation arises, hence $P(\vec{s}, \vec{\delta}) = P_0(\vec{s}, \vec{\delta})$. However, for a general spectrum, $\Omega_{s_1 s_2} \neq 0$ introduces additional contributions to the probability distribution, hence $P(\vec{s}, \vec{\delta}) \neq P_0(\vec{s}, \vec{\delta})$. We take $\omega_r(z) = \omega(z - z_r)$, corresponding to a set of localized, orthonormal temporal modes that are translations of one another—a typical choice in experimental implementations. Although $\phi(k, u, z, x)$ depends on k , we approximate it by its value at k_0 , but frequency dependence can be included via a Taylor expansion of $\phi(k, u, z, x)$ around k_0 if needed.

Clearly, $P_0(\vec{s}, \vec{\delta})$ is of order ϵ^Q , as ϵ corresponds to the mean photon number per mode—after accounting for losses from the stellar source to telescopes—while $P_0(\vec{s}, \vec{\delta})$ denotes the probability of detecting Q photons in the temporal modes indexed by $\vec{s}$. Note that the summation excludes the identity permutation, so as long as the detected photons are sufficiently separated in time, i.e., $|\Delta x| \gg 2\|P(k)\|_\infty + \Delta k \|P'(k)\|_\infty$ for all pairs s_i, s_j , the difference $|P(\vec{s}, \vec{\delta}) - P_0(\vec{s}, \vec{\delta})|$ becomes negligible. In

practice, $\vec{s}$ and $\vec{\delta}$ correspond to the detection outcomes from measuring N localized temporal modes and observing Q photons. $\xi(\Delta x)$ can be computed in practice by characterizing the spectral profile $f(k)$ through preliminary measurements that record the intensity at each frequency. This allows Eq. (11) to serve as an estimate for whether further correction is necessary. If a significant fraction of photons are detected in closely spaced temporal modes such that $\sum_{\mathbb{P} \neq I} \prod_{i=1}^Q \xi(x_{s_i} - x_{s_{\mathbb{P}_i}})$ cannot be assumed to be negligible, then data postprocessing must account for the resulting correlations, as detailed in the following proposition.

Proposition 2: Assuming $n_{im} = I_i f_m$, $\sum_m f_m = 1$, and $\omega_r(z) = \omega(z - z_r)$, and further approximating the phase $\phi(k, u, z, x)$ by its value at $k = k_0$, the actual probability distribution—accounting for correlations between different temporal modes under a general spectrum—is given by

$$P(\vec{s}, \vec{\delta}) = \frac{\epsilon^Q}{2^Q} \sum_{\mathbb{P}} \prod_{i=1}^Q [(\delta_i g^* + \delta_{\mathbb{P}_i} g + 1 + \delta_i \delta_{\mathbb{P}_i}) \Omega_{s_i, s_{\mathbb{P}_i}}], \quad (12)$$

where $\epsilon g = \sum_{i=1}^W \eta I_i \exp \{ik_0[(u_1^2 - u_0^2/2d)] + ik_0[x_i(u_0 - u_1)/d]\}$.

In interferometric imaging, the goal is to estimate the coherence function g , which is a function of the intensity distribution n_{im} . Given the probability distribution $P(\vec{s}, \vec{\delta})$, which explicitly accounts for temporal-mode correlations, the parameter $g = |g|e^{i\theta}$ can be estimated using a standard maximum likelihood approach by solving $[\partial P(\vec{s}, \vec{\delta}) / \partial |g|]_{|g|=|\hat{g}|} = [\partial P(\vec{s}, \vec{\delta}) / \partial \theta]_{\theta=\hat{\theta}} = 0$.

As an example, we consider a spectrum of the form $f_m \propto \exp[-(k_m - k_0)^2 / \Delta k^2]$ and a pulse shape $\tilde{\omega}(k_m) \propto \exp\{-(k_m - k_0)^2 / 2\Delta k^2\}$ for $k_m \in [k_0 - \Delta k/2, k_0 + \Delta k/2]$. This pulse shape can be approximately realized by applying a Gaussian-shaped bandpass filter in the frequency domain and detecting within a time window of duration roughly $1/\Delta k$. In the case where two photons are detected at positions x_{s_1} and x_{s_2} , Proposition 1 provides the bound

$$|P(\vec{s}, \vec{\delta}) - P_0(\vec{s}, \vec{\delta})| \leq (2\epsilon)^2 |\xi(\Delta x)|^2, \quad \xi(\Delta x) \leq \frac{4.93}{|\Delta x| \Delta k}, \quad (13)$$

where we define the distance between the two detected photons $\Delta x = x_{s_1} - x_{s_2}$. If $\Delta x \sim 1/\Delta k$, the correction provided by Proposition 2 becomes necessary. We consider the example where the detection outcome is $\vec{\delta} = [1, -1]$, which corresponds to a projection onto the state $(|01\rangle_{s_1} + |10\rangle_{s_1}) \otimes (|01\rangle_{s_2} + |10\rangle_{s_2})/2$. The probability is given by

$$P(\vec{s}, \vec{\delta}) = \epsilon^2 (1 - |g|^2 \cos^2 \theta \Omega_{s_1 s_2}^2 + |g|^2 \sin^2 \theta \Omega_{s_1 s_2} \Omega_{s_2 s_1}), \quad (14)$$

where $\Omega_{s_1 s_2} = C(\Delta x) \{[\exp(-(\Delta k^2 \Delta x^2/8))]/2\sqrt{2\pi}[\text{erf}(1/2)]^2\}$, $C(\Delta x) = \text{erf}[(2 - i\Delta x \Delta k)/2\sqrt{2}] + \text{erf}[(2 + i\Delta x \Delta k)/2\sqrt{2}]$. $P(\vec{s}, \vec{\delta})$ is the accurate formula that accounts for the correlation and can be used to estimate g using methods such as maximum likelihood estimation.

We emphasize that both Proposition 1 and 2 rely on the weak-source limit, meaning that most temporal modes contain only vacuum, and correlations between temporal modes need not be taken into account in many cases. As the source becomes stronger, single-photon detection at each temporal mode is no longer an appropriate measurement strategy; in such cases, local heterodyne detection may be more suitable. Our framework also allows us to explore the HBT effect and the operation of intensity interferometers under general spectral conditions and finite bandwidth. As a consistency check, we verify that the model reproduces the quantum statistical correlations expected for thermal light. For a flat spectrum, successful operation of the intensity interferometer requires simultaneous photon detection at both telescopes. However, in the case of a nonflat spectrum, we show that even when the photons are detected at different times, it is still possible to extract information about g . A discussion of stronger sources imaged via local heterodyne detection, the HBT effect, more details of the illustrative examples, and the proofs of the two propositions are provided in Sec. III of SM [25].

Conclusion—We have shown that thermal light with a flat spectrum is a direct product of independent localized pulses. In the weak-light limit, the first nonvacuum term of the expansion of thermal light with a flat spectrum in the Fock basis is a mixture of independent localized pulses. Our work provides a quantum derivation of the van Cittert–Zernike theorem that incorporates finite spectral bandwidth and a multimode description of thermal light. Building on this framework, we examined the effect of a general spectrum when an astronomical interferometer measures continuous-wave thermal light using localized temporal modes. We established criteria under which intermode correlations can be neglected, and proposed a corrected estimation strategy for cases where these correlations are significant. Our results serve as a prerequisite for quantum protocols involving thermal light emitted by naturally occurring sources, enabling the use of localized temporal modes that are convenient for experimental implementation. We expect this theoretical framework to be readily generalizable to other quantum protocols that involve operations on thermal light emitted by natural sources. Moreover, our formulation captures the interplay between spatial correlations across telescopes, temporal correlations from spectral structure, and the quantum statistical

properties of thermal light—revealing a rich and fundamentally interesting coupling between different aspects of the light.

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Data availability—The data that support the findings of this Letter are openly available [28]; embargo periods may apply.

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